Gauge invariant monopole and artifact in the lattice SU(2) gauge theory

T. Sekido[a], F.V. Gubarev[a,b], and T. Suzuki[a]

[a]*Institute for Theoretical Physical , Kanazawa University,Kanazawa 920-1192 , Japan*
[b]*Institute of Theoretical and Experimental Physical, B.Cheremushkinskaya 25,Moscow,117259,Russia*

The Dual Meissner effect is expected to describe the quark confinement mechanism in QCD. It is an analogy of the superconductor. Monopoles having color magnetic charges are considered to condense in the QCD vacuum and the linear potential appears due to the squeezed color electric field. But to find the monopoles in QCD the monopole is a problem. In 1974 'tHooft and Polyakov proved that the classical monopole solution exists in the Georgi-Glashow model[4]. 'tHooft suggested in 1981 that the monopoles which are extracted by a partial gauge fixing called abelian projection cause the dual Meissner effect[3]. On the lattice monopoles are defined after an abelian projection following DeGrand-Toussaint (DGT) [6]. The DGT monopoles have been studied numerically. For example it is indicated that the string tension extracted form the monopole part reproduces the original one (monopole dominance)(e.g.[5]). However such successful results have been obtained using a specific gauge fixing called Maximal Abelian gauge(MA gauge).

Recently one of the authors (F.G.) in 2002 proposed a gauge invariant monopole (GIM) which can be extracted without the abelian projection[1]. He also formulated the monopole on the lattice. It is very interesting that the new monopoles have an integer charge on the lattice as the DGT monopoles. Here we investigate the property of the GIM in details and compare them with the DGT monopoles.

In the continuum limit the GIM is defined through the analogy between the magnetic charge in the Georgi-Glashow model and the Berry phase on the Wilson loop. In the Georgi-Glashow model the magnetic charge is determined only by the direction of the Higgs field $\vec{n}$.

$$q_{GG} = \frac{1}{8\pi} \int \{-\partial \wedge (\vec{n}\vec{A}) + \vec{n} \cdot \partial\vec{n} \times \partial\vec{n}\} \tag{1}$$

In the GIM the role of the Higgs field is played by a Wilson loop. A Wilson loop's eigenstate can be expressed by a 3D unit vector $\vec{n}$ state for SU(2).

$$W|\vec{n}\rangle \;=\; e^{i\varphi}|\vec{n}\rangle \tag{2}$$

where φ is called the Berry phase. And the Berry phase is expressed by a unit vector $\vec{n}$.

$$\delta\varphi_x = \frac{1}{4}\{\partial \wedge (\vec{n}\vec{A}) - \vec{n} \cdot \partial\vec{n} \times \partial\vec{n}\}\delta\sigma_x, \tag{3}$$

where we consider the infinitesimal Wilson loop. Now we make a closed surface using the infinitesimal Wilson loop. Namely we execute the surface integral. Then we get the same form as in (1). We can identify the quantity with the magnetic charge.

$$q_{GIM} = \frac{1}{2\pi} \int_C \delta\varphi_x \tag{4}$$

On the lattice we can define a similar quantity. But we have to change the infinitesimal Wilson loop to a 1×1 Wilson loop which is the smallest one. We assume the Berry phase on the cube to be the magnetic charge called as GIM on the lattice here. However the lattice is not smooth. So new problems arise. 1) The Higgs field on a lattice depends on the path of the Wilson loop and so many Higgs fields are defined on the same lattice site. Hence we have to consider the difference of the internal space with respect to Higgs fields on the same lattice site. 2) A new surface in the internal space appears at the lattice site. The Berry phase on the new surface contributes to the magnetic charge. However, the magnetic charge is expected to vanish in the continuum, since such an internal space disappears. Hence it is the lattice artifact. 3) The Higgs field defined above has a sign problem on the lattice. In the continuum theory the problem is trivial because the sign is fixed except for the global unimportant one. But on the lattice the sign problem is nontrivial, since all sites are discrete. Since we expect the Higgs fields on the same site become identical in the continuum limit, the sign may be chosen by minimizing an Ising-like action so that the Higgs fields on the same site approach each other smoothly.

The simulations have been done using the Wilson action on lattice 24^3 for β=2.2, 2.3, 2.4, 2.5. First we have confirmed that the magnetic charge is integer within the computer accuracy. And the magnetic charge occasionally takes ± 3. For the DGT monopole the magnetic charge is restricted to $-2 \sim 2$. Hence both monopoles are different actually. Secondly the GIM and the artifact monopole density have been measured. We have seen that the GIM density is larger than that of the DGT monopole in the MA gauge. In the DGT monopole case it is known that such good behaviors as the monopole dominance are seen only in gauges giving a small monopole density. The lattice artifact density remains nonvanishing even at large β. However after a block spin transformation, the artifact density decreases more rapidly than the GIM one. It implies that the artifact vanishes in the continuum limit.

To summarize, we have shown that the GIM on the lattice is affected by the lattice artifact. However a block spin transformation seems to decrease the effect of the lattice artifact. We have to study more how to decrease the lattice artifact.

This work is supported by NEC SX-5 at Research Center for Nuclear Physics (RCNP) of Osaka University.

References

[1] F.V.Gubarev and V.I.Zakharov, hep-lat/0204017.
[2] F.V.Gubarev, hep-lat/0204018.
[3] G.'tHooft, Nucl.Phys.B190(1981) 455.
[4] G.'tHooft,Nucl.Phys.B79(1974) 276. A.M.Polyakov,JET.Lett.20 (1974) 194.
[5] H.Shiba and T.Suzuki,Phys.Lett.B 333(1994) 461
[6] T.A.DeGrand and D.Toussaint, Phys.Rev.D22(1980) 2478.