Lattice-QCD-based Schwinger-Dyson Approach for Chiral Symmetry Restoration at Finite Temperature

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Dynamical chiral-symmetry breaking (DCSB) is one of the most important nonperturbative features in QCD. For the study of DCSB, the Schwinger-Dyson (SD) formalism has been used as a powerful method. For QCD, the SD formalism consists of an infinite series of nonlinear integral equations which determine the n-point function of quarks and gluons, and therefore it includes the infinite-order effect of the QCD gauge coupling constant g. The SD equation for the quark propagator $S(p)$ is described with the nonperturbative gluon propagator $D_{\mu\nu}(p)$ and the nonperturbative quark-gluon vertex $g\Gamma_\nu(p,q)$ as

$$S^{-1}(p) = S_0^{-1} + g^2 \int \frac{d^4q}{(2\pi)^4} \gamma_\mu S(q) D_{\mu\nu}(p-q) \Gamma_\nu(p,q) \tag{1}$$

in the Euclidean metric[1]. In the practical calculation for QCD, however, the SD formalism is drastically truncated: the perturbative gluon propagator and the one-loop running coupling are used instead of the nonperturbative quantities. This simplification seems rather dangerous because some of nonperturbative effects are neglected.

In this paper, we formulate the SD equation based on the recent lattice QCD (LQCD) results, i.e., the LQCD-based SD equation, and aim to construct a useful and reliable analytic framework including the proper nonperturbative effect in QCD.

In the Landau gauge, Euclidean gluon and quark propagators are generally expressed as

$$D_{\mu\nu}(p^2) = \frac{d(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right), \qquad S(p) = \frac{Z(p^2)}{\not p + M(p^2)}. \tag{2}$$

Here, we refer to $d(p^2)$ as the gluon polarization factor. $M(p^2)$ is called as the quark mass function, and $Z(p^2)$ corresponds to the wave-function renormalization of the quark field. In the quark propagator, DCSB is characterized by the mass generation as $M(p^2) \neq 0$.

The quark mass function $M(p^2)$ in the Landau gauge is recently measured in lattice QCD at the quenched level[2], and the lattice data in the chiral limit is well reproduced by

$$M(p^2) = M_0/\{1 + (p/\bar{p})^\gamma\} \tag{3}$$

with M_0=260 MeV, $\bar{p}$=870MeV and γ=3.04. The infrared quark mass $M(0) = M_0 \simeq 260$MeV seems consistent with the constituent quark mass in the quark model. Using this lattice result of $M(p^2)$, the pion decay constant is calculated as $f_\pi \simeq 87$ MeV with the Pagels-Stokar formula, and the quark condensate is obtained as $\langle \bar{q}q \rangle_{\Lambda=1\mathrm{GeV}} \simeq -(220\mathrm{MeV})^3$. These quantities related to DCSB seem consistent with the standard values.

For the quark-gluon vertex, we assume the chiral-preserving vector-type vertex, $\Gamma_\mu(p,q) = \gamma_\mu \Gamma((p-q)^2)$, which properly keeps chiral symmetry. Then, the SD equation is expressed as

$$\frac{M(p^2)}{Z(p^2)} = 3C_F \int \frac{d^4q}{(2\pi)^4} \frac{Z(q^2)M(q^2)}{q^2 + M^2(q^2)} \frac{K((p-q)^2)}{(p-q)^2}, \tag{4}$$

where we define the kernel function $K(p^2) \equiv g^2\Gamma(p^2)d(p^2)$ as the product of the quark-gluon vertex $\Gamma(p^2)$ and the gluon polarization factor $d(p^2)$. $C_F = \frac{4}{3}$ is the color factor for quarks.

