

# Importance of pion and The tensor optimized shell model

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# Pion is important in Nuclear Physics !

- Yukawa(1934) predicted pion (size) as a mediator of nuclear interaction to form nucleus
- Mayer-Jensen(1949) introduced shell model (Phenomenological)  
beginning of Nuclear Physics
- Nambu(1960) introduced the chiral symmetry and its breaking produced mass and the pion as pseudo-scalar particle

# Challenge

- Describe nuclei from the first principle (pion)
- Construct nucleus using NN interaction



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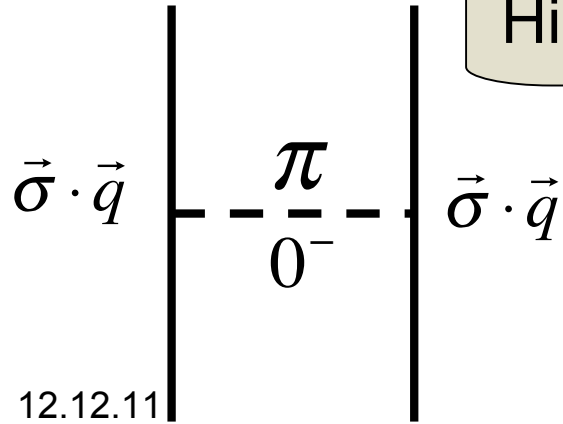
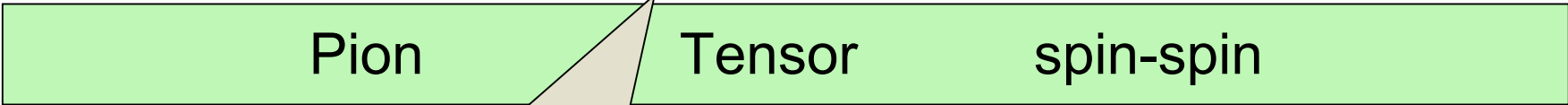
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# Pion is a pseudo-scalar particle (Nambu)

$$\begin{aligned} \vec{\sigma}_1 \cdot \vec{q} \frac{1}{m_\pi^2 + q^2} \vec{\sigma}_2 \cdot \vec{q} &= \frac{1}{3} \frac{q^2}{m_\pi^2 + q^2} S_{12}(\hat{q}) + \frac{1}{3} \frac{q^2}{m_\pi^2 + q^2} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ &= \frac{1}{3} \frac{q^2}{m_\pi^2 + q^2} S_{12}(\hat{q}) + \frac{1}{3} \left( 1 - \frac{m_\pi^2}{m_\pi^2 + q^2} \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \end{aligned}$$

Low momentum

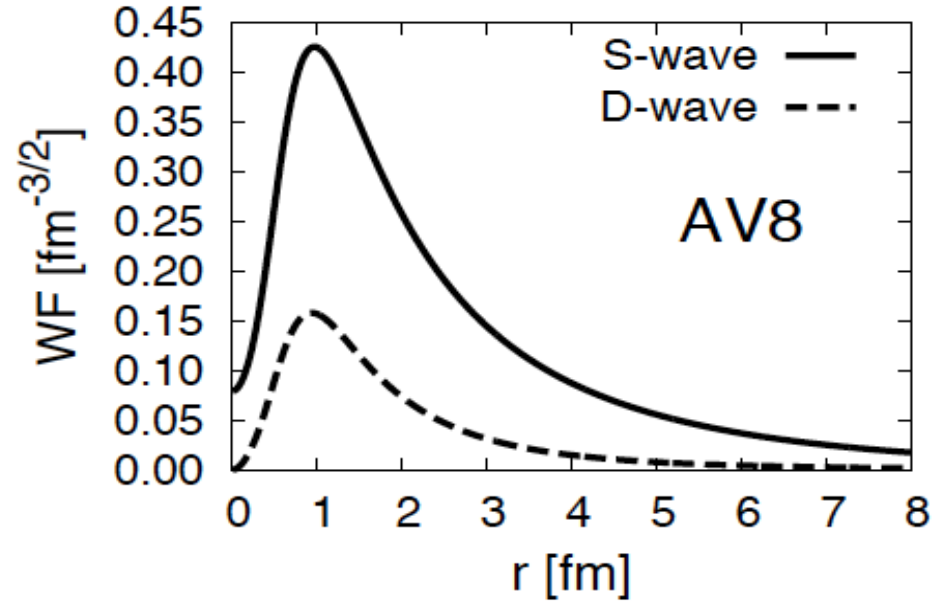
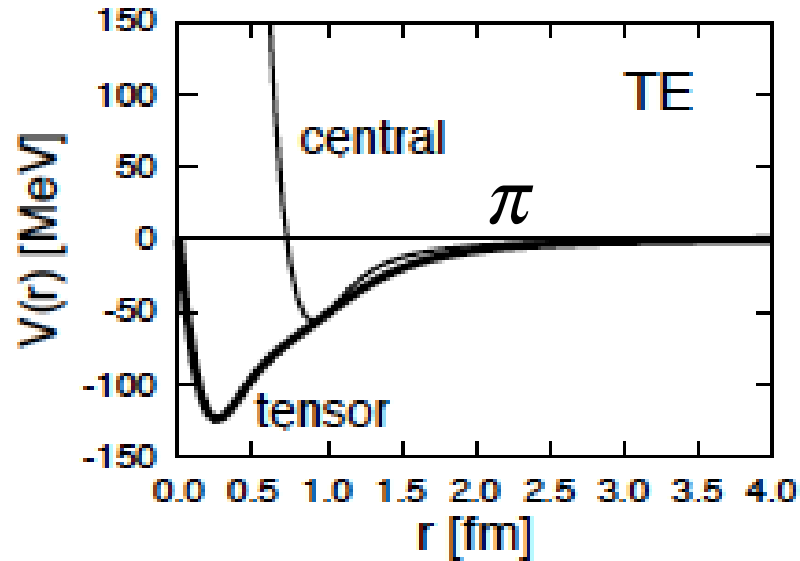


$$S_{12}(\hat{q}) = \sqrt{\frac{24\pi}{5}} [Y_2(\hat{q}) [\sigma_1 \sigma_2]_2]_0$$

Difficult to handle

# Deuteron ( $1^+$ )

NN interaction



$$\Psi_d = u(r)[Y_0(\hat{r}) \otimes \chi_1(\sigma_1 \sigma_2)]_{1M} + w(r)[Y_2(\hat{r}) \otimes \chi_1(\sigma_1 \sigma_2)]_{1M}$$

