Relativistic mean-field model and random-phase approximation with vacuum polarization

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The collective multipole excitations are studied in the framework of relativistic random-phase approximation with the vacuum polarization. First, we show for the nuclear ground state that the leading order of derivative expansion of the effective action arising from the vacuum correction agrees with the exact calculation using the Green function method very well. The derivative expansion makes us easy to perform a fully self-consistent calculation, even for the random-phase approximation. A remarkable effect of the inclusion of the vacuum polarization is the increase of the effective mass $m_{\text{eff}}=m_N^0$, which gives, for all multipole modes, smaller energy-weighted sum rule values than those of the typical relativistic model. Also, the large effective mass constrained by the vacuum polarization can give an excellent agreement with experimental data on the excitation energy for the isoscalar quadrupole resonances. It is shown, further, that the change of the shell structure due to the vacuum polarization plays an important role in the dipole compression modes.

Keywords: Relativistic mean field; random phase approximation; vacuum polarization.

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1. Introduction
The giant resonance of nuclei is of great interest because its energy can be directly related to the properties of nuclear matter, which places important constraints on the theoretical description of supernovae explosions, neutron stars, heavy ion collisions, and so on. The reliability of the model employed for the giant-resonance analyses may be thus confirmed by the comparison with the experimental energy-weighted sum rule (EWSR) distributions. A knowledge on the magnitude of the EWSR for the electromagnetic response in addition to its distribution will also impose a new constraint with regard to the nuclear model, and offer the valuable information for the study of a two-photon exchange process through the nuclear particle-hole excitations such as the dispersion correction to the elastic electron scattering and
nuclear-polarization correction to the atomic and muonic levels, which are approximately proportional to the EWSR strength.

A standard tool for the theoretical description of nuclear excitations is the random-phase approximation (RPA). In particular, many studies have been done by using the relativistic RPA (RRPA) on the relativistic mean-field (RMF) basis, wherein only positive-energy nucleons are taken into account and the Dirac sea is always regarded as unoccupied (this is sometimes called no-sea approximation\textsuperscript{1,2,3}). This approximation is very convenient because we do not have to worry about a renormalization procedure not only in the calculation of basis set under the full one-nucleon-loop contribution, which we refer to as the relativistic Hartree approximation (RHA), but also in the RRPA calculation.

In the no-sea approximation, the negative-energy states are not used for the construction of ground state at all. Nevertheless, these states, antinucleon states, which are bound strongly owing to the small effective mass, yields an important contribution for the calculation of excited states in the RPA framework in order to remove the spurious center-of-mass motion and to fulfill the current conservation. It is also known that using this approximation, there remain the discrepancies, presumably originated by the small effective mass, in the $\beta$-decay rates\textsuperscript{4} and the giant quadrupole resonances\textsuperscript{5,6}. On the other hand, the effect of the negative-energy states forces the effective nucleon mass large and hence, it is possible to solve these problems. These are the places where we thought important to reconsider the vacuum polarization effects.

In this work, we treat the one-loop vacuum correction explicitly in the Walecka model Lagrangian to study the effective mass effect in the nuclear excitations. The advantage of this approach is that, by fitting the ground-state properties, one can obtain the parameters which give the large effective mass without an elaborate modification of the Walecka model. It should be pointed out that, with the parameter set giving a large effective mass, the naive dimensional analysis\textsuperscript{7} gives naturalness of the one-loop vacuum correction. A problem in fitting the single-particle energies raised by the large effective mass can be solved by simply adding the tensor-coupling term for the meson-nucleon interaction. Then, we present a systematic study of the giant monopole, dipole, quadrupole resonances for the different effective mass in the fully consistent RRPA framework.

