Applicability of CDCC to the deuteron breakup at low energies

— Is CDCC an alternative to the Faddeev theory? —

RCNP Nuclear Physics Colloquium

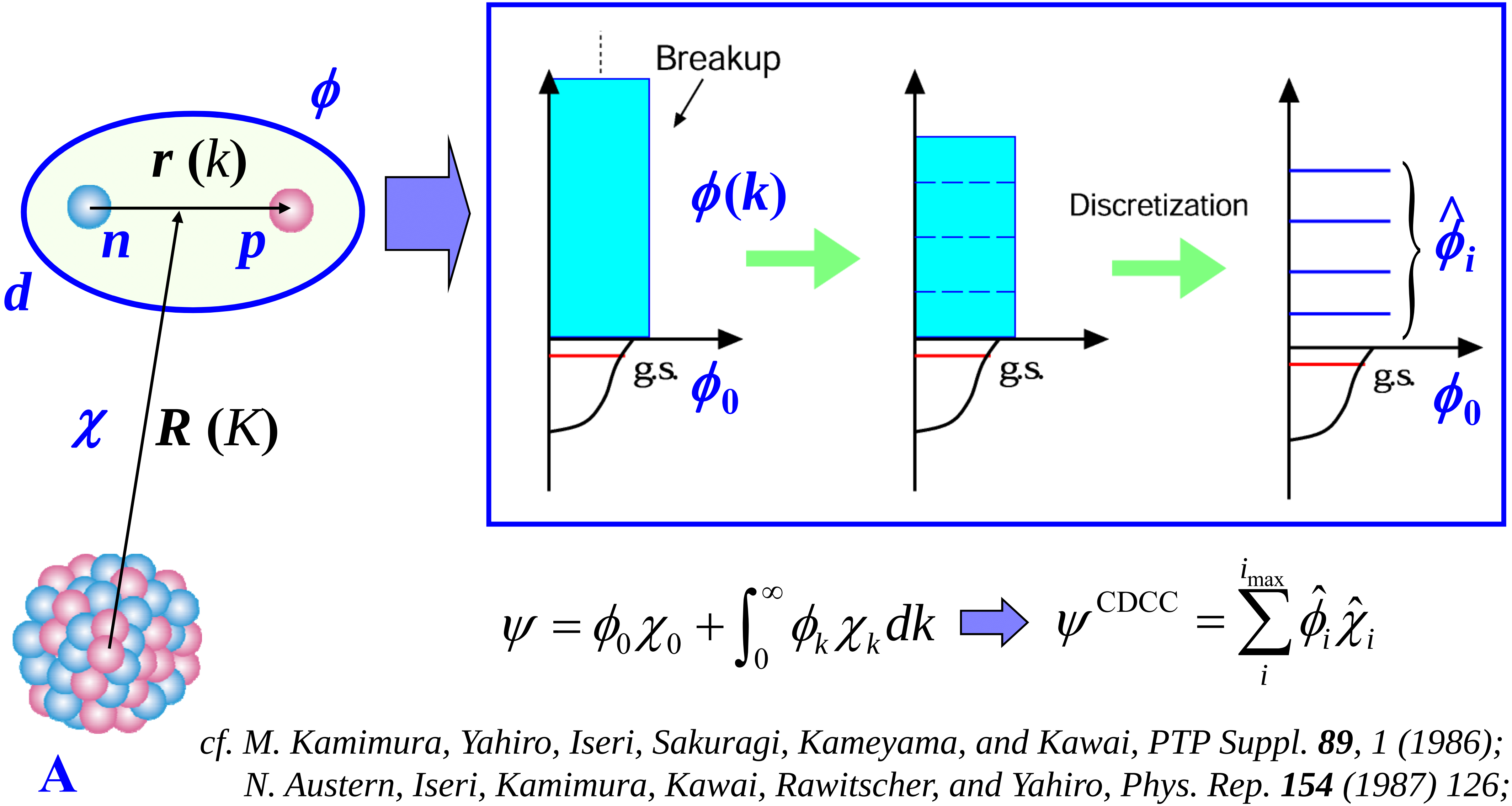
Kazuyuki Ogata

Research Center for Nuclear Physics (RCNP), Osaka University

in collaboration with

Kazuki Yoshida

The Continuum-Discretized Coupled-Channels method: CDCC (after *l*-truncation)



$$\psi = \phi_0 \chi_0 + \int_0^\infty \phi_k \chi_k dk \Rightarrow \psi^{\mathrm{CDCC}} = \sum_i^{i_{\max}} \hat{\phi}_i \hat{\chi}_i$$