S=1 and L=0 or 2

# Deuteron ( $1^+$ )

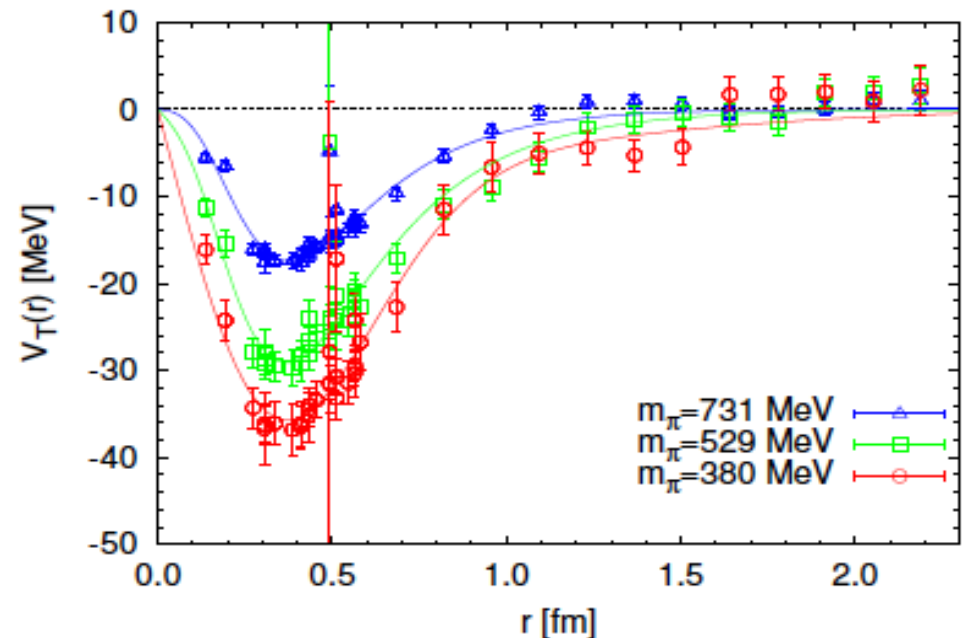
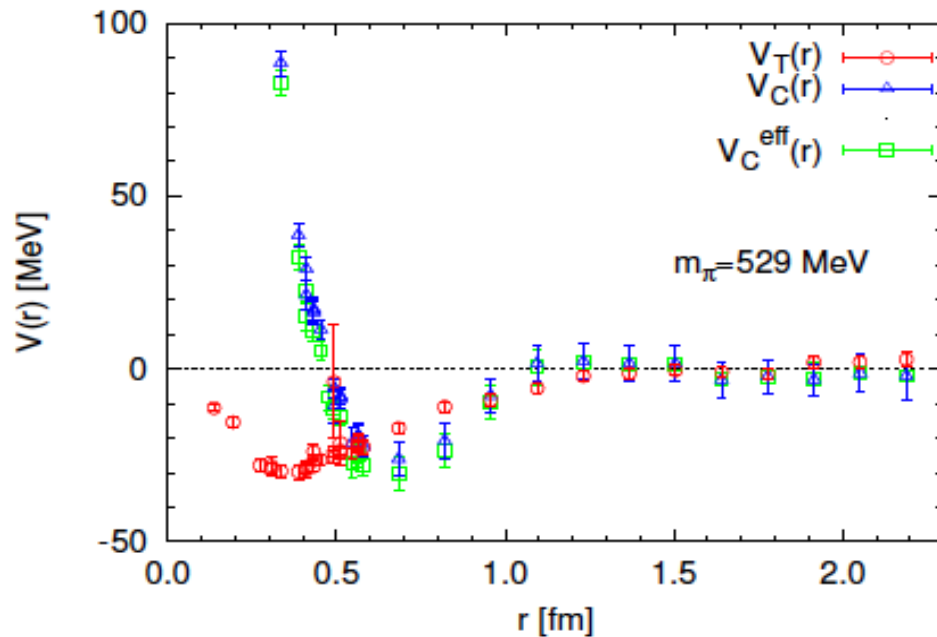
80% of attraction comes from tensor

D-wave component moves very fast

|         |             |
|---------|-------------|
| Energy  | -2.24 [MeV] |
| Kinetic | 19.88       |
| (SS)    | 11.31       |
| (DD)    | 8.57        |
| Central | -4.46       |
| (SS)    | -3.96       |
| (DD)    | -0.50       |
| Tensorc | -16.64      |
| (SD)    | -18.93      |
| (DD)    | 2.29        |
| LS      | -1.02       |
| P(D)    | 5.78 [%]    |
| Radius  | 1.96 [fm]   |
| (SS)    | 2.00 [fm]   |
| (DD)    | 1.22 [fm]   |

