Charge and Parity Projected Relativistic Chiral Mean Field Model for ^{4}He

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1. Introduction

One of the fundamental goal of nuclear physics is to understand the mutual relation between the nuclear structure and the nuclear force. Recently, variational calculations based on the realistic nuclear force in the real space by Argonne-Illinois Group were found successful in describing light nuclei (A$\leq$ 10) and showed that the pion plays a crucially important role to determine the nuclear structure [1]. The contribution of the pion exchange interaction to the total binding energy is about 70 $\sim$ 80 % of the net two-body interaction. This *ab initio* calculation is strongly motivated us to construct the theoretical framework to reflect the unique character of the pion in the nuclear structure on the same footing of other mesons. An introduction of the finite pion mean field breaks the parity and isospin symmetries in the intrinsic single-paritcle states, beacuse of the psuedescalar and isovector character which leads to coupling with the nucleon by spin-flip and changes in the parity and the charge number. The effect of the pion-nucleon intaraction appears to be large for jj closed-shell nuclei, while for the LS closed-shell nuclei the effect turned out to be very weak. The effect of pion-nucleon interaction inceases at the nuclear surface[2]. The single-particle energy level structure, especially for the splitting between the spin-orbit partners clearly appears for jj closed-shell nuclei[3]. This phenomenon is an important consequence of pionic correlation due to it's unique character. For the LS closed-shell nuclei, however, the result is not satisfactory. This problem suggests us to make serious care of the treatment of the finite pion mean field. We here discuss on the relativistic mean field framework which takes into account the pionic effect with the variation after parity and charge number projection. We call this new framework the charge and parity projected relativistic mean field (CPPRMF) model. We then apply this framework for the ground state of ^{4}He and discuss the relation between the mechanism of the pionic correlation and the nuclear ground state structure.

2. Chiral sigma model Lagrangian

We start with the linear σ model with the ω meson for the description of nuclei from a point of view of the chiral symmetry. The chiral symmetry is known to be the most important symmetry in the strong interaction. At the hadron level, the chiral symmetry is well described by using the linear σ model of Gell-Mann and Levy[4]. The pions emerge as Nambu-Goldstone bosons from the spontaneous SU(2) chiral symmetry breaking[5]. As for the pion-nucleon interaction, the pseudoscalar type leads to an unrealistically large attractive contribution from the negative-energy states, because γ_5 involve strong coupling between positive- and negative-energy states. We thus employ the nonlinear realization of the Lagrangian density for the finite nuclear system, which is obtained by the Weinberg transformation of the linear σ model[6]. The pseudovector type, $\gamma_5\gamma_\mu$, decouples the positive- and negative-energy states. We take the lowest-order term in the pion field, and the Lagrangian[3] is

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - \frac{g_A}{2f_\pi}\gamma_5\gamma_\mu\vec{\tau}\cdot\partial^\mu\vec{\pi} - g_\omega\gamma_\mu\omega^\mu)\psi \\ &+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}{m_\sigma}^2\sigma^2 + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{1}{2}{m_\pi}^2\vec{\pi}^2 - \lambda f_\pi\sigma^3 - \frac{\lambda}{4}\sigma^4 \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}{m_\omega}^2\omega_\mu\omega^\mu + \widetilde{g_\omega}^2 f_\pi\sigma\omega_\mu\omega^\mu + \frac{1}{2}\widetilde{g_\omega}^2\sigma^2\omega_\mu\omega^\mu.\end{aligned} \tag{1}$$

We have set $M = g_\sigma f_\pi$, $m_\pi^2 = \mu^2 + \lambda f_\pi^2$, $m_\sigma^2 = \mu^2 + 3\lambda f_\pi^2$, and $m_\omega = \widetilde{g_\omega} f_\pi$. We take the empirical values for the masses and the pion decay constant as $M = 939$ MeV, $m_\omega = 783$ MeV, $m_\pi = 139$ MeV, and $f_\pi = 93$MeV. The coupling constants of the σ-nucleon g_σ and the $\sigma\omega$-coupling constant $\widetilde{g_\omega}$ are automatically fixed by the relations $g_\sigma = M/f_\pi = 10.1$ and $\widetilde{g_\omega} = m_\omega/f_\pi = 8.42$, respectively. The strength of the σ-meson self-energy terms depends on the σ-meson mass, m_σ, through the relation $\lambda = (m_\sigma^2 - m_\pi^2)/2f_\pi$. The σ-meson mass and the ω-nucleon coupling constant, g_ω, are the free parameters. We introduce the pion-nucleon coupling constants g_A into this Lagrangian. The characteristic feature of this Lagrangian is the Higgs mechanism, where not only the nucleons but also the ω mesons obtain their masses by σ-meson condensation in vacuum[7].

