Bosonization of a two-color NJL model with a scalar diquark

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A hadronic effective Lagrangian is derived by bosonization of the quark fields and renormalized using the Eguchi method [1]. Although the bosonization technique has been used only for mesons, we can apply the technique not only for mesons (quark-antiquark) but also baryons (diquark) in two color scheme. Since baryons construct a matter, the bosonization of diquark-baryons is important to understand the behavior of the chiral symmetry in medium. We find the derived Lagrangian can be identified as an extended linear sigma model with meson and diquark-baryon fields. Hence our Lagrangian can describe both the quark and hadron dynamics.

We start from a two color and two flavor NJL Lagrangian with a scalar diquark fields as

$$\mathcal{L}_{NJL} = \bar{\psi}(i\partial\!\!\!/ - m_0 + \gamma_0\mu)\psi + \frac{G_0}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + \frac{H_0}{2}(\bar{\psi}i\gamma_5 t_2\tau_2 C\bar{\psi}^T)(\psi^T Ci\gamma_5 t_2\tau_2\psi). \tag{1}$$

The coupling constant for the mesonic channel G_0 and for diquark channel H_0 are uniquely fixed as $G_0 = H_0$ [2]. Hence mesonic channel and quark channel are transformed under SU(4) transformation at the same time, which is known as Pauli-Gürsey symmetry.

The bosonization technique is now used to write the Lagrangian in terms of auxiliary meson fields, $\sigma \sim \bar{\psi}\psi$, $\vec{\pi} \sim \bar{\psi}i\gamma_5\vec{\tau}\psi$ and diquark fields, $\Delta \sim \psi^T Ci\gamma_5 t_2\tau_2\psi$, $\Delta^* \sim \bar{\psi}i\gamma_5 t_2\tau_2 C\bar{\psi}^T$. The partition function of the NJL model including these auxiliary fields emerges an effective Lagrangian, after introducing the mean field approximation as $\sigma(x) = \sigma_0 + s(x)$ and $\Delta(x) = \Delta_0 + d(x)$ and $\Delta^*(x) = \Delta_0^* + d^*(x)$, and integrating out the quark fields, as

$$\begin{aligned}\mathcal{L} = &-\frac{i}{2}\mathrm{tr}(\ln\hat{S}^{-1} + \ln(1+\hat{S}\hat{K})) - \frac{1}{2}M_s^2\sigma_0^2 - \frac{1}{2}M_s^2(s^2(x) + \vec{\pi}^2(x)) - M_s^2\sigma_0 s(x) \\ &-\frac{1}{2}M_d^2\Delta_0^*\Delta_0 - \frac{1}{2}M_d^2(\Delta_0^* d(x) + d^*(x)\Delta_0) - \frac{1}{2}M_d^2 d^*(x)d(x)\ ,\end{aligned} \tag{2}$$

with the Nambu-Gorkov propagator matrix $\hat{S}$ and the auxiliary fields matrix $\hat{K}$ and the couplings are decomposed as $G_0 = \frac{g_0^2}{M_s^2}$ and $H_0 = \frac{g_d^2}{M_d^2}$. The dynamical properties of mesons and diquarks are generated from a expansion of $-\frac{i}{2}\mathrm{tr}\ln(1+\hat{S}\hat{K})$. Since the first four therms in the expansion provide divergent integrals in the limit of cut-off $\Lambda \to \infty$, we classify the terms as $U_{div} + U_{conv}$, where U_{div} indicates divergent integration terms and U_{conv} convergent terms in $U^{(k)}$ $(k = 1,2,3,4)$ and all other $\sum_{k=5}^{\infty} U^{(k)}$. The divergent terms can be absorbed into renormalization parameters and eventually we can obtain a renormalizable hadronic Lagrangian since the coupling constants g_0 and g_d are dimensionless and the order of the auxiliary fields are up to four.

The first order $k = 1$ generates the gap equations and the other terms $k = 2,3,4$ provide a generalized linear sigma model with inclusion of diquarks. The renormalization parameters are introduced as

$$\begin{aligned} g_0^2 I_0 =& Z_M^{-1}, \quad \sigma = Z_M^{\frac{1}{2}}\sigma_R \quad \vec{\pi} = Z_M^{\frac{1}{2}}\vec{\pi}_R, \quad \Delta = Z_M^{\frac{1}{2}}\Delta_R \quad \Delta^* = Z_M^{\frac{1}{2}}\Delta_R^* \\ & M_s^2 - 2g_0^2 I_2 + 4g_0^2 I_0\mu^2 = 0, \quad 2g_0^4 I_0 = Z_\lambda^{-1}\lambda_0\ , \quad \lambda = Z_M^2 Z_\lambda^{-1}\lambda_0\ , \\ & 2g_0 m_0(I_2 - 2\mu^2 I_0) = Z_{SB}^{-1}\varepsilon_0, \quad \varepsilon = Z_M^{\frac{1}{2}} Z_{SB}^{-1}\varepsilon_0\ ,\end{aligned} \tag{3}$$

with the second order and the logarithmic divergent integrals I_2 and I_0. The last line denotes the explicit chiral symmetry breaking term. Finally, we obtain:

$$\begin{aligned}\mathcal{L}_{E\sigma SB} =& \frac{1}{2}\left[(\partial_\nu\vec{\pi}_R(x))^2 + (\partial_\nu\sigma_R(x))^2 + |(\partial_\nu - 2i\mu\delta_{\nu 0})\Delta_R(x)|^2\right] \\ & - \frac{\lambda}{4}\left[\sigma_R^2(x) + \vec{\pi}_R^2(x) + \Delta_R^*(x)\Delta_R(x) - v^2\right]^2 + \varepsilon\sigma_R(x).\end{aligned} \tag{4}$$

To write this Lagrangian, we have assumed $g_0 = g_d$ and $M_s = M_d$ due to the Pauli-Gürsey symmetry. Since the diquark-baryon is constructed by two quarks (qq) and the antidiquark-baryon is by two antiquarks ($\bar{q}\bar{q}$), the baryon chemical potential can be written $\mu_B = 2\mu$ which appears in the kinetic term of the diquarks.

References

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