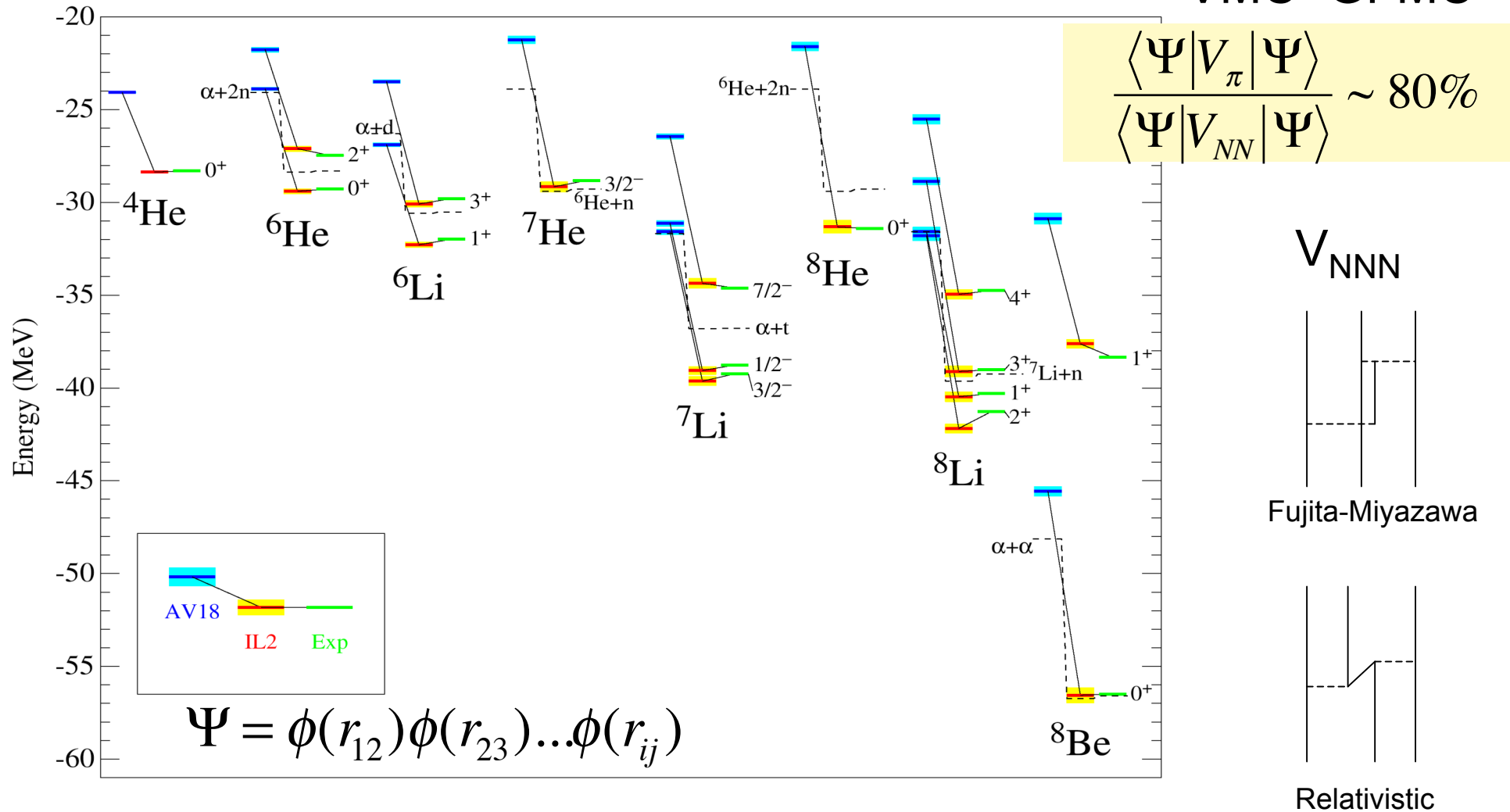
# Theoretical Foundation of the Nuclear Force in QCD and Its Applications to Central and Tensor Forces in Quenched Lattice QCD Simulations

Sinya AOKI,<sup>1</sup> Tetsuo HATSUDA<sup>2</sup> and Noriyoshi ISHII<sup>2</sup>



# Variational calculation of few body system with NN interaction

VMC+GFMC



C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)

Heavy nuclei (Super model)

Pion is key



## How to handle tensor interaction in heavy nuclei

- Transition from relative S-wave to D-wave provides large attraction
- In shell model, this is achieved by taking 2p-2h state (Myo et al.)

- Tensor optimized shell model (TOSM)

$$\Psi = C_0|0\rangle + \sum_{\alpha} C_{\alpha}|2p-2h:\alpha\rangle$$

- Short range correlation is treated by UCOM (Feldmeier et al.)

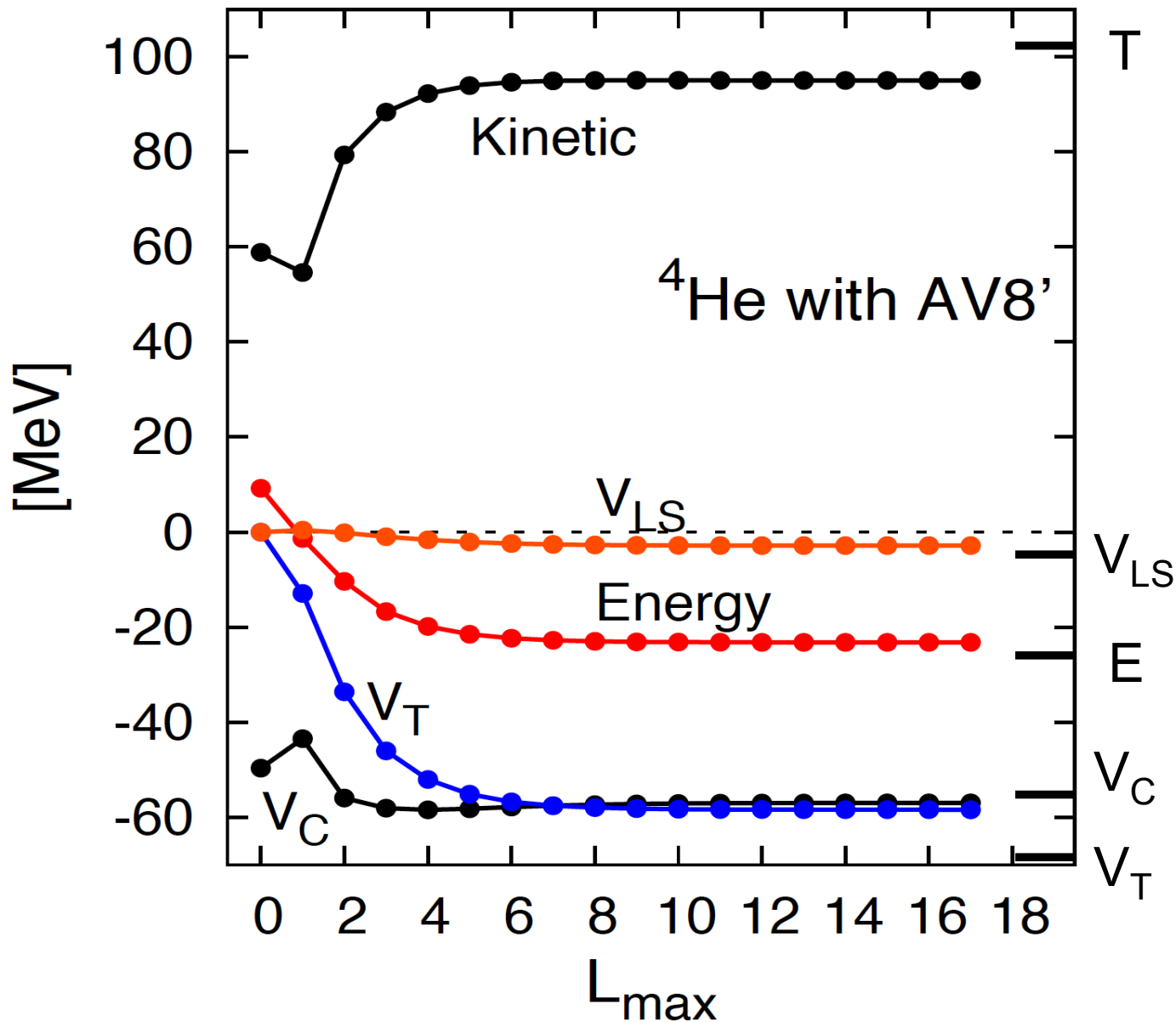
TOSM (+UCOM) with AV8'

$$\Psi = C_0|0\rangle + \sum_{\alpha} C_{\alpha}|2p2h:\alpha\rangle$$

(Myo Toki Ikeda)

