Relativistic Hartree model with vacuum polarization in finite nuclei

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Abstract

We study the effect of the negative energy states in the relativistic "mean field" model for finite nuclei using the rigorous Green function method. It turned out that the spin orbit splittings come out to be small as compared with the experimental values due to the necessity to change the nucleon mass from the free value not only for the positive energy nucleons but also for whole negative energy nucleons. This means that the Walecka model ought to be reconsidered as the sigma-omega ingredients are not enough to describe nucleus. We study also the goodness of the derivative expansion method, which is a powerful method to treat the negative energy states for the ground state and also for the excited states.

The existence of negative-energy solutions is one of the interesting characters in the relativistic picture. Using the relativistic mean-field (RMF) theory based on the quantum hadrodynamics (QHD)[1], it has been found that the negative-energy states of nucleons (antinucleon states) have a significant role in the various sum rules and in the transversal response[2]. The nuclear ground state as well as the excitation states should be also constructed with not only the nucleon field but also the antinucleon field within the one-nucleon loop correction, which is referred to as relativistic Hartree approach (RHA). The RHA calculation has been developed by several authors within the local-density approximation and the derivative-expansion method [3, 4, 5, 6, 7, 8]. However, the exact evaluation of one-loop corrections in a finite nuclear system has never been performed. Indeed, this is an exceedingly difficult task, in particular, for the finite system since the exact treatment of the vacuum polarization requires the computation of the infinite number of the negative-energy states. In this context, there is an excellent method developed in the quantum electrodynamics(QED); the summation over the eigenstates is replaced by the energy integral of the Dirac Green function along the imaginary energy axis[9]. In this report we apply this technique to the RHA calculation of the QHD model.

The Lagrangian density including isoscalar scalar meson (σ), isoscalar vector meson (ω), and photon (A) is employed;

$$\begin{aligned}\mathcal{L}_N &= \bar{\psi}_N(i\gamma^\mu\partial_\mu - m_N)\psi_N + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\ &\quad -\frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ &\quad -g_\sigma\bar{\psi}_N\sigma\psi_N - g_\omega\bar{\psi}_N\gamma_\mu\omega^\mu\psi_N - \frac{1}{2}e\bar{\psi}_N(1+\tau_3)\gamma_\mu A^\mu\psi_N - \delta\mathcal{L}, \end{aligned} \tag{1}$$

where $\delta\mathcal{L} = -\frac{1}{4}\zeta_\omega(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 + \frac{1}{2}\zeta_\sigma\partial_\mu\sigma\partial^\mu\sigma + \alpha_1\sigma + \frac{1}{2}\alpha_2\sigma^2 + \frac{1}{3}\alpha_3\sigma^3 + \frac{1}{4}\alpha_4\sigma^4$ denotes counterterms to renormalize the nucleon density which has a divergence from the vacuum. The renormalized baryon ($\rho_{\omega\ \mathrm{ren}}(\boldsymbol{r})$) and scalar ($\rho_{\sigma\ \mathrm{ren}}(\boldsymbol{r})$) densities are given by

$$\rho_{\omega\ \mathrm{ren}}(\boldsymbol{r}) = \int_C \frac{dz}{2\pi i}\mathrm{Tr}[\gamma_0 G^H(\boldsymbol{r},\boldsymbol{r};z)] + (\mathrm{CT}), \tag{2}$$

$$\rho_{\sigma\ \mathrm{ren}}(\boldsymbol{r}) = \int_C \frac{dz}{2\pi i}\mathrm{Tr}[G^H(\boldsymbol{r},\boldsymbol{r};z)] + (\mathrm{CT}), \tag{3}$$

where $G^H(\boldsymbol{r},\boldsymbol{r};z)$ is the single-particle Green function of the Hartree approximation with the potential terms. The z integrations are carried out along the modified Feynman contour which lies below the real axis up to the nuclear Fermi energy[9]. All divergences arising from these integrals are cancelled by the contributions of the counterterms denoted by CT. The integral along the Feynman contour can be changed to an integral over the imaginary z axis with the additional pole contribution of the positive-energy states up to Fermi level. Thus, we may write the unrenormalized baryon and scalar densities as

