Isospin-asymmetry matrix element in $T_z = \pm 3/2 \to \pm 1/2$ mirror Gamow-Teller transitions for $A = 41$ nuclei

Y. Fujita[1] S. Sasaki[2] T. Adachi[1] G.P.A. Berg[3,4] B.A. Brown[5] H. Fujita[1] K. Hatanaka[3] J. Kamiya[3] K. Nakanishi[3] Y. Sakemi[3] Y. Shimbara[1] Y. Shimizu[3] Y. Tameshige[3] M. Uchida[6] T. Wakasa[3] and M. Yosoi[6]

[1] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[2] *Shizuoka Institute of Technology, Fukuroi, Shizuoka 437-8555, Japan*
[3] *Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*
[4] *Kernfysisch Versneller Instituut, Zernikelaan 25, 9747 AA Groningen, The Netherlands*
[5] *NSCL and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824*
[6] *Department of Physics, Kyoto University, Sakyo, Kyoto 606-8502, Japan*

The $B(\mathrm{GT})$ strengths for the $T_z = \pm 3/2 \to \pm 1/2$ transitions in the $A = 41$ sytem, i.e., ^{41}K ($T_z = +3/2$) to ^{41}Ca ($T_z = +1/2$) and ^{41}Ti ($T_z = -3/2$) to ^{41}Sc ($T_z = -1/2$), were derived from the ^{41}K(^{3}He,t) measurement and the ^{41}Ti β-decay study, respectively, and they are shown in Figs. 1 and 2. By comparing them, we notice that the gross features of these two $B(\mathrm{GT})$ distributions for the isospin mirror transitions are similar, but the details are somewhat different. One of the interesting features is that the strengths of two $J = 5/2$ states at about 4.8 MeV and 4.9 MeV in ^{41}Ca and ^{41}Sc, respectively, are almost reversed. It is natural to think that these reversed strengths in the $T_z = \pm 3/2 \to \pm 1/2$ analog transitions are caused by the action of an interaction depending on T_z, i.e., an isospin asymmetric interaction. For simplicity, let us think only of the mixture between these $J = 5/2$ doublet states. Then we can deduce that: (A) the isospin-asymmetry matrix-elements acting in these isospin mirror states have similar strengths but opposite signs, and (B) without the isospin-asymmetry matrix-elements the transition strengths to these doublet states are almost the same. We estimate the approximate values of isospin-asymmetry matrix-elements on the bases of these assumptions.

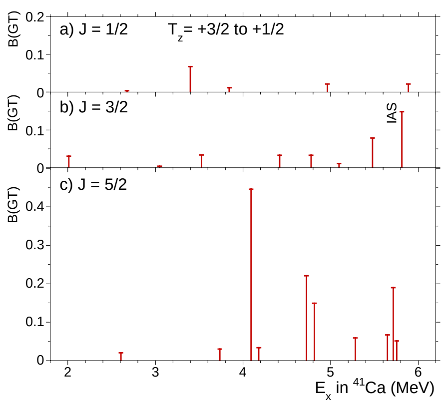

Figure 1: The $T_z = +3/2 \to +1/2$ $B(\mathrm{GT})$ distributions deduced from the ^{41}K(^{3}He, t) measurement for the ^{41}Ca states with (a) $J^\pi = 1/2^+$, (b) $J^\pi = 3/2^+$, and (c) $J^\pi = 5/2^+$.

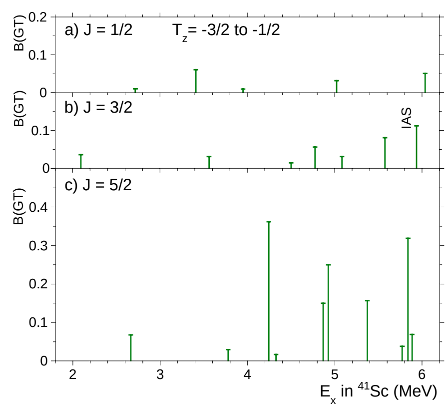

Figure 2: The $T_z = -3/2 \to -1/2$ $B(\mathrm{GT})$ distributions deduced from the ^{41}Ti β-decay measurement for the ^{41}Sc states with (a) $J^\pi = 1/2^+$, (b) $J^\pi = 3/2^+$, and (c) $J^\pi = 5/2^+$.

