Nonperturbative determination of spin-dependent potentials in QCD

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The spin-dependent corrections to the static quark-antiquark potential are phenomenologically relevant to describing the fine and hyper-fine splitting of the heavy quarkonium spectra, and thus, it is interesting to address these corrections from QCD. In QCD, these corrections show up at $O(1/m^2)$ with the expansion of the inverse of the quark mass m, which are summarized in the form [1]

$$\begin{aligned} V_{\rm SD}(r) &= \left(\frac{\vec{s}_1\cdot\vec{l}_1}{2m_1^2}-\frac{\vec{s}_2\cdot\vec{l}_2}{2m_2^2}\right)\left(\frac{V_0'(r)}{r}+2\frac{V_1'(r)}{r}\right)+\left(\frac{\vec{s}_2\cdot\vec{l}_1}{m_1m_2}-\frac{\vec{s}_1\cdot\vec{l}_2}{m_1m_2}\right)\frac{V_2'(r)}{r} \\ &+\frac{1}{m_1m_2}\left(\frac{(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2}-\frac{\vec{s}_1\cdot\vec{s}_2}{3}\right)V_3(r)+\frac{\vec{s}_1\cdot\vec{s}_2}{3m_1m_2}V_4(r)\,, \end{aligned} \tag{1}$$

where $\vec{r}_1$ and $\vec{r}_2$ ($r\equiv|\vec{r}_1-\vec{r}_2|$) denote the positions of quark and antiquark, m_1 and m_2 ($=m$) the masses, $\vec{s}_1$ and $\vec{s}_2$ the spins, $\vec{l}_1=-\vec{l}_2=\vec{l}$ the orbital angular momenta. $V_0(r)$ is the static potential and the prime denotes the derivative with respect to r. $V_i(r)$ (i = 1-4) are the so-called spin-dependent potentials. Given the field strength $F_{\mu\nu}$, where the electric and the magnetic fields are defined by $E_i=F_{4i}$ and $B_i=\frac{1}{2}\epsilon_{ijk}F_{jk}$, respectively, the spin-dependent potentials in Eq. (1) are expressed as

$$\frac{r_k}{r}V_1'(r)=\epsilon_{ijk}\lim_{\tau\to\infty}\int_0^\tau dt\; t\langle\langle B_i(\vec{0},0)E_j(\vec{0},t)\rangle\rangle\,,\quad \frac{r_k}{r}V_2'(r)=\epsilon_{ijk}\lim_{\tau\to\infty}\int_0^\tau dt\; t\langle\langle B_i(\vec{0},0)E_j(\vec{r},t)\rangle\rangle\,, \tag{2}$$

$$\left(\frac{r_ir_j}{r^2}-\frac{\delta_{ij}}{3}\right)V_3(r)+\frac{\delta_{ij}}{3}V_4(r)=2\lim_{\tau\to\infty}\int_0^\tau dt\;\langle\langle B_i(\vec{0},0)B_j(\vec{r},t)\rangle\rangle\,, \tag{3}$$

where the double bracket $\langle\langle\cdots\rangle\rangle$ is the expectation value of the field strength correlator on the q-$\bar{q}$ source.

We investigated these corrections using lattice QCD simulations by employing the multi-level algorithm [2] with a modification as applied to the flux-tube profile measurement [3]. We observed remarkably clean signals for all the spin-dependent potentials. In Figure 1 we show the spin-orbit potentials V_1' and V_2' measured at $\beta=6.0$ ($a=0.093$ fm) with the Wilson gauge action on the 20^4 lattice. Other potentials and simulation details are presented in our forthcoming publication. Preliminary results can be found in [4].

Simulations have been performed on the NEC SX5 at RCNP, Osaka University. We thank H. Togawa and A. Hosaka for technical supports.

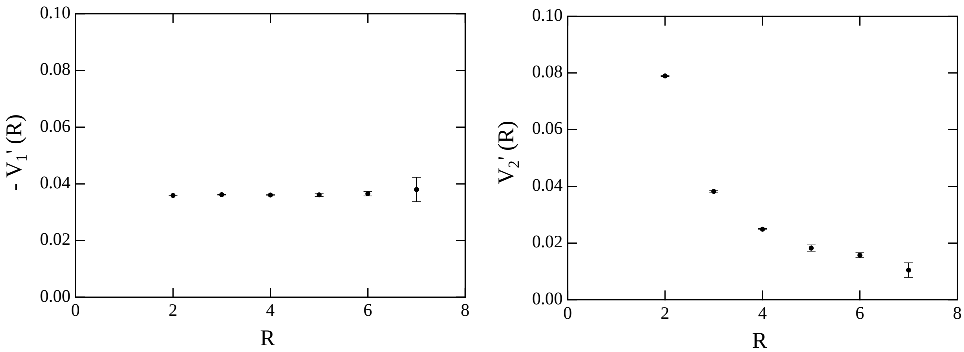

Figure 1: The spin-orbit potentials $V_1'(r)$ (left) and $V_2'(r)$ (right).

References

[1] E. Eichten and F. Feinberg, Phys. Rev. **D23**, 2724 (1981).
[2] M. Lüscher and P. Weisz, JHEP 09 (2001) 010.
[3] Y. Koma, M. Koma, and P. Majumdar, Nucl. Phys. **B692**, 209 (2004) [arXiv:hep-lat/0311016].
[4] M. Koma, Y. Koma, and H. Wittig, PoS **LAT2005**, 216 (2005) [arXiv:hep-lat/0510059].