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TWO-MESON CLOUD AROUND Θ^+

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The self-energy of the flavor SU(3) antidecuplet coming from two-meson cloud effect is studied. Assuming that the Θ^+ and the $N(1710)$ resonances belong to an antidecuplet representation, we construct effective Lagrangians for the decay of $N(1710)$ into $N\pi\pi$ with two pions in s- or p-wave based on the SU(3) symmetry. The self-energies for all members of the antidecuplet turn out to be attractive binding, and stronger binding is obtained as the strangeness increases. The two-meson cloud contributes at least 20% of the empirical mass splitting between states with different strangeness.

1 Introduction

In 2003, evidences of exotic pentaquark states $\Theta^+(1540)$[1] and $\Xi^{--}(1860)$[2] were reported. The spin, parity, and representation of flavor SU(3) of these particle are not yet determined experimentally. In general, once the representation of flavor SU(3) is specified, the mass spectrum within a flavor multiplet is described by the Gell-Mann–Okubo[GMO] rule up to linear order of strange quark mass. The GMO mass splitting is mainly originated from the mass differences between strange and light quarks. In this paper, we investigate the self-energy caused by the two-meson cloud, which contributes to the mass splitting in addition to the quark mass difference.

The two-meson cloud component in the Θ^+ is naturally expected, by observing that the energy of $K\pi N$ system is only 30 MeV above the Θ^+ state. Indeed, there were previous attempts to describe the Θ^+ as a $K\pi N$ state[3,4,5,6], and some attractive interaction is found for the $K\pi N$ states of $J^P = 1/2^+$, although the strength is not enough to bind the three-body system.

In the present work, we assume that the Θ^+ and $N(1710)$ belongs to an antidecuplet ($\overline{\mathbf{10}}$) representation of flavor SU(3). This implies the $J^P = 1/2^+$ for the Θ^+. Then we construct effective interaction Lagrangians which account for the $N(\pi\pi$ s-wave) and ρN decay channels of this resonance. Using the $SU(3)$ extended effective Lagrangians, we calculate the self-energy of all members of the antidecuplet. Details of the present study can be found in Ref. 7.

2 Formulation

We assume that the interaction Lagrangians are SU(3) symmetric. For the present purpose, we need the process $\mathbf{8}_M + \mathbf{8}_M + \mathbf{8}_B \to \overline{\mathbf{10}}_P$ where an octet baryon $\mathbf{8}_B$ and two octet mesons $\mathbf{8}_M$ couple to an antidecuplet baryon $\overline{\mathbf{10}}_P$. The group theoretical irreducible decomposition of the product of $\mathbf{8}_M$, $\mathbf{8}_M$, $\mathbf{8}_B$, and $\overline{\mathbf{10}}_P$ gives four independent singlets, in which two $\mathbf{8}_M$ mesons are combined into $\mathbf{8}^s_{MM}$, $\mathbf{8}^a_{MM}$, $\mathbf{10}_{MM}$ and $\mathbf{27}_{MM}$. This means that there are four independent structures of the interaction Lagrangians, which are SU(3) symmetric. However, two of them ($\mathbf{8}^a_{MM}$, $\mathbf{10}_{MM}$) are identically zero, due to additional symmetry under exchange of two mesons. Therefore, we can construct the following effective Lagrangians without derivatives:

$$\mathcal{L}^{8s} = \frac{g^{8s}}{2f} \overline{P}_{ijk} \epsilon^{lmk} \phi_l{}^a \phi_a{}^i B_m{}^j + h.c. \ , \tag{1}$$

$$\mathcal{L}^{27} = \frac{g^{27}}{2f} \left[4\overline{P}_{ijk} \epsilon^{lbk} \phi_l{}^i \phi_a{}^j B_b{}^a - \frac{4}{5} \overline{P}_{ijk} \epsilon^{lbk} \phi_l{}^a \phi_a{}^j B_b{}^i \right] + h.c. \ , \tag{2}$$

where P, B and ϕ are the baryon antidecuplet, baryon octet and meson octet fields, respectively. The superscripts $8s$ and 27 stand for the SU(3) structure of the combinations of two $\mathbf{8}^s$ mesons. A factor $1/2f$ is introduced to make g^{8s} and g^{27} dimensionless ($f = 93$ MeV is the pion decay constant). These Lagrangians are the two lowest ones with respect to the low energy expansion. In practice, however they are not sufficient to account for the experimental decay of $N(1710)$ into two pions correlated in the ρ-meson. In order to reproduce such decay mode, we introduce a Lagrangian with one derivative:

$$\mathcal{L}^{8a} = i \frac{g^{8a}}{4f^2} \overline{P}_{ijk} \epsilon^{lmk} \gamma^\mu (\partial_\mu \phi_l{}^a \phi_a{}^i - \phi_l{}^a \partial_\mu \phi_a{}^i) B_m{}^j + h.c. \ . \tag{3}$$

In section 4, we will discuss other possible Lagrangians.