The nuclear Hamiltonian is written as $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\mathrm{IA}}$, where $\mathcal{H}_0$ and $\mathcal{H}_{\mathrm{IA}}$ are isospin symmetry (conserving) term and isospin-asymmetry term, respectively. Two eigenstates of $\mathcal{H}_0$ are denoted by $\boldsymbol{\Phi_a}$ and $\boldsymbol{\Phi_b}$. Since the Hamiltonian $\mathcal{H}_0$ is isospin symmetric, these wave functions are the same in both $T_z = 1/2$ (^{41}Ca) and $T_z = -1/2$ (^{41}Sc) nuclei. The excitation energies E_a and E_b of observed states are the eigenvalues of the total Hamiltonian $\mathcal{H}$. Let us think of wave functions $\boldsymbol{\Psi_a}$ and $\boldsymbol{\Psi_b}$ that satisfy

$$(\mathcal{H}_0 + \mathcal{H}_{\mathrm{IA}})\boldsymbol{\Psi_a} = E_a \boldsymbol{\Psi_a} \quad \text{and} \quad (\mathcal{H}_0 + \mathcal{H}_{\mathrm{IA}})\boldsymbol{\Psi_b} = E_b \boldsymbol{\Psi_b}, \tag{1}$$

respectively. The form of these equations are formally the same for $T_z = \pm 1/2$ nuclei, but note here that the matrix-elements in the isospin-asymmetry term $\mathcal{H}_{\mathrm{IA}}$ are different in the $T_z = \pm 1/2$ nuclei [assumption (A)]. Therefore, the wave functions $\boldsymbol{\Psi_a}$ and $\boldsymbol{\Psi_b}$ are different in $T_z = \pm 1/2$ nuclei.

We can formally write states $\boldsymbol{\Psi_a}$ and $\boldsymbol{\Psi_b}$ in terms of linear combinations of the two states $\boldsymbol{\Phi_a}$ and $\boldsymbol{\Phi_b}$ as

$$\boldsymbol{\Psi_a} = \alpha \boldsymbol{\Phi_a} + \beta \boldsymbol{\Phi_b}, \quad \text{and} \quad \boldsymbol{\Psi_b} = \beta \boldsymbol{\Phi_a} - \alpha \boldsymbol{\Phi_b}, \tag{2}$$

where $\alpha^2 + \beta^2 = 1$. Note again here that the coefficients α and β are different in the $T_z = \pm 1/2$ nuclei. Using these relationships, the matrix element of the off-diagonal matrix $\mathcal{H}_{\mathrm{IA}}$, i.e., the isospin-asymmetry term, can be written as

$$\langle \mathbf{\Phi_a} | \mathcal{H}_{\mathrm{IA}} | \mathbf{\Phi_b} \rangle = \alpha\beta(E_a - E_b). \tag{3}$$

Let us think of the GT transitions caused by the operator $\mathcal{O} = \boldsymbol{\sigma\tau}$ starting from the g.s. $\mathbf{\Phi_0}$ of $T_z = \pm 3/2$ nuclei. For simplicity, we assume that the effect of isospin asymmetry in the g.s. is small. The GT transition strength $B(\mathrm{GT})$ is proportional to the squared value of the transition matrix element of $\sigma\tau$ type. Therefore, the ratios of the $B(\mathrm{GT})$ values R^0 and R^1 for the transitions to the two excited states before and after the mixing, respectively, can be expressed as

