Importance of pion and The tensor optimized shell model

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Pion is important in Nuclear Physics!

- Yukawa (1934) predicted pion (size) as a mediator of nuclear interaction to form nucleus
- Mayer-Jensen (1949) introduced shell model (Phenomenological) beginning of Nuclear Physics
- Nambu (1960) introduced the chiral symmetry and its breaking produced mass and the pion as pseudo-scalar particle
Challenge

• Describe nuclei from the first principle (pion)
• Construct nucleus using NN interaction
Pion is a pseudo-scalar particle (Nambu)

\[ \vec{\sigma}_1 \cdot \vec{q} \frac{1}{m_\pi^2 + q^2} \vec{\sigma}_2 \cdot \vec{q} = \frac{1}{3} \left( \frac{q^2}{m_\pi^2 + q^2} S_{12}(\hat{q}) \right) + \frac{1}{3} \left( 1 - \frac{m_\pi^2}{m_\pi^2 + q^2} \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \]

\[ = \frac{1}{3} \frac{q^2}{m_\pi^2 + q^2} S_{12}(\hat{q}) + \frac{1}{3} \left( 1 - \frac{m_\pi^2}{m_\pi^2 + q^2} \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \]

\[ S_{12}(\hat{q}) = \sqrt{\frac{24\pi}{5}} \left[ Y_2(\hat{q})[\sigma_1 \sigma_2]_2 \right]_0 \]

12.12.11
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Deuteron (1\(^+\))

NN interaction

\[
\Psi_d = u(r)[Y_0(\hat{r}) \otimes \chi_1(\sigma_1 \sigma_2)]_{1M} + w(r)[Y_2(\hat{r}) \otimes \chi_1(\sigma_1 \sigma_2)]_{1M}
\]

S=1 and L=0 or 2
Deuteron \((1^+)\)

80% of attraction comes from tensor

D-wave component moves very fast

<table>
<thead>
<tr>
<th>Energy</th>
<th>(-2.24) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic</td>
<td>19.88</td>
</tr>
<tr>
<td>((SS))</td>
<td>11.31</td>
</tr>
<tr>
<td>((DD))</td>
<td>8.57</td>
</tr>
<tr>
<td>Central</td>
<td>(-4.46)</td>
</tr>
<tr>
<td>((SS))</td>
<td>(-3.96)</td>
</tr>
<tr>
<td>((DD))</td>
<td>(-0.50)</td>
</tr>
<tr>
<td>Tensorc</td>
<td>(-16.64)</td>
</tr>
<tr>
<td>((SD))</td>
<td>(-18.93)</td>
</tr>
<tr>
<td>((DD))</td>
<td>2.29</td>
</tr>
<tr>
<td>LS</td>
<td>(-1.02)</td>
</tr>
<tr>
<td>(P(D))</td>
<td>5.78 (%)</td>
</tr>
<tr>
<td>Radius</td>
<td>1.96 [fm]</td>
</tr>
<tr>
<td>((SS))</td>
<td>2.00 [fm]</td>
</tr>
<tr>
<td>((DD))</td>
<td>1.22 [fm]</td>
</tr>
</tbody>
</table>
BE = 5MeV for $^4\text{He}$
Variational calculation of few body system with NN interaction

\[ \langle \Psi | V_{\pi} | \Psi \rangle \sim 80\% \]

\[ \langle \Psi | V_{NN} | \Psi \rangle \]

\[ \Psi = \phi(r_{12})\phi(r_{23})...\phi(r_{ij}) \]


Heavy nuclei (Super model)  Pion is key
How to handle tensor interaction in heavy nuclei

- Transition from relative S-wave to D-wave provides large attraction
- In shell model, this is achieved by taking 2p-2h state (Myo et al.)
- Tensor optimized shell model (TOSM)
  \[ \Psi = C_0 \left\langle 0 \right| + \sum_{\alpha} C_{\alpha} \left| 2p - 2h : \alpha \right\rangle \]
- Short range correlation is treated by UCOM (Feldmeier et al.)
TOSM (+UCOM) with AV8’

\[ \Psi = C_0 |0\rangle + \sum_{\alpha} C_{\alpha} |2p2h : \alpha\rangle \]

