Strong infrared divergence of the color-Coulomb potential

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The confinement mechanism in the Coulomb gauge has received a lot of attention recently. In the Coulomb gauge, Faddeev-Popov ghosts play a significant role in color confinement. It was shown by Zwanziger that the color-Coulomb potential which is an instantaneous interaction energy between quarks provides an upper bound for the static potential and the necessary condition for the static potential being a confining potential is that the color-Coulomb potential is also a confining potential, i.e., "no confinement without color-Coulomb confinement" [2]. The SU(3) lattice gauge simulation has revealed that the color-Coulomb potential rises linearly at large distances and it is stronger than the static potential [1]. Because the kernel of the instantaneous interaction contains the inverse of the Faddeev-Popov operator twice, it is expected that the strong confining feature of the instantaneous interaction originates from the near-zero modes of the Faddeev-Popov ghost operator.

Recently, Greensite et al. have proposed the necessary condition for color confinement. If the condition

$$\lim_{\lambda\to 0}\frac{\langle\rho(\lambda)F(\lambda)\rangle}{\lambda} > 0$$

is satisfied in the infinite volume limit, the color-Coulomb potential is more singular than the Coulomb potential ($\sim 1/\vec{p}^2$) in the infrared region. Here $\rho(\lambda)$ is the normalized eigenvalue density and $F(\lambda)$ is the expectation value of the negative Laplacian defined as follows:

$$\rho(\lambda) = \frac{N(\lambda,\lambda+\Delta\lambda)}{(N_c^2-1)V_3\Delta\lambda}, \quad F(\lambda) = \sum_a \int d^3x \phi_\lambda^{*a}(\vec{x})(-\nabla^2)\phi_\lambda^a(\vec{x}),$$

where $N(\lambda,\lambda+\Delta\lambda)$ is the number of eigenvalues in the interval $[\lambda,\lambda+\Delta\lambda]$, V_3 the spatial volume of a lattice, $\phi_\lambda(\vec{x})$ the eigenvectors of the Faddeev-Popov ghost operator. Since the color-Coulomb potential provides an upper bound for the static potential, the strong infrared divergence of the color-Coulomb potential is necessary for color confinement.

We have investigated the eigenvalue density of the Faddeev-Popov ghost operator in the Coulomb gauge using quenched SU(3) lattice gauge simulations [3]. We observed the accumulation of the near-zero modes of the ghost operator and confirmed that the necessary condition for color confinement is satisfied. Accordingly, we conclude that in the Coulomb gauge the enhancement of the low-lying eigenvalue density of the Faddeev-Popov ghost operator leads to the confining color-Coulomb potential.

Our simulations were performed on SX-5(NEC) vector-parallel computer at the RCNP of Osaka University.

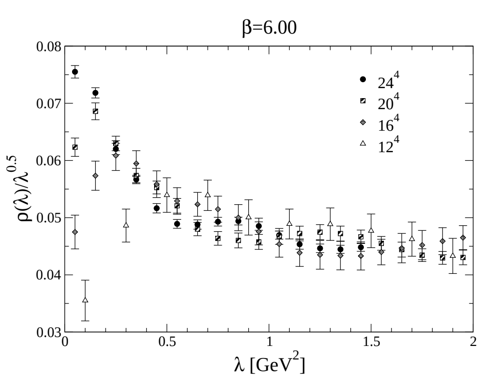

Figure 1: Eigenvalue density of the Faddeev-Popov ghost operator (divided by that in an Abelian gauge theory, $\rho(\lambda) \sim \sqrt{\lambda}$).

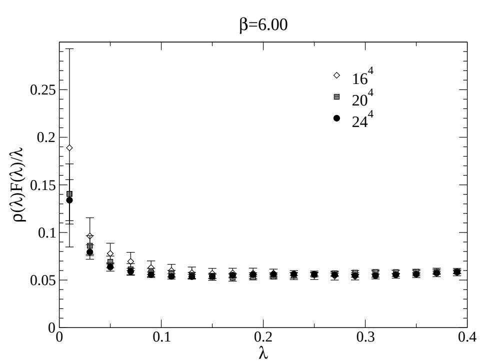

Figure 2: $\rho(\lambda)F(\lambda)/\lambda$ in the confinement phase. It seems that $\rho(\lambda)F(\lambda)/\lambda$ diverges or have a finite (positive) value as $\lambda \to 0$ in the infinite volume limit.

References

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