$$\int_C \frac{dz}{2\pi i}\mathrm{Tr}[\gamma^0 G^H(\boldsymbol{r},\boldsymbol{r};z)] = \sum_{\epsilon_i>0}^{F}\psi_i^\dagger(\boldsymbol{r})\psi_i(\boldsymbol{r}) - \int_{-i\infty}^{+i\infty}\frac{dz}{2\pi i}\mathrm{Tr}[\gamma^0 G_V^H(\boldsymbol{r},\boldsymbol{r};z)], \tag{4}$$

$$\int_C \frac{dz}{2\pi i}\mathrm{Tr}[G^H(\boldsymbol{r},\boldsymbol{r};z)] = \sum_{\epsilon_i>0}^{F}\bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r}) + \int_{-i\infty}^{+i\infty}\frac{dz}{2\pi i}\mathrm{Tr}[G_V^H(\boldsymbol{r},\boldsymbol{r};z)], \tag{5}$$

respectively. Here, G_V^H is the vacuum part of the single-particle Green function of the relativistic Hartree approximation. The numerical integration for G_V^H along the imaginary z axis can be carried out straightforwardly,

Table 1: The total binding energies, the rms charge radii, and the single-particle energies in ^{16}O and ^{40}Ca.

		Present RHA	TM2[10]	Experiment[11]
^{16}O	$E_{\rm total}/A(E_{VP}/A)$ [MeV]	8.05(1.69)	7.93(−)	7.98(−)
	r_{ch} [fm]	2.65	2.67	2.74
	Single particle state of proton			
	$1s_{1/2}$[MeV]	31.0	38.2	40 ± 8
	$1p_{3/2}$[MeV]	15.6	18.6	18.4
	$1p_{1/2}$[MeV]	13.3	11.1	12.1
	Single particle state of neutron			
	$1s_{1/2}$[MeV]	35.6	42.3	45.7
	$1p_{3/2}$[MeV]	19.7	22.4	21.8
	$1p_{1/2}$[MeV]	17.4	14.8	15.7
^{40}Ca	$E_{\rm total}/A(E_{VP}/A)$ [MeV]	8.47(2.23)	8.48(−)	8.55(−)
	r_{ch} [fm]	3.42	3.50	3.45
	Single particle state of proton			
	$1s_{1/2}$[MeV]	36.5	45.2	50 ± 11
	$1p_{3/2}$[MeV]	25.5	30.7	
	$1p_{1/2}$[MeV]	24.0	36.2	34 ± 6
	Single particle state of neutron			
	$1s_{1/2}$[MeV]	45.5	53.1	
	$1p_{3/2}$[MeV]	33.8	38.3	
	$1p_{1/2}$[MeV]	32.3	33.8	

since there are no poles in the integrand. Although the second terms of right-hand sides of (4) and (5) have divergences, an expansion of the total vacuum correction in the coupling constants g_ω and g_σ of the meson fields verifies that all divergences are contained in the first order of g_ω for baryon density, and are contained in terms up to the third order of g_σ for the scalar density. These divergences can be cancelled by taking the proper counterterms $\delta\mathcal{L}$ into account.

Now, we show the results of the relativistic Hartree calculation with a rigorous treatment of the one-nucleon loop in ^{16}O and ^{40}Ca. The numerical procedure of the present RHA calculation is similar to that used in the conventional RMF calculation; firstly, the Dirac equation is solved under the external ω- and σ-fields, for the valence nucleons only. Secondly, using the same potential, we calculate the vacuum densities by the Green function numerically. Thirdly, the equations of motion of mesons are solved with the source terms due to the valence nucleons and vacuum contributions. Substituting these results in the Dirac equation, we complete one iteration step. The iteration is continued until the total binding energy of the nucleus converges, showing self-consistency. The QHD parameter set is chosen so as to reproduce reasonably well the experimental values of the total binding energies, the rms radii, and the single-particle energies for both of ^{16}O and ^{40}Ca. In the second column of Table I, we give the results with the coupling constants and masses $g_\sigma = 7.38$, $g_2 = 7.90$, $g_3 = 3.20$, and $m_\sigma = 458.0$ MeV for the σ-meson, and $g_\omega = 9.18$ and $m_\omega = 783.0$ MeV for the ω-meson. We see that our total binding energies including the vacuum correction and rms radii are similar to those of RMF[10] and agree with the experimental data[11] well.

