Description of three-body scattering states using complex scaling method and its application to Coulomb breakup reactions of two-neutron halo nuclei

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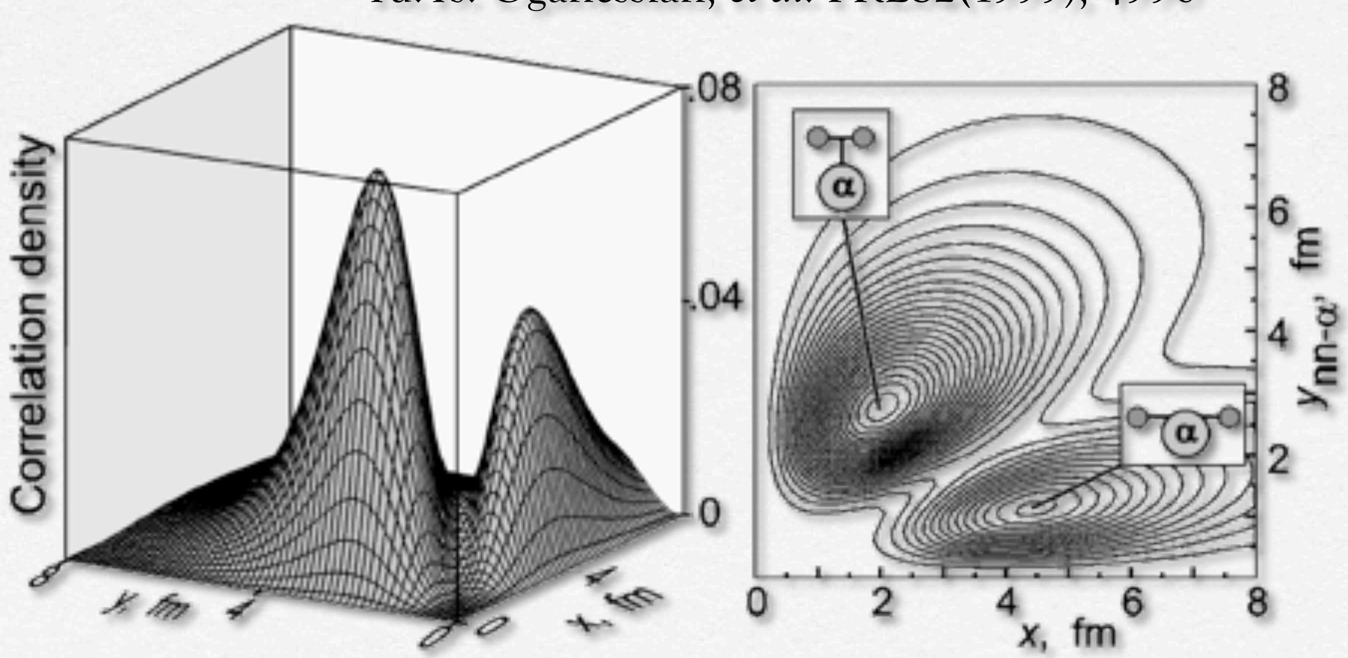
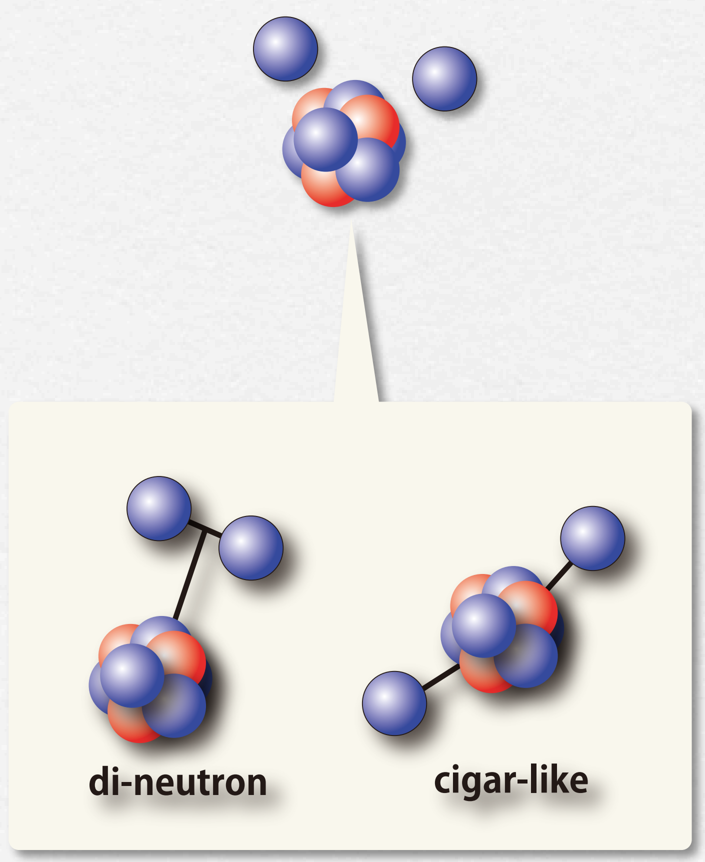
Two-neutron halo nuclei

- Two-neutron halo nuclei have exotic structure in which the weakly-bound neutrons are spread out beyond the core nucleus.
- From the theoretical calculation using the core+n+n three-body model, the exotic correlation between two halo neutrons, the so-called dineutron, plays a significant role in reproducing the observed two-neutron separation energies and the large matter radii of ^{6}He and ^{11}Li.

