${}^{4}_{\Lambda\Lambda}$H in Halo EFT

Shung-Ichi Ando

Daegu University,

Republic of Korea

Contents

- Introduction:
  Light double Λ hyper nuclei and $a_{\Lambda\Lambda}$
- ${}^{4}_{\Lambda\Lambda}$H in Halo EFT
- Results and discussion

Introduction

- Light double Λ hyper nuclei, ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, ${}^{11}_{\Lambda\Lambda}$Be, ...
- Λ-Λ interactions, H-dibaryon, Lattice QCD simulations ...
- Scattering length $a_{\Lambda\Lambda}$, [Gasparyan et al. PRC85,015204(2012)]

$$a_{\Lambda\Lambda} = -1.2 \pm 0.6\,\text{fm}\,,$$

deduced from ^{12}C$(K^-, K^+\Lambda\Lambda X)$ data, whereas the $\Lambda\Lambda$ model potentials give

$$a_{\Lambda\Lambda} \simeq -0.27 \sim -3.8\,\text{fm}\,, \quad r_{\Lambda\Lambda} \simeq 0.34 \sim 15.0\,\text{fm}\,,$$

where those values are consistent with ${}^{6}_{\Lambda\Lambda}$He.

${}^{4}_{\Lambda\Lambda}$H: References

- Experiment:
  - BNL-AGS E906, Ahn *et al.*, PRL87, 132504 (2001)
- Theory:
  - Filikhin, Gal, PRL89, 172502(2002)
  - Nemura, Akaishi, PRC67, 051001(2003)
  - Shoeb, PRC69, 054003 (2004)
  - Nemura, Shinmura, Akaishi, Myint, PRL94, 202502 (2005)
  - Sharma, Usmani, Bodmer, CPL30, 032101 (2013)

${}^{4}_{\Lambda\Lambda}$H in Halo EFT

- Effective Field Theories
  - Model independent approach
  - Separation scale, perturbation scheme
  - Parameters should be fixed by experiments
- ${}^{4}_{\Lambda\Lambda}$H in Halo EFT
  - We choose a typical energy, $B_\Lambda = 0.13$ MeV of ${}^{3}_{\Lambda}$H and a high energy, $B_2 = 2.22$ MeV of the deuteron,
  - ${}^{4}_{\Lambda\Lambda}$H as $\Lambda\Lambda d$ cluster system,
  - consider S-wave Λ-${}^{3}_{\Lambda}$H scattering at leading order; 2 parameters in $S = 0$ and 4 in $S = 1$ channel

Specifications

- A typical momentum, $\gamma_{\Lambda d} = \sqrt{2\mu_{\Lambda d} B_\Lambda} \simeq 13.5$ MeV,
  a large momentum, $\gamma = \sqrt{m_N B_2} \simeq 45.7$ MeV,
  thus the expansion parameter $\gamma_{\Lambda d}/\gamma \sim 1/3$
- Consider S-waves and leading order terms only
- Two cluster channels; Λ-$t({}^3_\Lambda\mathrm{H})$ and d-$s(\Lambda\Lambda({}^1S_0))$
- Λ-${}^3_\Lambda$H scattering for $S = 0$ channel
  Cutoff Λ_c insensitive, no three-body interaction,
  described by effective range parameters in ${}^3_\Lambda$H channel
- Λ-${}^3_\Lambda$H scattering for $S = 1$ channel
  Cutoff Λ_c sensitive, three-body interaction is needed,
  described by $\gamma_{\Lambda d}$, $a_{\Lambda\Lambda}$, $g_1(\Lambda_c)$, Λ_c

Calculation

- Lagrangian

$$\mathcal{L} = \mathcal{L}_\Lambda + \mathcal{L}_d + \mathcal{L}_s + \mathcal{L}_t + \mathcal{L}_{\Lambda t} \, ,$$

$$\mathcal{L}_\Lambda = B_\Lambda^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2m_\Lambda} \right] B_\Lambda + \cdots \, ,$$

$$\mathcal{L}_d = d_i^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2m_d} \right] d_i + \cdots \, ,$$