(Myo Toki Ikeda)

PTP 121 (2009)

Few body Calculation
(Kamda et al (2001))
Configurations in TOSM

particle states

Gaussian expansion

C₀

C₁

C₂

C₃

nlj

proton neutron

hole states

(harmonic oscillator basis)

Application to Hypernuclei by Umeya to investigate $\Delta N-\Sigma N$ coupling
$^{4-8}$He with TOSM+UCOM

- Excitation energies in MeV
- Bound state app.
- No continuum
- No $V_{\text{NNN}}$

Excitation energy spectra are reproduced well

T. Myo, A. Umeya, H. Toki, K. Ikeda
PRC84 (2011) 034315
We cannot treat the tensor interaction in HF space.

\[ \langle 0 | S_{12} | 0 \rangle = 0 \]

\[ S_{12} = \sqrt{\frac{24\pi}{5}} \left[ Y_2(\hat{r}) \times [\sigma_1 \times \sigma_2]_2 \right]^{(0)}. \]

We improve Brueckner-Hartree-Fock theory
Comparison of BHF and EBHF

Hartree-Fock equations look very similar

**BHF**

\[
\langle 0 | T + G | 0 \rangle = \langle 0 | T + V | 0 \rangle - \sum_{\alpha\beta} \langle 0 | V | \alpha \rangle \langle \alpha | \frac{1}{H_{HF} - E_{HF}^h + V} | \beta \rangle \langle \alpha | V | 0 \rangle
\]

**EBHF**

\[
\langle 0 | H_{eff} | 0 \rangle = |C_0|^2 \langle 0 | T + V | 0 \rangle - |C_0|^2 \sum_{\alpha\beta} \langle 0 | V | \alpha \rangle \langle \alpha | \frac{1}{H - E} | \beta \rangle \langle \beta | V | 0 \rangle
\]
\[ |\Psi\rangle = C_0 |0\rangle + \sum_{\alpha} C_{\alpha} |2p - 2h : \alpha\rangle \]

\[ \langle \Psi | \Psi \rangle = |C_0|^2 + \sum_{\alpha} |C_{\alpha}|^2 = 1 \]

\[ \langle \Psi | \hat{O} | \Psi \rangle = |C_0|^2 \langle 0 | \hat{O} | 0 \rangle + \sum_{\alpha \beta} C_{\alpha}^* C_{\beta} \langle \alpha | \hat{O} | \beta \rangle \]

Shell model

80~90%
Shell model

High momentum Component

TOSM wave function

Matrix element

Shell model

Tensor state

\[ \text{Distribution} \]

\[ 300 \text{MeV} / c(k_F) \]

Momentum
Variational calculation of few body system with NN interaction

\[ \langle \Psi | V_\pi | \Psi \rangle / \langle \Psi | V_{NN} | \Psi \rangle \approx 80\% \]

\[ \Psi = \phi(r_{12}) \phi(r_{23}) ... \phi(r_{ij}) \]


Heavy nuclei (Super model)

Pion is key
Importance of delta

We treat delta explicitly for three body interaction.

NN channel  NΔ channel  ΔΔ channel

Two body potential including delta

→ Argonne delta model potential (AV28)


Three body interaction
Effect of delta in deuteron

### Chart Details

- **Energy Levels**
  - AV14: 21.37 + 3.16
  - AV28: 8.90
  - AV14: 8.90
  - AV28: 8.90