2. Vacuum Polarization

Although it is inevitable that the negative-energy nucleon states contribute to the nuclear structure as the vacuum fluctuation within a relativistic framework, all recent investigations of the RRPA in the QHD model have neglected the actual antinucleon degrees of freedom. On the other hand, the rigorous calculation of the basis set (the Hartree level) under the full one-nucleon-loop contribution has been developed by using the Green function method\textsuperscript{8}. With this technique, however, the estimation of the vacuum-polarization effect is too hard even in a ground state to
extend it to the RRPA formalism including the particle-hole correlations. For the present RHA + RPA calculation, therefore, we consider to apply the local-density approximation (LDA) and/or the derivative expansion (DE), which describe the vacuum contribution by the additional new terms of meson fields in the effective Lagrangian. As a result, it makes easy to perform the RRPA calculation with vacuum polarization because the meson propagators are constructed including the vacuum part of nuclear response as well as self-interaction terms. First, we shall compare the baryon and scalar densities of LDA and DE induced by the vacuum polarization with those of exact Green function method. The effective action of one-loop correction with LDA and leading-order DE is given by

$$\Gamma^{(1)} = \int d^4x - V_F(\sigma) + \frac{1}{2} Z^\sigma_F(\sigma) (\partial_\mu \sigma)^2 + \frac{1}{4} Z^\sigma_F(\sigma) \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{4} Z^A_F(\sigma) F_{\mu\nu} F^{\mu\nu} + \cdots,$$

(1)

where the functional coefficients are given by

$$V_F(\sigma) = -\frac{1}{4\pi^2} \left[ (m_N - g_\sigma \sigma)^4 \ln \left( 1 - \frac{g_\sigma \sigma}{m_N} \right) + m_N^3 g_\sigma \sigma - \frac{7}{2} m_N^2 g_\sigma^2 \sigma^2 + \frac{13}{3} m_N g_\sigma^3 \sigma^3 - \frac{25}{12} g_\sigma^4 \sigma^4 \right],$$

(2)

$$Z^\sigma_F(\sigma) = -\frac{g_\sigma^2}{2\pi^2} \ln \left( 1 - \frac{g_\sigma \sigma}{m_N} \right),$$

(3)

$$Z^\sigma_F(\sigma) = \frac{g_\sigma^2}{3\pi^2} \ln \left( 1 - \frac{g_\sigma \sigma}{m_N} \right),$$

(4)

$$Z^A_F(\sigma) = \frac{e^2}{6\pi^2} \ln \left( 1 - \frac{g_\sigma \sigma}{m_N} \right).$$

(5)

The LDA corresponds to the use of $V_F(\sigma)$ only, while the leading order DE corresponds to the use of the first four terms in Eq. (1). The baryon and scalar densities are obtained by the functional derivatives of the effective action (1) by the omega and the sigma mesons, respectively. The baryon and scalar densities induced by the vacuum polarization are given in Fig. 1, together with those from the exact calculation using the Green function method. There, for purpose of comparison, we assume the same potential in evaluating the vacuum polarization. We can see that the densities obtained by the LDA are corrected significantly; not only for the baryon density, which vanishes in this approximation, but also for the scalar density. Both the scalar and baryon density-profiles obtained by the Green function method are in a surprisingly good agreement with those of the leading-order DE, and thus we find that it is sufficient to expand up to the leading order of DE in the vacuum polarization.

Next, we shall consider the vacuum contribution to the particle-hole correlation. In general, the Hartree response function, $\Pi_H$, is decomposed by two parts; the density part, $\Pi_D$, and the Feynman part, $\Pi_F$. Using the derivative expansion, we can simply obtain the Feynman part by the second derivatives of the meson fields...
\[ \Pi_F^{\sigma} = \delta(x-y) \left[ V''(\sigma) - \frac{1}{2}Z''(\sigma)(\partial_\mu \sigma)^2 - \frac{1}{2}Z''(\sigma)(\partial_\mu \omega_0)^2 - \frac{1}{2}Z''(\sigma)(\partial_\mu \vec{A}_0)^2 \right] / g_\sigma^2 \\
+ \delta(x-y)[\partial_\mu \partial^\mu Z_F(\sigma)] / g_\sigma^2 + \partial^\mu [Z_F(\sigma) \partial_\mu \delta(x-y)] / g_\sigma^2, \]
\[ \Pi_F^{\omega} = \partial^\mu [Z_F^\omega(\sigma)(\partial_\mu \omega_0)\delta(x-y)] / g_\sigma^2, \]
\[ \Pi_F^{\omega} = -Z_F^\omega(\sigma)(\partial_\mu \omega_0)(\partial^\mu \delta(x-y)) / g_\sigma^2, \]
\[ \Pi_F^{\omega} = \partial^\mu [Z_F(\sigma)(\partial_\mu \delta(x-y))] / g_\sigma^2, \]
\[ \Pi_F^{A} = \partial^\mu [Z_F^A(\sigma)(\partial_\mu \vec{A}_0)\delta(x-y)] / e^2, \]
\[ \Pi_F^{A} = -Z_F^A(\sigma)(\partial_\mu \vec{A}_0)(\partial^\mu \delta(x-y)) / g_\sigma^2, \]
\[ \Pi_F^{A} = \partial^\mu [Z_F^A(\sigma) \partial_\mu \delta(x-y)] / e^2, \]