cf. M. Kamimura, Yahiro, Iseri, Sakuragi, Kameyama, and Kawai, PTP Suppl. **89**, 1 (1986);
N. Austern, Iseri, Kamimura, Kawai, Rawitscher, and Yahiro, Phys. Rep. **154** (1987) 126;
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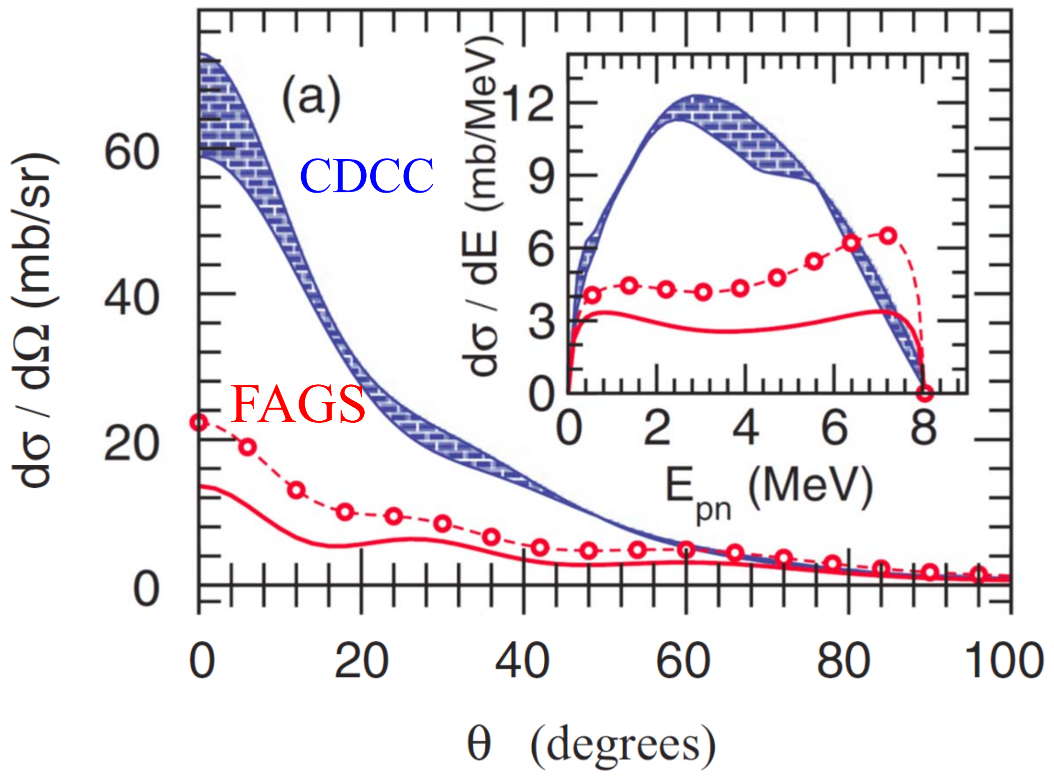
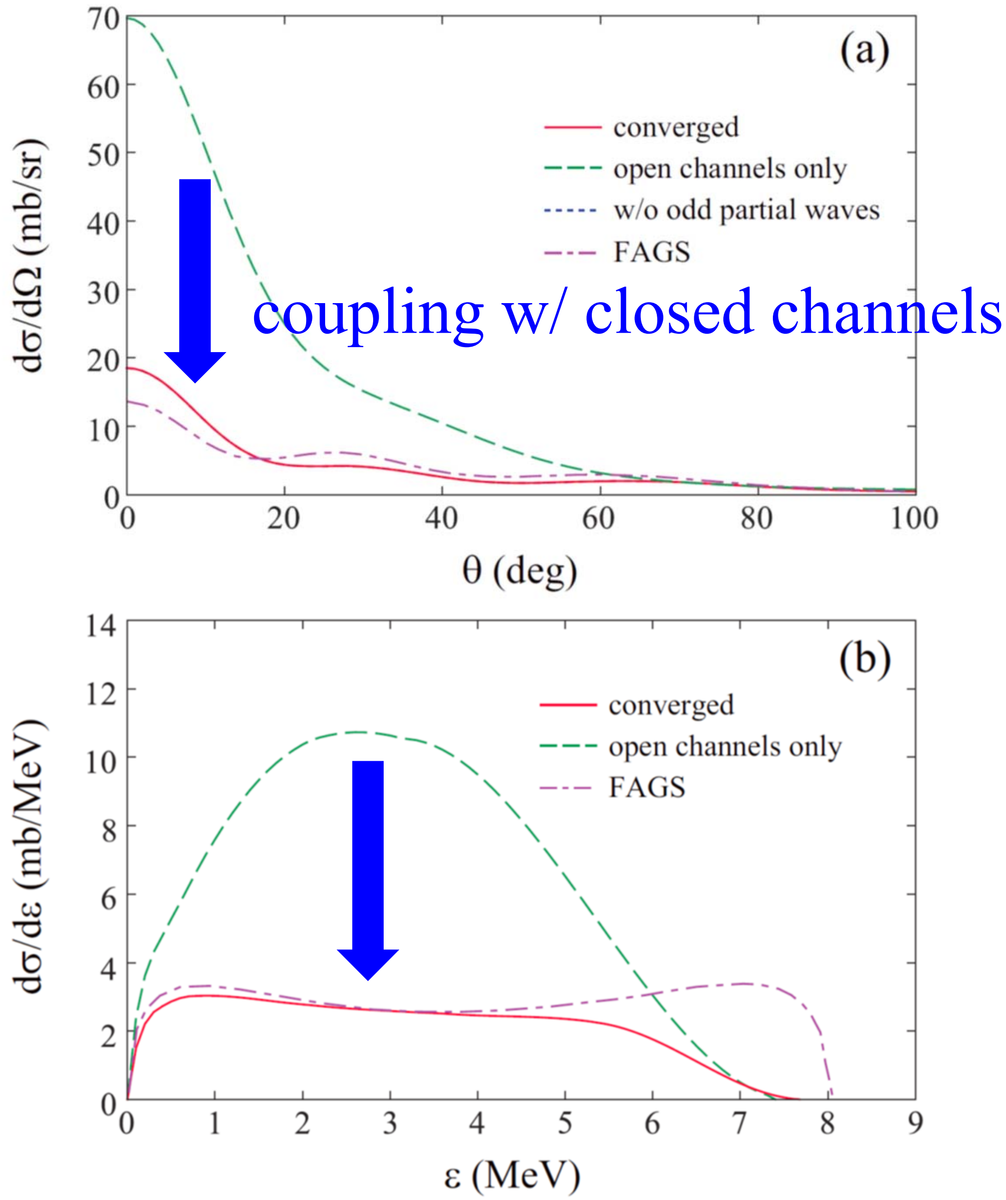
*N. J. Upadhyay et al., PRC **85**, 054621 (2012).*

$^{12}C(d,pn)^{12}C_{gs}$ at E_d = 12 MeV



*KO and K. Yoshida, PRC **94**, 051603(R) (2016).*



CDCC severely overshoots the result of FAGS, <u>if closed-channels are neglected</u>.

Plan of this talk

I. Three-body exact theory

L. D. Faddeev, Zh. Eksp. Theor. Fiz. ***39****, 1459 (1960) [Sov. Phys. JETP* ***12****, 1014 (1961)].*

- ✓ Three-body scattering problem
- ✓ Shortcomings of the Lippmann-Schwinger equation

II. Three-body “exact” theory in a model space

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- ✓ The l-truncation, the center of CDCC
- ✓ k-truncation and discretization

