Two step nonlinear acceleration theory for cyclic particle accelerators

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1. Introduction

Ultra precise particle beams are produced at the RCNP cyclotron complex due to the stabilization of magnetic fields within 2 ppm. This stabilization is achieved by controlling the temperature of iron cores of cyclotron magnets within $0.01°c$ for a long time. Several years ago, however, the temperature of iron cores of the Ring cyclotron was stabilized already but that of the AVF cyclotron, the injector cyclotron of Ring cyclotron, was not yet stabilized. When the strength of the magnetic fields of the AVF cyclotron changed by a tiny amount, i.e. 2 ppm, the energy spread of the particle beams extracted from the Ring cyclotron was deteriorated largely by a factor of 3. This unexpectedly large response of the energy spread to the small change of the magnetic field was difficult to understand within the present knowledge of the cyclotron.

In order to understand why the energy spread is so sensitive to the change of the magnetic field, we start to work on a nonlinear acceleration theory for cyclotron accelerators. If the beam quality depends, for example, on a square root of the change of the magnetic field, the square root of 2 ppm is as large as about 1 ppt. The consideration of nonlinearity can provide some hint as described here for the understanding of the above unexpectedly large response.

A concept of fixed field alternating gradient (FFAG) accelerator and a spiral-sector cyclotron were proposed in 1956 [1]. The effect of the radio frequency manipulation in fixed field accelerators was studied in the same year and it was shown that the longitudinal motion at the transition energy (isochronous point) was nonlinear [2]. The cyclotron is designed to utilize the isochronous condition and hence we have to consider much more seriously the nonlinearity in the longitudinal motion.

2. Nonlinear acceleration theory

In order to construct a nonlinear acceleration theory, a simple model for the cyclic particle accelerator is better expressed in the cylindrical coordinates [3]. The magnetic field is constructed to depend only on the radial coordinate, r, and the particle beam is accelerated by the surf-riding rf acceleration process. In addition, the electric field of the surf-riding rf acceleration consists of the tangential component, Et, and the normal component, En, along the particle path; the tangential component accelerates particles along the particle path and the normal component bends the particle path itself without acceleration like a magnetic field. The effect of the normal component depends on the self-consistent solution of the particle path, which is determined by the relation between the components of the electric field. This relation is given with the help of a factor a which is defined from the direction of the electric field at the acceleration gap as

$$E_{gap,r} = \alpha E_{gap,\theta}. \quad (1)$$

A particle motion on the median plane at z=0 is expressed as an independent variable of a turn number instead of time; the derivative of a physical quantity X with respect to the turn number is expressed as X', X'' and so on. Both the radial coordinate, r, and its derivative, r', can be eliminated from the equations of motion without any approximation and we obtain the following three coupled equations of motion which are expressed with variables of Lorenz factor, γ, angular velocity, $\dot{\theta}$, and the particle phase ϕ with respect to the acceleration field. The differential equations are obtained as,

$$\phi' = 2\pi h \frac{\omega_{rf} - h\dot{\theta}}{h\dot{\theta}}. \quad (2)$$

$$\gamma' = \begin{cases} \dfrac{q}{m_0 c^2} V \sin\phi & for\ \ synchrotrons\ \ \ and\ \ \ synchro-cyclotrons \\ \dfrac{q}{m_0 c^2} V \cos\phi & for\ \ \ cyclotrons \end{cases} . \tag{3}$$

$$\frac{1}{4\pi^2}\frac{a_1}{\gamma\dot{\theta}}\left(\gamma'\frac{\partial\omega_B^{eff}}{\partial\gamma} + \dot{\theta}'\frac{\partial\omega_B^{eff}}{\partial\dot{\theta}} + \gamma''\frac{\partial\omega_B^{eff}}{\partial\gamma'} + \dot{\theta}''\frac{\partial\omega_B^{eff}}{\partial\dot{\theta}'}\right)$$
$$= \left(1 - \frac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}}\right)^3 - \left(1 - \frac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}}\right)^2 + \frac{1}{4\pi^2}a_2\left(1 - \frac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}}\right) + \frac{1}{4\pi^2}a_1 a_3 , \tag{4}$$