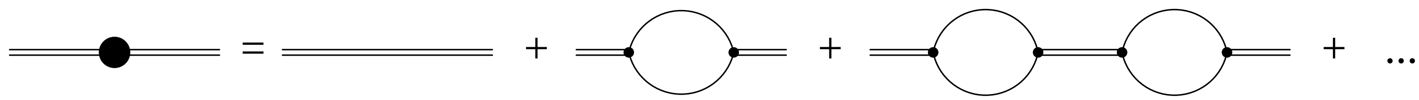
$$\mathcal{L}_s = \sigma_s s^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{4m_\Lambda} + \Delta_s \right] s - y_s \left[s^\dagger \left(B_\Lambda^T P^{(^1S_0)} B_\Lambda \right) + \text{H.c.} \right] + \cdots \, ,$$

$$\mathcal{L}_t = \sigma_t t^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2(m_d + m_\Lambda)} + \Delta_t \right] t + \frac{y_t}{\sqrt{3}} \left[t^\dagger \vec{\sigma}\cdot\vec{d} B_\Lambda + \text{H.c.} \right] + \cdots \, ,$$

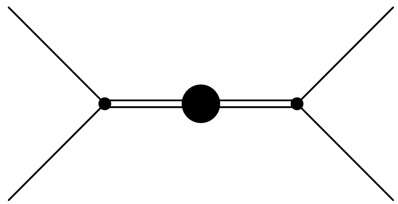
$$\mathcal{L}_{\Lambda t} = -\frac{g_1(\Lambda_c)}{\Lambda_c^2} \left(B_\Lambda^T P_i^{(^3S_1)} t \right)^\dagger \left(B_\Lambda^T P_i^{(^3S_1)} t \right) + \cdots \, ,$$

Two-body part: $\Lambda\Lambda$ in $^1S_0$ state

- Dressed dibaryon propagator



- S-wave scattering amplitude



- Renormalized dressed dibaryon propagator

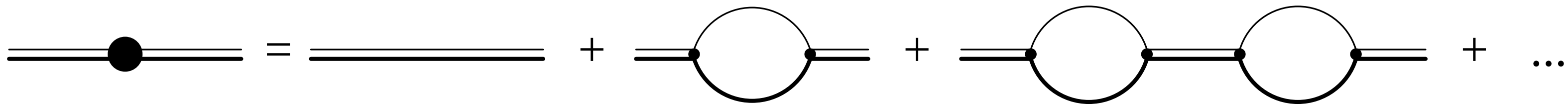
$$D_s(p_0,\vec{p}) \quad = \quad \frac{4\pi}{m_\Lambda y_s^2}\frac{1}{\frac{1}{a_{\Lambda\Lambda}}+\frac{1}{2}r_{\Lambda\Lambda}\left(-m_\Lambda p_0+\frac{1}{4}\vec{p}^2-i\epsilon\right)-\sqrt{-m_\Lambda p_0+\frac{1}{4}\vec{p}^2-i\epsilon}},$$

and

$$y_s=-\frac{2}{m_\Lambda}\sqrt{\frac{2\pi}{r_{\Lambda\Lambda}}}\,.$$

Two-body part: Λd *in* ${}^3_\Lambda$*H channel*

- Dressed ${}^3_\Lambda$H propagator



- Renormalized dressed ${}^3_\Lambda$H propagator

$$D_t(p_0, \vec{p}) = \frac{2\pi}{\mu_{\Lambda d} y_t^2} \frac{1}{\frac{1}{a_{\Lambda d}} + \frac{1}{2} r_{\Lambda d} \left[-2\mu_{\Lambda d} \left(p_0 - \frac{1}{2(m_\Lambda + m_d)} \vec{p}^2 \right) \right] - \sqrt{-2\mu_{\Lambda d} \left(p_0 - \frac{1}{2(m_\Lambda + m_d)} \vec{p}^2 \right)}},$$

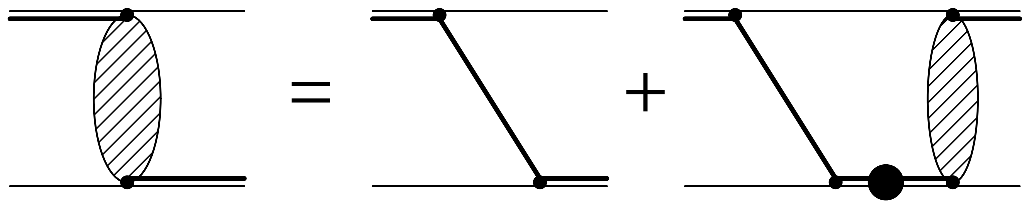
with

$$y_t = -\frac{1}{\mu_{\Lambda d}} \sqrt{\frac{2\pi}{r_{\Lambda d}}},$$

and $\gamma_{\Lambda d} \simeq \frac{1}{a_{\Lambda d}} + \frac{1}{2} r_{\Lambda d} \gamma_{\Lambda d}^2$.