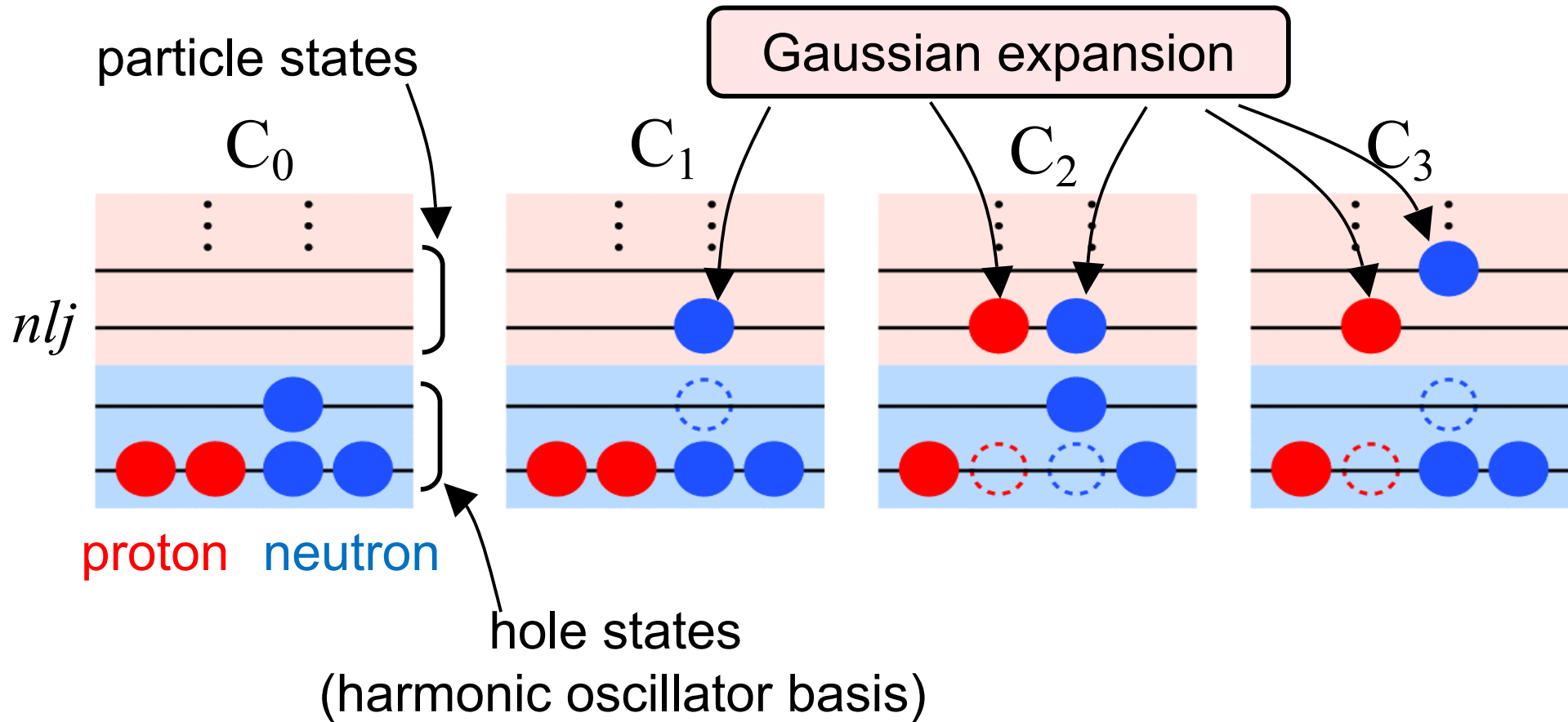
PTP 121 (2009)

$$|2p\rangle = |[l_1 l_2]_L\rangle$$



Few body  
Calculation  
(Kamada et al  
(2001))

