Systematic measurement of monopole transition strengths at low excitation energies using α inelastic scattering

KADOYA Tomosuke
YOKOTA Naoki
KAWABATA Takahiro
Department of Physics, Kyoto University
Large E0 strength could be a signature of spatially developed α cluster states.


$0^+_{2}$ state in $^{12}$C: $B(E0; IS) = 121 \pm 9$ fm$^4$

Single Particle Unit: $B(E0; IS)_{s.p.} \sim 40$ fm$^4$

- SM-like compact GS w.f. is equivalent to the CM w.f. at SU(3) limit.
- GS contains CM-like component due to possible alpha correlation.

- SM-like Compact GS.
- Developed Cluster State

Monopole operators excite inter-cluster relative motion.


E0 strength is a key observable to examine α cluster structure.
Inelastic Alpha Scattering

Inelastic $\alpha$ scattering is a good probe for nuclear excitation strengths.

- Simple reaction mechanism
  - Good linearity between $d\sigma/d\Omega$ and $B(\hat{o})$.
  \[
  \frac{d\sigma}{d\Omega}(\Delta J^\pi) \approx KN|J(q)|^2 B(\hat{O})
  \]
  - Folding model gives a reasonable description of $d\sigma/d\Omega$.

- Selectivity for the $\Delta T = 0$ and natural-parity transitions.

- Multiple decomposition analysis is useful to separate $\Delta J^\pi$.

\[
\frac{d\sigma}{d\Omega}^{\text{exp}} = \sum_{\Delta J^\pi} A(\Delta J^\pi) \frac{d\sigma}{d\Omega}(\Delta J^\pi)^{\text{calc}}
\]

We are measuring inelastic $\alpha$ scattering to extract IS E0 strengths and to search for the $\alpha$ condensed states.
Missing Monopole Strength

Monopole strengths for the Hoyle state from hadron scattering is 50% smaller than that from electron scattering.

\[ \frac{\sqrt{4\pi} B(CO,q)}{q^2} = \frac{ME}{6} \left(1 - \frac{q^2}{20} R_{1f}^2 + \frac{q^4}{840} \langle r^6 \rangle_{1f} \right) \]

\[ ^{12}\text{C}(e,e') \]

\[ ME = 5.37 (22) \text{ efm}^2 \]

15.0(13)% of EWSR

EWSR fraction extracted from \((e,e')\) seems to be reliable.

Why is the monopole strength in \((\alpha,\alpha')\) missing?

\[ ^{12}\text{C}(\alpha,\alpha') \]

EWSR 7.6(9)% of EWSR

<table>
<thead>
<tr>
<th>ME (efm²)</th>
<th>FMD</th>
<th>BEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.62</td>
<td>6.53</td>
<td>6.45</td>
</tr>
</tbody>
</table>


EWSR fraction extracted from \((e,e')\) seems to be reliable.

Why is the monopole strength in \((\alpha,\alpha')\) missing?
Double Folding Model Analysis

Microscopic analysis was done by D. T. Khoa and D. C. Cuong.


- CDJLM (modified version of CDM3Y)
- $3\alpha$ RGM or Breathing Mode (BM) transition density.
- DWBA or CC ($0^+_1\rightarrow 2^+_1\rightarrow 0^+_2\rightarrow 0^+_1$)

Both DWBA and CC systematically overestimate at all energies.
$3\alpha$ RGM and BM give similar results.
Consistent to the previous results.

$N_I$ for the $\alpha + ^{12}\text{C}(0^+_2)$ channel was adjusted to obtain a reasonable CC result ($N_I \sim 2.5—3.4$).

Strong absorption due to the dilute and weakly bound natures of the Hoyle state ???
Missing monopole strengths might be evidence of the alpha condensed states ???
Experiment

Experiment was performed at RCNP, Osaka University.

Background-free measurement at extremely forward angles

(\alpha, \alpha') @ 130 MeV
\theta_{lab} = 0^\circ \sim 19^\circ
^{12}\text{C}, \ ^{16}\text{O}, \ ^{24}\text{Mg}, \ ^{28}\text{Si}, \ ^{40}\text{Ca}, \ ^{58}\text{Ni}
Single Folding Model Analysis

Experimental data at RCNP is analyzed by single folding model.

Single folding by phenomenological $\alpha$N interaction.

\[ U_0(r) = \int d\bar{r}' \rho_0(r') V(|\bar{r} - \bar{r}'|, \rho_0(r')) \]

- GS densities are taken from electron scattering assuming
  \[ \rho_{0n}(r) = \rho_{0p} (r'), \quad r' = (Z/N)^{1/3} r \]

- Two choices of $\alpha$N interaction to fit $d\sigma/d\Omega$
  \[ V(|\bar{r} - \bar{r}'|, \rho_0(r')) = -V \left( 1 + \beta_V \rho_0(r')^{2/3} \right) \exp\left(-|\bar{r} - \bar{r}'|^2/\alpha_V \right) \]
  \[ -iW \left( 1 + \beta_W \rho_0(r')^{2/3} \right) \exp\left(-|\bar{r} - \bar{r}'|^2/\alpha_W \right) \]

Density-independent (DI, $\beta_V = \beta_W = 0$)
Density-dependent (DD, $\beta_V = \beta_W = -1.9$)

Due to the lack of backward data, there are so-called “deep-shallow” ambiguities.

Backward data for $^{12}$C, $^{24}$Mg, and $^{28}$Si are extrapolated by using the existing 140-MeV data.