III. Numerical verification of the AYK/AKY paper

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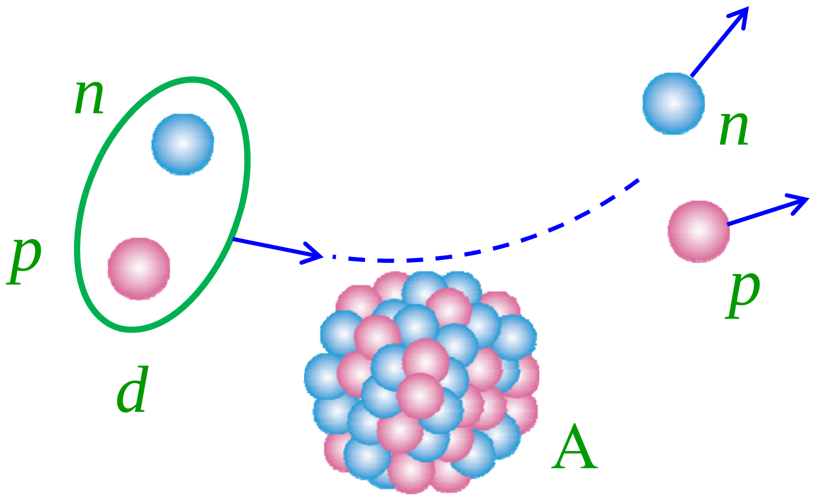
Three-body scattering problem



Assumptions for simplicity:

- No spins
- No Coulomb
- No absorption (imaginary pot.)
- 2-body problem solved

Schroedinger Equation

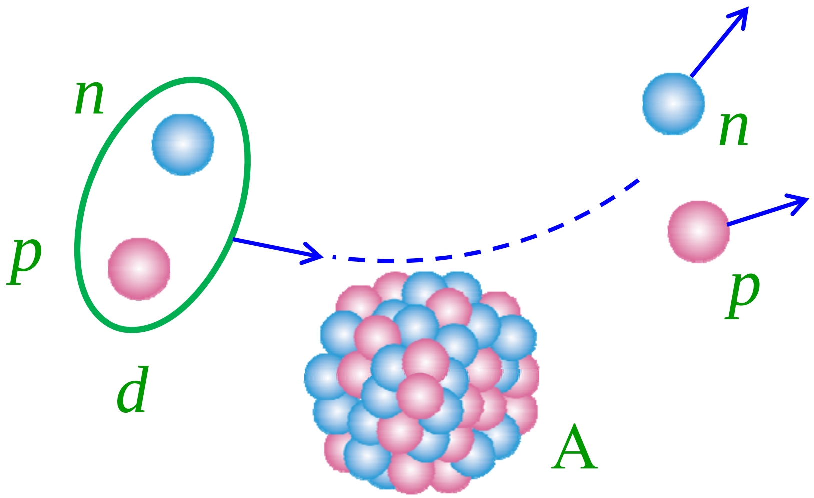
boundary condition (b.c.) not specified

$$[E - K - V_{pn} - V_{pA} - V_{nA}]\,\Psi = 0.$$

- Solution 1: $\Psi = e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_d + \frac{1}{E - H + i\varepsilon}(V_{pA} + V_{nA})\, e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_d$

$$= \frac{i\varepsilon}{E - H + i\varepsilon} e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_d \equiv \Omega^{(+)} e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_d.$$

Three-body scattering problem



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Schroedinger Equation

$$[E - K - V_{pn} - V_{pA} - V_{nA}]\,\Psi = 0.$$

- Solution 2:

$$\Psi = e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_d + \frac{1}{E - (K + V_{pn}) + i\varepsilon}(V_{pA} + V_{nA})\,\Psi.$$

$\equiv H_d$ (channel Hamiltonian)

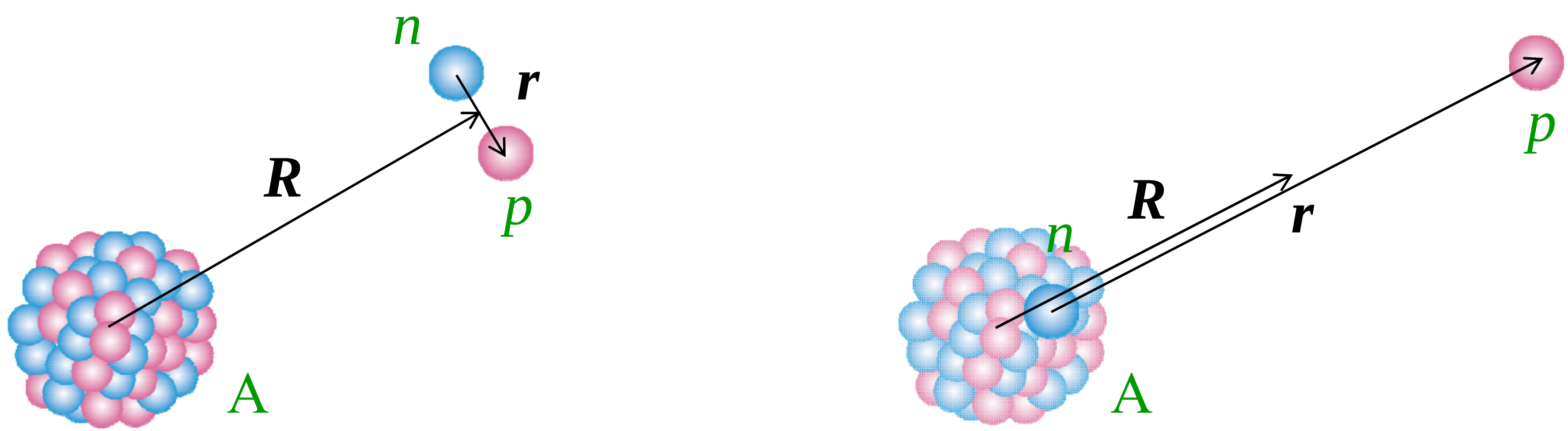
Lippmann-Schwinger (LS) equation

Problems of the LS equation

$$\Psi = e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_d + \frac{1}{E - H_d + i\varepsilon}\left(V_{pA} + V_{nA}\right)\Psi.$$

1. Absence of the rearrangement channels
2. Divergence problem due to the disconnected diagram
3. Nonuniqueness of the solution

The b.c. of the LS Eq. is not appropriate.