3. Charge and parity projected relativistic mean field theory

Pion has the character of pseudoscalar and isovector, which violates the parity and isospin symmentry. We thus demand the single-particle wave function consists of the four component with different parity and charge so that the pion is able to contribute to the finite mean field as other mesons. The nucleon Dirac spinor is given as

$$\psi_i = \psi_i(p,+) + \psi_i(p,-) + \psi_i(n,+) + \psi_i(n,-). \tag{2}$$

The intrinsic mixed parity and charge number total wave function is defined by a Slater determinant of a set $\{\psi_i, \langle\psi_i|\psi_j\rangle = \delta_{ij}\}$;

$$\Psi = \prod_{i=1}^{A} a_i^\dagger|0\rangle, \qquad\qquad i = n\tau jm. \tag{3}$$

Here the creation operator $a_i^\dagger$ creates a nucleon state with the quantum number i.

Since the nuclear state is a good eigenstate of parity and charge number, it is necessary to restore the parity and charge number of the total wave function. We obtaine the charge number and parity pprojected total wave function;

$$\Psi^{[Z,\pm]} = \mathcal{P}^c(Z)\mathcal{P}^p(\pm)\Psi, \tag{4}$$

by using the following charge number, $\mathcal{P}^c(Z)$, and parity, $\mathcal{P}^p(\pm)$, projection operators as,

$$\mathcal{P}^c(Z) = \frac{1}{2\pi}\int_0^{2\pi} d\theta e^{i(\hat{Z}-Z)\theta}, \qquad \hat{Z} = \sum_{i=1}^{A}\frac{1+\tau_i^3}{2}, \tag{5}$$

$$\mathcal{P}^p(\pm) = \frac{1\pm\hat{P}}{2}, \qquad \hat{P} = \prod_{i=1}^{A}\hat{p}_i, \qquad \hat{p}_i\psi_i(\vec{r},\xi) = \gamma_0\psi_i(-\vec{r},\xi). \tag{6}$$

An essential point of this study is that we adopt the charge number and parity projected wave function given in Eq.(4) as a trail function. This variational scheme is named the variation after projection(VAP). We will show the importance of this variational scheme in the next section. The variation with respect to the unknown functions included in the total energy $E^{[Z,\pm]}$, the meson fields and occupied single-particle states;

$$\delta E^{[Z,\pm]} = \delta\frac{\langle\Psi^{[Z,\pm]}|\hat{H}|\Psi^{[Z,\pm]}\rangle}{\langle\Psi^{[Z,\pm]}|\Psi^{[Z,\pm]}\rangle} = 0, \tag{7}$$

leads to the field equations for mesons and charge and parity projected relativistic mean field equations for nucleon. Here the total hamiltonian is given through the relation between the Lagrangian density and the Hamiltonian density;

$$\hat{H} = \int d^3x\mathcal{H} \qquad \mathcal{H} = \sum_{\phi}\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\dot{\phi} - \mathcal{L}, \tag{8}$$

where ϕ denotes the nucleon field ψ_i, and $\pi-$, $\sigma-$, and ω-meson fields.

4. Parity projection and variational method

Let us consider the positive and negative parity states which are obtained by operating the parity projection from the intrinsic parity mixed state to understand the mechanism of the pion-nucleon interaction. We represent the parity mixed single-particle state as, $|jm\rangle = \alpha_j|jm,\kappa\rangle + \beta_j|jm,\bar{\kappa}\rangle$. The intrinsic parity mixed total wave function which is fully occupied up to the Fermi level defined as just multiple state of single particle states for simplicity,

$$\begin{aligned}|\Psi\rangle &= \prod_{jm}\Big(\alpha_j|jm,\kappa\rangle + \beta_j|jm,\bar{\kappa}\rangle\Big) \\ &= \prod_{jm}\alpha_j|jm,\kappa\rangle + \sum_{j_1m_1}\beta_{j_1}|j_1m_1,\bar{\kappa}\rangle \prod_{jm\neq j_1m_1}\alpha_j|jm,\kappa\rangle \\ &+ \sum_{j_1m_1}\sum_{j_2m_2}\beta_{j_1}|j_1m_1,\bar{\kappa}\rangle\beta_{j_2}|j_2m_2,\bar{\kappa}\rangle \prod_{jm\neq j_1m_1,j_2m_2}\alpha_j|jm,\kappa\rangle + \cdots\end{aligned} \tag{9}$$