# Configurations in TOSM

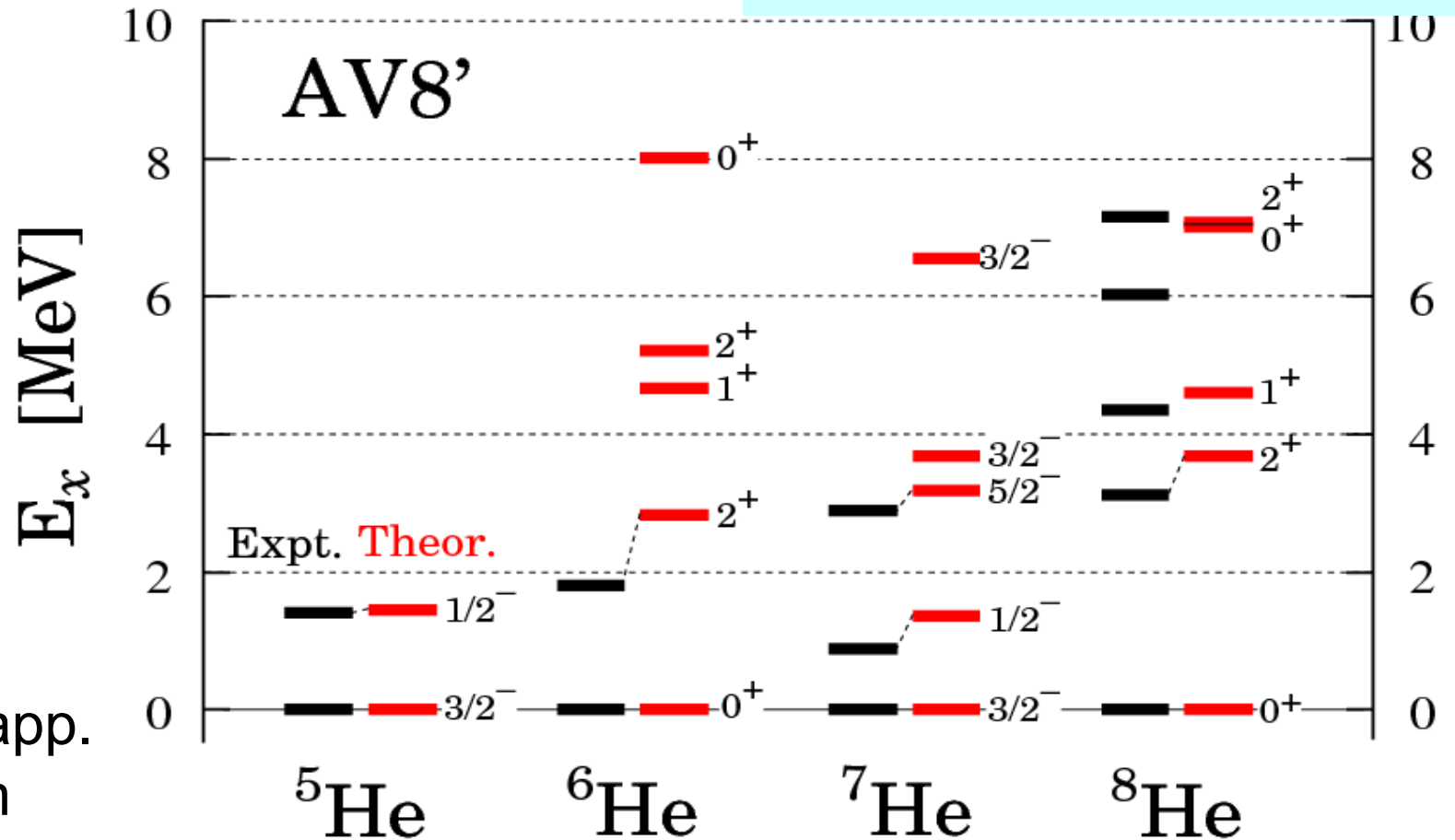


Application to Hypernuclei by Umeya  
to investigate  $\Lambda N$ - $\Sigma N$  coupling

# $4\text{-}^8\text{He}$ with TOSM+UCOM

T.Myo,A.Umeya,H.Toki,K.Ikeda  
PRC84 (2011) 034315

- Excitation energies in MeV



- Bound state app.
- No continuum
- No  $V_{\text{NNN}}$

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- Excitation energy spectra are reproduced well

toki@pionrcnp

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## Extension of Hartree-Fock theory with TOSM

Y.Ogawa H.Toki

Annals of Physics 326 (2011) 2039

$$\langle 0 | S_{12} | 0 \rangle = 0 \quad S_{12} = \sqrt{\frac{24\pi}{5}} [Y_2(\hat{r}) \times [\sigma_1 \times \sigma_2]_2]^{(0)}.$$

We cannot treat the tensor interaction in HF space.

TOSM ansatz

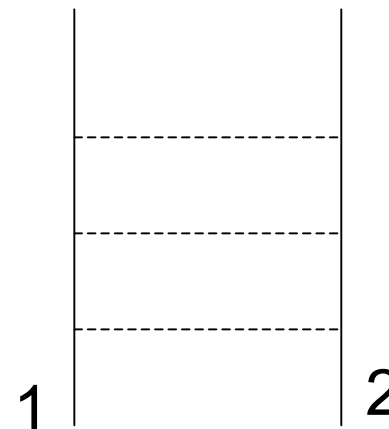
$$|\Psi\rangle = C_0 |0\rangle + \sum_{\alpha} C_{\alpha} |2p - 2h : \alpha\rangle$$

$$\delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0 \quad \langle \Psi | \Psi \rangle = |C_0|^2 + \sum_{\alpha} |C_{\alpha}|^2 = 1$$

We improve Brueckner-Hartree-Fock theory

## Comparison of BHF and EBHF

Hartree-Fock equations look very similar



BHF

$$\langle 0 | T + G | 0 \rangle = \langle 0 | T + V | 0 \rangle - \sum_{\alpha\beta} \langle 0 | V | \alpha \rangle \langle \alpha | \frac{1}{H_{HF} - E_{HF}^h + V} | \beta \rangle \langle \beta | V | 0 \rangle$$

EBHF

$$\langle 0 | H_{eff} | 0 \rangle = |C_0|^2 \langle 0 | T + V | 0 \rangle - |C_0|^2 \sum_{\alpha\beta} \langle 0 | V | \alpha \rangle \langle \alpha | \frac{1}{H - E} | \beta \rangle \langle \beta | V | 0 \rangle$$