The Faddeev theory

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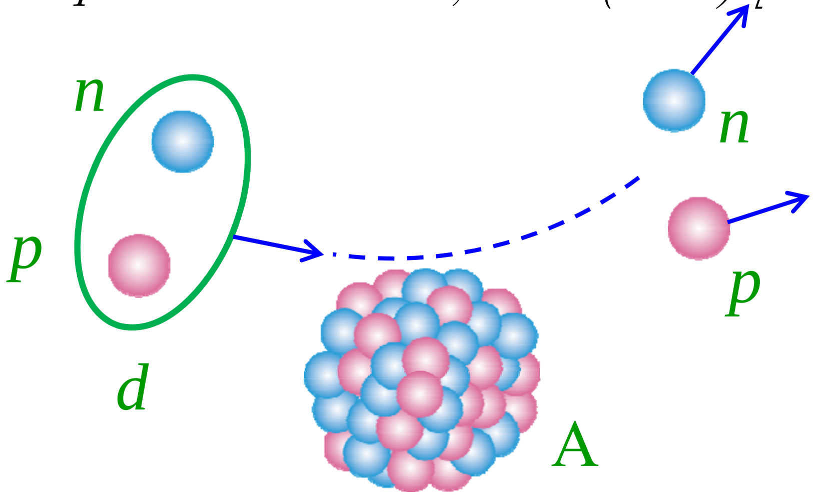

$$[E - K - V_{pn} - V_{pA} - V_{nA}]\,\Psi = 0, \quad \Psi = \Psi_d + \Psi_p + \Psi_n.$$

Faddeev Eqs.

$$[E - K - V_{pn}]\,\Psi_d = V_{pn}\left(\Psi_p + \Psi_n\right),$$

$$[E - K - V_{nA}]\,\Psi_n = V_{nA}\Psi_d + V_{nA}\Psi_p,$$

$$[E - K - V_{pA}]\,\Psi_p = V_{pA}\Psi_d + V_{pA}\Psi_n.$$

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$$[E - K - V_{pn} - V_{pA} - V_{nA}]\,\Psi = 0, \quad \Psi = \Psi_d + \Psi_p + \Psi_n.$$

Faddeev Eqs.

not pair int. but 3-body int.

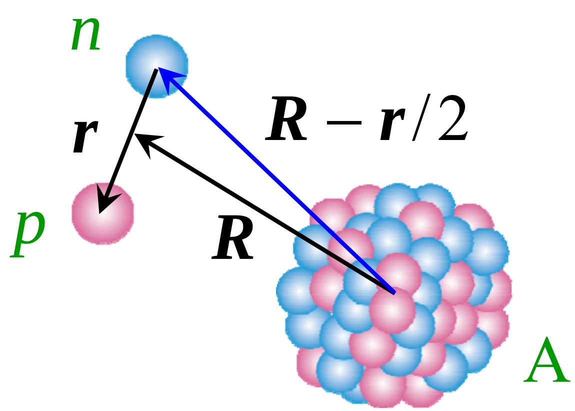
$$[E - K - V_{pn} - \mathcal{P}_{l_{\max}} (V_{nA} + V_{pA})\, \mathcal{P}_{l_{\max}}]\,\Psi_d = V_{pn} (\Psi_p + \Psi_n)\,,$$

$$[E - K - V_{nA}]\,\Psi_n = (V_{nA} - \mathcal{P}_{l_{\max}} V_{nA} \mathcal{P}_{l_{\max}})\,\Psi_d + V_{nA}\Psi_p,$$

$$[E - K - V_{pA}]\,\Psi_p = (V_{pA} - \mathcal{P}_{l_{\max}} V_{pA} \mathcal{P}_{l_{\max}})\,\Psi_d + V_{pA}\Psi_n.$$

$$\mathcal{P}_{l_{\max}} = \int d\hat{\boldsymbol{r}}' \sum_{l \le l_{\max}} \sum_{m} Y_{lm}(\hat{\boldsymbol{r}})\, Y^*_{lm}(\hat{\boldsymbol{r}}')$$

$$\mathcal{P}_0 e^{-\mu(\boldsymbol{R}-\boldsymbol{r}/2)^2} \to e^{-\mu R^2} e^{-\mu r^2/4}$$



Problems of the LS equation

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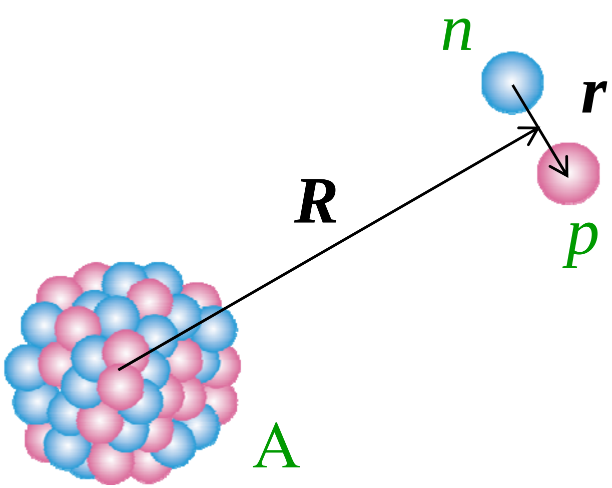

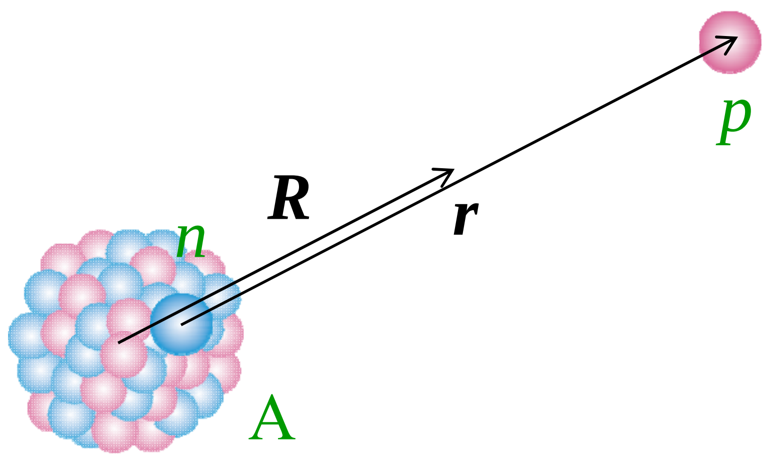

l-truncation, the center of CDCC

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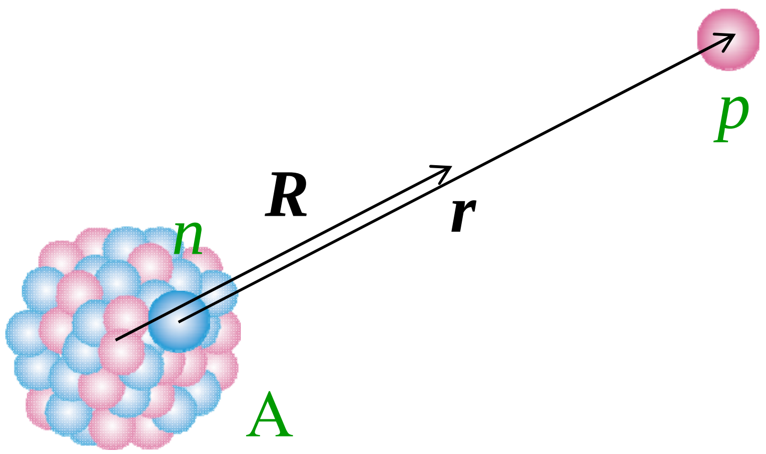

$$\mathcal{P}_{l_{\max}} = \int d\hat{\boldsymbol{r}}' \sum_{l \le l_{\max}} \sum_{m} Y_{lm}(\hat{\boldsymbol{r}}) Y^{*}_{lm}(\hat{\boldsymbol{r}}')$$