Three-body part: $S = 0$ *channel*

- S-wave Λ-${}^3_\Lambda$H scattering for $S = 0$ channel



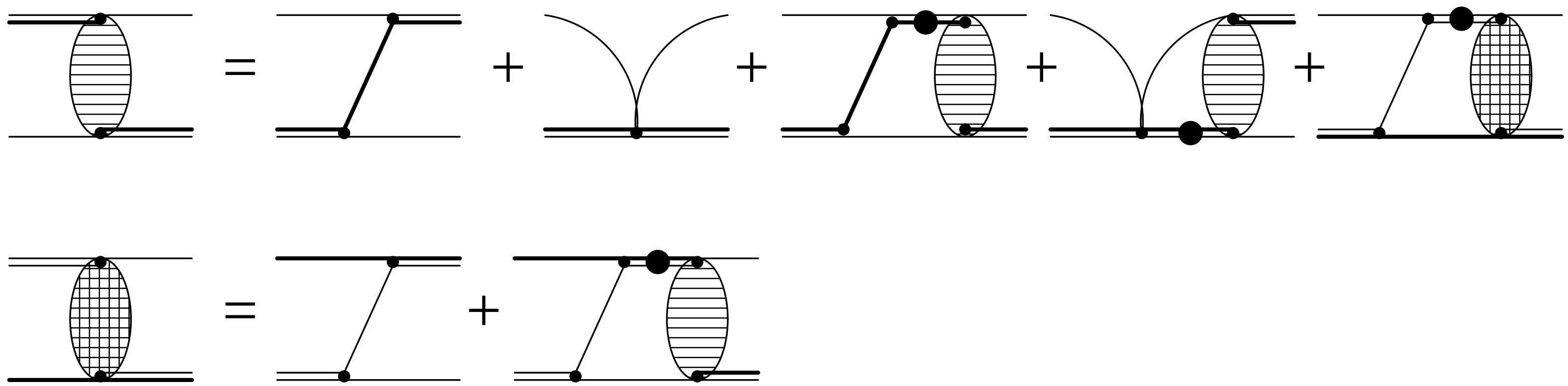
- Integral equation

$$
\begin{aligned}
t(p,k;E) &= -3K_{(a)}(p,k;E) \\
&\quad + \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 3K_{(a)}(p,l;E) D_t\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) t(l,k;E)\,,
\end{aligned}
$$

with

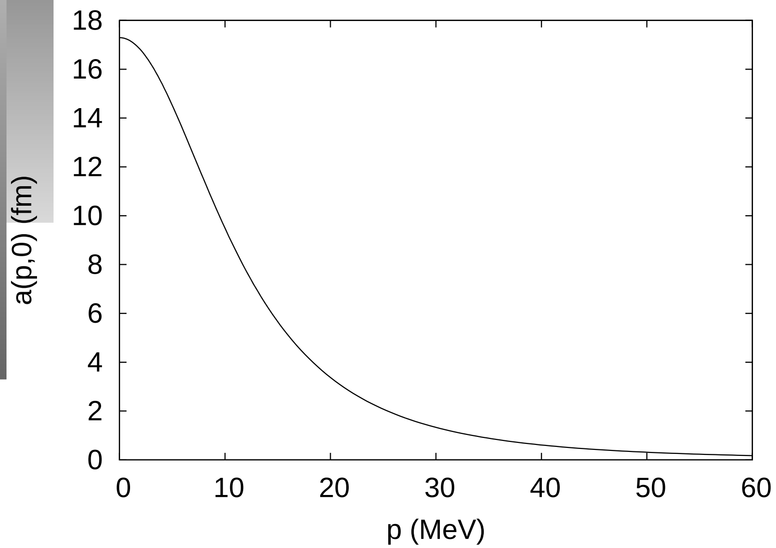
$$
K_{(a)}(p,l;E) = \frac{1}{3} m_d y_t^2 \frac{1}{2pl} \ln\left(\frac{\frac{m_d}{2\mu_{\Lambda d}}(p^2+l^2) + pl - m_d E}{\frac{m_d}{2\mu_{\Lambda d}}(p^2+l^2) - pl - m_d E}\right)\,,
$$

