Phenomenology of spin 3/2 baryons with pentaquarks

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RCNP, Osaka\textsuperscript{a}
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Introduction: Flavor SU(3) symmetry

Existence of $\Theta^+$ + Flavor SU(3) symmetry

- Existence of flavor partners of $\Theta^+$

Assuming the flavor multiplet that $\Theta^+$ belongs to, we examine its properties by symmetry relation, in connection with known baryon resonances.

- to determine the $J^P$ of $\Theta^+$

Phenomenological but model independent analysis up to $O(m_s)$
Pure antidecuplet case

Simplest assignment for $\Theta^+$

Test the masses and widths of partners via flavor SU(3) symmetry relations
**Pure antidecuplet case**

**Mass and decay width [MeV]**

\[
M(10; Y) = M_{10} - aY \\
g_{\Theta KN} = \sqrt{6} g_{N^*} \pi N
\]

<table>
<thead>
<tr>
<th>(J^P)</th>
<th>(M_{\Theta})</th>
<th>(M_N)</th>
<th>(M_{\Sigma})</th>
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<th>(\Gamma_{\Theta})</th>
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<td>1/2(^-)</td>
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**Pure antidecuplet case**

**Mass and decay width [MeV]**

\[ M(1\bar{1}0; Y) = M_{1\bar{1}0} - aY \]

\[ g_{\Theta KN} = \sqrt{6}g_{N^* \pi N} \]

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### Pure antidecuplet case

**Mass and decay width [MeV]**

\[ M(\bar{10} \; Y) = M_{\bar{10}} - aY \]

\[ g_{\Theta KN} = \sqrt{6} g_{N^* \pi N} \]

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### Pure antidecuplet case

**Mass and decay width [MeV]**

\[
M(\overline{10}; Y) = M_{\overline{10}} - aY
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<td>1540 (\Theta(1540))</td>
<td>1700 (N(1700))</td>
<td>1860</td>
<td>2020 (\Xi(2030))</td>
<td>1.3</td>
</tr>
</tbody>
</table>

are not reproduced simultaneously.
Octet-antidecuplet mixing

Second simplest assignment for $\Theta^+$

Mixing is induced by the SU(3) breaking in mass term.
Octet-antidecuplet mixing

Mass formulae

\[ M_{\Theta} = M_{10} - 2a \]
\[ M_{\Xi_{10}} = M_{10} + a \]
\[ M_{\Lambda} = M_{8} \]
\[ M_{\Xi_{8}} = M_{8} + b + \frac{1}{2}c \]
\[ M_{N_{1}} = \left( M_{8} - b + \frac{1}{2}c \right) \cos^2 \theta_{N} + (M_{10} - a) \sin^2 \theta_{N} - \delta \sin 2\theta_{N} \]
\[ M_{N_{2}} = \left( M_{8} - b + \frac{1}{2}c \right) \sin^2 \theta_{N} + (M_{10} - a) \cos^2 \theta_{N} + \delta \sin 2\theta_{N} \]
\[ M_{\Sigma_{1}} = (M_{8} + 2c) \cos^2 \theta_{\Sigma} + M_{10} \sin^2 \theta_{\Sigma} - \delta \sin 2\theta_{\Sigma} \]
\[ M_{\Sigma_{2}} = (M_{8} + 2c) \sin^2 \theta_{\Sigma} + M_{10} \cos^2 \theta_{\Sigma} + \delta \sin 2\theta_{\Sigma} \]

8 masses v.s. 6 parameters

\[ J^P = 1/2^- : \text{too wide width} \]
\[ J^P = 3/2^+ : \text{states are not well established} \]
Mass spectra

1/2^+

1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0

1/2^+

predicted

3/2^-

3/2^-

predicted
Decay width of $\Theta$

Relation between coupling constants

$$g_\Theta = \sqrt{6}(g_{N_2} \cos \theta_N - g_{N_1} \sin \theta_N)$$

C.G. Coeff.  N* decay  from masses

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$\theta_N$ [deg]</th>
<th>$\Gamma_\Theta$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^+$</td>
<td>29</td>
<td>29.1</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>33</td>
<td>3.1</td>
</tr>
</tbody>
</table>
Two-meson coupling

Then, what about **two-meson coupling**?

SU(3) relation enable us to calculate

the cross section of

from the decay of nucleons into two pions.
Two-meson coupling

Contact interaction:

\[ pK + nK + \pi + pK + nK + \pi + \Delta + \rho_0 \]
Two-meson coupling

Branching fraction [%]

<table>
<thead>
<tr>
<th>J^P</th>
<th>state</th>
<th>πN</th>
<th>ππN(s)</th>
<th>ππN(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2^+</td>
<td>N(1440)</td>
<td>65</td>
<td>7.5</td>
<td>&lt;8</td>
</tr>
<tr>
<td></td>
<td>N(1710)</td>
<td>15</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>3/2^-</td>
<td>N(1520)</td>
<td>55</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>N(1700)</td>
<td>10</td>
<td>&lt;85-95</td>
<td>&lt;35</td>
</tr>
</tbody>
</table>

Still large uncertainty
Constraints on the coupling

Self-energy: not too large, but not too small

\[ \pi^- p \rightarrow K^- \Theta^+ \text{ at KEK: upper limit is } \sim 4.1 \mu b \]

~ 100 MeV

It is necessary to use the interference effect among two terms, \( s \) and \( v \).
$\Theta$ production

$\pi^- p \rightarrow K^- \Theta^+$

$K^+ p \rightarrow \pi^+ \Theta^+$

$1/2^+$

$\sigma(K^+) \sigma(\pi^-) \sim 50$

large interference

$3/2^-$

$\sigma(K^+) \sigma(\pi^-) \sim 3$

small interference
We examine 8–10 mixing scheme for the exotic and non-exotic baryon resonances.

Masses of $\Theta(1540)$ and $\Xi(1860)$ are well fitted in the 8–10 mixing scheme with $J^P = 1/2^+$ or $3/2^-$ baryons.

The width of $\Theta$ is very narrow for the $J^P = 3/2^-$ case.

For both cases, the mixing angle is close to the ideal angle.

Summary 2: Two-meson coupling and $\Theta$ production

Based on the mixing scheme, we evaluate the two-meson coupling of $\Theta$, and calculate the reaction process for $\Theta$ production.

There is an interference effect between two amplitudes, which is prominent for $1/2^+$ case and rather moderate for $3/2^-$ case.

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$g^s$</th>
<th>$g^v$</th>
<th>$\sigma_{\pi^-}$</th>
<th>$\sigma_{K^+}$</th>
<th>Re$\Sigma_{\Theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^+$</td>
<td>1.59</td>
<td>-0.27</td>
<td>4.1 $\mu$b</td>
<td>$&lt;1928$ $\mu$b</td>
<td>-78 MeV</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>0.104</td>
<td>0.209</td>
<td>4.1 $\mu$b</td>
<td>$&lt;113$ $\mu$b</td>
<td>-23 MeV</td>
</tr>
</tbody>
</table>