→ Deep-shallow ambiguities are solved.
Results for $2^+$ transitions

$^{12}$C($\alpha,\alpha'$) 4.44 MeV ($2^+_1$)

$^{16}$O($\alpha,\alpha'$) 6.92 MeV ($2^+_1$)

$^{24}$Mg($\alpha,\alpha'$) 1.37 MeV ($2^+_1$)

$^{28}$Si($\alpha,\alpha'$) 1.78 MeV ($2^+_1$)

$^{40}$Ca($\alpha,\alpha'$) 3.90 MeV ($2^+_1$)

$^{58}$Ni($\alpha,\alpha'$) 1.45 MeV ($2^+_1$)
Results for $0^+$ transitions

- $^{12}\text{C}(\alpha,\alpha')$
  - 7.65 MeV ($0_2^+$)
  - $\beta$:DD ref
  - $\beta$:DI ref

- $^{16}\text{O}(\alpha,\alpha')$
  - 6.05 MeV ($0_2^+$)
  - $\beta$:DD
  - $\beta$:DI

- $^{16}\text{O}(\alpha,\alpha')$
  - 12.05 MeV ($0_3^+$)
  - $\beta$:DD
  - $\beta$:DI

- $^{24}\text{Mg}(\alpha,\alpha')$
  - 6.43 MeV ($0_2^+$)
  - $\beta$:DD ref
  - $\beta$:DI ref

- $^{28}\text{Si}(\alpha,\alpha')$
  - 4.98 MeV ($0_2^+$)
  - $\beta$:DD ref
  - $\beta$:DI ref

- $^{40}\text{Ca}(\alpha,\alpha')$
  - 3.35 MeV ($0_2^+$)
  - $\beta$:DD
  - $\beta$:DI
Transition strengths deduced from electromagnetic transitions.

Normalization factors for $B(E\lambda)$ should be unity because the transition densities used in the single folding calculation are taken from electron scattering.

Calculated cross sections are normalized to fit the experimental data.

$B(E\lambda;IS) = R \cdot B(E\lambda;IS)_\text{ele}$

$R$ should be unity because the transition densities used in the single folding calculation are taken from electron scattering.
Normalization factors for $2^+$ transitions

- $R$ is close to unity for all transitions.
- DD and DI give similar results.

Error bars:
Thick from accuracy of $B(E\lambda)_{ele}$
Thin from the DWBA analysis
Normalization factors for $0^+$ transitions

- $R$ is systematically much smaller than unity.
- Result with DI is relatively better than that with DD.

Error bars:
Thick from accuracy of $B(E\lambda)_{\text{ele}}$
Thin from the DWBA analysis
Normalization factors for $3^-$ transitions

- $R$ is close to unity for all transitions.
- DD and DI give similar results.

Error bars:
- Thick from accuracy of $B(E\lambda)_{\text{ele}}$
- Thin from the DWBA analysis
Transition pot. for $0^+$ and $2^+$ transitions

- Too strong density dependence in the inner region of the $0^+$ transition density.
- Density dependence of the effective interaction should be improved.

\[
\begin{align*}
V\left(\vec{r} - \vec{r}', \rho_0(r')\right) &= -V\left(1 + \beta \rho(r)^{2/3}\right) \exp\left(-|\vec{r} - \vec{r}'|^2 / \alpha_v\right) \\
&\quad - iW\left(1 + \beta \rho(r)^{2/3}\right) \exp\left(-|\vec{r} - \vec{r}'|^2 / \alpha_w\right)
\end{align*}
\]
Uncertainties in DWBA calculation

I. Distorting potentials
II. Transition densities
   I. Macroscopic models
   II. Microscopic models
III. Coupled Channel effects
   I. Comparison between DWBA and coupled channel calculations
      →Examined for the $2_1^+$ (4.44 MeV), $0_2^+$ (7.65 MeV), and $3_1^-$ (9.64 MeV) in $^{12}$C
Uncertainties in Distorting Potential

- Calculated cross sections decrease and R is slightly improved for the DI interaction. Uncertainties in the distorting potential should be solved.
  → Very recently, a new measurement of $\alpha$ elastic scattering was done.
- DD calculation does not change.
  → DD calculation gives better description at backward angles.
Uncertainties in transition densities give no significant changes.
Coupled Channel Effects

Strong coupling between the $0_2^+ - 2_2^+$ states.

5 states calculation: $0_1^+, 2_1^+, 4_1^+, 0_2^+, \text{ and } 3_1^+$
6 states calculation: $0_1^+, 2_1^+, 4_1^+, 0_2^+, 3_1^+, \text{ and } 2_2^+ (10.3 \text{ MeV})$

☑ The 5 state calculation gives no significant change.
☑ Inclusion of the $2_2^+$ state decreases the cross section for the $0_2^+$ state only, the 6 state calculation with the DI interaction gives reasonable result.
Summary

- Excitation strengths for the low-lying states in $^{12}$C, $^{16}$O, $^{24}$Mg, $^{28}$Si, $^{40}$Ca, and $^{58}$Ni are systematically studied by measuring alpha inelastic scattering at 130 MeV.
- DWBA analysis gives reasonable results for B(E2; IS), but systematically underestimates B(E0; IS).
  - DI interaction gives better description for B(E0; IS).
- “Missing monopole strength” is not special for the Hoyle state. It is a universal problem in the monopole transitions.
  - Strong coupling between the $0^+_{2}$ and $2^+_{2}$ states partially solve the problem, but B(E0; IS) is still overestimated.
  - Density dependence in the effective interaction might be a key to solve the problem.