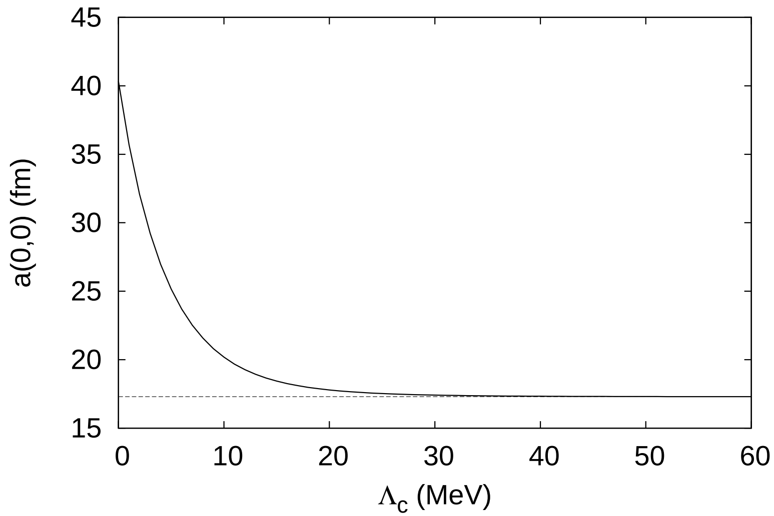
Three-body part: $S = 1$ ***channel***

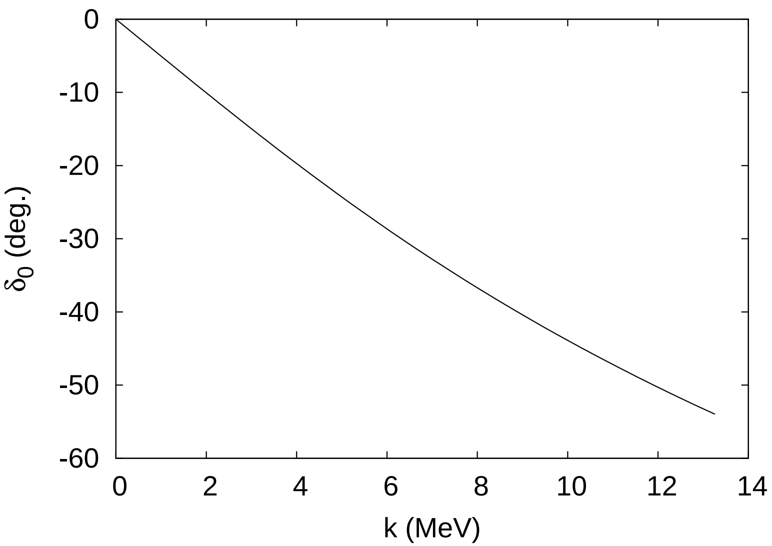


$$
\begin{aligned}
a(p,k;E) &= K_{(a)}(p,k;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 \left[K_{(a)}(p,l;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2}\right] D_t^{LO}\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b1)}(p,l;E) D_s^{LO}\left(E - \frac{1}{2m_d}l^2, \vec{l}\right) b(l,k;E)\,, \\
b(p,k;E) &= K_{(b2)}(p,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b2)}(p,l;E) D_t^{LO}\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E)\,,
\end{aligned}
$$