The first term corresponds to the state where all the single-particle states are occupied by the normal parity state and it is $|0p-0h\rangle$ state. This state has the 0^+ state. In the second term, $|j_1m_1,\kappa\rangle$ state is replaced with the opposite parity state, $|j_1m_1,\bar{\kappa}\rangle$. It means that in the $|0p-0h\rangle$ ground state, a particle moves from the normal parity single particle state, $|j_1m_1,\kappa\rangle$, to the abnormal parity state, $|j_1m_1,\bar{\kappa}\rangle$. Thus the second term means the sum of $|1p-1h\rangle$ states, which has 0^- spin-parity due to the $\pi(0^-)$-nucleon coupling. In the same manner, the third term means two 0^- $|1p-1h\rangle$ states, namely the $|2p-2h\rangle$ states which has the 0^+ parity.

Therefore, the wave functions which are projected out to the positive and negative parity state, respectively are written as,

$$\mathcal{P}^c(+)|\Psi\rangle = |(0p-0h)\rangle + |(2p-2h)\rangle + |(4p-4h)\rangle + \cdots, \quad (10)$$

$$\mathcal{P}^c(-)|\Psi\rangle = |(1p-1h)\rangle + |(3p-3h)\rangle + \cdots. \quad (11)$$

The character of the parity projected wave function is that the positive parity state consists of even number of 1p-1h pairs with 0^-. It means that the positive parity projection provides 2p-2h states as major correction terms. The matrix element of Hamiltonian,

$$\langle 0p-0h|\hat{H}|2p-2h\rangle, \quad (12)$$

gives the dominant component for the π^a-nucleon interaction, while for other mesons, σ, and ω-nucleon interactions the $\langle 0p-0h|\hat{H}|0p-0h\rangle$, $\langle 2p-2h|\hat{H}|2p-2h\rangle$, are the dominant components.

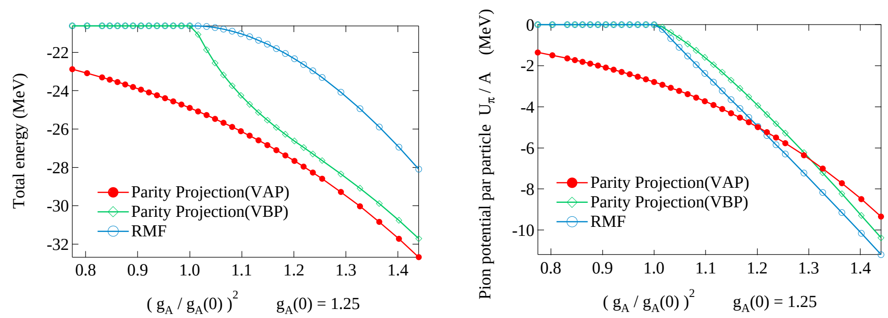

Figure 1: The total energy is shown in the left-hand panel and the pion energy per particle is shown in the right-hand panel as a function of the pion-nucleon coupling constant for the cases of variation before projection (diamonds), variation after projection (solid circle) and parity mixed RMF (open circle). $g_A(0)$ is the axial vector coupling constant in the free space πNN scattering.

Figure 1 shows the total energy and the pion energy per particle as a function of the pion-nucleon coupling constant squared. In the parity mixed relativistic mean field framework there is a critical coupling constant where the pion mean field starts to become finite. In the weak coupling region at around $(g_A/g_A(0))^2 \leq 1$, we do not get any energy gain by the VBP method. On the other hand, we obtain a large energy gain by the VAP method in this region. It is very important that the critical coupling constant is sufficiently small as compared with that of the free space πNN coupling, $g_A = 1.25$, and it means that such state where the pion mean field becomes finite exists as more stable state. In the parity mixed relativistic mean field framework, the LS-shell closed nuclei which have small contributions from the pion. In the parity projected relativistic mean field framework based on VAP scheme, we can take into account the effect of pion-nucleon interaction, namely 2p-2h correlations. The LS-closed shell nuclei also have sufficiently large effect of the pion-nucleon interaction. It is indispensable to solve the finite pion mean field based on the VAP scheme, especially in case of small pion-nucleon coupling.

5. Results

We apply the new relativistic mean field framework (CPPRMF) constructed in the previous section to ^{4}He nucleus. We assume that the intrinsic ground state is a fully occupied state as, $\{n\tau jm\} = \{0, 1, 1/2, \pm 1/2\}$ and $\{0, 2, 1/2, \pm 1/2\}$. The intrinsic total wave function (3) is a mixed state of charge number, $Z = 0 \sim 4$ and positive and negative parity states. The total wave function of the ^{4}He ground state (0^+, $Z = 2$) is obtained by projecting out the positive parity and charge state, $Z = 2$, according to Eq.(4).

