Overlap of QRPA states for calculation of nuclear matrix elements of neutrinoless double-beta decay

J. Terasaki

Division of Physics and Center for Computational Sciences, University of Tsukuba, Tsukuba, 305-8577, Japan

Neutrino has been assumed to be massless in the standard model, however, it has been proven recently that the neutrino has a mass [1]. Thus, it is an important subject in physics to determine the neutrino mass. One of few methods for this determination is to use the decay probability of the neutrinoless double-beta decay with an assumption that the neutrino is a Majorana particle, and a few dozens of projects are in progress in order to measure the decay probability. Clearly, we are now in the exciting era of the neutrino-mass problem.

The measured decay probability and the calculated transition matrix element are independent information for determining the neutrino mass. A challenging task in the theoretical procedure is to calculate the nuclear matrix element of that decay, to which accurate nuclear wave functions are necessary. One of methods to calculate the nuclear wave functions is the proton-neutron quasiparticle random-phase approximation (pnQRPA) (see Ref. [2] and references therein). We use a version of the QRPA slightly different from the usual way for avoiding near-instability due to the pairing correlations which often occurs in the pnQRPA; we use the like-particle QRPA. This approach is possible under the closure approximation (e.g. Ref. [3]) by using the equation of the nuclear matrix element

$$\mathcal{M}^{(0\nu)} = \sum_{b_f b_i} \sum_{ii'} \sum_{jj'} \langle ii'|V|jj'\rangle \langle 0_f^+|c_i^\dagger c_{i'}^\dagger|b_f\rangle \langle b_f|b_i\rangle \langle b_i|c_j c_{j'}|0_i^+\rangle. \quad (1)$$

ii' (jj') are proton (neutron) states, and their creation and annihilation operators are denoted by $c_i^\dagger, c_i$ $(c_j^\dagger, c_j)$. V is the transition operator arising from the neutrino potential. $|0_i^+\rangle$ and $|0_f^+\rangle$ denote the initial and final nuclear ground states of the decay, respectively. $|b_i\rangle$ and $|b_f\rangle$ are the intermediate states, which are obtained by the like-particle QRPA based on the initial and final ground states, respectively. The intermediate state is defined by

$$|b_i\rangle = O_{b_i}^\dagger |0_i^+\rangle_{\mathrm{QRPA}},\ O_{b_i}|0_i^+\rangle_{\mathrm{QRPA}} = 0, \quad (2)$$

where $O_{b_i}^\dagger$ is the creation operator of the QRPA state, and this operator can be obtained by solving the QRPA equation. The second equation defines the QRPA ground state $|0_i^+\rangle_{\mathrm{QRPA}}$. Needless to say, it is necessary to calculate the four factors in Eq. (1) with comparable accuracy. Therefore, we have to obtain the QRPA ground states explicitly using the definition (2) for $\langle b_f|b_i\rangle$. This calculation is, however, not easy at all, and simplified methods have been used (see references in Ref. [2]).

Our major achievement in 2012 is that the overlap $\langle b_f|b_i\rangle$ has been calculated using the QRPA ground state wave functions explicitly for the first time [4, 5] as

$$\begin{aligned}\langle b_f|b_i\rangle \ \simeq\ & \frac{1}{N_I N_F}\Big\{\langle f|O_{b_f}O_{b_i}^\dagger|i\rangle + \sum_{K\pi}\left(\langle f|v_F^{(K\pi)\dagger}O_{b_f}O_{b_i}^\dagger|i\rangle + \langle f|O_{b_f}O_{b_i}^\dagger v_I^{(K\pi)}|i\rangle\right) \\ & + \sum_{K\pi}\langle f|v_F^{(K\pi)\dagger}O_{b_f}O_{b_i}^\dagger v_I^{(K\pi)}|i\rangle\Big\}. \quad (3)\end{aligned}$$

The operator to transform the HFB initial ground state $|i\rangle$ to the QRPA ground state $|0_i^+\rangle_{\mathrm{QRPA}}$ is given by $\exp[v_F^{(K\pi)}]/N_I$ with the normalization factor N_I, and the analogous equation is applied to the final state. Here, $v_F^{(K\pi)}$ is the generator of the QRPA correlation, which is proportional to the QRPA backward amplitudes and expressed in terms of products of four quasiparticle creation operators. The K quantum number and the parity π are good quantum numbers in our calculations. We have shown that the second-order terms with respect to $v_F^{(K\pi)}$ and $v_I^{(K\pi)}$ in Eq. (3) are negligible in test calculations using $A = 26$ nuclei [4, 5]. The calculation of the nuclear matrix elements in the $A = 150$ region is in progress.

References

[1] S. Abe *et al.*, The KamLAND Collaboration, Phys. Rev. Lett. **100**, 221803 (2008)

[2] M. T. Mustonen and J. Engel, Phys. Rev. C **87**, 064302 (2013)

[3] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. No. 83, 1 (1985)

[4] J. Terasaki, Phys. Rev. C **86**, 021301(R) (2012)

[5] J. Terasaki, Phys. Rev. C **87**, 024316 (2013)