Four-Body Faddeev-Yakubovsky Calculation Using the Finite Range Expansion Method

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A four-nucleon scattering calculation with modern two- and three-nucleon interactions is one of the most challenging calculations in few-body physics. The four-body Faddeev-Yakubovsky (FY) equations are equivalent to the Schödinger or Lippmann-Schwinger (LS) equation applied to a four-body system, and the equations are employed in order to study four-nucleon systems accurately. One of the hurdles to solve the FY equations is the size of the matrices due to three variables to express the dynamics of a four-body system.

To clear this hurdle, we often represent the three-body and two-two subamplitudes in separable form, as well as two-body potentials, which is one of the standard approaches to three-body calculations (see, e.g. Ref. [1]). It transforms an integration into a sum of separable terms. If the number of separable terms needed to obtain converged results is considerably fewer in the separable expansion, it would be very useful to clear this hurdle, reducing the size of the matrices to be solved. But most of the methods require to solve the LS equation or eigenequation based on it, and numerical errors are accumulated with incleasing the number of separable terms. Therefore, in this paper, we propose to extend the finite range expansion[2] (FRE) method, which don't need to solve the eigenequations. This method was originally introduced to represent two-body potentials in separable form. It gives well converged solutions of the three-body Faddeev equations (see, e.g. [1]).

The basic idea of FRE for the two-body subsystem is as follows. We start from the two-body LS equation. In this equation, wave functions are always multiplied by the potentials. Thus, if potentials are of finite range in configuration space, we can expand the wave functions with a complete set of basis functions within that range. In the three-body subsystem, Alt-Glassberger-Sandhas[3] or Amado-Mitra-Faddeev-Lovelace (see, e.g. [4]) equations are the start corresponding to the LS equation in the two-body subsystem, and there are also the same kind of equations in the two-two subsystem. The Born terms of these equations correspond to the potential in the LS equation, and, if the terms are of finite range, we can extend FRE to the three-body and two-two subsystems. Details of this extension are described in Ref. [5].

This paper reports to check numerical convergence of the solutions of the FY equations applying to a four-nucleon system. We employ the Yamaguchi potentials as the nucleon-nucleon interactions for the $^1S_0$ and $^3S_1$-$^3D_1$ states, performed calculations for the $J^\pi = 0^+$, $T = 0$ state of the four-nucleon system, and omit the Coulomb force. Defining the energy as 0 at the four-body break-up threshold, we check the convergence in the negative energy region, because the Born terms of the three-body and two-two subsystems are of finite range and we can extend FRE directly. More details of the calculations for this feasibility study are also in Ref. [5].

Table 1 shows the convergence in the phase shift for 3N+N elastic scattering. The calculations are performed at −3.45 MeV, which is in between 2N+2N and 2N+N+N break-up thresholds. We found that FRE needs only 18 and 14 ranks for four-digit convergence in the three-body and two-two subsystems, respectively. We were able to realize the convergence up to 6 digits with FRE. In the three-body subsystem, for instance, 18 ranks implies 6 basis functions per channel. If we solve the FY equations without separable expansion, typically we need 30-40 sites per channel to perform a numerical integration. Thus, we found that FRE can reduce the size of the matrices of the FY equations without spoiling the numerical stability.

Table 1: Convergence of the phase shift (in degrees) for 3N+N elastic scattering at −3.45 MeV. The first column and row list the numbers of ranks for the three-body and two-two subsystems, respectively.

	2	8	14	20
1	-9.8741	-6.2460	−6.2334	−6.2334
6	13.7064	16.8625	16.8918	16.8919
12	13.1850	16.3836	16.4128	16.4128
18	13.1872	16.3850	16.4142	16.4142
24	13.1787	16.3766	16.4057	16.4057
30	13.1782	16.3761	16.4053	16.4053
36	13.1782	16.3761	16.4052	16.4053

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