The antidecuplet self-energies are given by

$$\Sigma_P^{(j)}(p^0) = \sum_{B,m_1,m_2} \left(F^{(j)} C^{(j)}_{P,B,m_1,m_2} \right) I^{(j)}(p^0; B, m_1, m_2) \left(F^{(j)} C^{(j)}_{P,B,m_1,m_2} \right) \ , \tag{4}$$

where the index j labels the interaction Lagrangians, the argument p^0 is the energy of the antidecuplet baryon, $F^{(j)}$ are coupling constants appearing in the Lagrangian, and $C^{(j)}_{P,B,m_1,m_2}$ are SU(3) coefficients which are compiled in Appendix of Ref. 7. The function $I^{(j)}(p^0; B, m_1, m_2)$ is the two-loop integral with two mesons and one baryon as shown in the upper panel of Fig. 1:

$$\begin{aligned} & I^{(j)}(p^0; B, m_1, m_2) \\ &= - \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} |t^{(j)}|^2 \frac{1}{k^2 - m_1^2 + i\epsilon} \frac{1}{q^2 - m_2^2 + i\epsilon} \\ & \times \frac{M}{E} \frac{1}{p^0 - k^0 - q^0 - E + i\epsilon} \ , \end{aligned} \tag{5}$$

where $t^{(j)}$ are the amplitudes derived from the Lagrangian (j), M and m_i are the masses of the intermediate baryon and mesons, and E is the energy of the intermediate baryon. The real part of this integral is divergent, and therefore we introduce the three momentum cutoff Λ in the range $700 - 800$ MeV.

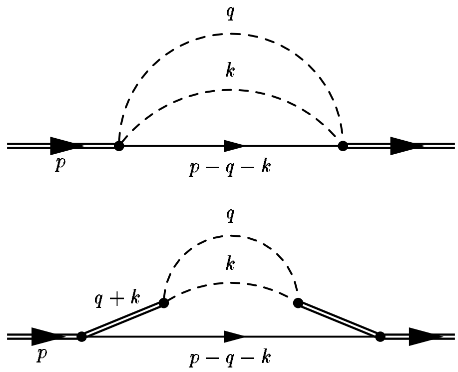

Fig. 1. Diagrams for self-energy of baryon antidecuplet due to two-meson cloud. Lower panel : inclusion of vector meson propagator.

The imaginary part of the diagram represents the decay width, in accordance with the optical theorem. The total decay width is given by $\Gamma_P^{(j)}(p^0) = -2\mathrm{Im}\Sigma_P^{(j)}(p^0)$, while the partial decay width to a particular channel is given by

$$\begin{aligned}&\Gamma_P^{(j)}(p^0; B, m_1, m_2)\\ &= -2\mathrm{Im}\left(F^{(j)}C^{(j)}_{P,B,m_1,m_2}\right) I^{(j)}(p^0; B, m_1, m_2)\left(F^{(j)}C^{(j)}_{P,B,m_1,m_2}\right) ,\end{aligned} \tag{6}$$

It is known that $N(1710) \to N\pi\pi$(p-wave) occurs through the $N\rho$ decay. In order to keep the closest contact to the experimental information, we replace the contact interaction of the $\mathcal{L}^{8a}$ to account for the vector meson propagator (Fig. 1, lower), and include the factor $m_v^2/[(q+k)^2 - m_v^2]$ in each $P \to BMM$ vertex.

3 Numerical results

In this section we perform calculations using $\mathcal{L}^{8s}$ and $\mathcal{L}^{8a}$. We will address $\mathcal{L}^{27}$ and other possible Lagrangians in section 4. The coupling constants in the Lagrangians are fixed so as to reproduce the partial decay widths of the $N(1710)$ to $N\pi\pi$(s-wave, isoscalar) and $N\rho \to N\pi\pi$(p-wave, isovector) respectively. These are controlled by the imaginary part of the self-energies, which are finite and independent of the cutoff. The central values in the PDG[8] are $\Gamma(N\pi\pi, s\text{-wave}) = 25$ MeV and $\Gamma(N\pi\pi, p\text{-wave}) = 15$ MeV. A fit to these values gives us $g^{8s} = 1.9$ and $g^{8a} = 0.32$. With these couplings we calculate the real part of the self-energies for all the antidecuplet. For the bare antidecuplet mass p^0 as input, we take an average value of $p^0 = 1700$ MeV. We have checked the dependence of p^0 and found that the results have the same qualitative trend, but the depth of the binding varies.