Numerical results: $S = 0$ ***channel***





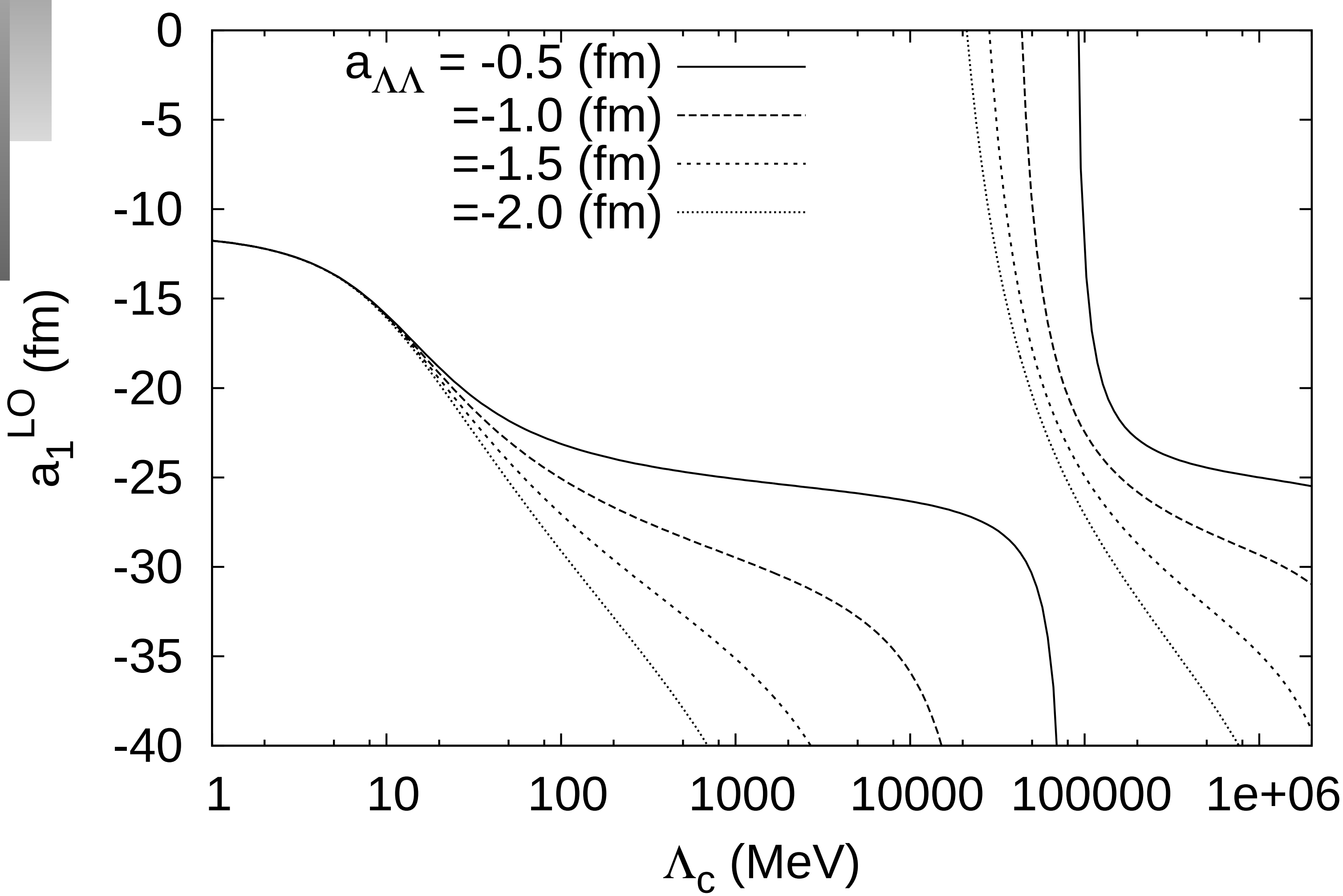


We chose $\Lambda_c = 170$ MeV,
and the input parameters are $\gamma_{\Lambda d} \simeq 13.5 \pm 2.6$ MeV, $r_{\Lambda d} = 2.3 \pm 0.3$ fm.