$$R^0 = \frac{B^0(\mathrm{GT})_b}{B^0(\mathrm{GT})_a} = \frac{|\langle \mathbf{\Phi_b} | \mathcal{O} | \mathbf{\Phi_0} \rangle|^2}{|\langle \mathbf{\Phi_a} | \mathcal{O} | \mathbf{\Phi_0} \rangle|^2}, \tag{4}$$

and

$$R^1 = \frac{B^1(\mathrm{GT})_b}{B^1(\mathrm{GT})_a} = \frac{|\langle \mathbf{\Psi_b} | \mathcal{O} | \mathbf{\Phi_0} \rangle|^2}{|\langle \mathbf{\Psi_a} | \mathcal{O} | \mathbf{\Phi_0} \rangle|^2}, \tag{5}$$

where $B^0(\mathrm{GT})$ and $B^1(\mathrm{GT})$ are the $B(\mathrm{GT})$ values before the mixing of states and the observed $B(\mathrm{GT})$ values after the mixing of states, respectively. By putting Eq. (2) into Eq. (5), and using R^0, the ratio R^1 can be written as

$$R^1 = \frac{|\langle \beta\mathbf{\Phi_a} - \alpha\mathbf{\Phi_b} | \mathcal{O} | \mathbf{\Phi_0} \rangle|^2}{|\langle \alpha\mathbf{\Phi_a} + \beta\mathbf{\Phi_b} | \mathcal{O} | \mathbf{\Phi_0} \rangle|^2} \tag{6}$$

$$\simeq \frac{|\beta - \alpha\sqrt{R^0}|^2}{|\alpha + \beta\sqrt{R^0}|^2}. \tag{7}$$

The transformation from Eq. (6) to Eq. (7) is not exact when the associated phases are different in the matrix elements $\langle \mathbf{\Phi_a} | \mathcal{O} | \mathbf{\Phi_0} \rangle$ and $\langle \mathbf{\Phi_b} | \mathcal{O} | \mathbf{\Phi_0} \rangle$.

There is no way to study the ratio R^0 experimentally. However, according to the assumption (B), we can put $R^0 \approx 1$, i.e., the doublet states have almost equal $B(\mathrm{GT})$ values without the influence of $\mathcal{H}_{\mathrm{IA}}$. Using the experimental $B(\mathrm{GT})$ values of Figs. 1 and 2, the values of R^1 are obtained for the doublet states observed at 4.8 MeV in the $(^3\mathrm{He}, t)$ study and also for the doublet states observed at 4.9 MeV in the β-decay study. A set of α and β is obtained for each of these R^1 values using Eq. (7) and the relationship $\alpha^2 + \beta^2 = 1$. The isospin-asymmetry matrix-element is calculated by putting a set of α and β and the difference of the excitation energies into Eq. (3). As a result, values of -8.3 keV and 7.5 keV are obtained for the $T_z = +3/2 \rightarrow +1/2$ and $T_z = -3/2 \rightarrow -1/2$ transitions, respectively. The signs of them are opposite and the absolute values are nearly the same, which is consistent with the assumption (A). It is interesting that relatively small isospin-asymmetry interactions of ≈ 8 keV can make the reversed transition strengths observed for the isospin mirror transitions to the doublet states with $\Delta E \approx 70$ keV.

Isospin mixing was studied at a similar mass of $A = 37$ for a pair of $J^\pi = 3/2^+$ states. One of them was the $T = 3/2$ IAS in ^{37}K at $E_x = 5.051$ MeV and the other was a $T = 1/2$ state lying 31 keV below. The relative proton widths of the two levels measured in a $^{36}\mathrm{Ar}(\vec{p}, p_0)$ resonance reaction implied an isospin-mixing matrix-element of 4.8 keV [1]. In addition the analysis of the β^+-decay study from the g.s. of ^{37}Ca to these two states suggested a value of 5.9 keV [1, 2]. It is interesting that similar values are deduced for the isospin-asymmetry matrix-element obtained here and the isospin-mixing matrix-element.

For details see Ref. [3].

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