The present result is produced by using small coupling constants in comparison with those of RMF. Hence, the scalar and the vector potentials in the present RHA differ considerably from those of RMF. The results of the present RHA in ^{16}O and ^{40}Ca are compared with the results of RMF in Figs. 1 and 2 for the scalar and the vector potentials, respectively. The vacuum contribution plays a crucial role for the generation of such the weak

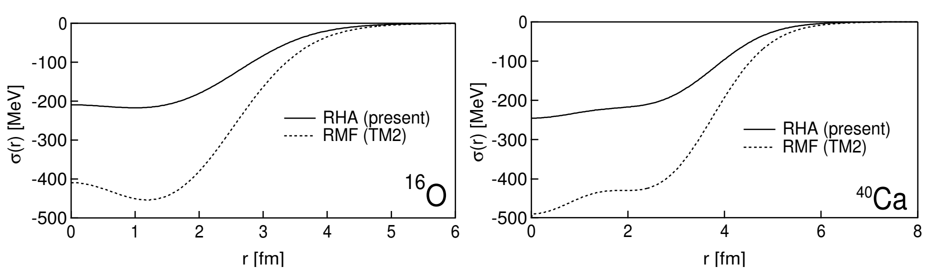

Figure 1: Scalar potentials.

meson fields. In Fig. 3, we plot the σ-meson field in nuclear matter as a function of the coupling constant g_σ while keeping the other parameters fixed. The figure shows that the vacuum correction contributes destructively to the valence contribution, and it is impossible to obtain a strong meson field unless a very large coupling constant

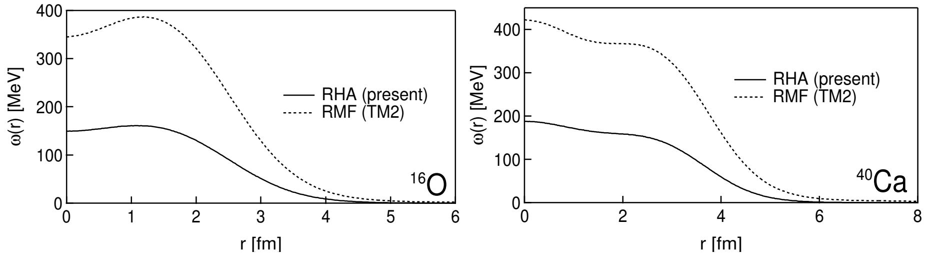

Figure 2: Vector potentials.

is used. We could of course choose such a parameter set with large coupling constants. However, this would have produced an unstable solution in the RHA calculation, since the deeply-bound antinucleon states produced by the strong ω- and σ-fields with large coupling constants, imply to produce a large vacuum effect, which works in the opposite direction. Such a RHA solution is not realistic, because it is unstable even for trivial fluctuations in the nucleon density. Hence, we have to choose a parameter set which produces a weak field in the self-consistent iteration.

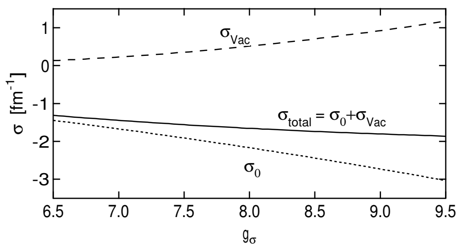

Figure 3: The scalar potential in nuclear matter. A σ-meson mass $m_\sigma = 458.0$ and a Fermi momentum $k_F = 1.42$ are employed. σ_0 denotes the ordinary σ-meson field, generated from the valence nucleons while σ_{Vac} denotes the contribution from the vacuum. Due to the cancellation between them, the net σ-meson field does not increase smoothly with the coupling constant, g_σ.