Yu.Ts. Oganessian, *et al.* PRL82(1999), 4996

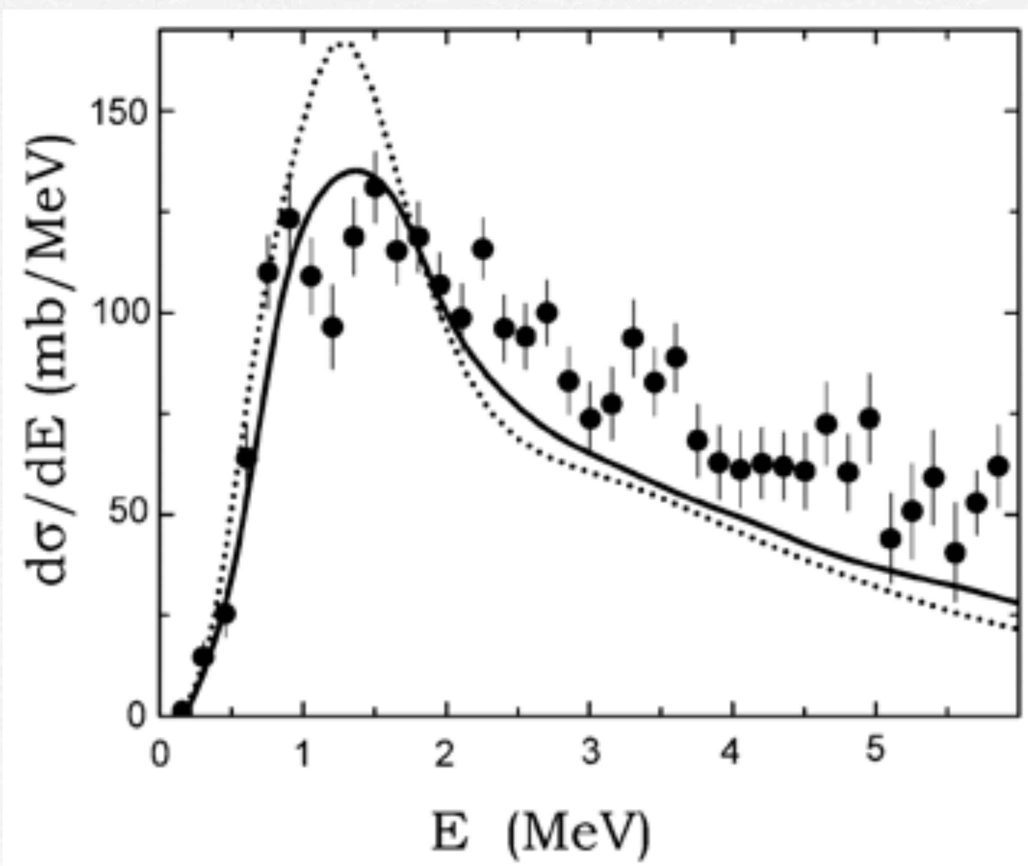




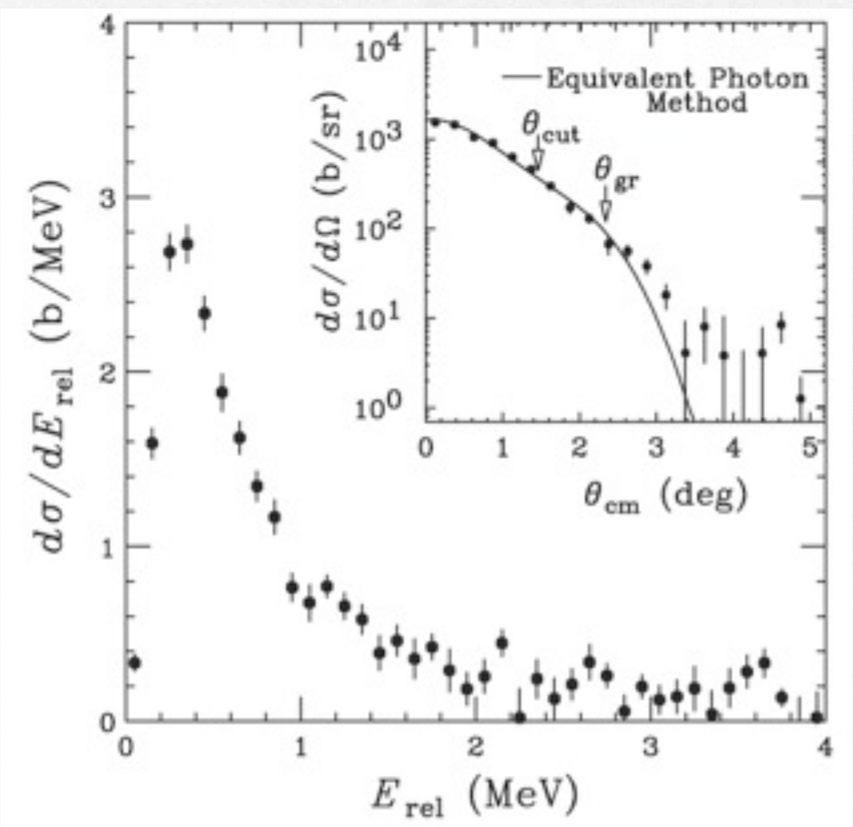
Coulomb breakup reactions of two-neutron halo nuclei

- Coulomb breakup reactions have been performed to investigate the electric response of weakly-bound halo neutrons.
- The observed cross sections show the low-lying enhancements, and those enhancements have been expected to be responsible to the exotic structure of two-neutron halo nuclei.

^{6}He breakup: T. Aumann et al., PRC 59, 1252 (1999).

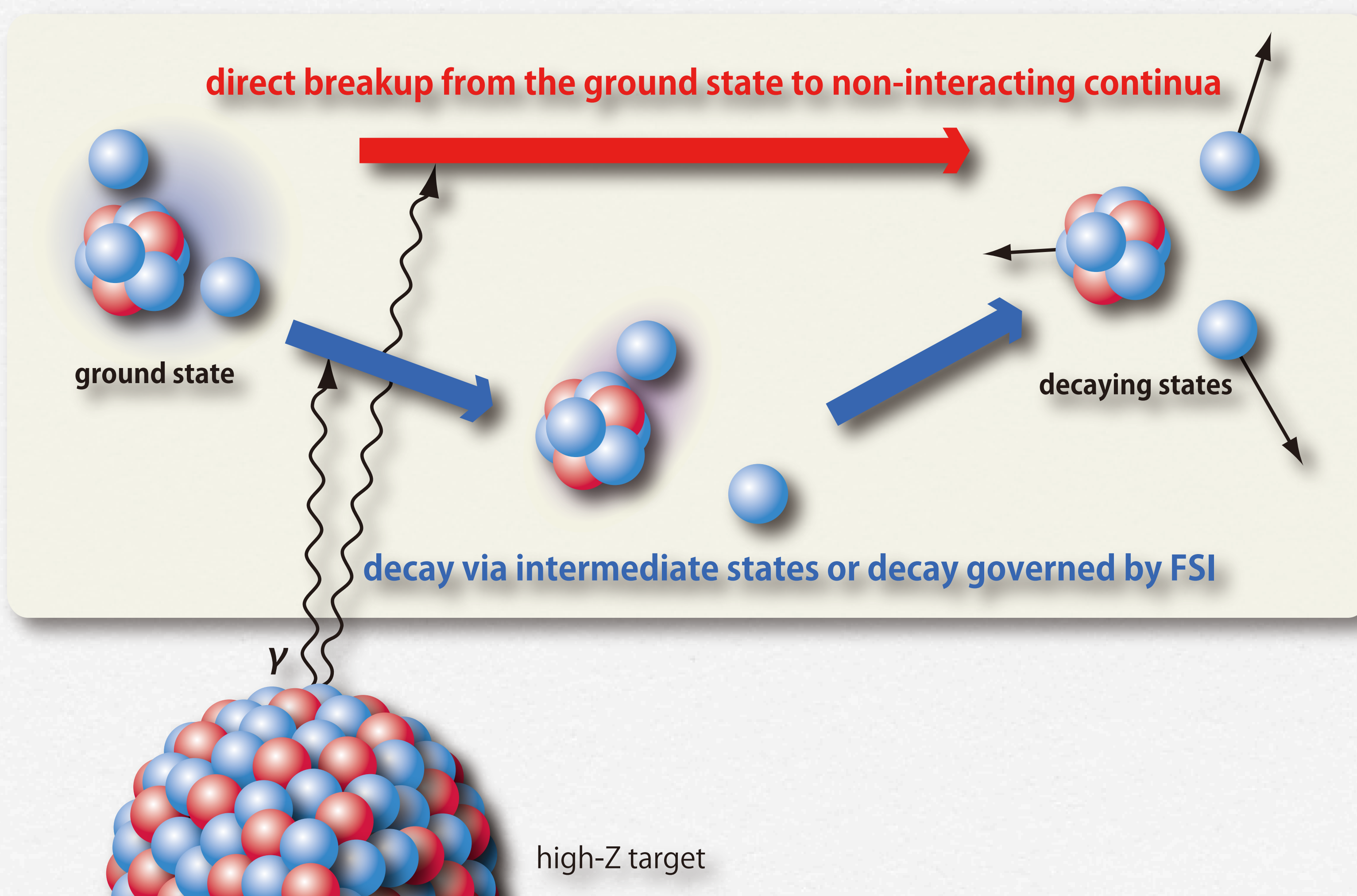


^{11}Li breakup: T. Nakamura et al., PRL 96, 252502 (2006).



To investigate the Coulomb breakups of two-neutron halo nuclei

- Two-neutron halo nuclei are Borromean systems in which any binary subsystems have no bound states, and hence, they are broken up to the core+n+n three-body scattering states.
  - To understand the properties of two-neutron halos from the Coulomb breakups, it is necessary to describe the core+n+n three-body scattering states accurately and to clarify the breakup mechanisms.



In this talk...

- We investigate the Coulomb breakup mechanisms of two-neutron halo nuclei, ^{6}He and ^{11}Li, by using the core+n+n three-body models.
  - To describe the final scattering states of the core+n+n system, we use the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).
  - Using CSLS, we discuss the mechanism of the Coulomb breakup reactions of two-neutron halo nuclei.
    - What kinds of processes dominate the Coulomb breakup reactions?
      - simultaneous three-body breakup or sequential breakup?
    - What kinds of correlations play roles in the reactions?
      - How the differences in the configurations reflects on the cross section?
        - ^{6}He : $(p_{3/2})^2$ configuration is dominant
        - ^{11}Li: $(p_{1/2})^2 + (s_{1/2})^2$ due to the breaking of N=8 magicity

Complex scaling method (CSM)

- CSM is a powerful tool to investigate the many-body resonances on the same footing as the bound-state cases.
  - In CSM, the relative coordinates and momenta are transformed as follows.

$$U(\theta) : \mathbf{r} \to \mathbf{r}e^{i\theta}, \qquad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

  - Applying the above transformation to the Schroedinger equation, we obtain the complex-scaled Schroedinger equation as follows.

$$\hat{H}\chi(\mathbf{r}) = E\chi(\mathbf{r}) \to \hat{H}^{\theta}\chi^{\theta}(\mathbf{r}) = E^{\theta}\chi^{\theta}(\mathbf{r})$$