In Fig. 2 we show the results for the contributions from $\mathcal{L}^{8s}$ and total contributions of $\mathcal{L}^{8a}$ and $\mathcal{L}^{8s}$, with cutoffs 700 and 800 MeV. We see that all the self-energies are attractive, and that the interaction is more attractive for the larger the strangeness, hence the $\Theta_{\overline{10}}$ is always more bound. $\mathcal{L}^{8s}$ provides more binding than $\mathcal{L}^{8a}$ for the same cutoff. The splitting between the $\Theta_{\overline{10}}$ and $\Xi_{\overline{10}}$ states is about 45 MeV for a cutoff of 700 MeV and 60 MeV for a cutoff of 800 MeV. Since the experimental splitting is 320 MeV for the $\Theta(1540)$ and $\Xi(1860)$, the splitting provided by the two-meson cloud is of the order of 20 % of the experimental one.

We show the results that we obtain for the partial decay widths in Table 1. To calculate the decay, we have taken the observed masses, $M_{N_{\overline{10}}} = 1710$, $M_{\Sigma_{\overline{10}}} = 1770$ and $M_{\Xi_{\overline{10}}} = 1860$ MeV as p^0, because the phase space is essential for the imaginary part. We can see that the widths are not very large for all channels. When compared with the experimental data, indeed, $\Sigma(1770)$ and $\Xi(1860)$ would have total widths into two-meson and a baryon of about 24 and 2 MeV, which are compatible with the total width of about 70 and 18 MeV, respectively[2,8]. Detailed information of the partial decay widths of these resonances to three body channels will give us more understanding of the $PBMM$ interaction.

4 Other possible Lagrangians

In Ref. 7, in addition to $\mathcal{L}^{8s}$ and $\mathcal{L}^{8a}$, we examined $\mathcal{L}^{27}$ and two more terms, namely chiral symmetric Lagrangian $\mathcal{L}^{\chi}$ and $SU(3)$ breaking Lagrangian $\mathcal{L}^{M}$. However, it was found that the contributions from these additional terms are either absorbed into the $\mathcal{L}^{8s}$ contribution or restricted to be small from physical consideration. A possible small contribution from these terms would be considered as a theoretical uncertainty in our analysis.

First we address the chiral symmetric Lagrangian;

$$\mathcal{L}^{\chi} = \frac{g^{\chi}}{2f}\overline{P}_{ijk}\epsilon^{lmk}(A_\mu)_l{}^a(A^\mu)_a{}^i B_m{}^j + \text{h.c.}, \tag{7}$$

where $A_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$ is the axial current written in terms of the chiral field $\xi = e^{i\phi/\sqrt{2}f}$. For the process of our interest, we can just replace

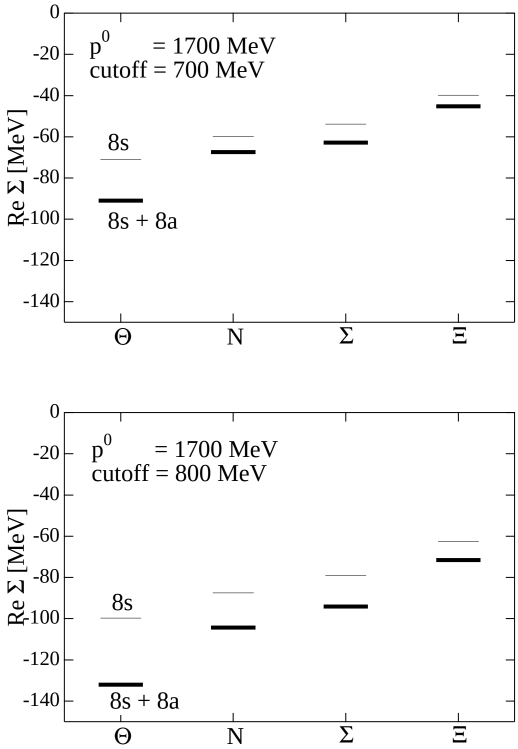

Fig. 2. Mass shifts of baryon antidecuplet ($\mathrm{Re}\Sigma_P$) due to two-meson cloud with $p^0 = 1700$ MeV.

as follows.

$$(A_\mu)_l{}^a (A^\mu)_a{}^i \rightarrow \frac{1}{2f^2} \partial_\mu \phi_l{}^a \partial^\mu \phi_a{}^i. \tag{8}$$

As we see, since the SU(3) structure of $\mathcal{L}^\chi$ is identical to that of the $\mathcal{L}^{8s}$, we can naively expect that the result should not change very much. Indeed, by setting $\Lambda = 525$ MeV, the mass shifts obtained from $\mathcal{L}^\chi$ are similar to those of $\mathcal{L}^{8s}$. There are some deviation in decay width of the order of a few MeV, due to the meson momenta appearing in the $\mathcal{L}^\chi$ loop.