Here, a series of functions a_i $(i = 0,1,2,3)$ are defined as follows,

$$a_0(\gamma,\gamma') = \frac{\gamma'}{\gamma}\frac{1}{1 - \dfrac{1}{\gamma^2}} , \tag{5}$$

$$a_1(\gamma,\dot{\theta},\gamma',\dot{\theta}') = \frac{\gamma'}{\gamma}\frac{1}{\gamma^2\left(1 - \dfrac{1}{\gamma^2}\right)} - \frac{\dot{\theta}'}{\dot{\theta}} , \tag{6}$$

$$a_2(\gamma,\dot{\theta},\gamma',\dot{\theta}',\gamma'',\dot{\theta}'') = \frac{\gamma'^2}{\gamma^2}\frac{2 + \dfrac{1}{\gamma^2}}{\gamma^2\left(1 - \dfrac{1}{\gamma^2}\right)^2} + \frac{\gamma'}{\gamma}\frac{\dot{\theta}'}{\dot{\theta}}\frac{1 - 4\dfrac{1}{\gamma^2}}{1 - \dfrac{1}{\gamma^2}} + \frac{\dot{\theta}'^2}{\dot{\theta}^2} - \frac{\gamma''}{\gamma}\frac{1}{\gamma^2\left(1 - \dfrac{1}{\gamma^2}\right)} + \frac{\dot{\theta}''}{\dot{\theta}} , \tag{7}$$

$$a_3(\gamma,\dot{\theta},\gamma',\dot{\theta}') = \frac{\gamma'}{\gamma}\frac{2 - 3\dfrac{1}{\gamma^2}}{1 - \dfrac{1}{\gamma^2}} + 3\frac{\dot{\theta}'}{\dot{\theta}} , \tag{8}$$

and ω_B^{eff} consists of the actual magnetic field and the bending effect of En and is called as an effective magnetic field in this paper for simplicity and is given as

$$\omega_B^{eff} = \omega_B^{eff}(\gamma,\dot{\theta},\gamma',\dot{\theta}') = \omega_B(\gamma,\dot{\theta}) - \frac{1}{4\pi^2}a_0\frac{\alpha 2\pi\left(1 - \dfrac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}}\right) - a_1}{2\pi\left(1 - \dfrac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}}\right) + \alpha a_1} , \tag{9}$$

where $\omega_B(r) = -\dfrac{q}{m_0}B_z(r)$ is defined on the basis of the actual magnetic field. We comment here several points of the above coupled equations. A remarkable point is that the equations are written in a compact form and they do not include the radial coordinate r explicitly. The non-linear effects are contained in the coefficients, a.

Hence, the above equations are applicable not only to cyclotrons but also to synchrotrons because none of the radial coordinate r appears in the above expressions, although the motivation of the present study comes from the unusual behavior of the cyclotron.

3. Reference beam orbit method

Three differential coupled equations, Eqs. (2), (3) and (4), are an extended version of the well-known expressions for the synchrotron oscillation. We propose now to solve the above coupled equations in two steps. In the first step, a proper motion corresponding to a reference particle motion is introduced as a reference motion by solving the above equations; a proper motion is given in a combination of γ'_{ref}=const. and θ'_{ref}=0 or $\dot{\theta}'_{ref}$=const. with a simplified assumption. A partial differential equation derived from Eq. (4) can be integrated out to obtain a reference effective magnetic field ω_{Bref}^{eff}.

In the second step, the accelerator conditions as the actual magnetic field and rf accelerating voltage (frequency and amplitude) are deviated slightly from the reference values obtained in the first step. The equations for the small deviation of arbitrary particles from the reference motion are then obtained by approximating the higher order terms while keeping some nonlinear expressions. We note that the non-linearity remains in the longitudinal motion of the small deviation. The resulting equations for the small deviation are then solved for a combination of the longitudinal motion and the transverse motion. Of course the longitudinal motion is a sum of linear motion and nonlinear motion for Lorentz factor γ. The transverse motion of the betatron oscillation is a linear motion and obtainable for angular velocity $\dot{\theta}$ instead of the radial coordinate r because the term of $\dot{\theta}''$ appears in Eq. (4) through Eq. (7). In order to solve the expressions for the deviation motion actually, we need one more relation,

$$\omega_B' = \gamma' \frac{\partial \omega_B}{\partial \gamma} + \dot{\theta}' \frac{\partial \omega_B}{\partial \dot{\theta}} = -K_B \frac{a_1}{1 - \dfrac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}}} \left\{ \omega_B^{eff} + \frac{1}{4\pi^2} a_0 \frac{\alpha 2\pi \left(1 - \dfrac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}} \right) - a_1}{2\pi \left(1 - \dfrac{\omega_B^{eff} - \gamma\dot{\theta}}{\gamma\dot{\theta}} \right) + \alpha a_1} \right\} \quad (10)$$

Here, $\boldsymbol{K_B} = -\dfrac{\boldsymbol{r}}{\boldsymbol{\omega_B}} \dfrac{\boldsymbol{d\omega_B}}{\boldsymbol{dr}}$ is a field index of actual magnetic field.