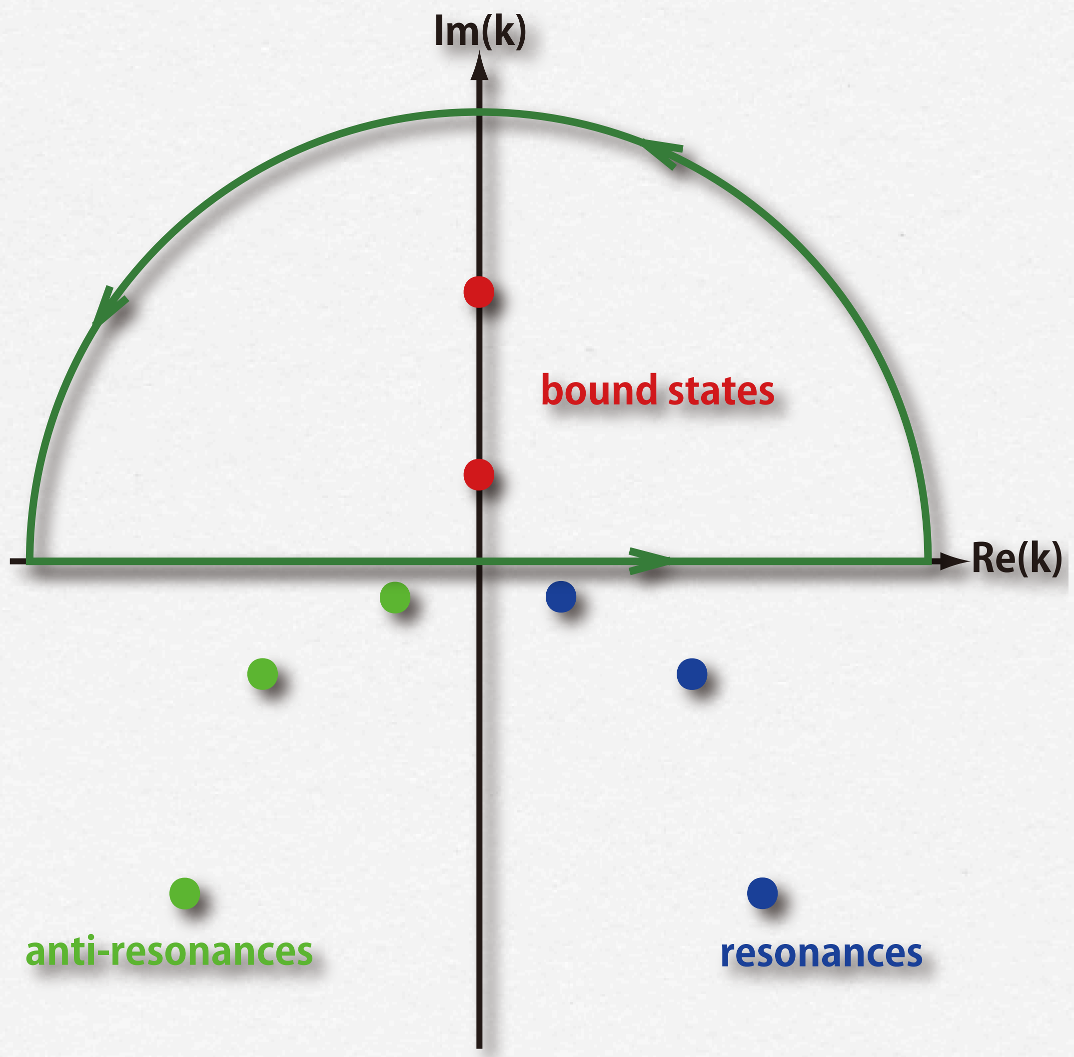
where the complex-scaled wave functions and Hamiltonian are given as

$$\chi^{\theta}(\mathbf{r}) = U(\theta)\chi(\mathbf{r}) = e^{\frac{3}{2}i\theta}\chi(\mathbf{r}e^{i\theta})$$

$$\hat{H}^{\theta} = U(\theta)\hat{H}U^{-1}(\theta)$$

Complex scaling method (CSM)

- Under the transformation in CSM, the resonance poles are obtained as the discretized states as well as the bound states.
  - By rotating the contour of the integral pass in the momentum plane with outgoing boundary condition, the resonance poles in the S-matrix are found as the residues.



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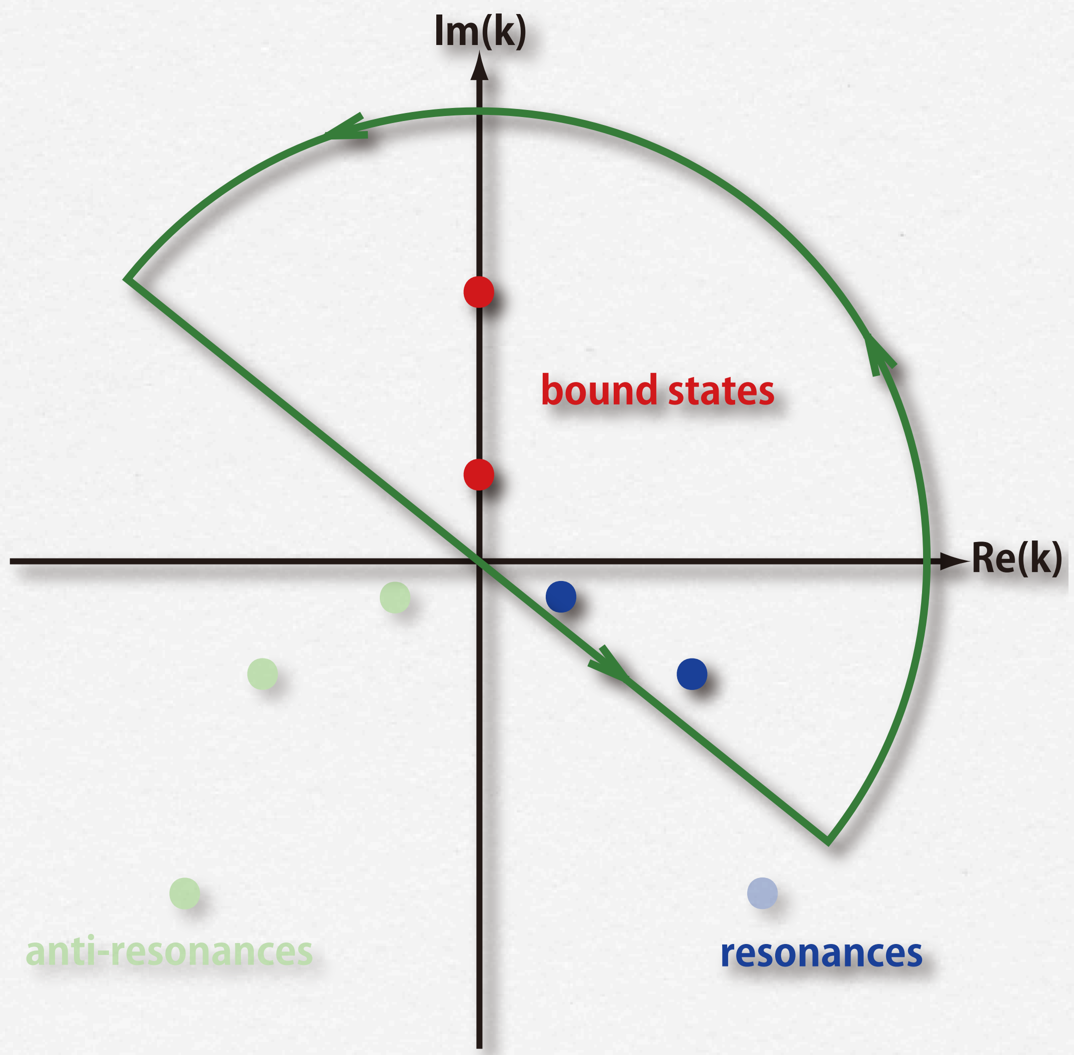