$\mathcal{P}_{l_{\max}}$ smears out $\hat{\boldsymbol{r}}$ w/ the resolution of $1/l_{\rm max}$.

[If $l_{\rm max} \to \infty$, it means $\delta(\boldsymbol{r}' - \boldsymbol{r})$.]

- We have no rearrangement-like channel in the asymptotic region because of $\mathcal{P}_{l_{\max}}$.
- As $l_{\rm max}$ increases, the coupling between the 1st Eq. and the other two becomes weaker.

Three-body theory in a model space

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Faddeev Eqs.

not pair int. but 3-body int. $\rightarrow 0$

$$[E - K - V_{pn} - \mathcal{P}_{l_{\max}}(V_{nA} + V_{pA})\,\mathcal{P}_{l_{\max}}]\,\Psi_d = V_{pn}(\Psi_p + \Psi_n),$$

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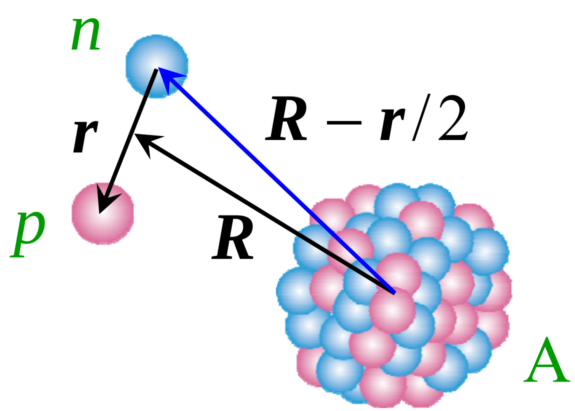

CDCC, as an alternative to the Faddeev theory

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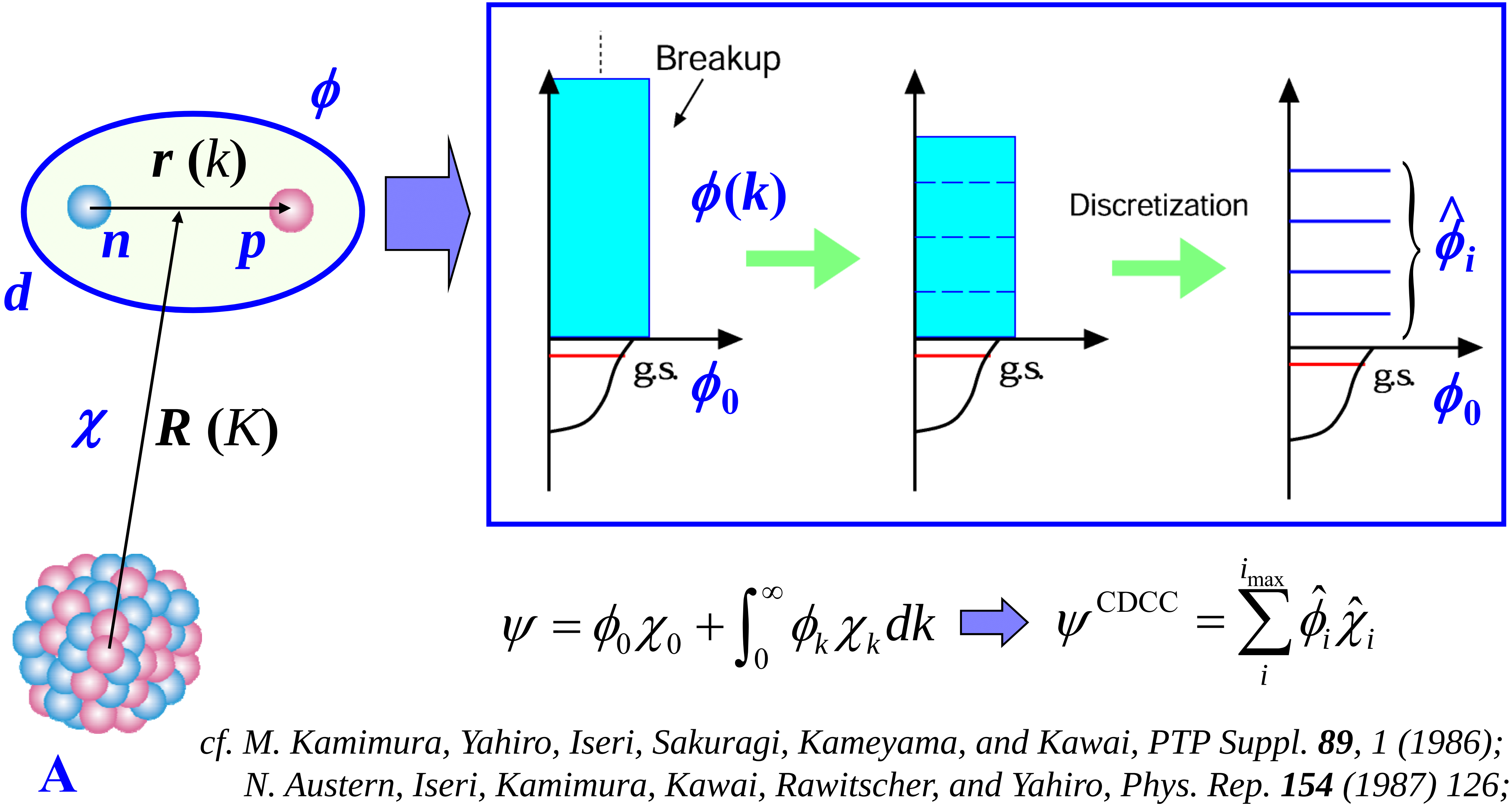
CDCC solves the following LS eq.:

$$\Psi^{\mathrm{CDCC}} = e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_d + \frac{1}{E - H_d + i\varepsilon}\mathcal{P}_{l_{\max}}\left(V_{nA} + V_{pA}\right)\mathcal{P}_{l_{\max}}\Psi^{\mathrm{CDCC}}.$$

CDCC gives a proper solution to a three-body scattering problem
if the solution converges w/ respect to *l*.