We extract the SD kernel function $K(p^2) \equiv g^2\Gamma(p^2)d(p^2)$ from the quark propagator obtained in lattice QCD. Shifting the integral variable from q to $\tilde{q} \equiv p - q$ in Eq.(4), we get

$$\frac{M(p^2)}{Z(p^2)} = \frac{3C_F}{8\pi^3}\int_0^\infty d\tilde{q}^2 K(\tilde{q}^2)\int_0^\pi d\theta \sin^2\theta \frac{Z(p^2+\tilde{q}^2-2p\tilde{q}\cos\theta)M(p^2+\tilde{q}^2-2p\tilde{q}\cos\theta)}{p^2+\tilde{q}^2-2p\tilde{q}\cos\theta+M^2(p^2+\tilde{q}^2-2p\tilde{q}\cos\theta)}. \quad (5)$$

We shown in Fig.1 the kernel function $K(p^2) \equiv g^2\Gamma(p^2)d(p^2)$ extracted from the lattice QCD result of the quark propagator in the Landau gauge. As remarkable features, we find "infrared vanishing" and "intermediate enhancement" in the SD kernel function $K(p^2)$. In fact, $K(p^2)$ seems consistent with zero in the very infrared region as $K(p^2 \sim 0) \simeq 0$, and $K(p^2)$ exhibits a large enhancement in the intermediate-energy region around $p \sim 0.5$GeV. Note that the simple SD equation using the perturbative gluon propagator and the one-loop running coupling fails to reproduce these features and would be too crude to study QCD.

Finally, we apply the LQCD-based SD equation to chiral symmetry restoration in finite-temperature QCD. At a finite temperature T, the field variables obey the (anti-)periodic boundary condition in the imaginary-time direction, which leads to the SD equation for the thermal quark mass $M_n(\mathbf{p}^2)$ of the Matsubara frequency $\omega_n \equiv (2n+1)\pi T$ as

$$M_n(\mathbf{p}^2) = T\sum_{m=-\infty}^{\infty}\int\frac{d^3q}{(2\pi)^3}\frac{3C_F M_m(\mathbf{q}^2)}{\omega_m^2+\mathbf{q}^2+M_m^2(\mathbf{q}^2)}\frac{K((\omega_n-\omega_m)^2+(\mathbf{p}-\mathbf{q})^2)}{(\omega_n-\omega_m)^2+(\mathbf{p}-\mathbf{q})^2}. \quad (6)$$

Using the obtained kernel function $K(p^2)$, we solve Eq.(6) for the thermal quark mass $M_n(\mathbf{p}^2)$. Figure 2 shows the thermal infrared quark mass $M_0(\mathbf{p}^2=0)$ plotted against the temperature T. We thus find chiral symmetry restoration at a critical temperature $T_c \sim 100$MeV.

References

[1] H. Iida, M. Oka and H. Suganuma, Nucl. Phys. **B** (Proc. Suppl.) **129-130** (2004) 566.
[2] P.O.Bowman, U.M.Heller, A.G.Williams, Nucl. Phys. **B** (Proc.Suppl.) **109** (2002) 163.

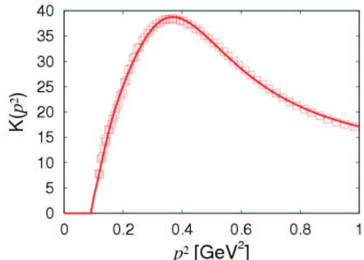

Figure 1: The kernel function in the SD equation, $K(p^2) \equiv g^2\Gamma(p^2)d(p^2)$, extracted from the lattice QCD result of the quark propagator in the Landau gauge. The calculated data are denoted by the square symbols, and the solid curve denotes a fit function for them. In the very infrared region, the kernel seems consistent with zero. As remarkable features, $K(p^2)$ exhibits infrared vanishing and intermediate enhancement.

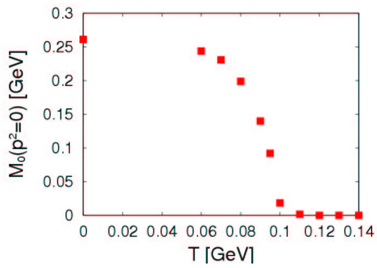

Figure 2: The thermal infrared quark mass $M_0(\mathbf{p}^2=0)$ plotted against the temperature T. The critical temperature T_c is found to be about 100MeV.