Study of $1/m$ corrections in HQET

S. Negishi[1], H. Matsufuru[2], T. Onogi[3] and T. Umeda[4]
[1] *Department of Physics, Kyoto University, Kyoto 606-8501, Japan*
[2] *High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan*
[3] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[4] *Brookhaven National Laboratory, Upton, New York, 11973, USA*

Simulation of lattice QCD is one of the most effective approaches in order to test the standard model and find a clue to the physics beyond the standard model. But in the case of the bottom quark, the mass in lattice unit is so large $ma \approx 1 \sim 4$ that the conventional lattice fermion action has no control over the discretization errors. In order to avoid this problem which has been raised of the heavy-quark calculation, we must take the rational approaches such as the heavy-quark effective theory (HQET) [1], which is an effective theory that reproduces the low energy mode of a heavy quark in the static limit. However HQET has unknown coefficients for $1/m$ corrections terms and the ultraviolet behavior is essentially different from QCD. Therefore we must match HQET with QCD, which is composed of three steps: (a) to execute the matching condition $(\langle \mathcal{O}_{\rm QCD}(1/z)\rangle = \langle \mathcal{O}_{\rm HQET}(1/z)\rangle\,,\ z \equiv ML \gg 1$ ($\mathcal{O}$: an observable), where M is defined by [2]), (b) to match coefficients of $1/m$-correction terms from the matching condition and (c) to evaluate the step scaling functions [1]. In this report we give study on (a) as the first step for (b) and also computing the static limit; especially on the search for new observables which are efficient for the determination of $1/m$ term.

In Ref. [3], Heitger *et al.* propose we can evaluate some HQET parameters by combining observables in varied kinematical conditions and execute heavy quark lattice simulations of both QCD and HQET in $L_0 \approx 0.2$fm. Defining effective energies as $E^{\rm eff}_{\rm PS}(x_0,\theta)$, $E^{\rm eff}_{\rm V}(x_0,\theta)$, we can determine various observables in kinematical conditions such as $\Gamma_{\rm av}$, Ξ, to assess δm (mass term), $\omega_2^{(1)}$ (kinetic term) respectively. It is known that it is difficult to evaluate the kinetic term of $1/m$ correction $\omega_2^{(1)}$ with good accuracy, because the cancellation between power divergences of effective energies at $T/2$ and $T/4$ causes large systematic errors. Therefore in this work we propose an alternative observable:

$$\Xi^{\rm new} = \Gamma_{\rm av}(1/z)|_{\theta=1.0} - \Gamma_{\rm av}(1/z)|_{\theta=0.5},\ (\theta\text{: twisted boundary condition}). \tag{1}$$

Results display as follows. Fig. 1 shows the $1/z$ dependence of the observable $\Xi^{\rm new}$ at $\theta = 0.5,\ 1.0$, $\beta = 7.4802$. Assuming that the data for $z = 3.0,\ 3.8,\ 5.15$ are very close to the continuum limit, we made a linear fit in $1/z$ as $L\Xi^{\rm new} = a_0 + a_1/z$. Our preliminary results are $a_0 = 0.750(12)$ and $a_1 = 0.479(47)$, where the errors are statistical only. The data for $z = 9.0$ again deviate from the $1/z$ scaling, which is probably due to the discretization error. Therefore further study is needed to see if this discrepancy vanishes in the continuum limit. Nevertheless, it is encouraging that this observable has clear $1/z$ dependence so that it has sensitivity to determine the coefficient of $1/m$-correction terms $\omega_2^{(1)}$ in HQET.

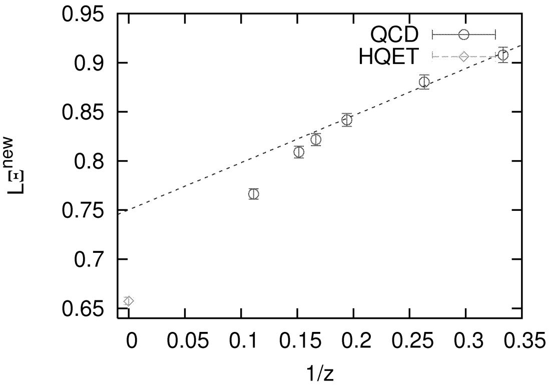

Figure 1: $1/z$ dependence of the new observable $L\Xi^{\rm new}$. The circle symbols denote our data for QCD and the diamond denotes the HQET result. The dashed line is the linear fit using three lighter points.

References

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