where the superscript in \( \Pi_F \) indicates mesons at initial and final vertices. It may be also expected in the RPA calculation that it is sufficient to expand up to the leading order of DE in the vacuum polarization as far as the low-momentum transfer region are concerned. The verification of the self-consistency provide a check that the vacuum polarizations in the RPA level are sufficiently described by Eqs. (6)-(12). In fact, we emphasize here that the spurious state in the isoscalar dipole mode is decoupled and the transition density is conserved by using \( \Pi_F \) of Eqs. (6)-(12).

3. Results and Discussion

3.1. EWSR

Energy-weighted sum rules depend on the completeness of the particle-hole basis and are preserved in the RPA calculation provided a certain appropriate condition is satisfied. Therefore, the EWSR is often used as an indicator of the strength of the nuclear response in the calculation which requires the summation over all nuclear intermediate states. Generally, we find that all relativistic results of the EWSR are somewhat larger than the classical EWSR\(^{11}\) in any multipole states, and the reason has been previously presumed as being due to the effective mass\(^{1,12}\). The model including the vacuum polarization cannot shift the effective mass to less
than \( m_{\text{eff}}/m_N = 0.8 \) due to the feedback effect from the vacuum. As a result, the EWSR values are to be suppressed in comparison with the values calculated with TM1\(^{13}\) and NL3\(^{14}\) (Fig. 2). Nevertheless, they are still enhanced by 10 – 30\% of the nonrelativistic classical results. Experimentally, the excess strength has been reported in the analysis of the giant monopole resonance for the 13.7 keV peak in \(^{208}\text{Pb}\)^{15}. The present results are also consistent with the scattering analysis. In addition, it would be interesting to perform the analysis of anomaly in the nuclear-polarization corrections for the \(^{90}\text{Zr}\), \(^{208}\text{Pb}\), and Sn isotope by the present extended relativistic nuclear models; the nuclear-polarization correction is considerably sensitive to the EWSR strength and the anomaly seems to require the effective mass effect\(^{16,17}\). As regards the effect of the large effective mass, we will reconsider in the subsection of the giant quadrupole resonance.

In passing, we mention that since the charge operator \( Q \gamma_0 \) commutes with the single-particle Dirac Hamiltonian of nucleus, the EWSR value should vanish if the "RRPA negative-energy states" are also taken into account in the sum. This is due to the fact that the RRPA states satisfy a completeness relation by including them.

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**3.2. ISGMR**

The compressibility of nuclear matter is important in the description of the properties of nuclei. It is generally accepted that the best procedure to determine the compression modulus is to calculate isoscalar monopole energies regarded as the "breathing mode" of the nucleus by using microscopic models. In the present analysis we examine the predictions of the various parameter sets of the effective Lagrangian within the framework of the self-consistent RRPA. In Fig. 3, we show the ISGMR centroid for \(^{90}\text{Zr}\), \(^{116}\text{Sn}\), and \(^{208}\text{Pb}\) nuclei as a function of the compressibility. Evidently we see that the centroid energies with the similar compressibility agree with each other, and are irrelevant to the inclusion of the vacuum polarization. This means that the inclusion of the vacuum polarization has a negligible effect in the position of the monopole excitation energy if the compressibility is fixed to a
3.3. ISGDR

The isoscalar-giant dipole resonance (ISGDR) contains the spurious component caused by the center-of-mass motion. We have indicated that the spurious state appears at zero excitation energy if the fully self-consistent calculation is achieved\textsuperscript{10}. The ISGDR strengths for \(^{116}\text{Sn}\) and \(^{208}\text{Pb}\) are depicted in Fig. 4. Experimentally, it is observed to split into upper- and lower-energy components in the ISGDR strengths (the experimental peaks are indicated by two arrows in the figure\textsuperscript{18,5}).