We have $a_0^{th} = 17.3 \pm 2.9$ fm. ($r_0 \simeq 4$ fm)

Numerical results: $S = 1$ ***channel***

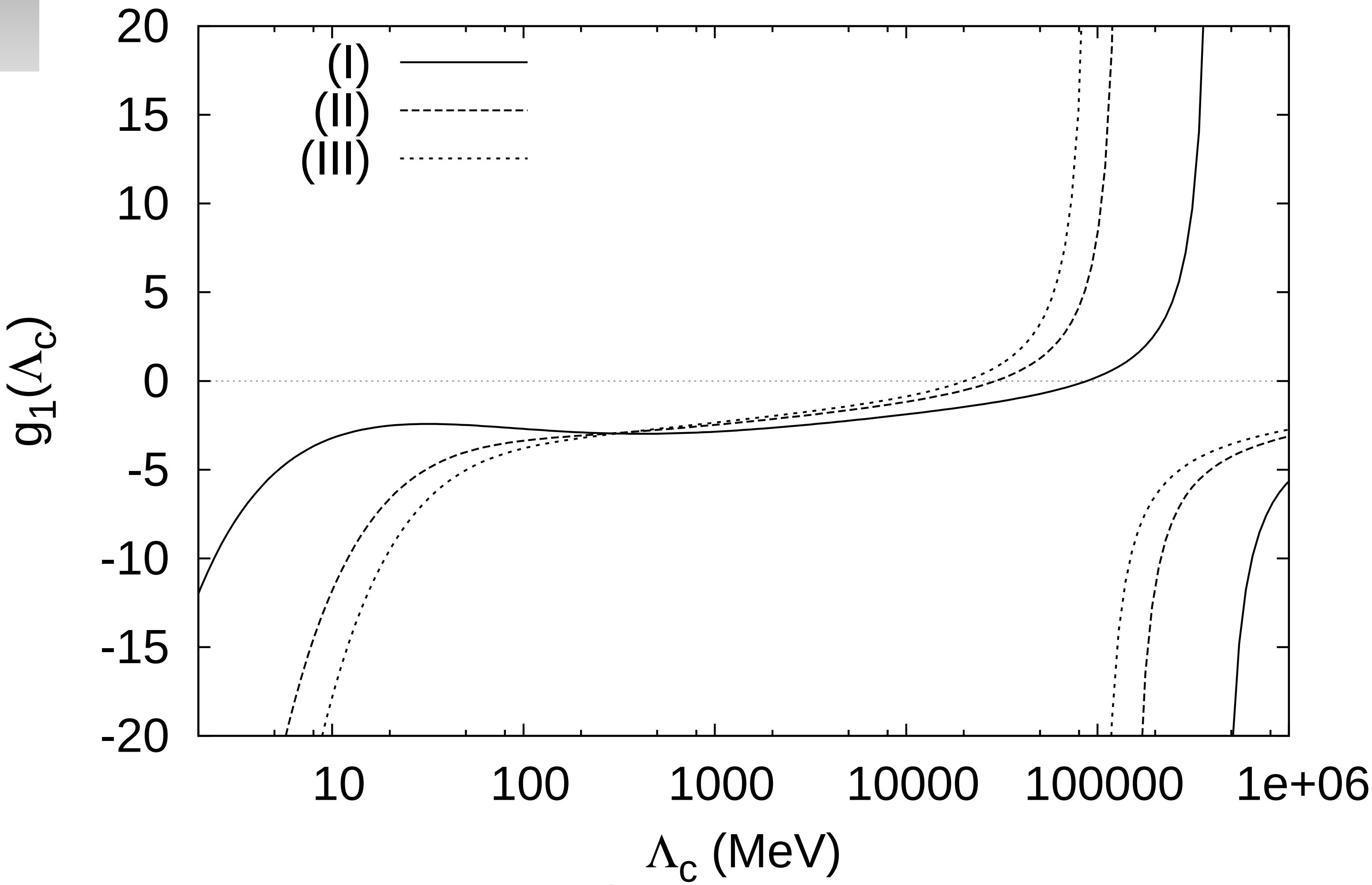