- Continuum-Discretization has nothing to do w/ the justification of CDCC.
- *l*-truncation allows one to truncate also *r* and *k*.
- Convergence for other quantities ($r_{\max}$, $k_{\max}$, and Δk, etc.) must be confirmed to obtain a proper solution to the LS Eq.

The Continuum-Discretized Coupled-Channels method: CDCC (after *l*-truncation)



$$\psi = \phi_0 \chi_0 + \int_0^\infty \phi_k \chi_k dk \Rightarrow \psi^{\mathrm{CDCC}} = \sum_i^{i_{\max}} \hat{\phi}_i \hat{\chi}_i$$

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Faddeev-AGS vs. CDCC

N. J. Upadhyay, A. Deltuva, F. M. Nunes, PRC ***85****, 054621 (2012).*

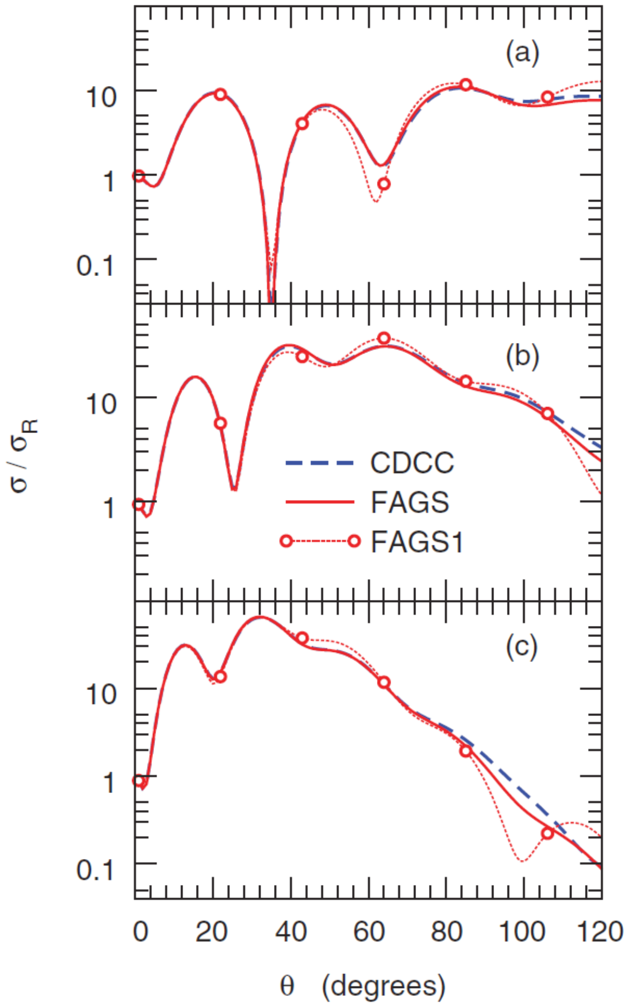


FIG. 2. (Color online) Elastic cross section for $d+^{10}$Be: (a) $E_d = 21.4$ MeV, (b) $E_d = 40.9$ MeV, and (c) $E_d = 71$ MeV.

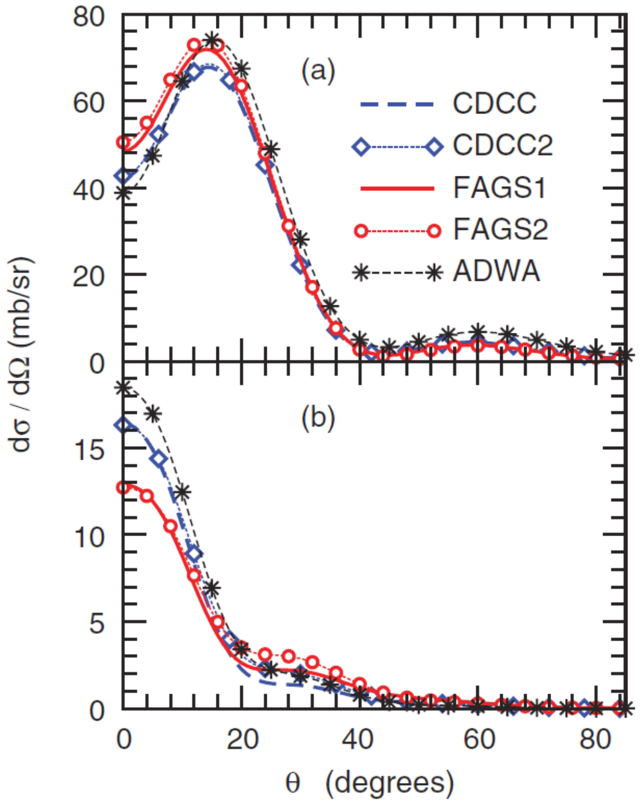


FIG. 6. (Color online) Angular distribution for ^{12}C$(d, p)^{13}$C: (a) $E_d = 12$ MeV and (b) $E_d = 56$ MeV.