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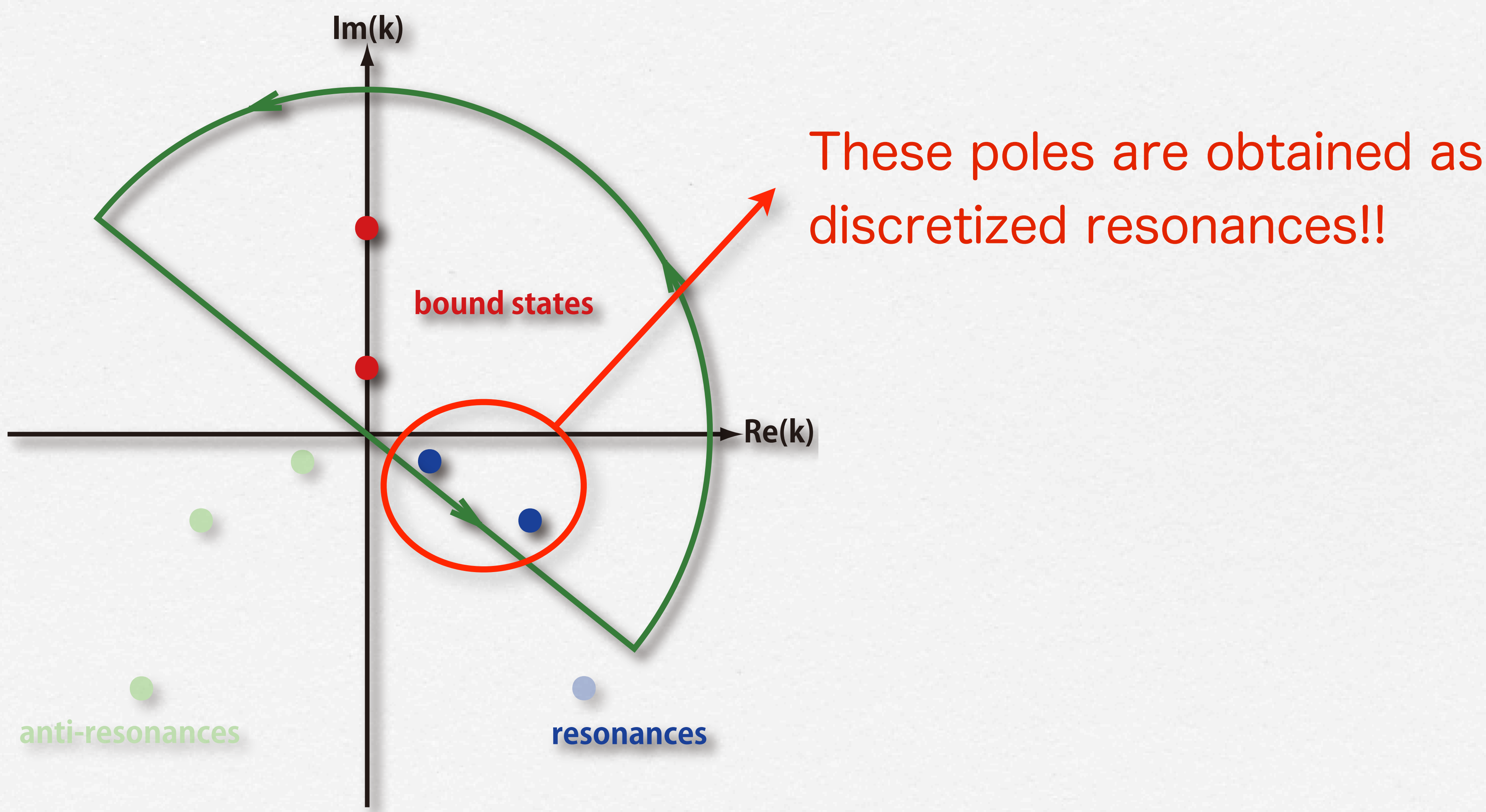
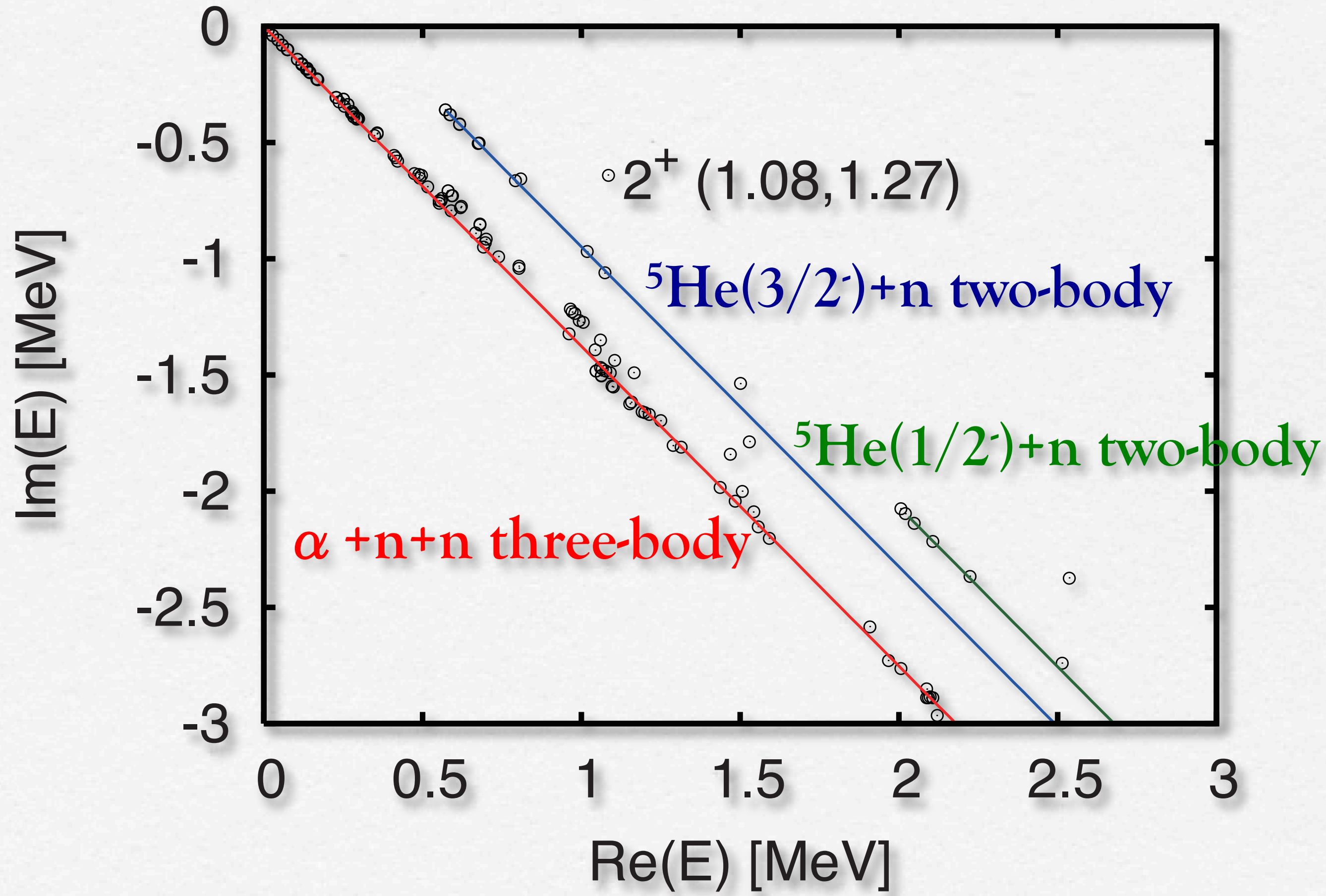


Energy spectra in CSM

- In CSM, the energy eigenvalues are obtained as complex numbers, and their imaginary parts represent the outgoing boundary conditions.
  - The resonance has the energy of $E = E_r - i\Gamma/2$, where Er and Γ are the resonance energy and decay width, respectively.
  - The continuum states are classified into several families corresponding to decay channels.



ex) obtained spectra of 2^+ states of ^{6}He

CSLS and complex-scaled Green's function

- The behaviors of the energy eigenvalues in CSM is useful to describe the scattering of many-body systems since outgoing boundary conditions are imposed.
- Using this characteristic, we describe the many-body scattering states by using the Lippmann-Schwinger formalism.
  - We start with the formal solution of the Lippmann-Schwinger equation.

$$\Psi^{(\pm)} = \Phi_0 + \lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} \pm i\varepsilon} \hat{V} \Phi_0$$

  - To take into account the outgoing boundary condition in the Green's function, we apply CSM to the Green's function.

$$\lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} + i\varepsilon} = U^{-1}(\theta) \frac{1}{E - \hat{H}^\theta} U(\theta)$$