## TOSM wave function

## Matrix element

$$|\Psi\rangle = C_0|0\rangle + \sum_{\alpha} C_{\alpha}|2p-2h:\alpha\rangle$$

$$\langle\Psi|\Psi\rangle = |C_0|^2 + \sum_{\alpha} |C_{\alpha}|^2 = 1$$

80~90%  
Shell model

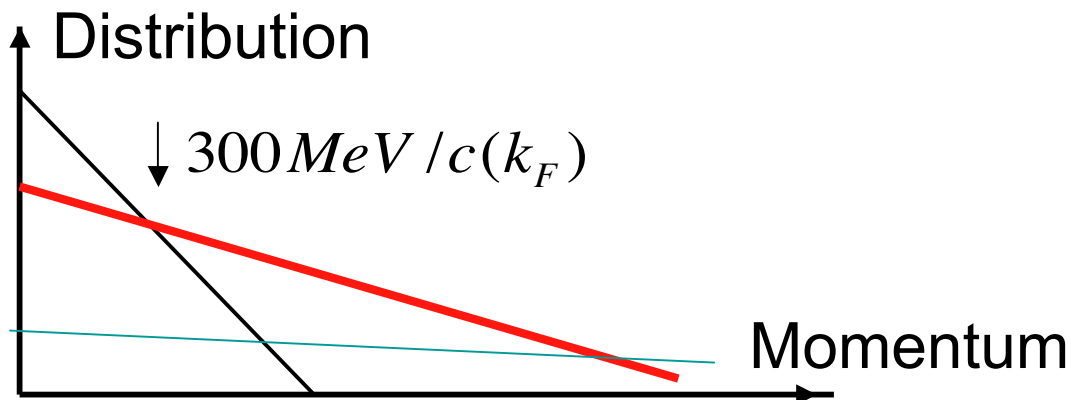
High momentum  
Component

Shell model  
↓

$$\langle\Psi|\hat{O}|\Psi\rangle = |C_0|^2 \langle 0|\hat{O}|0\rangle$$

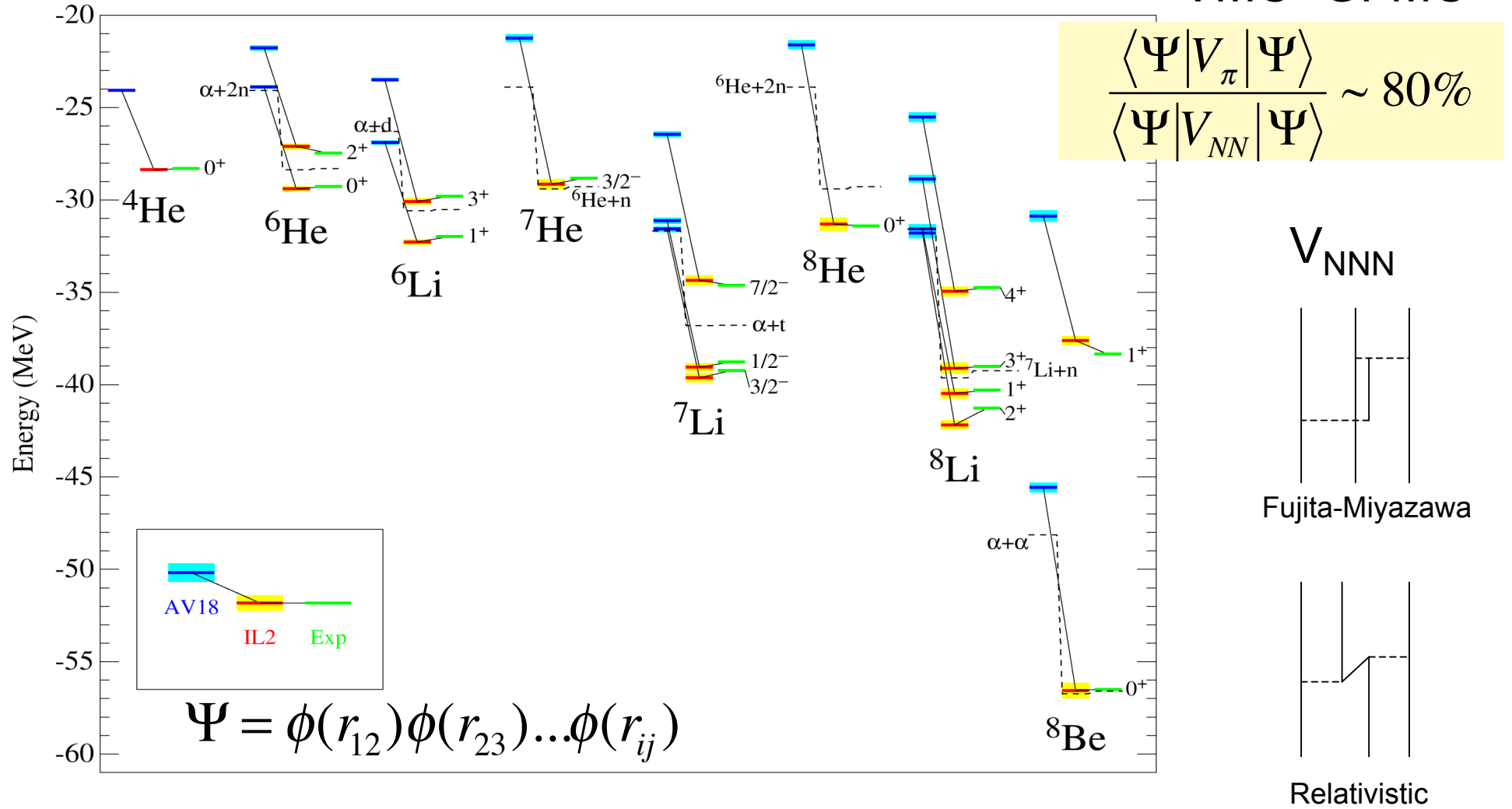
$$+ \sum_{\alpha\beta} C_{\alpha}^* C_{\beta} \langle \alpha|\hat{O}|\beta\rangle$$

↑  
Tensor state



# Variational calculation of few body system with NN interaction

VMC+GFMC



C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)

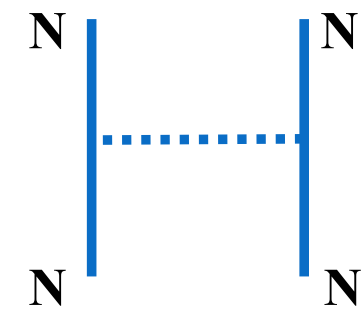
Heavy nuclei (Super model)

Pion is key

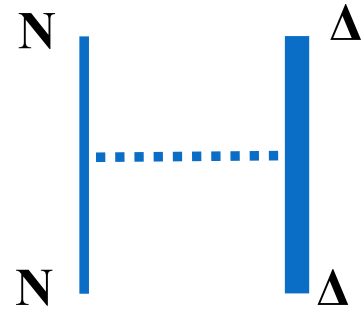


# Importance of delta

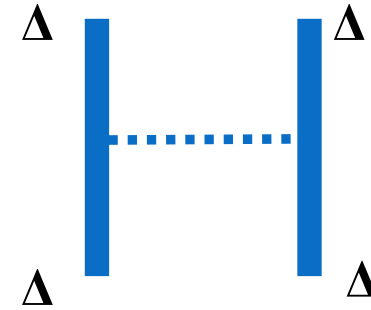
We treat delta explicitly for three body interaction.