It should be noted for cyclotrons that such a reference of the first step as magnetic field is not an ideal one to be realized because of the vicinity of the transition energy. The deviation can provide fixed points preferably and thus nonlinear longitudinal oscillation occurs to generate a stable beam bunch. Observed large deterioration of the energy spread described above may possibly be related to the change of the nonlinear longitudinal oscillation motion.

4. Reconsideration of the common mode and the normal mode of power supply for accelerator magnets

We want to discuss here another important element for the stable operation of accelerators, which is a power supply. A rectifier device of power supply for accelerator magnets changes from the thyristor type to the IGBT type due to the recent technological advance. The power supply of the IGBT rectifier type, however, suffers from the noise at high frequency of around 1 MHz. Similarly, the power supply of the thyristor type has a noise at around 5 kHz. The suppression of this noise was achieved in the power supplies of thyristor rectifier type for the synchrotron magnets of HIMAC more than 10 years ago so that all the processes of injection, acceleration, slow extraction and deceleration in pulse operation were carried out successfully without introduction of any beam feedback. All these successes are caused by the ultra-stable magnetic fields of the HIMAC system.

The present performance of HIMAC with the 2270 V and 2260 A power supply for bending magnets at the flat top with the duration of around 500 ms was 1 ppm for dc stability and reproducibility without the use of an active filter as a series dropper and 0.3 ppm for ac noise covering 5 kHz component with an active filter of reactor-transformer type. The reason why the noise was suppressed so much was because common mode noise was suppressed by a common mode static filter in a special electric configuration of both power supply and magnet load, in the similar way as the normal mode noise was suppressed by a normal mode static filter.

The power supply and the magnet configuration of the common mode and the normal mode is given in Fig. 1, which shows the concept of the HIMAC power supply together with the static filters and magnet loads. It should be noted that all the components in 3-line electric circuit (6 terminals with 3 input terminals and 3 output terminals) are placed in a symmetric manner around the center (common) line and the iron yokes are connected to the common line from magnet to magnet. The current, I, flows at the upper line and the current, J, at lower line. The normal mode current and common mode current are then defined as I+J and I-J, respectively.

The common mode current flows just in the common line. Currents I and J have the same sign of normal mode current but have the opposite sign of common mode current. A data was taken for KEK 12 GeV PS and INS TARN2 for the common mode. The data showed that the main component of the noise was coming from the common mode current because of the opposite sign.

The origin of the common mode current is considered to come from the alternate switching of the thyristor rectifier of upper and lower halves of the power source, U and V in Fig. 1 so that a difference voltage U-V changes its sign and causes common mode current I-J. In order to suppress common mode current, a middle point of upper and lower halves of the power source should be connected to the common line so as to make the common mode filter effective.

If we do not connect between the middle point of the rectifier and the common line, the common mode filter turns out to be ineffective. In this case, we find a big noise in the magnetic fields coming from the common mode current. We conclude that the connection of the middle point of the power supply is important for stable operation of synchrotrons. The configuration of the IGBT power supply, however, does not have the middle point as the thyristor rectifier. Hence, usually we do not connect the middle point of the IGBT rectifier with the common line. This seems to be the reason of big noise. Hence, we recommend the use of the symmetric configuration to upper and lower halves and make the common mode filter effective by connecting the middle point of the rectifier with the common line. The detail is written in the recent publication [4].

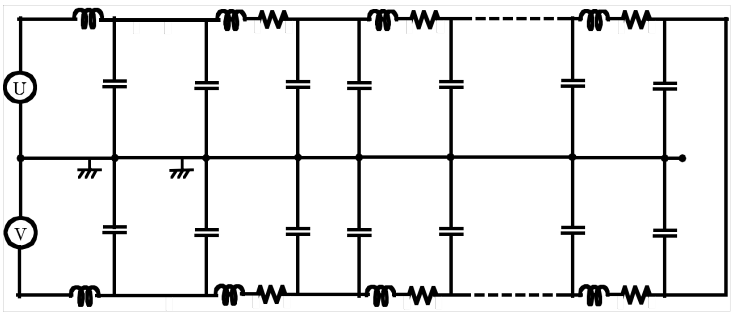

Fig. 1 The concept of the HIMAC power supply with the static filters and magnet loads.

References

[1] K.R. Symon, D.W. Kerst, L.W. Jones, L.J. Laslett, and K.M. Terwilliger, Fixed-Field Alternating-Gradient Particle Accelerators, Phy. Rev. 103 (1956) 1837-1859.

[2] K.R. Symon and A.M. Sessler, Method of Radio Frequency Acceleration in Fixed Field Accelerators with Applications to High Current and Intersecting Beam Accelerators, Proc. of CERN Symp., (1956) 44-58.

[3] K. Sato, J. Particle Accelerators Society of Japan, 1 (200) 1

[4] K. Sato and H. Toki, To appear in NIM (2006)