The strange quark matter could be the true ground state of the finite density QCD by many theoretical researches. The stability of the strange matter would suggest a wide range of sizes of the strange matter from strangelets to strange stars [1]. Recently, a candidate of a quark star was reported as a compact star with the radius 5–6 km [3]. It is discussed that such a small star cannot be a neutron star from the previous theoretical studies.

By using the effective models of QCD, the stability of the quark star has been discussed. The MIT bag model predicts the stable strange stars in which the ingredients are the uniform strange matter [1, 2]. However, the MIT bag model cannot describe dynamical chiral symmetry breaking. It was shown that the strange matter appears, not in the ground state, but only in the high density state in the NJL model [4]. We discuss the mass and radius of the quark stars and the possibility that the strange matter exists in the quark stars by considering chiral symmetry breaking.

To obtain the mass and the radius of the strange stars, assuming a static and spherical configuration in the Einstein equation, we use the Tolman-Oppenheimer-Volkov (TOV) equations,

\[
\frac{dp}{dr} = -G(\epsilon + p) \frac{m + 4\pi r^3 p}{r(r - 2Grm)}, \tag{1}
\]

\[
\frac{dm}{dr} = 4\pi r^2 \epsilon, \tag{2}
\]

where \( r \) is the distance from the center of the star, \( m \) is the mass in the sphere of the radius \( r \), and \( G \) is the gravitational constant. The radius of the star is determined by no pressure condition on the surface; \( p(R) = 0 \). The mass of the star is given by \( M = m(R) \).

In the TOV equations, the energy density \( \epsilon \) and the pressure \( p \) have to be given by the equation of state of the quark matter. Our interest is to investigate the effects on the stability of the strange stars by dynamical chiral symmetry breaking. Therefore we use the equation of state of the quark matter which is obtained by the mean field approximation in the NJL model,

\[
\mathcal{L} = \bar{\psi}(i\partial - m^0)\psi + \frac{g}{2} \sum_{a=0}^{8} \left[ \left( \bar{\psi} \frac{\lambda^a}{2} \psi \right)^2 + \left( \bar{\psi} i\gamma_5 \frac{\lambda^a}{2} \psi \right)^2 \right], \tag{3}
\]

where \( \lambda^a \) are Gell-Mann matrices normalized by \( \text{tr} \lambda^a \lambda^b = 2\delta^{ab} \), \( g \) the coupling constant. The parameters are determined to reproduce the properties of the hadrons in the vacuum.

We assume the \( \beta \)-equilibrium and the charge neutrality for \( u, d \) and \( s e^- \), namely \( u \leftrightarrow d + e^- \), \( u + d \leftrightarrow u + s \) and \( \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0 \). The neutrinos are not taken into account, since the mean free path of the neutrinos is considered to be much longer than the size of the quark star. These conditions reduce the number of the free chemical potentials to one, and we take a baryon chemical potential \( \mu_B \) as one free chemical potential. Note that the charge neutrality is global condition, in which the quark number density is expressed by a fraction of the stable density. We see a mixed phase of the quarks because of the first order phase transition of chiral symmetry breaking in the NJL model.
The results are shown in Fig. 1. The mass and the radius of the quark star begins from the origin on the $M - R$ plot, which comes from the stability of the quark matter. The relation $M = \epsilon_0 \frac{4\pi}{3} R^3$ of the $ud$ matter without gravitation is also plotted, where $\epsilon_0$ is the energy density in the stable $ud$ matter. The correspondence of the quark matter without gravitation and the quark star with small mass and radius could suggest the continuity from the quark droplets to the quark star. Namely, it is possible that any small quark star could exist due to the stability of the quark matter. On the other hand, the neutron star cannot have an arbitrary small radius, since the neutron matter is not stable without external force. We obtain the maximum mass $M_{\text{max}} = 1.3 - 1.4 M_{\text{sun}}$ and the radius $R \sim 8\text{km}$. These values imply that the quark stars are more compact than the neutron stars. Our result is consistent with the observed value $R = 5 - 6 \text{ km}$ of the compact star reported by [3].

The distribution of the quarks in the quark star with maximum mass is shown in Fig. 1. There are several layers of quark matter, such as (a) the mixed phase of the $u$ quark matter, (b) the uniform $ud$ matter and (c) the mixed phase of the $s$ quark matter. In the mixed phase, the number density of the flavor $f$ is represented by $n_f = \chi n^*_f$, where $n^*_f$ is the stable density of the quark matter of flavor $f$. The $s$ quark can appear in a mixed phase embedded in the uniform $ud$ matter only around the center of the star. This result by the NJL model is much different from the uniform strange matter in any places in the quark star by the MIT bag model.

References