NN channel



NΔ channel

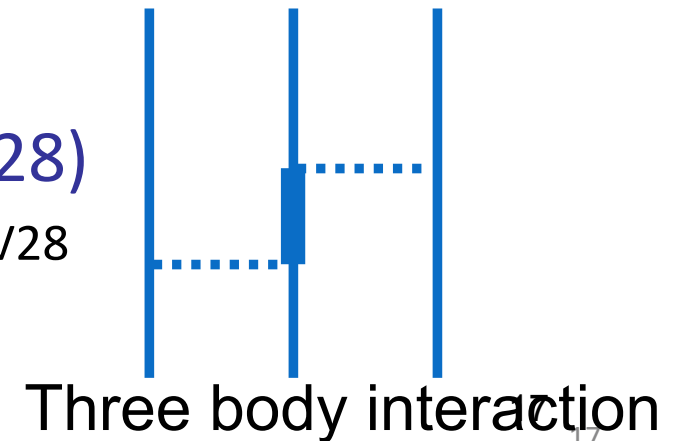


ΔΔ channel

Two body potential including delta

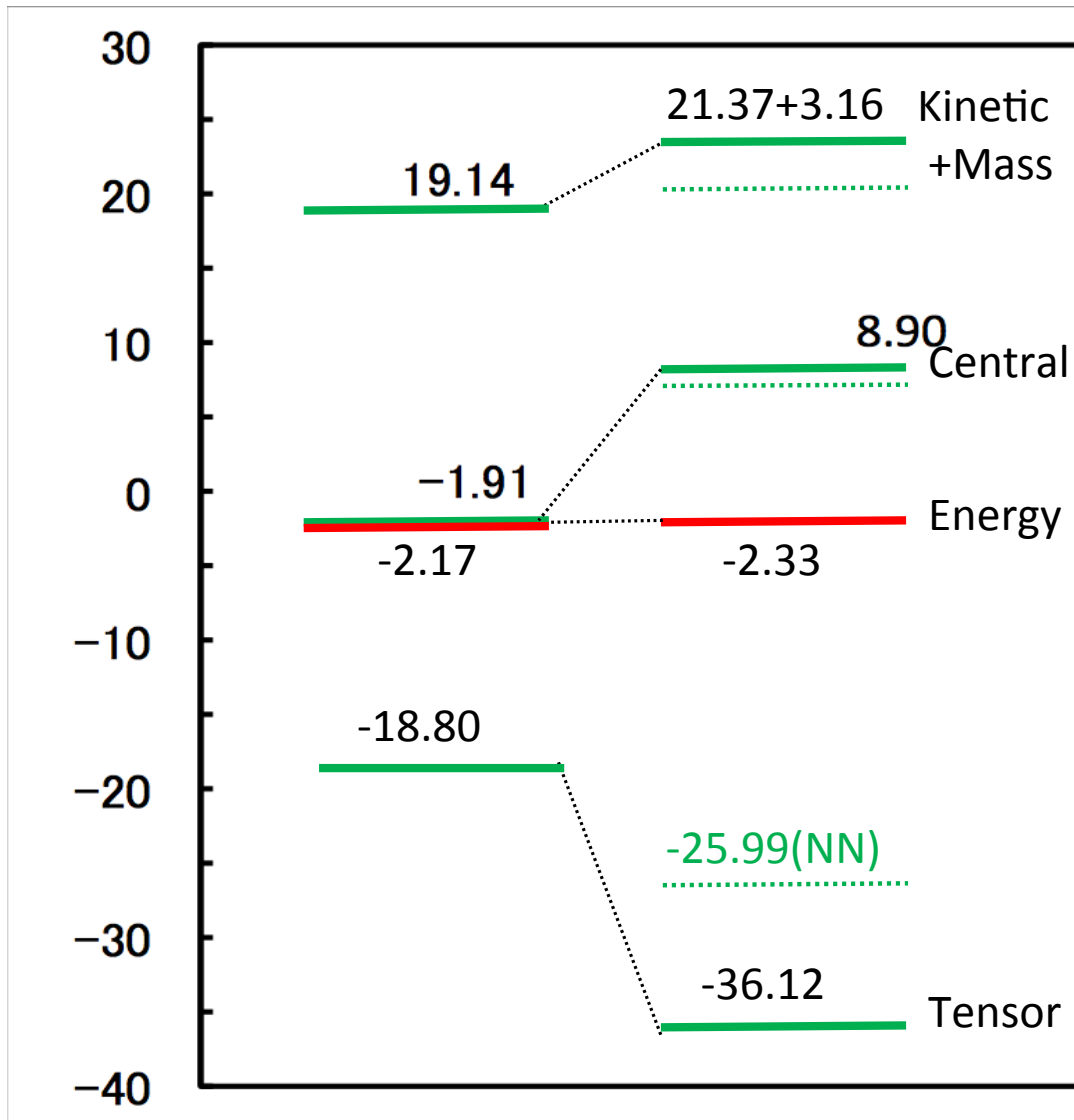
→ Argonne delta model potential (AV28)

R.B. Wiringa et al, PRC 29, 1207(1984) AV14 & AV28



Three body interaction

# Effect of delta in deuteron



| Deuteron 1 <sup>+</sup>                           | AV14  | AV28  |
|---|-------|-------|
| L·S   | 0.36  | 0.86  |
| L <sup>2</sup>                                    | 3.07  | 3.63  |
| (L·S) <sup>2</sup>                                | -4.03 | -4.14 |
| P <sub>NN</sub> [ <sup>3</sup> S <sub>1</sub> ] % | 93.96 | 93.22 |
| P <sub>NN</sub> [ <sup>3</sup> D <sub>1</sub> ]   | 6.04  | 6.23  |
| P <sub>ΔΔ</sub> [ <sup>3</sup> S <sub>1</sub> ]   |       | 0.04  |
| P <sub>ΔΔ</sub> [ <sup>3</sup> D <sub>1</sub> ]   |       | 0.02  |
| P <sub>ΔΔ</sub> [ <sup>7</sup> D <sub>1</sub> ]   |       | 0.43  |
| P <sub>ΔΔ</sub> [ <sup>7</sup> G <sub>1</sub> ]   |       | 0.04  |

# Prior studies for nuclei with delta

No calculations for  $A \geq 4$   
Too many states are necessary.

TOSM

$A=3$

$$\Psi = |S\rangle + |D\rangle + |D_{N\Delta}\rangle + |D_{\Delta\Delta}\rangle$$

1. Hannover group (Germany)

Bonn potential + single delta

Sauer, PHYS. REV. C68, 024005 (2003)

2. Los Alamos group (Fadeev calculations)

AV28 potential → Not enough binding for  ${}^3\text{H}$

Approximation : double  $\Delta$  up to  $L=2$

A. Picklesimer et al. Phys. Rev. C46 (1992)

Wave function in deuteron

$$\Psi_{NN} = |{}^3S_1\rangle + |{}^3D_1\rangle$$

$$\Psi_{\Delta\Delta} = |{}^3S_1\rangle + |{}^3D_1\rangle + |{}^7D_1\rangle + |{}^7G_1\rangle \leftarrow \text{about } 0.04 \%$$