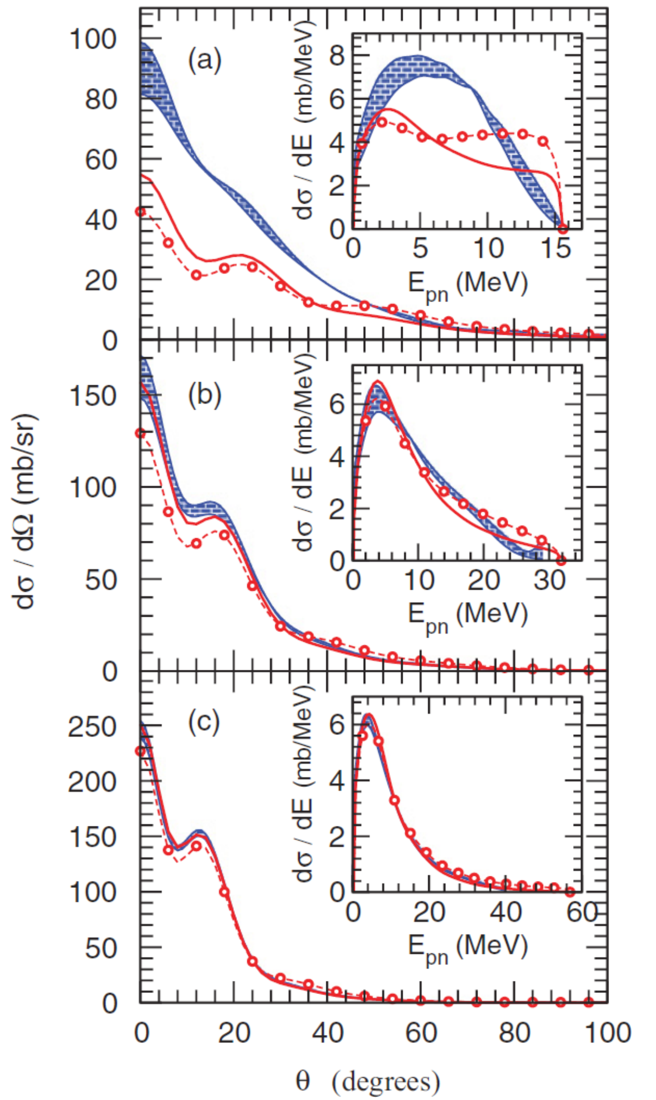
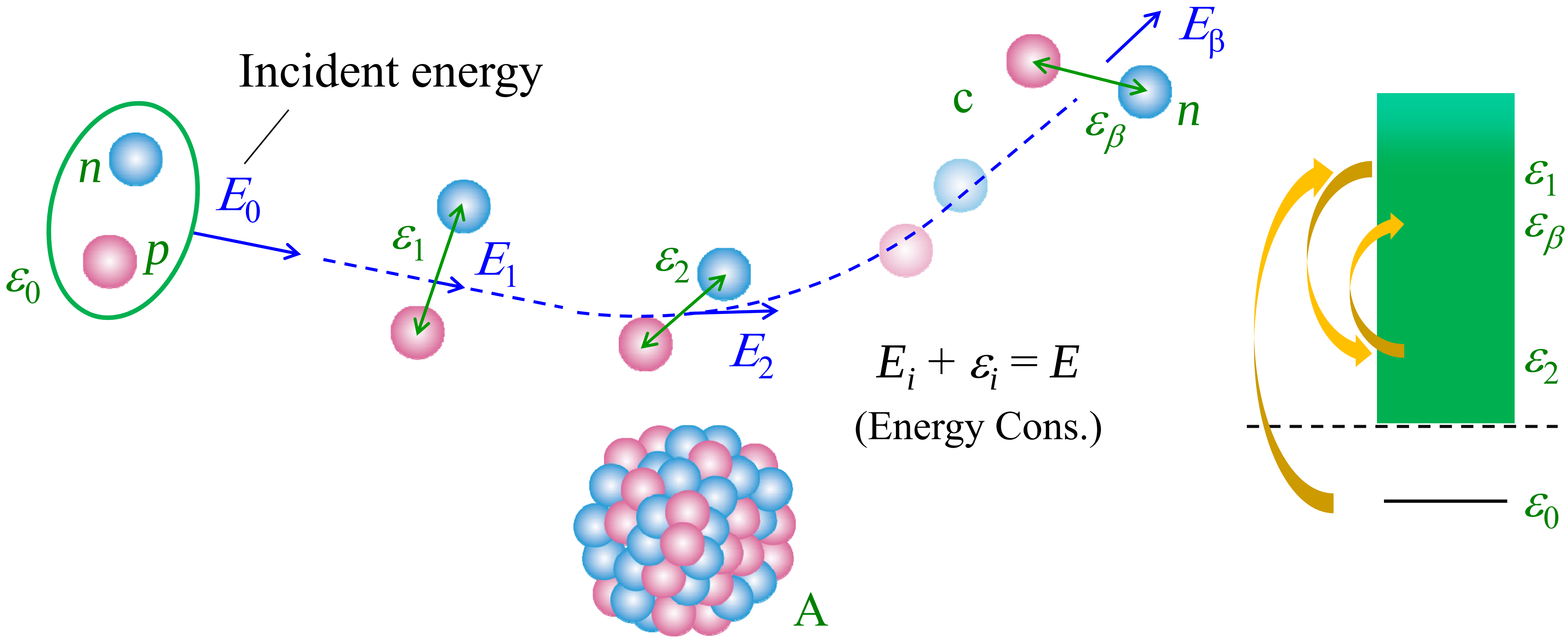


FIG. 8. (Color online) Breakup distributions for the ^{10}Be$(d, pn)^{10}$Be reaction at (a) $E_d = 21$ MeV, (b) $E_d = 40.9$ MeV, and (c) $E_d = 71$ MeV. Results for CDCC (hatched band), FAGS (solid), and FAGS1 (circles).

Description of deuteron breakup process by CDCC



- Open channels ($E_i > 0$): directly connected to observables
- Closed channels ($E_i < 0$): virtual breakup channels