CSLS and complex-scaled Green's function

- We expand the complex-scaled Green's function with the complete set constructed with the solved eigenstates of complex-scaled Hamiltonian H^θ.
  - Here, we solve the eigenvalue problem of H^θ by using the L^2-type basis functions as similar to the bound-state cases.

$$U^{-1}(\theta)\frac{1}{E-\hat{H}^\theta}U(\theta)=\sum_n U^{-1}(\theta)|\chi_n^\theta\rangle\frac{1}{E-E_n^\theta}\langle\tilde{\chi}_n^\theta|U(\theta)$$

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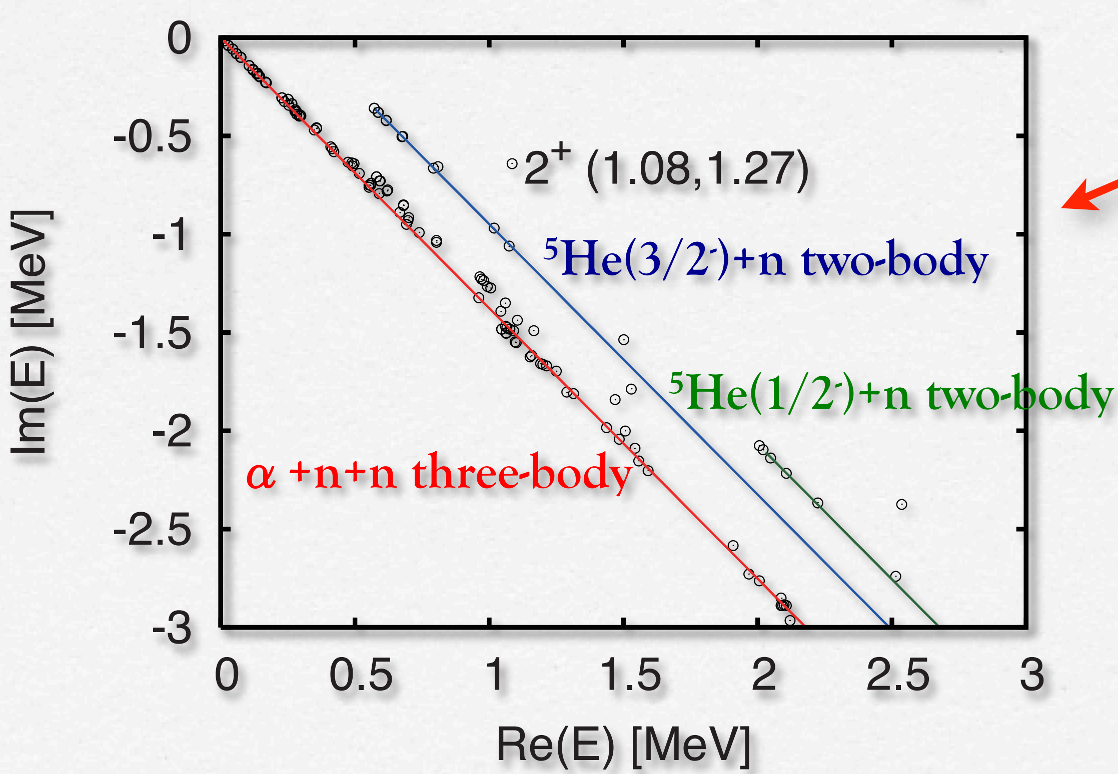


Outgoing boundary conditions are taken into account by imaginary parts of energy eigenvalues without any explicit enforcement.

CSLS and complex-scaled Green's function

- Using the complex-scaled Green's function, we obtain the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).

$$|\Psi^{(+)}\rangle = |\Phi_0\rangle + \sum_n U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E - E_n^\theta} \langle\tilde{\chi}_n^\theta|U(\theta)\hat{V}|\Phi_0\rangle$$

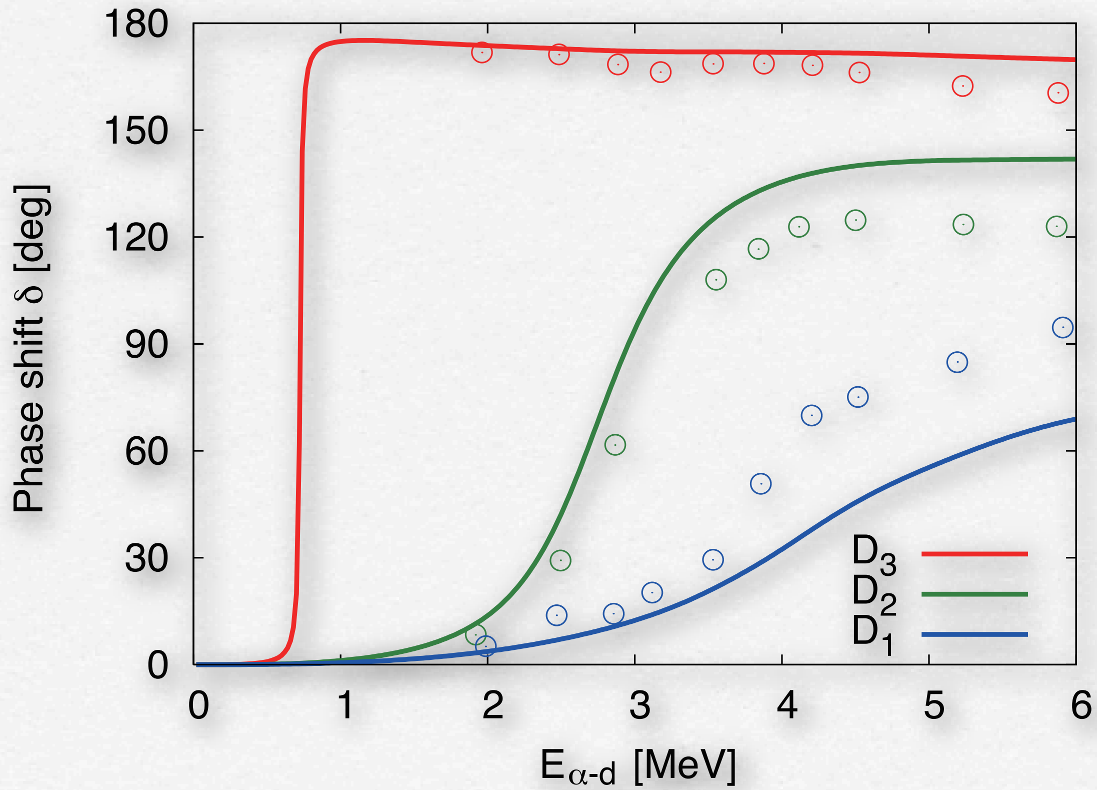
$$\langle\Psi^{(-)}| = \langle\Phi_0| + \sum_n \langle\Phi_0|\hat{V}U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E - E_n^\theta} \langle\tilde{\chi}_n^\theta|U(\theta)$$

  - The advantages in CSLS is that we can solve the scattering problems of many-body systems
    - in similar way to the bound-state cases and
    - without explicit enforcement of outgoing boundary conditions.

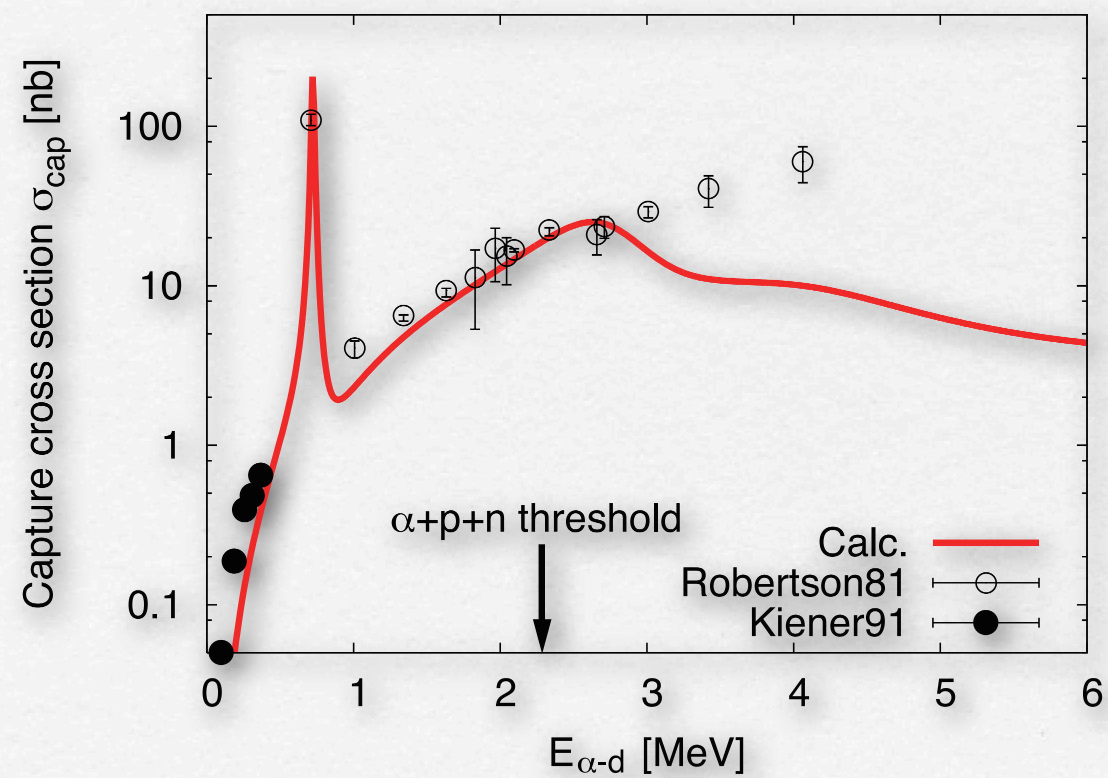
Reliability of CSLS - three-body scattering problem -