The RMF reproduces the observed tendency of the single-particle spectra reasonably well, due to the small effective mass of the nucleon, $m_N^*(r) = m_N + g_\sigma \sigma(r)$. The fact that the scalar field is suppressed in the RHA results in the large effective mass. As a result, it raises a problem in fitting the single-particle energies. As seen in Table I, the energy splittings in the single-particle states of the present RHA are very small, and they are unlikely to agree with the experimental values. This is a known problem, from previous RHA calculations, using the local-density approximation and the derivative expansion to estimate the vacuum correction[5, 7]. Thus, the QHD models require a mechanism for producing the spin-orbit splittings, other than the small effective mass. One suggestion is made in Refs. [14, 13], where a tensor-coupling of the ω-meson is introduced in order to provide the spin-orbit splittings. Another candidate to solve this problem may be the possibility of the finite pion mean field in the relativistic Hartree framework, which was suggested to provide the spin up and spin down partners with large energy separations[12]. It is interesting to extend the present RHA calculation by taking these effects into account. This is certainly a subject to be worked out in a future study.

Lastly, we compare our densities induced by the vacuum polarization with previous results. The effect of the negative-energy nucleons for finite nuclei was first estimated by the local-density approximation[3, 6]. It was developed further by applying the derivative-expansion method [4, 5, 7, 14]. In the local-density approximation, the vacuum correction is given by

$$\rho_{\sigma,ren}^{VP(LDA)}(r) = -\frac{1}{\pi^2}\left[m_N^{*3}\ln(m_N^*(r)/m_N) + 1/3m_N^3 - 3/2m_N^2 m_N^*(r) + 3m_N m_N^{*2}(r) - 11/6m_N^{*3}(r)\right], \tag{6}$$

and the scalar density decreases in the nuclear interior. The vacuum does not change the baryon density, because of the conservation of the baryon number. In the derivative-expansion method, on the other hand, the presence of the derivative term allows a non-vanishing correction for the baryon density, as well as for the scalar density:

$$\rho_{\omega,ren}^{VP(DE)}(r) = -\frac{g_\omega}{3\pi^2}\boldsymbol{\nabla}\cdot\ln\left(\frac{m_N^*(r)}{m_N}\right)\boldsymbol{\nabla}\omega_0(r), \tag{7}$$

$$\begin{aligned}\rho_{\sigma,ren}^{VP(DE)}(r) &= \rho_{\sigma,ren}^{VP(LDA)}(r) - \frac{g_\sigma}{2\pi^2}\boldsymbol{\nabla}\cdot\ln\left(\frac{m_N^*(r)}{m_N}\right)\boldsymbol{\nabla}\sigma(r) \\ &\quad -\frac{g_\sigma^2}{4\pi^2 m_N^*(r)}(\boldsymbol{\nabla}\sigma(r))^2 + \frac{g_\omega^2}{6\pi^2 m_N^*(r)}(\boldsymbol{\nabla}\omega(r))^2,\end{aligned} \tag{8}$$

where only the leading order of the derivative terms is taken into account. The baryon and scalar densities induced by the vacuum polarization are given in Fig. 6, together with those from the local-density approximation and the derivative expansion. We can see that the densities obtained by the local-density approximation are corrected significantly; not only for the baryon density, which vanishes in this approximation, but also for the scalar density. Both the scalar and baryon density-profiles obtained by the present calculation are in a surprisingly good agreement with those of the derivative expansion.

