Residual Interaction Effects on Deeply Bound Pionic states in Sn isotopes

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Deeply bound pionic atoms predicted theoretically in Ref. [1] were observed first in 1996 in (d,^{3}He) reactions [2] . Very recently, K. Suzuki *et al.* performed the (d,^{3}He) reactions on the Sn isotopes and succeeded to observe deeply bound $1s$ pionic states in Sn isotopes very precisely [3][4]. Experimental errors for the binding energies of the $1s$ state are around $\Delta E \sim 20$keV . It is very interesting to determine the s-wave potential parameters from deeply bound pionic atoms, because the s-wave parameters relate closely to chiral dynamics in the finite nuclear density. Acocordingly, the accurate new data leads to have animated discussions about s-wave parameters and its accompanied chiral parameters [3].

However, since we make use of the single neutron pickup (d,^{3}He) reaction, the final pionic states are the one-pion-particle-one-neutron-hole $[\pi \otimes n^{-1}]_J$ states with respect to the target nuclei [5]. So far all theoretical calculations and analyses of the data postulate that the residual interaction effect between the pion and the neutron hole is small and can be neglected except for the evaluation in Ref. [6]. It is true that the errors of data are significantly larger than the estimated residual interaction effects for Pb case [6]. But it is not trivial whether the residual interaction effects are negligible or not in Sn isotopes cases. Thus we calculate the residual interaction effects on pionic states in Sn isotopes.

We apply the same theoretical model described in Ref. [6] to evaluate the residual interaction effects and we take account of not only s-wave terms but also p-wave terms as the residual πN interaction. We assume the one-neutron-hole configuration as the final nuclear state. The matrix elements of the residual interaction can be written as

$$
\begin{aligned}
& < \pi', \beta; J|\overline{V}|\pi, \alpha; J > \\
= & -\frac{1}{2m_\pi}(-1)^{-J+j_\alpha+j_\beta+1/2} \\
& \times\sqrt{(2j_\alpha+1)(2j_\beta+1)(2\ell_\alpha+1)(2\ell_\beta+1)(2\ell'_\pi+1)(2\ell_\pi+1)} \\
& \times\sum_L(-1)^L \left\{ \begin{array}{ccc} \ell'_\pi & j_\beta & J \\ j_\alpha & \ell_\pi & L \end{array} \right\} \left\{ \begin{array}{ccc} \ell_\alpha & j_\alpha & \frac{1}{2} \\ j_\beta & \ell_\beta & L \end{array} \right\} (\ell_\beta 0 \ell_\alpha 0 \mid L0)(\ell_\pi 0 \ell'_\pi 0 \mid L0) \\
& \times \left[\begin{array}{l} (b_0+b_1)\ \int_0^\infty dr r^2 R^*_{\ell_\beta}(r) R_{\ell_\alpha}(r) R_{\ell'_\pi}(r) R_{\ell_\pi}(r) \\ +(c_0+c_1)\ \int_0^\infty dr r^2 R^*_{\ell_\beta}(r) R_{\ell_\alpha}(r) \\ \quad \times \left\{ \left(\frac{\mathrm{d}R_{\ell'_\pi}(r)}{\mathrm{d}r}\right)\left(\frac{\mathrm{d}R_{\ell_\pi}(r)}{\mathrm{d}r}\right) + \frac{\ell_\pi(\ell_\pi+1)+\ell'_\pi(\ell'_\pi+1)-L(L+1)}{2}\frac{R_{\ell'_\pi}(r)R_{\ell_\pi}(r)}{r^2} \right\} \end{array} \right] . \qquad (1)
\end{aligned}
$$

where m_π is the pion mass. We fix the interaction strength as $b_0 = -0.0283 m_\pi^{-1}$,$b_1 = -0.12 m_\pi^{-1}$, $c_0 = 0.223 m_\pi^{-3}$ and $c_1 = 0.25 m_\pi^{-3}$ which are taken from the pion-nucleus optical potential parameters [7] and considered to be an effective πN interaction strength in nucleus. We include the neutron-hole states $s^{-1}_{1/2}$, $d^{-1}_{3/2}$, $h^{-1}_{11/2}$, $g^{-1}_{7/2}$ and $d^{-1}_{5/2}$. We can evaluate the residual interaction effects by diagonalizing the matrix elements of the whole Hamiltonian describing the pion-nucleus system.