- Application of CSLS to the α+d scattering using α+p+n three-body model.
  - The observed elastic phase shift and the capture cross section are nicely reproduced.
  - CSLS is reliable method to describe the three-body scattering problem.

elastic phase shifts for α+d



radiative capture cross section $^2H(\alpha,\gamma)^6Li$



R.G.H. Robertson et al., PRL**47**(1981), 1867.
J. Kiener et al., PRC**44**(1991), 2195.

Core+n+n three-body models

- The Hamiltonians are given as follows.

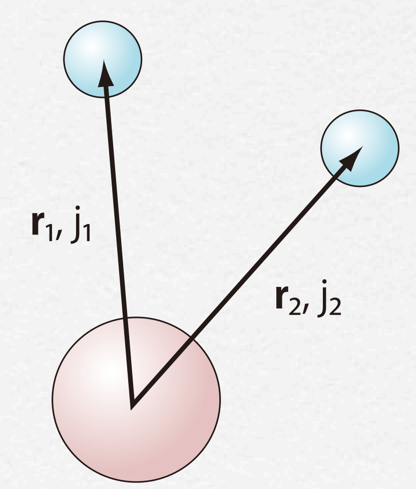
$$\hat{H} = \sum_{i=1}^{3} t_i - T_{\rm cm} + \sum_{i=1}^{2} V_{\text{core-}n}(\mathbf{r}_i) + V_{n\text{-}n}$$

- We use the following interactions in the calculation
  - Core-n interaction
    - ^{6}He: effective KKNN potential, which reproduce the scattering data of ^{4}He+n.
    - ^{11}Li: folding G-matrix int. with the ^{9}Li density + phenomenological LS force.
  - n-n interaction
    - ^{6}He: Minnesota force
    - ^{11}Li: AV8' force

- The wave functions are expanded with the Gaussian basis functions.

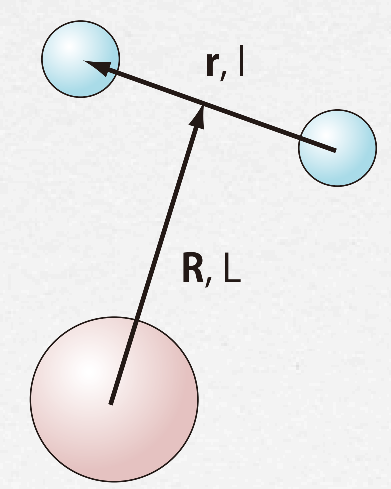
T. Myo, et al, PRC**63**, 054313 (2001).
T. Myo et al., PRC**76**, 024305 (2007).
H. Kanada, T. Kaneko, S. Nagata, and M. Nomoto, PTP**61**, 1327 (1979).
E. Hiyama et al., Prog. Part. Nucl. Phys. **51**, 223 (2003).

COSM (V-type)



shell model-like

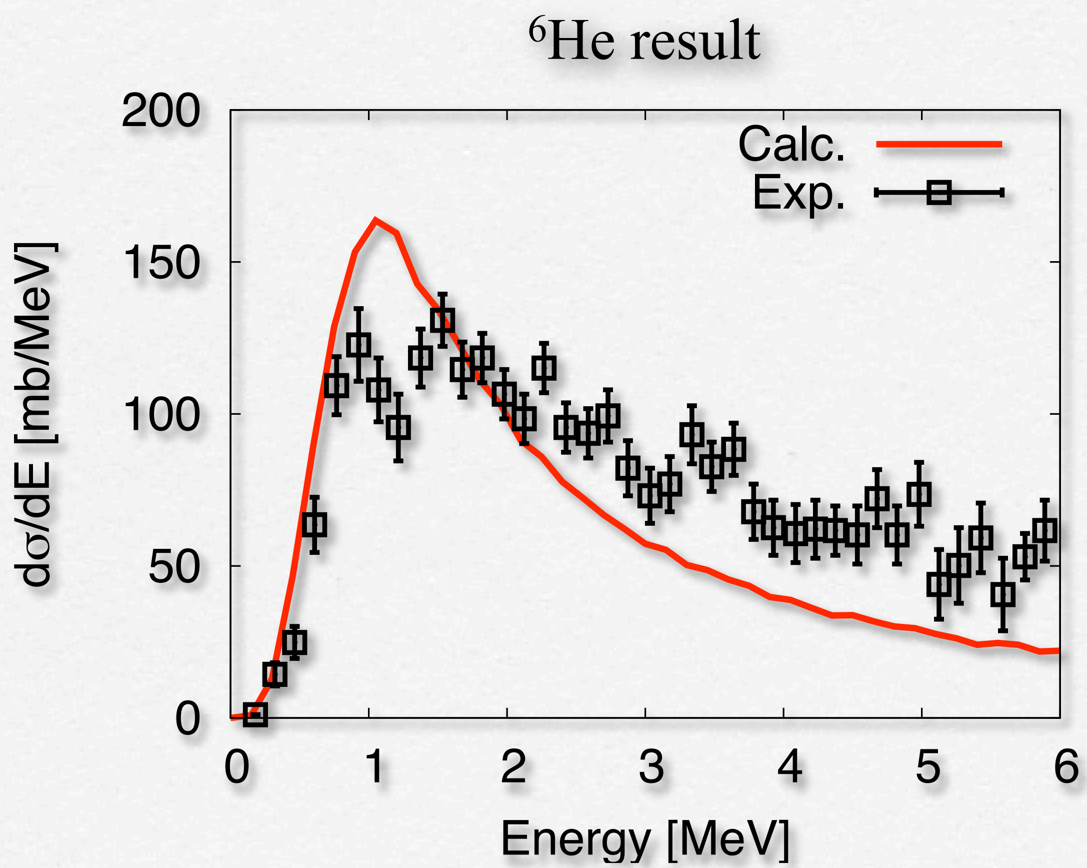
ECM (T-type)