Both in the model with (the solid curve) and without the vacuum polarization (the dashed and dash-dotted curves), we see that there are two components similar to observed ones. In comparison with the experiments, however, the splittings between the upper and lower peaks are somewhat large in all calculations employed here. On the other hand, we find that calculations reproduce the peaks of the lower-energy region. Moreover, the results for the upper component with the vacuum polarization are different from the results of the models without the vacuum polarization, and are nearer to the observed data, although the predicted peaks are still 1 – 4 MeV higher than the data. It has been reported that the upper component is the compression mode, whose energy depends on the compression modulus, while the energy of the lower component changes with the effective mass but is essentially independent of compression modulus\textsuperscript{19,20}. However, in the present model with the parameter set RHAT1, which has a similar compression modulus to that with TM1 and NL3, the energies in upper region for all nuclei certainly shift to the lower energy than those with TM1 and NL3. This suggests that not only the compression modulus but also the shell structure is important to resolve the discrepancy in the position of the peak for the upper-energy component.

3.4. ISGQR

The giant quadrupole resonance is induced by the \(2\hbar\omega\) particle-hole correlation in the harmonic oscillator potential model. In relativistic models the shell structure
is significantly affected by the effective mass. As a result, the models with and without the vacuum polarization may yield the different results from each other. The EWSR distributions of the Coulomb response of the ISGQR mode are shown in Fig. 5 for $^{90}$Zr and $^{208}$Pb. The ISGQR strength exhibits a strong collectivity. The peaks seen in the region of less than 8 MeV for the ISGQR of $^{208}$Pb come from the low-lying discrete states. In both nuclei, the downward shift of the model with vacuum polarization (RHAT1) from the model without vacuum polarization (TM1 and NL3) is clearly observed both in the ISGQR modes. Also, it is found that the RHAT1 results are in excellent agreement with the experimental data indicated by the arrow for each nucleus$^5,21$. The centroid energies calculated by the relativistic model with the parameter set NL3 and the relativistic point-coupling model were approximately 1-2 MeV above the experimental energies$^5,6$. These disagreements are drastically improved by the present model including the vacuum polarization. Thus, the effective mass plays an important role in the improvement of the results in the ISGQR. This is contrast to the ISGMR case with compressibility fixed to a certain value. From the present analysis, it is indicated that the effective mass $m_{eff}/m_N \sim 0.9$ is required experimentally. In the present model the large effective mass is theoretically caused by the vacuum effect. It is a quite interesting result that the inclusion of the vacuum polarization gives much better results for nuclear excitations through the effective mass effect.

Fig. 4. The isoscalar-dipole strength in $^{116}$Sn and $^{208}$Pb. Arrows indicate experimental energies.

Fig. 5. The isoscalar-quadrupole strength in $^{90}$Zr and $^{208}$Pb. Arrows indicate experimental energies.
4. Summary

We have studied the nuclear excitations by a self-consistent RRPA method with the vacuum-polarization contribution incorporating with tensor-coupling term for the $\omega$ meson. With the vacuum polarization, it is impossible to obtain the effective mass as small as that of the conventional relativistic model, since the vacuum polarization works to reduce the scalar-meson field. As a result, the enhancement of the EWSR strength due to the effective mass is suppressed by the inclusion of the vacuum polarization. In addition, we stress here that the present extended relativistic model is able to give better predictions in almost all giant resonances than the relativistic model without the vacuum polarization. In particular, we have shown that the calculated ISGQR centroids reproduce the experimental data for each nucleus nicely. Thus the present study suggests that the vacuum-polarization effect plays an important role in the excitations through the change of the single-particle structure by means of the effective mass effect. Further testing of the present models by observables involving nuclear excitations, such as nuclear-polarization corrections in muonic atoms and distributions of $\beta$-decay strengths, would be very interesting.

References