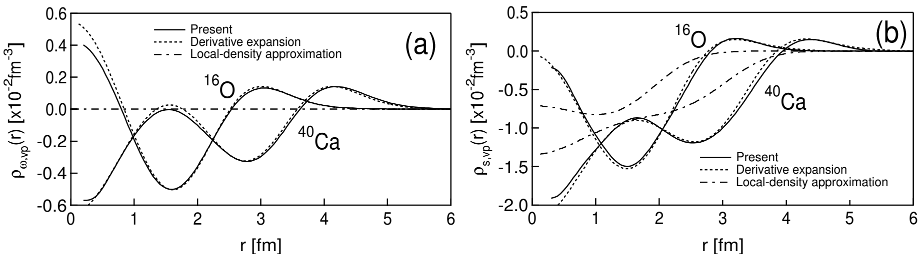

Figure 4: Vacuum correction for (a) baryon and (b) scalar densities.

However, this cannot always be the case[15], and the excellent agreement between our method and the leading-order derivative expansion can be attributed to the specifics of the $\sigma - \omega$ model. Consider, for example, the vacuum correction in the baryon density without σ-meson. Using the present method, the vacuum correction in this situation can turn out to be significant with a large coupling constant of the ω-meson. As found in Eq. (7), on the other hand, the vacuum correction from the derivative expansion vanishes exactly for $m_N^*(r) \to m_N$. Hence, we find that σ-meson plays an important role in the agreement between our method and the leading-order derivative expansion. Thus, the present calculation supports that the leading-order derivative expansion is greatly useful for the estimation of the vacuum correction in the RHA.

In summary, we have developed a rigorous calculation of vacuum-polarization effects in the relativistic Hartree approach based on the Green functional method. Our results, exploiting the Walecka model, have reproduced the experimental binding energies and rms radii of ^{16}O and ^{40}Ca nicely. However, it was impossible to find a QHD parameter set capable of reproducing the spin-orbit splittings in accordance with the observed data. In the Walecka model, the main attraction is caused by the large σ-mean field, which provides a small nucleon effective mass in finite nuclei. However, the negative-energy nucleons will acquire a mass differing from that of the free nucleon only reluctantly. On the whole, the effective nucleon mass remains quite large, implying that the spin-orbit splittings in the single particle spectra come out very small. The QHD type effective theory based on the $\sigma - \omega$ mesons, then, needs to include new types of interaction terms and/or go beyond the RHA approximation to solve this problem.

We have found that our results from the RHA calculation are very similar to those in Refs. [4, 7], where the derivative-expansion method was used to estimate the vacuum polarization. In particular, it has been shown that the agreement of the density-profiles of the vacuum correction is quite good. Thus, the validity of this approximation has been confirmed by the present calculation. The calculation with the self-consistent particle-hole correlation using the Lagrangian reconstructed by the derivative expansion is now in progress.

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References

[1] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[2] A. Haga, Y. Horikawa, Y. Tanaka, and H. Toki, Phys. Rev. C**69**, 044308 (2004).

[3] C. J. Horowitz and B. D. Serot, Phys. Lett. B **140**, 181 (1984).

[4] R. J. Perry, Phys. Lett. B **182**, 269 (1986).

[5] D. A. Wasson, Phys. Lett. B **210**, 41 (1988).

[6] W. R. Fox, Nucl. Phys. A**495**, 463 (1989).

[7] G. Mao, H. Stöcker, and W. Greiner, Int. J. Mod. Phys. E**8**, 389 (1999).

[8] P. G. Blunden, Phys. Rev. C**41**, 1851 (1990).

[9] M. Gyulassy, Phys. Rev. Lett. **33**, 921 (1974).

[10] Y. Sugahara and H. Toki, Nucl. Phys. A**579**, 557 (1994).

[11] D. Vautherin and D. M. Brink, Phys. Rev. C**5**, 626 (1972).

[12] Y. Ogawa, H. Toki, S. Tamenaga, H. Shen, A. Hosaka, S. Sugimoto, and K. Ikeda, Prog. Theore. Phys. **111**, 75 (2004).

[13] Y. Sugahara and H. Toki, Prog. Theor. Phys. **92**, 803 (1994).

[14] G. Mao, Phys. Rev. C**67**, 044318 (2003).

[15] I. W. Stewart and P. G. Blunden, Phys. Rev. D**55**, 3742 (1997).