- **Kinetic Energy**
  - AV14: 19.14
  - AV28: 19.14

- **Central Energy**
  - AV14: -1.91
  - AV28: -1.91

- **Energy**
  - AV14: -2.17
  - AV28: -2.33

- **Tensor Energy**
  - AV14: -18.80
  - AV28: -18.80

- **Table of Results**

<table>
<thead>
<tr>
<th>Deuteron 1^+</th>
<th>AV14</th>
<th>AV28</th>
</tr>
</thead>
<tbody>
<tr>
<td>L \cdot S</td>
<td>0.36</td>
<td>0.86</td>
</tr>
<tr>
<td>L^2</td>
<td>3.07</td>
<td>3.63</td>
</tr>
<tr>
<td>(L \cdot S)^2</td>
<td>-4.03</td>
<td>-4.14</td>
</tr>
<tr>
<td>P_{NN} [^3S_1] %</td>
<td>93.96</td>
<td>93.22</td>
</tr>
<tr>
<td>P_{NN} [^3D_1]</td>
<td>6.04</td>
<td>6.23</td>
</tr>
<tr>
<td>P_{\Delta\Delta} [^3S_1]</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>P_{\Delta\Delta} [^3D_1]</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>P_{\Delta\Delta} [^7D_1]</td>
<td>0.43</td>
<td>0.43</td>
</tr>
<tr>
<td>P_{\Delta\Delta} [^7G_1]</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Prior studies for nuclei with delta

No calculations for $A \geq 4$
Too many states are necessary.

$A=3$

1. Hannover group (Germany)
Bonn potential + single delta
2. Los Alamos group (Fadeev calculations)
AV28 potential $\rightarrow$ Not enough binding for $^3\text{H}$
Approximation: double $\Delta$ up to $L=2$

Wave function in deuteron

$$\Psi_{NN} = |^3S_1\rangle + |^3D_1\rangle$$
$$\Psi_{\Delta\Delta} = |^3S_1\rangle + |^3D_1\rangle + |^7D_1\rangle + |^7G_1\rangle$$ ← about 0.04 %
### Results in $^{1}$Even channel

**$^{1}$E channel**  \( L=\text{even}, S=\text{even}, T=1 \)

<table>
<thead>
<tr>
<th>( J=0 )</th>
<th>( d=1.70 \text{ fm} )</th>
<th>no ( N\Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy [MeV]</td>
<td>5.7</td>
<td>8.2</td>
</tr>
<tr>
<td>Kinetic</td>
<td>10.51 (NN=9.23)</td>
<td>9.51 (NN=9.30)</td>
</tr>
<tr>
<td>Central</td>
<td>0.90 (NN=0.70)</td>
<td>-0.12 (NN=-0.12)</td>
</tr>
<tr>
<td>Tensor</td>
<td>-7.92</td>
<td>-1.75</td>
</tr>
<tr>
<td>( L\cdot S )</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>( L^2 )</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>( (L\cdot S)^2 )</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>( P_{NN} ) ([^{1}S_0]) %</td>
<td>99.37</td>
<td>99.89</td>
</tr>
<tr>
<td>( P_{\Delta\Delta} ) ([^{1}D_0])</td>
<td>0.01</td>
<td>0.007</td>
</tr>
<tr>
<td>( P_{\Delta\Delta} ) ([^{5}D_0])</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>( P_{N\Delta} ) ([^{5}D_0])</td>
<td>0.49</td>
<td>------</td>
</tr>
</tbody>
</table>

**Odd channels**

Delta contribution is about 1/5~1/10 in odd channels

\[
\Psi_{NN} = |^{1}S_0\rangle
\]
\[
\Psi_{\Delta\Delta} = |^{1}S_0\rangle + |^{5}D_0\rangle
\]
\[
\Psi_{N\Delta} = |^{5}D_0\rangle
\]
Delta with and without $^7G_1$
Pion (Tensor force) in finite nuclei

- Pion (Tensor force) is important in finite nuclei
- Tensor optimized shell model (TOSM) is used to treat the tensor force -- K computer
- Tensor force has a strong influence on the excitation spectra (TOSM)
- The wave function contains high momentum components
- Extended HF theory: BHF is reformulated
- Three body interaction by explicit treatment of delta
- Delta effect is very large