Next we draw our attention to the $\mathcal{L}^{27}$ and $\mathcal{L}^M$ Lagrangians. Explicit

Table 1. Partial decay widths for the allowed channels and total width for any BMM channel, at the masses of the antidecuplet members. All values are listed in units of MeV.

Decay widths [MeV]	$\Gamma^{(8s)}$	$\Gamma^{(8a)}$	Γ^{tot}_{BMM}
$N(1710) \to N\pi\pi$ (inputs)	25	15	40
$N(1710) \to N\eta\pi$	0.58	-	
$\Sigma(1770) \to N\overline{K}\pi$	4.7	6.0	24
$\Sigma(1770) \to \Sigma\pi\pi$	10	0.62	
$\Sigma(1770) \to \Lambda\pi\pi$	-	2.9	
$\Xi(1860) \to \Sigma\overline{K}\pi$	0.57	0.46	2.1
$\Xi(1860) \to \Xi\pi\pi$	-	1.1	

SU(3) breaking mass term $\mathcal{L}^M$ is defined as

$$\mathcal{L}^M = \frac{g^M}{2f}\overline{P}_{ijk}\epsilon^{lmk}S_l{}^iB_m{}^j + \text{ h.c.,} \tag{9}$$

with $S = \xi M\xi + \xi^\dagger M\xi^\dagger$, $M = \text{diag}(\hat{m}, \hat{m}, m_s)$. In the same way as Eq. (7), we expand the chiral field to extract the vertex we need. We note that it is unrealistic to make these Lagrangians by themselves responsible for the N(1710) decay width into $N\pi\pi$(s-wave) channel. This would lead to some unphysical results such as large binding energy of several hundreds MeV. Thus, assuming that one can not have a large fraction of these Lagrangians, we can study to what extent we can allow the fraction of the contributions from $\mathcal{L}^{27}$ and $\mathcal{L}^M$. The results are shown in Ref. 7. It turns out that $\mathcal{L}^{27}$ tends to contribute to make the binding energy deeper, and $\mathcal{L}^M$ also contributes to attractive binding energy, and the splitting of $\Theta_{\overline{10}}$ and $N_{\overline{10}}$ becomes large compared with the other splittings.

5 Discussion and conclusion

We have studied the two-meson cloud effect to the baryon antidecuplet. The assumptions made throughout the paper and the uncertainties in the experimental input make the nature of our analysis qualitative. We assume that the Θ^+ is a $1/2^+$ state with $I = 0$ and that it belongs to an antidecuplet. In addition to these minimal assumptions, we consider that the $N(1710)$ also belongs to this same antidecuplet. The meson cloud mechanism proposed in this work leads to the following conclusions:

1. The two-meson cloud yields an attractive self-energy for all members of

the antidecuplet. The observation of attraction is consistent with the previous attempts to describe the Θ^+ as a $K\pi N$ state[3,4,5,6].

2. It also contributes to the splitting between antidecuplet members, which is only moderately cutoff dependent and provides about 20% of the total splitting to a stronger effect for reasonable values of the cutoff. The role played by the two-meson cloud is therefore of relevance for a precise understanding of the nature of the Θ^+ and the antidecuplet.

3. The magnitude of 20% is also in agreement quantitatively with the strength of attraction found in the previous study of BMM three-body system[5]. The values of the mass splitting are such that they still leave some room for quark correlation effects after the GMO mass splitting coming from the mass difference between u, d and s constituent quarks is considered. The contribution to the splitting from the meson cloud is of the same order of magnitude as the one provided by these quark correlations.

4. From the experimental point of view, it is clear that the investigation of the decay channels into two mesons and a baryon of the resonances $N(1710)$, $\Sigma(1770)$ and $\Xi(1860)$ deserves renewed interest.

For future perspective, we are going to apply the effective interactions obtained here to the reaction mechanism of the Θ^+ production. It is also important to include the representation mixing of $\overline{\mathbf{10}}$ with $\mathbf{8}$, with the different J^P assignment for the Θ^+ in order to construct the effective interaction[9].

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