# Results in $^1\text{E}$ channel

$^1\text{E}$  channel L=even, S=even, T=1

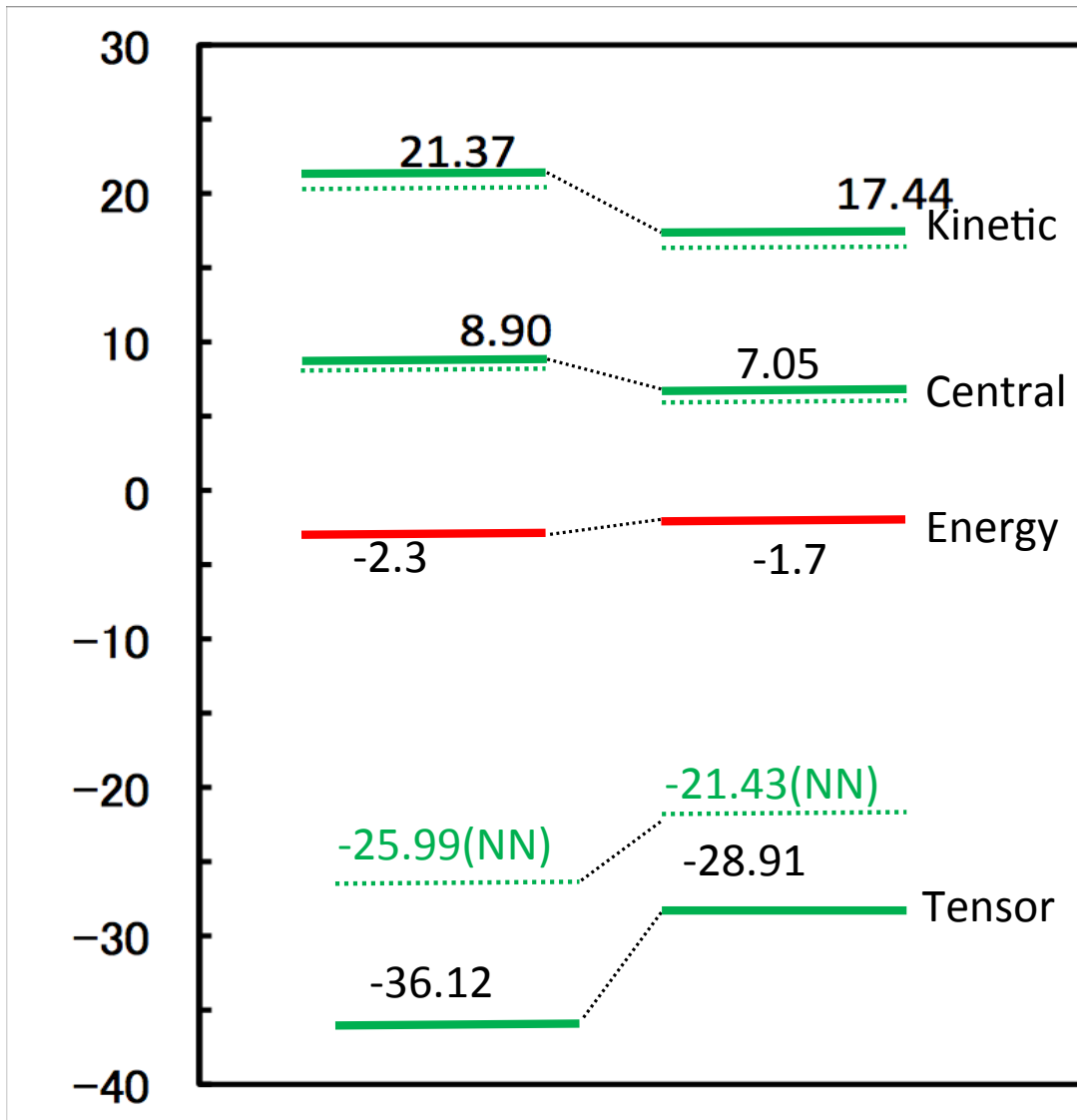
| <b>J=0</b>  | <b>d=1.70 fm</b> | <b>no N<math>\Delta</math></b> |
|---|------------------|--------------------------------|
| Energy [MeV]  | 5.7              | 8.2                            |
| Kinetic   | 10.51(NN=9.23)   | 9.51 (NN=9.30)                 |
| Central   | 0.90(NN=0.70)    | -0.12 (NN=-0.12)               |
| Tensor  | <b>-7.92</b>     | <b>-1.75</b>                   |
| L·S   | -----            | -----                          |
| L <sup>2</sup>  | -----            | -----                          |
| (L·S) <sup>2</sup>  | -----            | -----                          |
| P <sub>NN</sub> [ $^1\text{S}_0$ ] %                      | 99.37            | 99.89                          |
| P <sub><math>\Delta\Delta</math></sub> [ $^1\text{D}_0$ ] | 0.01             | 0.007                          |
| P <sub><math>\Delta\Delta</math></sub> [ $^5\text{D}_0$ ] | 0.12             | 0.10                           |
| P <sub>N<math>\Delta</math></sub> [ $^5\text{D}_0$ ]      | <b>0.49</b>      | -----                          |

$$\begin{aligned}\Psi_{NN} &= |^1S_0\rangle \\ \Psi_{\Delta\Delta} &= |^1S_0\rangle + |^5D_0\rangle \\ \Psi_{N\Delta} &= |^5D_0\rangle\end{aligned}$$

Odd channels

Delta contribution is about 1/5~1/10 in odd channels

# Delta with and without ${}^7G_1$



| Deuteron<br>$1^+$            | Full  | No<br>G-wave |
|------------------------------|-------|--------------|
| $L \cdot S$                  | 0.86  | 0.66         |
| $L^2$                        | 3.63  | 2.82         |
| $(L \cdot S)^2$              | -4.14 | -3.21        |
| $P_{NN} [{}^3S_1]$<br>%      | 93.22 | 94.29        |
| $P_{NN} [{}^3D_1]$           | 6.23  | 5.28         |
| $P_{\Delta\Delta} [{}^3S_1]$ | 0.04  | 0.03         |
| $P_{\Delta\Delta} [{}^3D_1]$ | 0.02  | 0.01         |
| $P_{\Delta\Delta} [{}^7D_1]$ | 0.43  | 0.37         |
| $P_{\Delta\Delta} [{}^7G_1]$ | 0.04  | -----        |

12.12.11 Full No G-wave @ki@pionrcnp

# Pion (Tensor force) in finite nuclei

- **Pion** (Tensor force) is important in finite nuclei
- **Tensor optimized shell model (TOSM)** is used to treat the tensor force -- **K computer**
- Tensor force has a strong influence on the excitation spectra (**TOSM**)
- The wave function contains **high momentum components**
- Extended HF theory: **BHF** is reformulated
- **Three body interaction** by explicit treatment of delta
- **Delta effect** is very large