In the CPPRMF method, not only the $\vec{\sigma}\cdot\tau^0\vec{\nabla}\pi^0$ type, but also the $\vec{\sigma}\cdot\tau^-\vec{\nabla}\pi^+$ and $\vec{\sigma}\cdot\tau^+\vec{\nabla}\pi^-$ type interactions are active and we can take into account this effect by the variation after charge number projection. Thus the amount of the expectation value of the pion energy, U_π, is around 3 times as large as that obtained in the case of the parity projected relativistic mean field method. This fact shows that the critical point, where the pion mean field arises, is sufficiently reduced. It means that more stable state is realized when the pion mean field becomes finite. Therefore, variation after projection method is important to construct the mean field framework with mixed parity and charge number to take properly into account the effect of the pion-nucleon interaction.

We study the constituents of the total energy for the case of finite pion mean field in the CPPRMF method as shown in Fig. 2. We adjust the ω-nucleon coupling constant to reproduce the total energy. We set pion-nucleon coupling constant $g_A = 1.15$. In general, as the pion mean field becomes larger, the kinetic energy

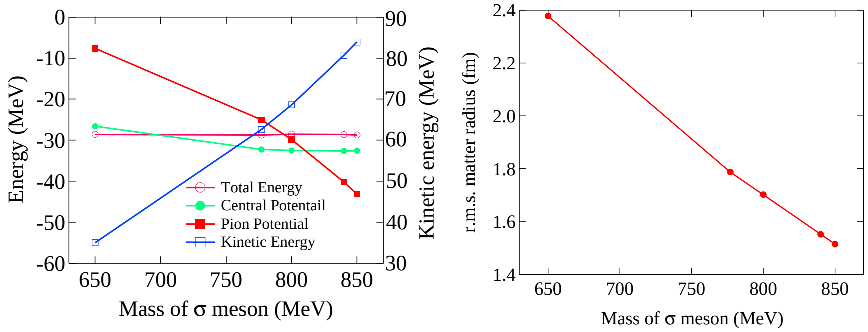

Figure 2: Constituents of the total energy for the ground state with 0^+ of ^{4}He in the CPPRMF method. On the left-hand panel are the pion potential (solid squares), $\sigma+\omega$-meson potentail (solid circles) and the kinetic energy (open squares, the scale is shown on the right axis). The total energy is represented by open circles. The matter r.m.s. radii are shown on the right-hand panel.

becomes larger and the central potential, $U_\sigma + U_\omega$, becomes smaller. This is the general tendency when the pion mean field arises. The mechanism of the energy gain due to the pion-nucleon interaction is shown in Eq.(16). To make the 2p-2h state for ^{4}He nucleus, for example, since two nucleons jump from the $0s_{1/2}$-orbital into the $0p_{1/2}$-orbital across the major shell in the shell model language, it needs large kinetic energy. The matter r.m.s. radius becomes small as the pion mean field increases. There is the strong correlation between the pion potential and the kinetic energy.

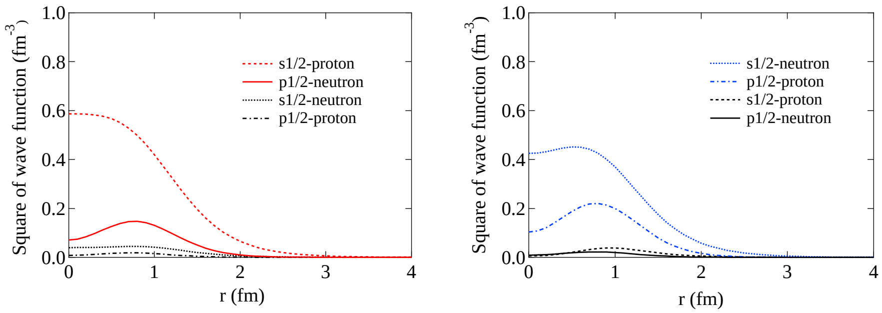

Figure 3: Square of components of the single particle wave function for the case with m_σ = 850 MeV. The dashed line represents the positive parity($0s_{1/2}$) proton state, the solid line represents the negative parity($0p_{1/2}$) neutron state, the dotted line represents the positive parity neutron state, and the dot-dashed line represents the negative parity proton state, respectively. The left-hand panel shows the proton dominant state, and right-hand panel shows the neutron dominant state.

