Finite size mass shift formula for stable particles

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Nowadays the control of finite volume effects in unquenched lattice QCD simulations becomes more important for the precise determination of the hadron spectrum. In this context, Lüscher's formula, relating the mass shift in finite volume with periodic boundary conditions to forward elastic scattering amplitudes in infinite volume, provides us with an interesting tool [1, 2]. In Ref. [3], we thus investigated the mass shift formula for the interacting two stable particle system along the lines of Lüscher's derivation for a self-interacting bosonic theory. In this report, we present the resulting formula and an application to the nucleon mass shift.

The physical mass of a stable particle is given by the position of the pole of the propagator of an asymptotic field, which is shifted from the bare one due to the self energy arising from virtual polarization effects. Using this fact, the finite size mass shift $\Delta m(L) = M(L) - m$ is represented as the difference of the self energies between the finite and infinite volumes $\Delta m(L) = -(\Sigma_L(p) - \Sigma(p))/2m + O((\Delta m)^2)$. In finite spacial volume L^3, $\Sigma_L(p)$ has a form quite similar to $\Sigma(p)$ with an exponential factor $e^{-iL\vec{m}\cdot\vec{q}}$, where $\vec{q}$ denotes the loop momentum, and the summation over $\vec{m} \in \mathbb{Z}^3$. Then $|\vec{m}| = 1$ yields the leading order contribution to $\Delta m(L)$.

The boson mass shift formula for ϕ_A in the ϕ_A-ϕ_B system is then found to be [1]

$$\Delta m_A(L) = -\frac{3}{8\pi m_A L}\left[\frac{\lambda^2}{2\nu_B}e^{-L\sqrt{m_B^2-\nu_B^2}} + \int_{-\infty}^{\infty}\frac{dq_0}{2\pi}\left\{e^{-L\sqrt{m_A^2+q_0^2}}F_{AA}(iq_0) + e^{-L\sqrt{m_B^2+q_0^2}}F_{AB}(iq_0)\right\}\right] + O(e^{-L\bar{m}}), \tag{1}$$

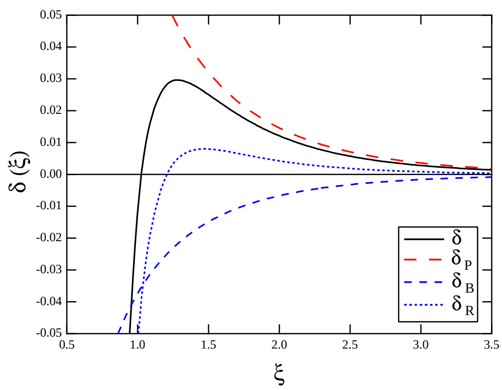
where $F_{AB}(\nu)$ and $F_{AA}(\nu)$ denote the forward scattering amplitudes of the processes $A + B \to A + B$ and $A + A \to A + A$ in the infinite volume, respectively. λ is an effective renormalized coupling defined from the residue of $F_{AB}(\nu)$ at $\nu = \pm\nu_B = \pm m_B^2/2m_A$ as $\lim_{\nu\to\pm\nu_B}(\nu^2 - \nu_B^2)\, F_{AB}(\nu) = \lambda^2/2$. The error term is defined by $\bar{m} \geq \sqrt{2(m_B^2 - \nu_B^2)}$, which is consistent with the order of $|\vec{m}| \geq \sqrt{2}$ contributions. The fermion mass shift formula for Ψ_A in the Ψ_A-ϕ_B system can be obtained by replacing $F_{AA} \to -(F_{AA} + F_{A\bar{A}})$ in Eq. (1). It should be emphasized that this formula is model independent and is valid to all orders in perturbation theory. All model dependence enters once the forward scattering amplitudes are specified.

As an application of the mass shift formula, we discuss the nucleon mass shift in the N-π system for the physical case. Since the formula is expected to hold nonperturbatively, we may insert the empirical N-π scattering amplitude [4]:

$$F_{N\pi}(\nu) = 6g^2\frac{\nu_B^2}{\nu_B^2 - \nu^2} + 6m_N(d_{00}^+\, m_\pi^{-1} + d_{10}^+\, m_\pi^{-3}\nu^2 + d_{20}^+\, m_\pi^{-5}\nu^4) + O(\nu^6)\,, \tag{2}$$

where $m_N = 938$ MeV and $m_\pi = 140$ MeV ($m_\pi/m_N = 0.149$) and $g^2/4\pi = 14.3$. The first term is identified with the pseudovector nucleon Born term with $\nu_B = m_\pi^2/2m_N \approx 0.07m_\pi$. The coefficients of the other terms are given by $d_{00}^+ = -1.46(10)$, $d_{10}^+ = 1.12(2)$ and $d_{20}^+ = 0.200(5)$. In Eq. (2) the isospin sum is carried out, neglecting the effect of isospin symmetry breaking. In this case we can ignore the terms F_{NN} and $F_{N\bar{N}}$ as $m_N \gg m_\pi$. The effective coupling is then computed as $\lambda^2 = -12g^2\nu_B^2$.

In the right figure we plot $\delta(\xi = Lm_\pi) \equiv \Delta m_N/m_N = \delta_P(\xi) + \delta_B(\xi) + \delta_R(\xi)$, where $\delta_P(\xi)$ corresponds to the first term of Eq. (1), and $\delta_B(\xi)$ and $\delta_R(\xi)$ correspond to the contributions of the pseudovector Born term and the rest in $F_{N\pi}(\nu)$, respectively. Note that $\xi = 1$ corresponds to $L = 1.4$ fm. We find that $\delta(\xi)$ seems to be mostly described by $\delta_P(\xi)$ as ξ increases. For the range $\xi = 2 \sim 3$ the mass shift is at least found to be about one percent (~ 10 MeV).



References

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[2] M. Lüscher, Commun. Math. Phys. **104**, 177 (1986).

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[4] G. Höhler, in Landolt-Börnstein, vol. I/9b2, edited by H. Schopper (Springer, Berlin, 1983), p.275, table 2.4.7.1.

[1] we assume that only A particle carries a conserved charge.