Coulombic Transformation in Momentum Space

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In a few-body system consisting of pair charged particles, Coulomb problem arises in momentum space because the diagonal components of the potential become singularities. Coulomb-Fourier transformation (CF) is known as a prescription for the Coulomb problem [1]. In a two-charged particle system, it is a unitary transformation which eliminates Coulomb potential from the original Hamitonian H .

Coulomb Wave Function in momentum space is given as

$$\langle \mathbf{p} \, |\psi_{\mathbf{k}}^{C(\pm)}\rangle = -\frac{1}{2\pi^2}\, e^{-\frac{\pi\eta}{2}}\, \Gamma(1\pm i\eta) \lim_{\varepsilon\to 0} \frac{\partial}{\partial\varepsilon} \frac{\left(p^2-(k\pm i\varepsilon)^2\right)^{\pm i\eta}}{\left((\mathbf{p}-\mathbf{k})^2+\varepsilon^2\right)^{1\pm i\eta}} \tag{1}$$

where the Sommerfeld parameter is $\eta = \frac{\mu e^2}{k}$ and the superscript $+$ $(-)$ denotes the physical boundary condition of outgoing (incoming) wave function. Using this expression, We have to evaluate the Coulombic transformation,

$$\mathcal{V}^{(++)}(\mathbf{k}',\mathbf{k}) = \langle \psi_{\mathbf{k}'}^{C(+)}|V^S|\psi_{\mathbf{k}}^{C(+)}\rangle = \int d\mathbf{p}' \int d\mathbf{p} \langle \psi_{\mathbf{k}'}^{C(+)}|\mathbf{p}'\rangle\langle \mathbf{p}'|V^S|\mathbf{p}\rangle\langle \mathbf{p}|\psi_{\mathbf{k}}^{C(+)}\rangle, \tag{2}$$

where V^S is a short range potential in the few-body system.

In numerical calculation, obviously, we have to establish the treatment of ε-limit in Eq.(2). Here, we adopt the point method[2] for this limitation. Namely, we evaluate the ε-limit by the analytic continuation.

For example, We calculate the CF-transformation for the Malfliet-Tjon potential [3] which has a Yukawa form as

$$V^S(\mathbf{p}',\mathbf{p}) = \frac{V_0}{2\pi^2}\frac{1}{(\mathbf{p}-\mathbf{p}')^2+\mu^2}. \tag{3}$$

The Coulombic transformed Malfliet-Tjon potentials are showed in Fig. 1 and Fig. 2, where $x = \hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}$ and we use the parameters as $V_0 = -65.1090[\text{MeV}\cdot\text{fm}^2]$, $\mu = 0.633[\text{fm}^{-1}]$.

We checked the high accuracy of numerical calculations of Coulombic transformation in momentum space, compared with the analytic expression in [1]

References

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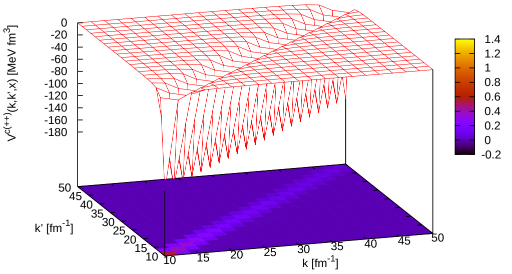

Figure 1: The Coulombic transformed Malfliet-Tjon potential at $x = 1$.

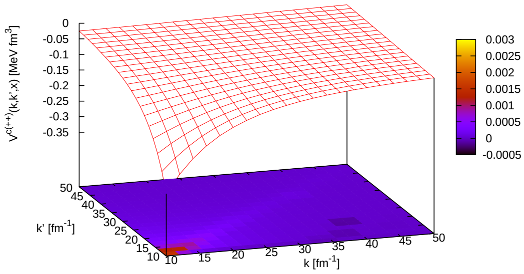

Figure 2: The Coulombic transformed Malfliet-Tjon potential at $x = 0$.