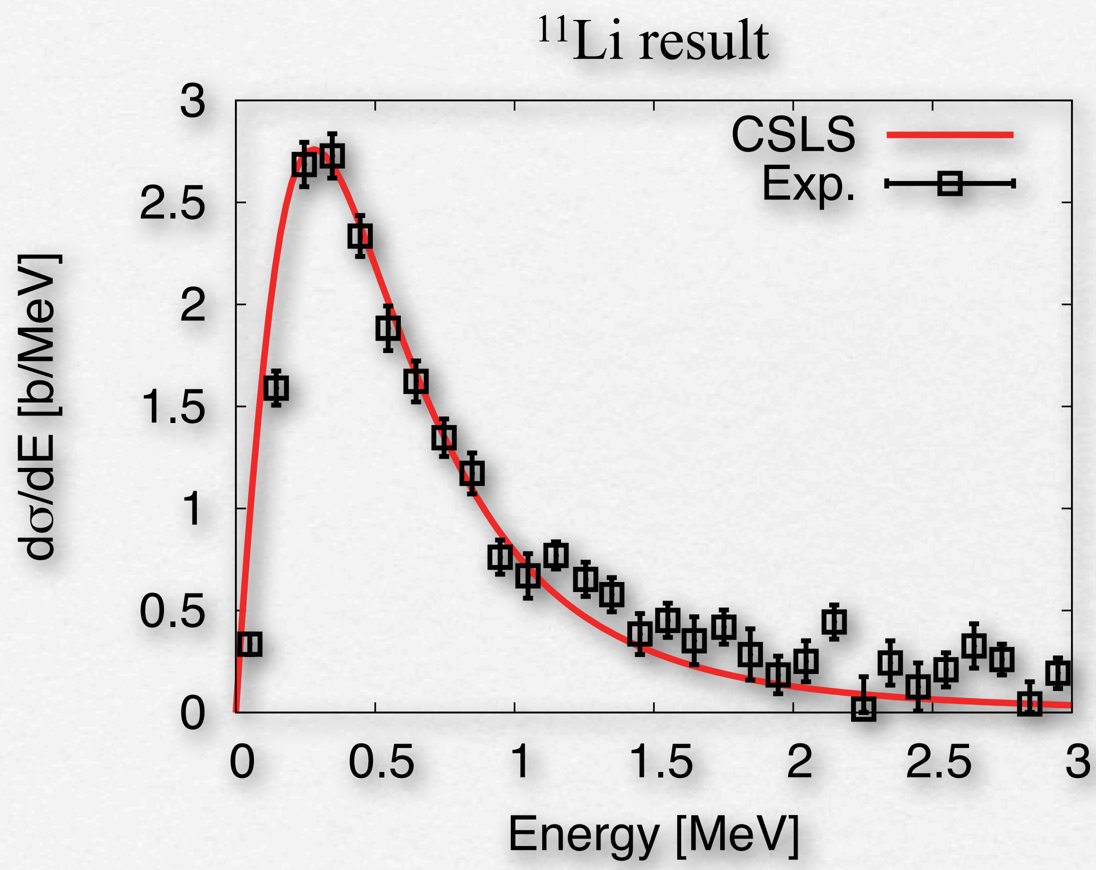


di-neutron-like

Coulomb breakup cross sections of ^{6}He and ^{11}Li

- We calculate the Coulomb breakup cross sections of ^{6}He and ^{11}Li, and compare the calculated results with experimental data.



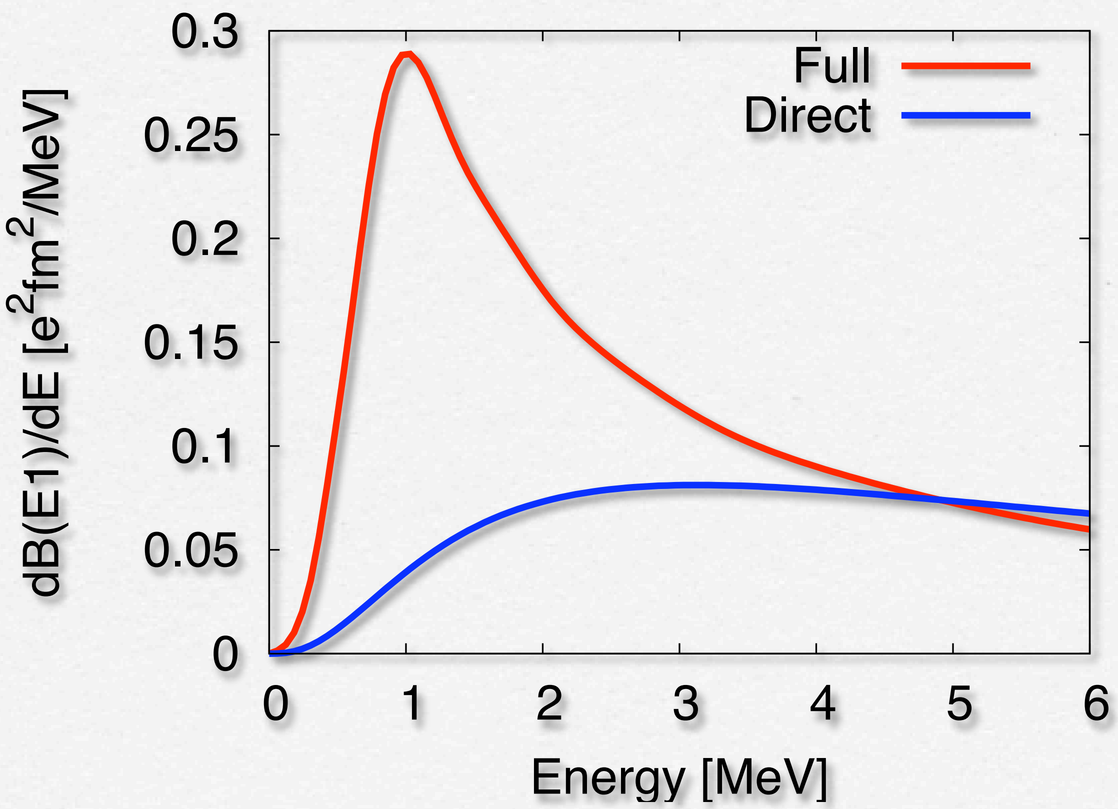


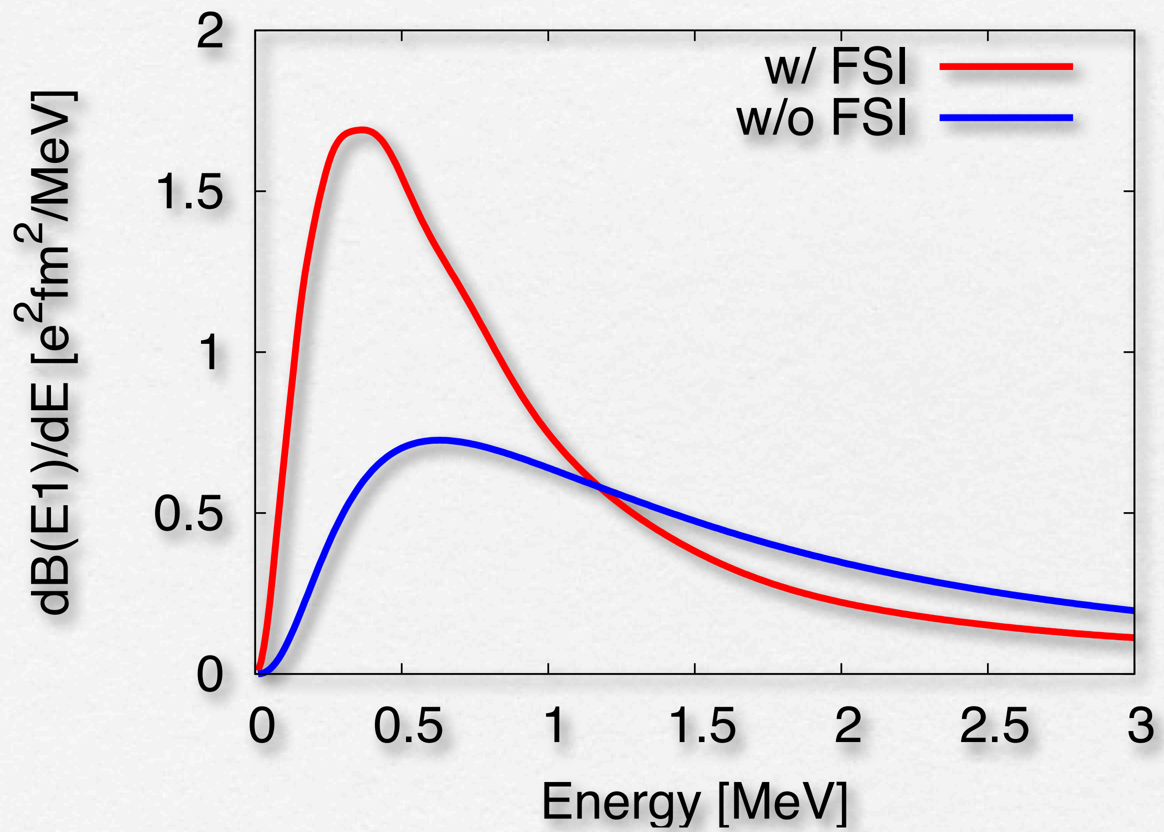
- Our results well reproduce the observed data.
- We next investigate the breakup mechanism of ^{6}He and ^{11}Li.