Figure 3 shows the square of the intrinsic single-particle wave function in CPPRMF method. The proton dominant single-particle wave functions (τ =1) is shown in the left-hand panel. The dominant component is the positive parity ($s_{1/2}$) proton state. This state couples with the negative parity ($p_{1/2}$) neutron state through the pion-nucleon interaction. The negative parity ($p_{1/2}$) neutron component has its peak at around 0.8 fm. This component becomes quite compact as compared with that of normal harmonic-oscillator p-shell wave function, which has its peak at around 1.5 fm. We calculate the overlap between the ($p_{1/2}$) proton component(upper part) in the CPPRMF method and the 0p-shell state of harmonic-oscillator wave function with various width, then the oscillator length at b= 0.85 fm gives the maximum amount of overlap. This tendency has been clearly shown in the non-relativistic treatment for the case of the tensor force [8]. The 2p-2h state in the CPPRMF method is not to be expressed in terms of the usual simple $0p$ state.

The point proton density distribution of ^{4}He ground state is shown in left-hand panel of Fig. 4. The density distribution in the CPPRMF method is depressed at the central part. It has the peak of the distribution corresponds to that of the negative parity ($p_{1/2}$) proton component in Fig.4. This is because the pion-nucleon interaction induces the admixture of $p_{1/2}$ and $s_{1/2}$ components. There is no depression at central part unless pion-nucleon interaction works. The form factor of ^{4}He is obtained by Fourier transformation and shown in right-hand panel. The form factor obtained in CPPRMF method has a dip at around the momentum transfer squared $q^2 = 10$ fm^{-2}. This position is related with the depression of the density distribution. Without the pion-nucleon interaction, the form factor has the dip at larger momentum region, around $q^2 = 16$ fm^2. As the pion

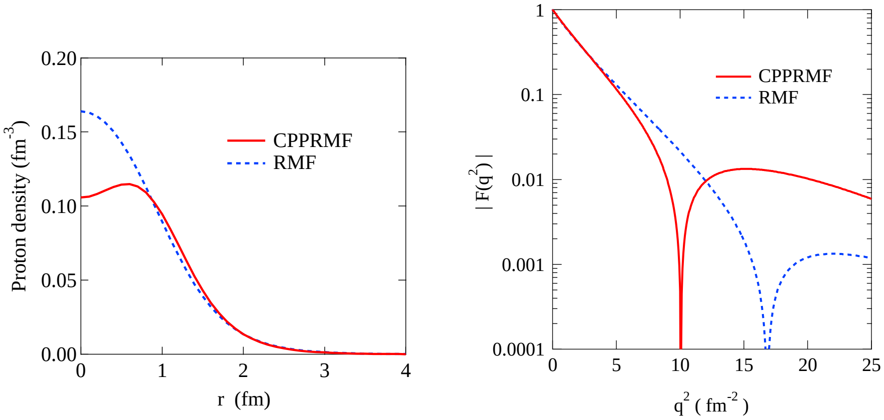

Figure 4: Density distribution (left-hand panel) and form factor (right-hand panel) for the ^{4}He ground state in the case with $m_\sigma = 850$ MeV, $g_A = 1.15$(solid curve). Those who obtained by the usual relativistic mean field calculation are also shown by the dashed curve, which corresponds to the $(0s_{1/2})^4$ configuration.

mean field becomes stronger the dip position gradually approaches to around $q^2 = 10$ fm^{-2}. Another critical feature of the form factor in CPPRMF method has a large amount of second maximum at high momentum region. It is related with the increase of the kinetic energy as the pion mean field works strongly. This fact means that the pionic correlation needs higher momentum components. In this calculation the amount of the second maximum significantly grows up from the case without pion mean field. The dip position and the second maximum at higher momentum clearly indicate the pion effect in the nucleus. The effect of the meson-exchange current is included naturally as a relativistic effect in the CPPRMF method. This is to be contrasted with any non-relativistic models[9].

6. Summay

We have discussed the role of the pion for the nuclear ground state structure by constructing the relativistic mean field with variation after charge number and parity projection scheme. We have shown that the state where the pion mean field becomes finite is realized more stable. In the VAP scheme, the LS closed-shell nuclei also have sufficiently large pionic effect. The finite pion mean field is obtained by the delicate balance between the energy loss due to the kinetic energy and the profit due to the pionic correlation in the total energy. We have obtained the ground state wave function of ^{4}He in the CPPRMF method. The pionic correlation leads to the large admixture of the $0s_{1/2}$ state and $0p_{1/2}$ state accompanied with the increase of the kinetic energy. The pion potential comes out to be close to about 70 %, which agrees with the variational calculation by Argonne-Illinois Group. The resulting wave function provides quite a satisfactory results for the form factor of ^{4}He.

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