Neglected in the preceding study

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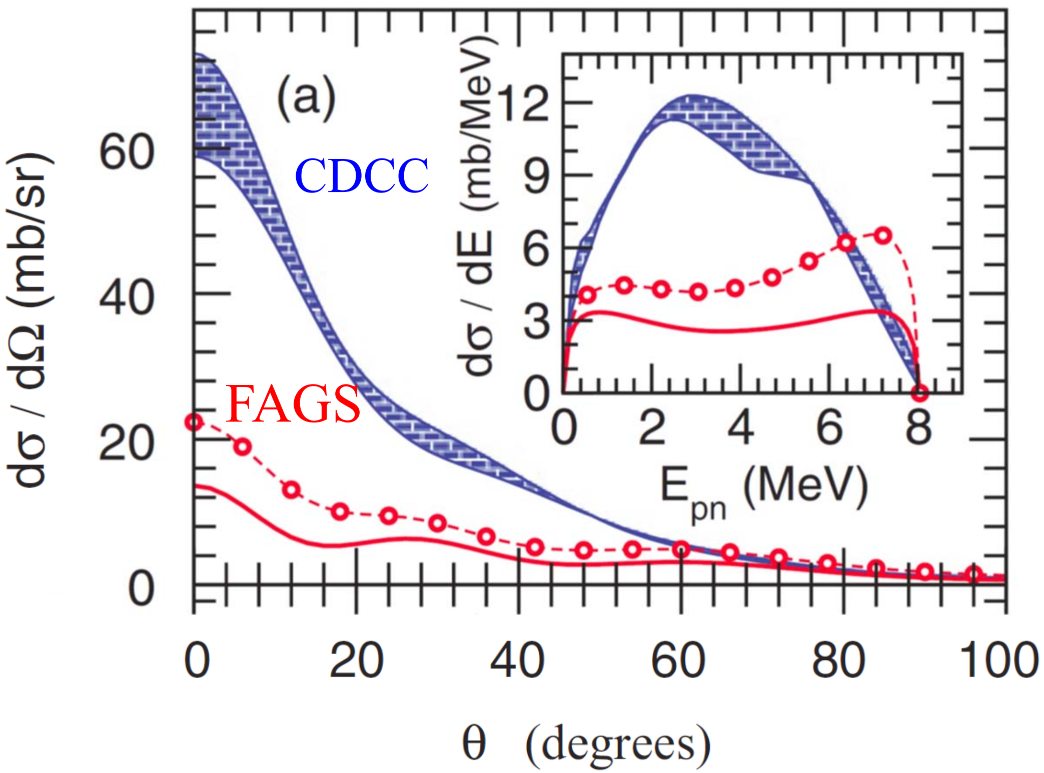

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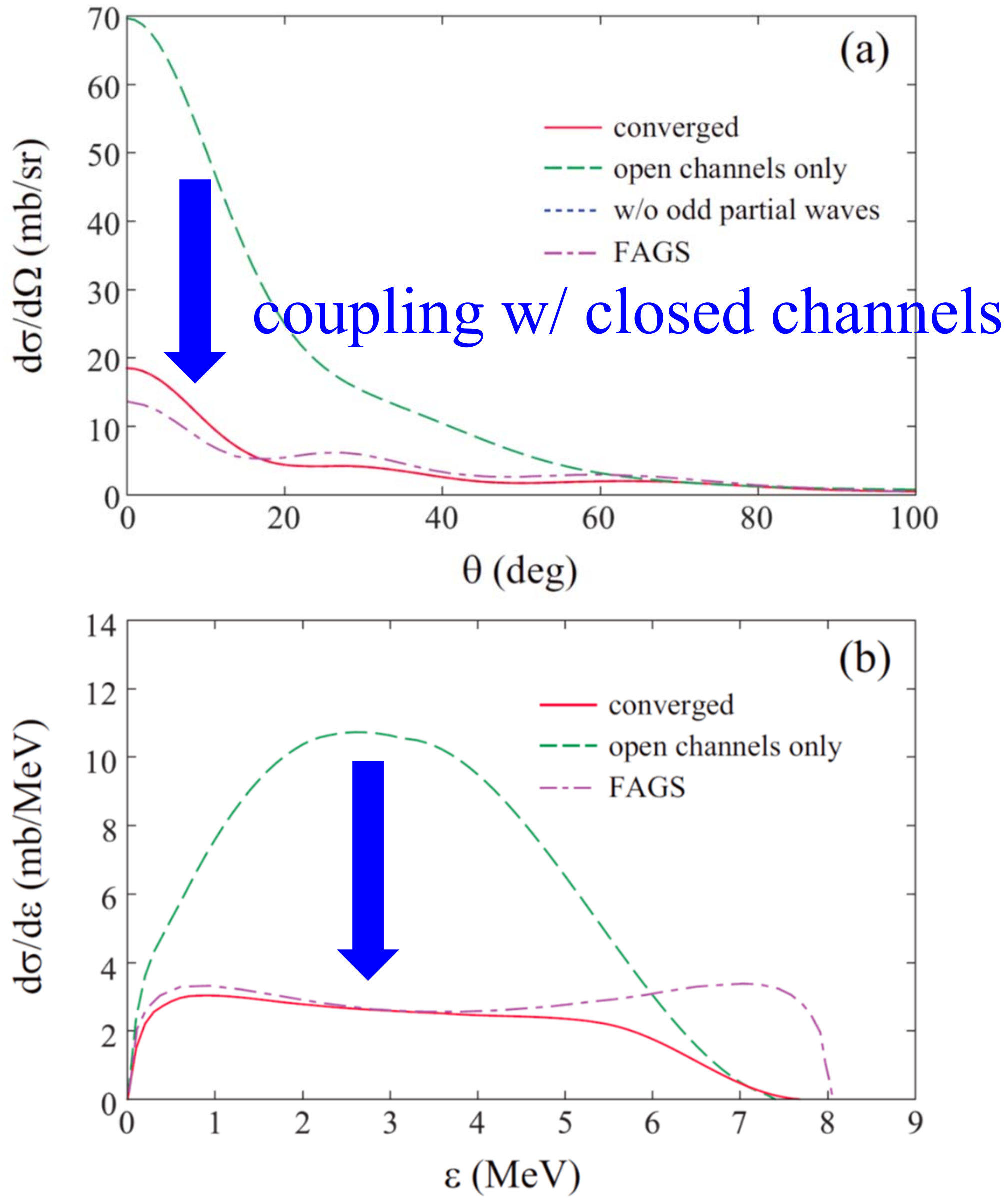

CDCC severely overshoots the result of FAGS, <u>if closed-channels are neglected</u>.

cf. triple-alpha study

Summary

- I have recapitulated the three-body scattering theory.
  - ✓ To use a single LS Eq. is not allowed; we have to rely on the Faddeev theory.
  - ✓ To use a single LS Eq. w/ the *l*-truncation is allowed, which gives a proper solution to the three-body scattering problem (for non-rearrangement processes).
  - ✓ Thus, CDCC is shown to be an alternative to the Faddeev theory.

- We have demonstrated the applicability of CDCC to deuteron breakup at low energies.
  - ✓ The failure of CDCC reported by the MSU group is shown to be due to the neglect of the closed-channels; their CDCC model space was not converged.
  - ✓ The coupling to the closed-channels are crucially important, as suggested by, e.g., Austern *et al*. in 1987.
  - ✓ The converged CDCC gives a result that agrees w/ that of FAGS, by which the theoretical foundation of CDCC has been re-established.