- Without $g_1(\Lambda_c)$



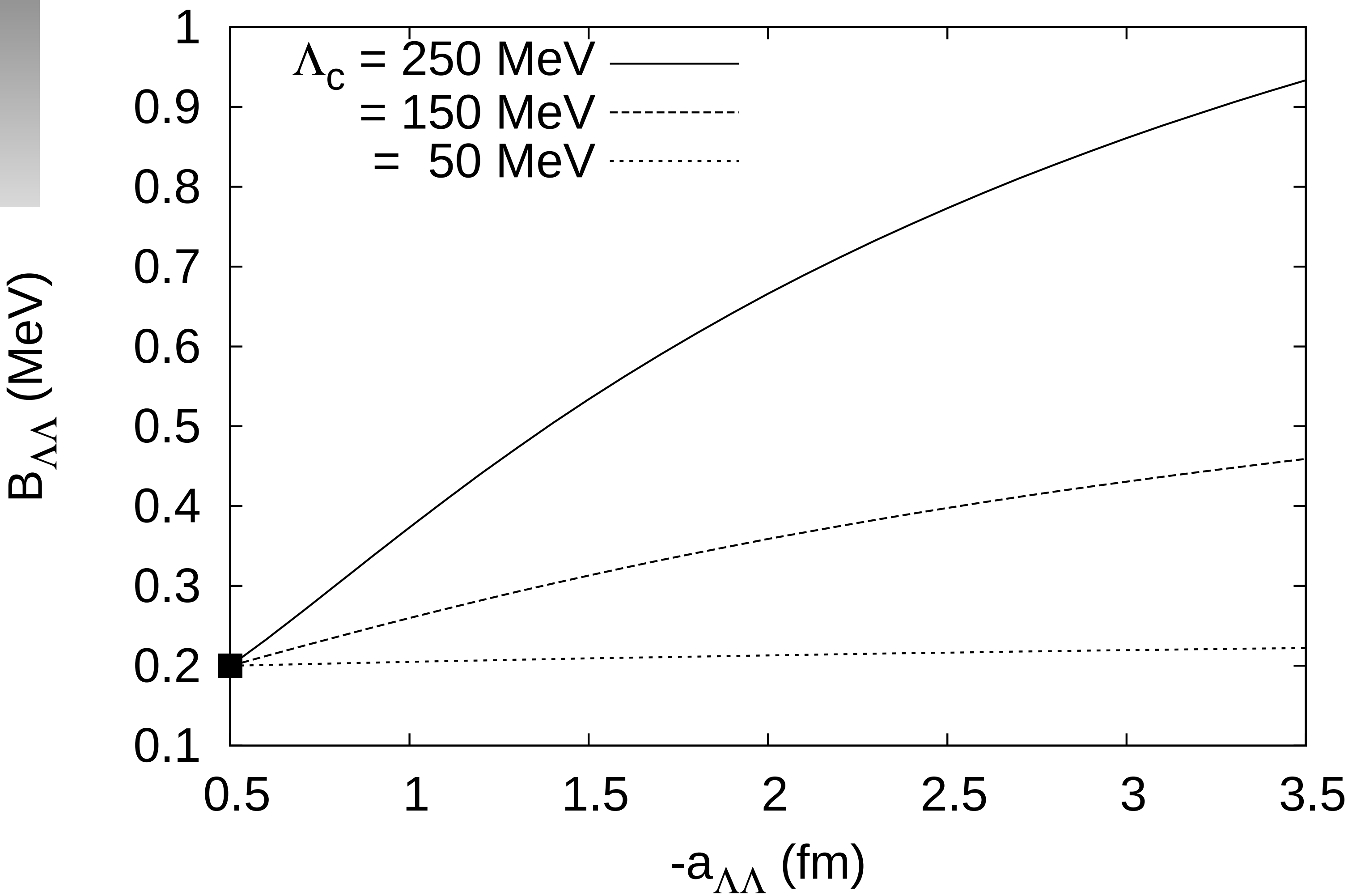
Numerical results: $S = 1$ ***channel***