The calculated results are shown in Table 1 for the $[(1s)_\pi \otimes j_n^{-1}]_J$ and $[(2p)_\pi \otimes j_n^{-1}]_J$ configurations for Sn isotopes. We found that the complex eigenenergies vary about 11-18 keV in the $[(1s)_\pi \otimes j_n^{-1}]_J$ and 0-9 keV in $[(2p)_\pi \otimes j_n^{-1}]_J$. These effects are relatively smaller than the natural widths of the deeply bound pionic $1s$- and $2p$-states. However, the shifts are comparable to the experimental errors [3] and the residual interaction effects are not negligible.

	^{115}Sn			^{119}Sn			^{123}Sn		
	1s	2p		1s	2p		1s	2p	
$s_{1/2}^{-1}$	-15.4	J=1/2	$-4.0-1.1i$	-13.5	J=1/2	$-5.2-2.0i$	-12.3	J=1/2	$-3.2-0.7i$
	$-4.2i$	J=3/2	$-4.0-1.1i$	$-3.3i$	J=3/2	$-3.8-1.1i$	$-2.4i$	J=3/2	$-3.5-0.8i$
$d_{3/2}^{-1}$	-15.9	J=1/2	$-9.1-3.1i$	-14.3	J=1/2	$-7.0-1.6i$	-12.8	J=1/2	$-8.1-2.5i$
	$-4.8i$	J=3/2	$0.3+0.3i$	$-3.7i$	J=3/2	$0.4+0.3i$	$-2.9i$	J=3/2	$0.2+0.1i$
		J=5/2	$-5.2-1.8i$		J=5/2	$-4.6-1.4i$		J=5/2	$-4.3-1.2i$
$g_{7/2}^{-1}$	-15.4	J=5/2	$-6.0-3.8i$	-13.0	J=5/2	$-5.5-3.3i$	-11.1	J=5/2	$-4.9-2.8i$
	$-7.3i$	J=7/2	$1.5+0.8i$	$-5.8i$	J=7/2	$1.3+0.7i$	$-4.6i$	J=7/2	$1.2+0.6i$
		J=9/2	$-4.4-2.9i$		J=9/2	$-3.9-2.4i$		J=9/2	$-3.5-2.0i$
$h_{11/2}^{-1}$	-18.3	J=9/2	$-7.7-4.0i$	-16.0	J=9/2	$-6.9-3.5i$	-14.1	J=9/2	$-6.3-3.0i$
	$-7.2i$	J=11/2	$1.7+0.8i$	$-6.0i$	J=11/2	$1.5+0.7i$	$-5.1i$	J=11/2	$1.4+0.6i$
		J=13/2	$-6.2-3.3i$		J=13/2	$-5.6-2.8i$		J=13/2	$-5.1-2.5i$
$d_{5/2}^{-1}$	-15.1	J=3/2	$-7.6-2.6i$	-13.6	J=3/2	$-7.1-2.2i$	-12.2	J=3/2	$-6.5-1.9i$
	$-4.8i$	J=5/2	$1.0+0.6i$	$-3.7i$	J=5/2	$0.9+0.4i$	$-2.8i$	J=5/2	$0.8+0.3i$
		J=7/2	$-5.0-1.7i$		J=7/2	$-4.6-1.4i$		J=7/2	$-4.3-1.2i$
exp. error	± 24 $\pm 44i$	——		± 18 $\pm 40i$	——		± 18 $\pm 36i$	——	

Table 1: Calculated complex energy shifts due to the residual interaction in ^{115,119,123}Sn. The results are shown in units of keV for $[(1s)_\pi \otimes j_n^{-1}]_J$ and $[(2p)_\pi \otimes j_n^{-1}]_J$ configurations. The experimental values are taken from Ref.[3].

In summary, we have calculated the complex energy shifts of the deeply bound pionic states in Sn isotopes and shown the results including s- and p-wave residual interaction effects. We assume the one-neutron-hole configuration in the final open-shell nucleus implicitely. The present results show that the sizes of the residual interaction effect are comparable to the experimental errors, and hence, we need to evaluate them using realistic theoretical formula. We believe that it is essential to evalaute these effects precisely in order to deduce the chiral parameters from the accurate pionic atom data.

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