T. Aumann et al., PRC **59**, 1252 (1999).
T. Nakamura et al., PRL **96**, 252502 (2006).

Effects of final-state interactions on the cross sections

- To clarify the breakup mechanism, we investigate the effects of FSI on the cross sections.
  - When the FSI dominate the cross sections, the sequential processes might play roles in the Coulomb breakup reactions.

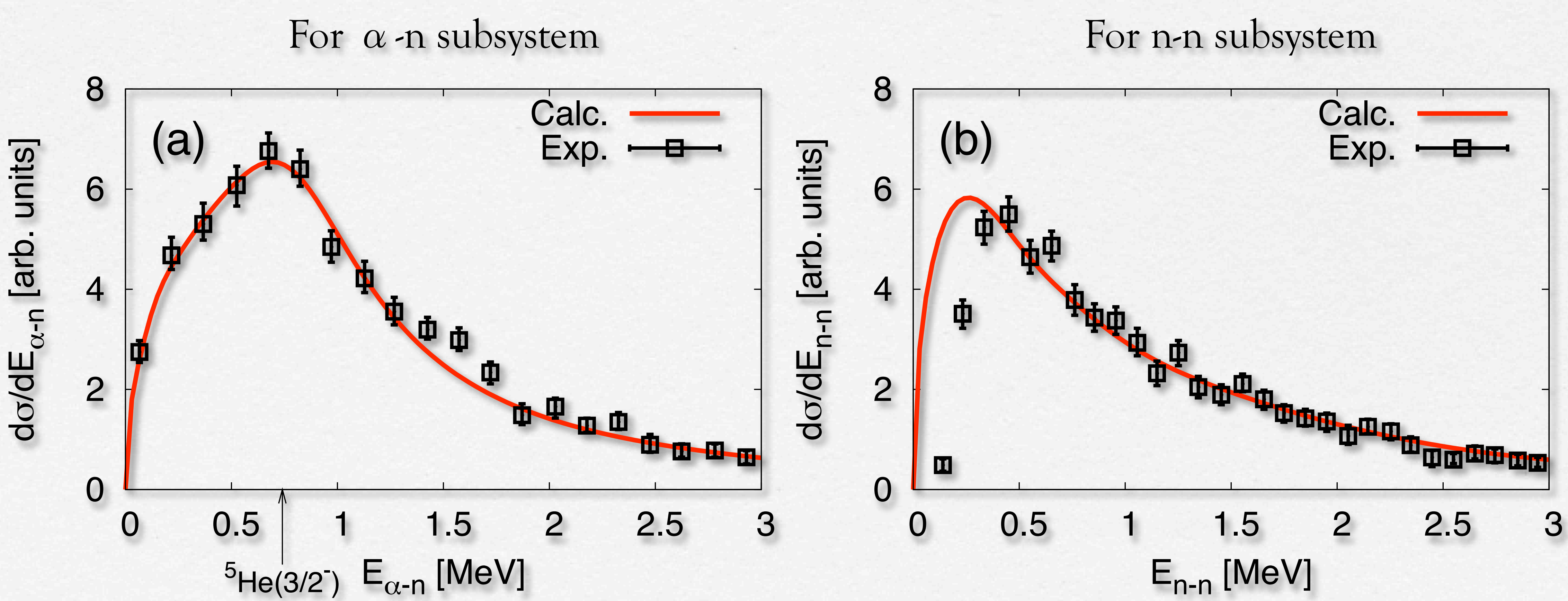




  - In both cases of ^{6}He and ^{11}Li, FSI have significant effects on the cross sections.
  - What kinds of FSI does contribute to the breakup cross sections?

Invariant mass spectra for binary subsystems

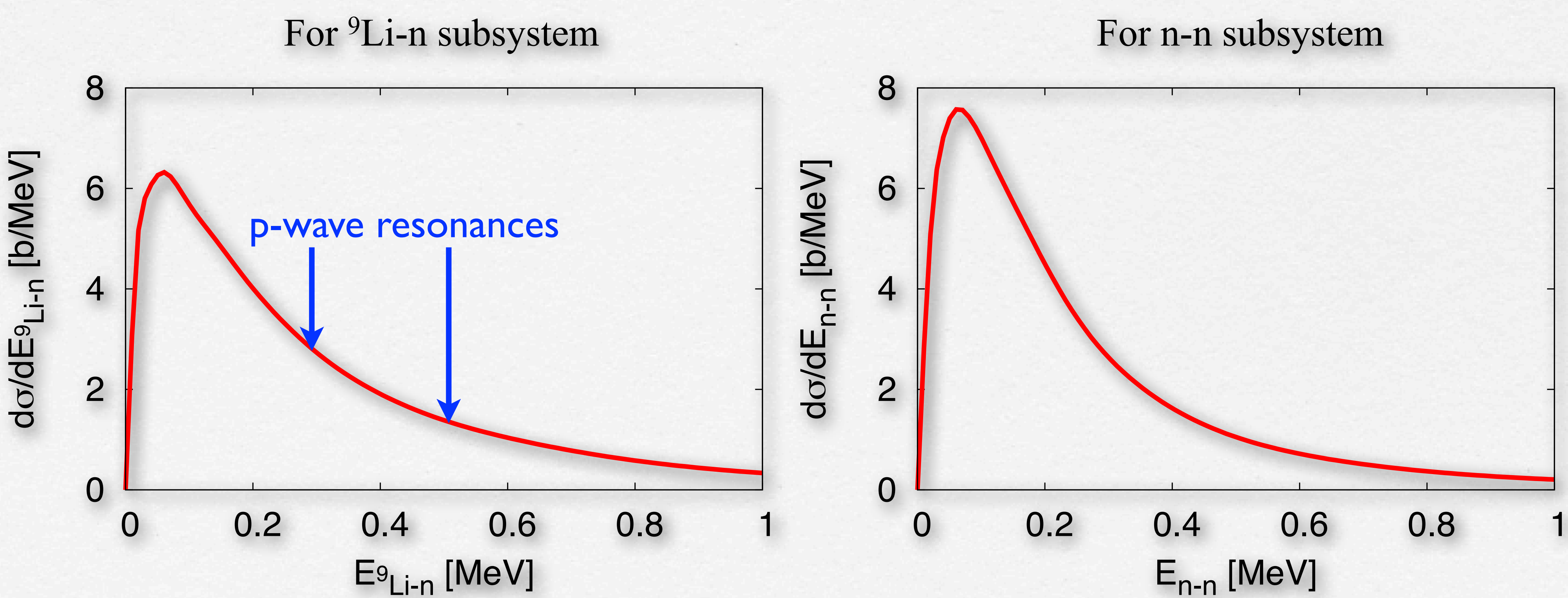
- Invariant mass spectra - ^{6}He case -
  - For α-n, the spectrum shows the peak at around 0.7 MeV, which corresponds to the ^{5}He(3/2$^-$) resonance energy.
    - The sequential decay via the ^{5}He(3/2$^-$)+n channel dominates the breakup of ^{6}He.
  - For n-n, the virtual-state correlation generates the low-lying peak in the spectra.



T. Aumann et al., PRC**59**, 1252 (1999).

Invariant mass spectra for binary subsystems

- Invariant mass spectra - ^{11}Li case -
  - In both cases of ^{9}Li-n and n-n, the sharp peaks appear below 0.1 MeV.
    - For n-n, the virtual-state correlation plays a significant role in breakup as similar to ^{6}He case.
    - For ^{9}Li-n, the peak comes from the virtual state of ^{10}Li, although the p-wave resonances of ^{10}Li cannot be found in the spectra.



Summary

- We investigated the mechanisms of Coulomb breakup reactions of two-neutron halo nuclei ^{6}He and ^{11}Li based on the core+n+n three-body models.
  - To describe the three-body scattering states of the core+n+n systems, we use the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).
    ➡ CSLS is capable method of describing the three-body Coulomb breakups.
  - We calculate the Coulomb breakup cross section, and investigate
    - what is the effects of FSI in the Coulomb breakup reaction
      ➡ FSI dominates the cross sections in both cases of ^{6}He and ^{11}Li.
    - what kinds of correlations play roles in the Coulomb breakup reactions of two-neutron halo nuclei.
      ➡ We confirmed that binary subsystem correlations have significant contributions to the breakups.
      - For ^{6}He, ^{5}He($3/2^-$) resonance and n-n virtual state are important.
      - For ^{11}Li, virtual-state correlations in ^{10}Li and n-n subsystems are clearly seen in the invariant mass spectra, while the p-wave resonances cannot be identified.