- With $g_1(\Lambda_c)$,

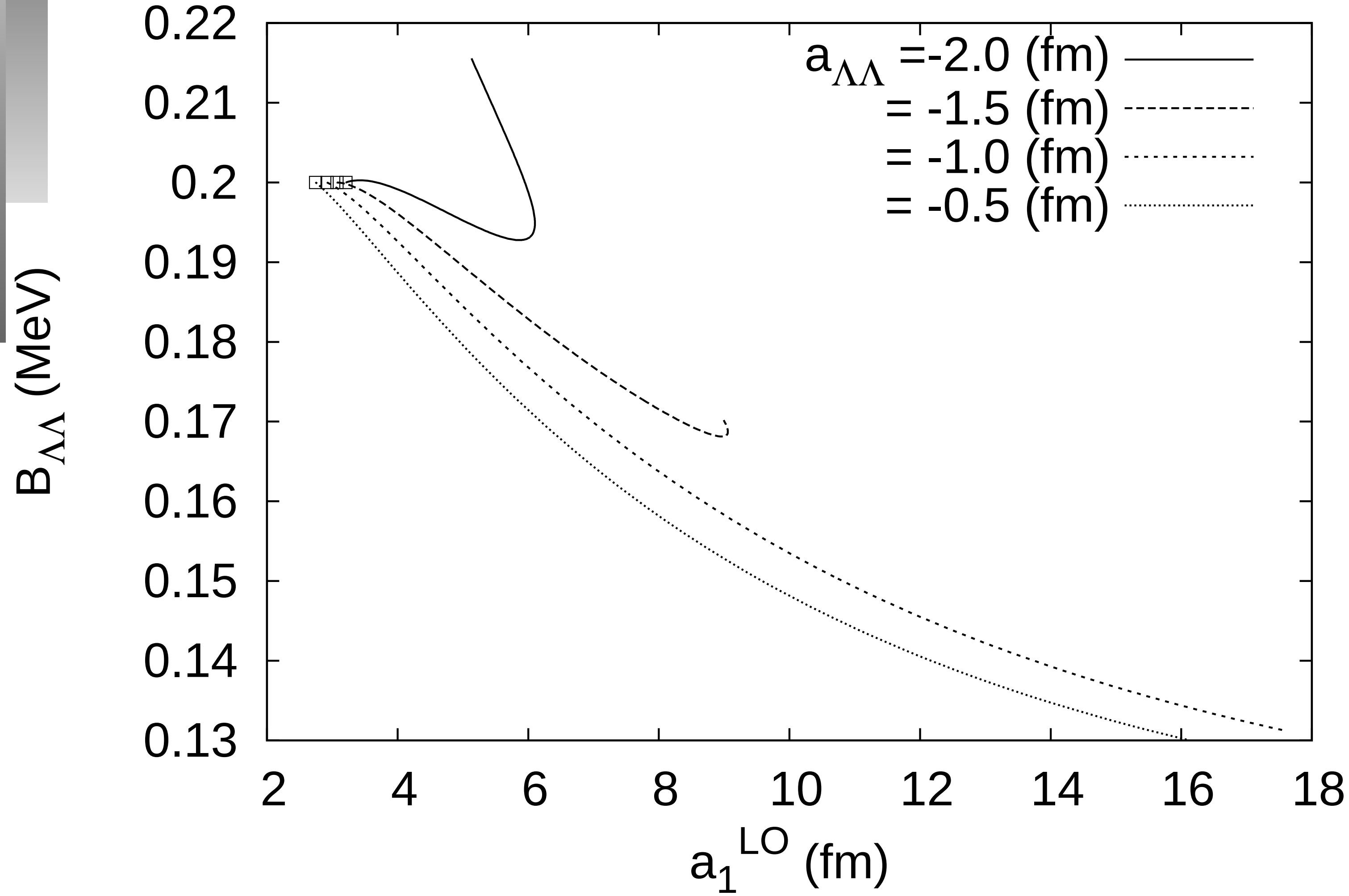
$(B_{\Lambda\Lambda}, a_{\Lambda\Lambda})$ = (I) (0.2 MeV, -0.5 fm), (II) (0.6, -1.5), (III) (1.0, -2.5).



Numerical results: $S = 1$ ***channel***



Numerical results: $S = 1$ *channel*



Results and discussion

- We studied ${}^{4}_{\Lambda\Lambda}$H in Halo EFT, as $d\Lambda\Lambda$ system, at LO.
- We find that the S-wave scattering of Λ and hypertriton in $S = 0$ channel is well described by the effective range parameters in the hypertriton channel.
- On the other hand, in $S = 1$ channel, the results are sensitive to the cutoff, and the three-body contact interaction is needed to introduce. The LO result is described by the four parameters; $\gamma_{\Lambda d}$, $a_{\Lambda\Lambda}$, $g_1(\Lambda_c)$, Λ_c.
- Because there is no experimental data for ${}^{4}_{\Lambda\Lambda}$H, we fixed $g_1(\Lambda_c)$ phenomenologically, and study the sensitivities of $B_{\Lambda\Lambda}$ and a_1^{LO} to $a_{\Lambda\Lambda}$ and Λ_c.