Many-body resonances and continuum states in He isotopes and their mirror nuclei

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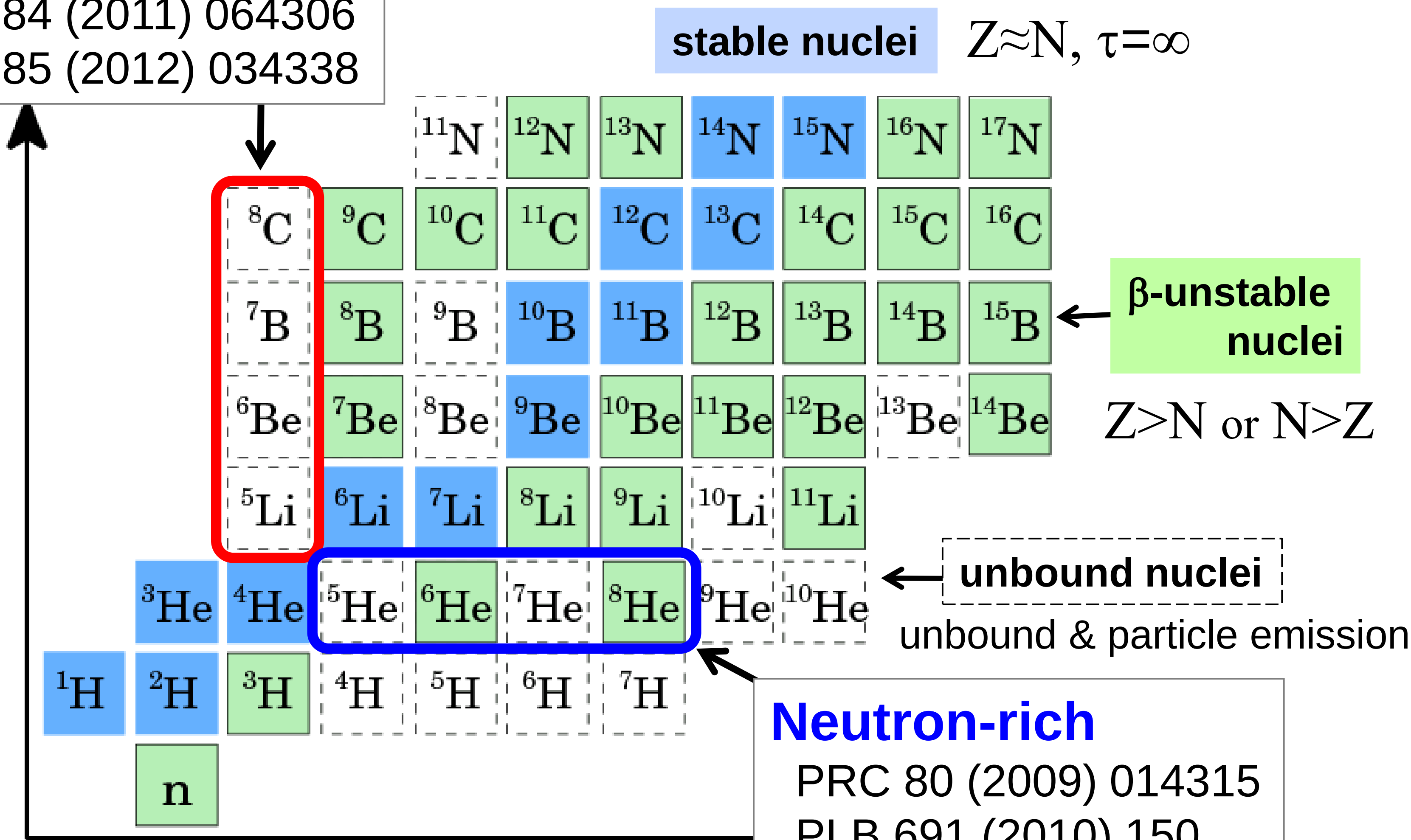
"Multi-particle resonances and astrophysical reaction problems in few-body systems" 2013.10

Outline

- Structure of Light Unstable Nuclei
  - He isotopes (neutron-rich)
  - mirror nuclei (proton-rich)
- Cluster Orbital Shell Model (**COSM**)
  - core nuclei + valence protons / neutrons
- Complex Scaling Method (**CSM**)
  - many-body resonances & continuum states
  - continuum level density, Green's function
  - strength functions, breakup reactions

Nuclear Chart

Proton-rich
PRC 84 (2011) 064306
PRC 85 (2012) 034338

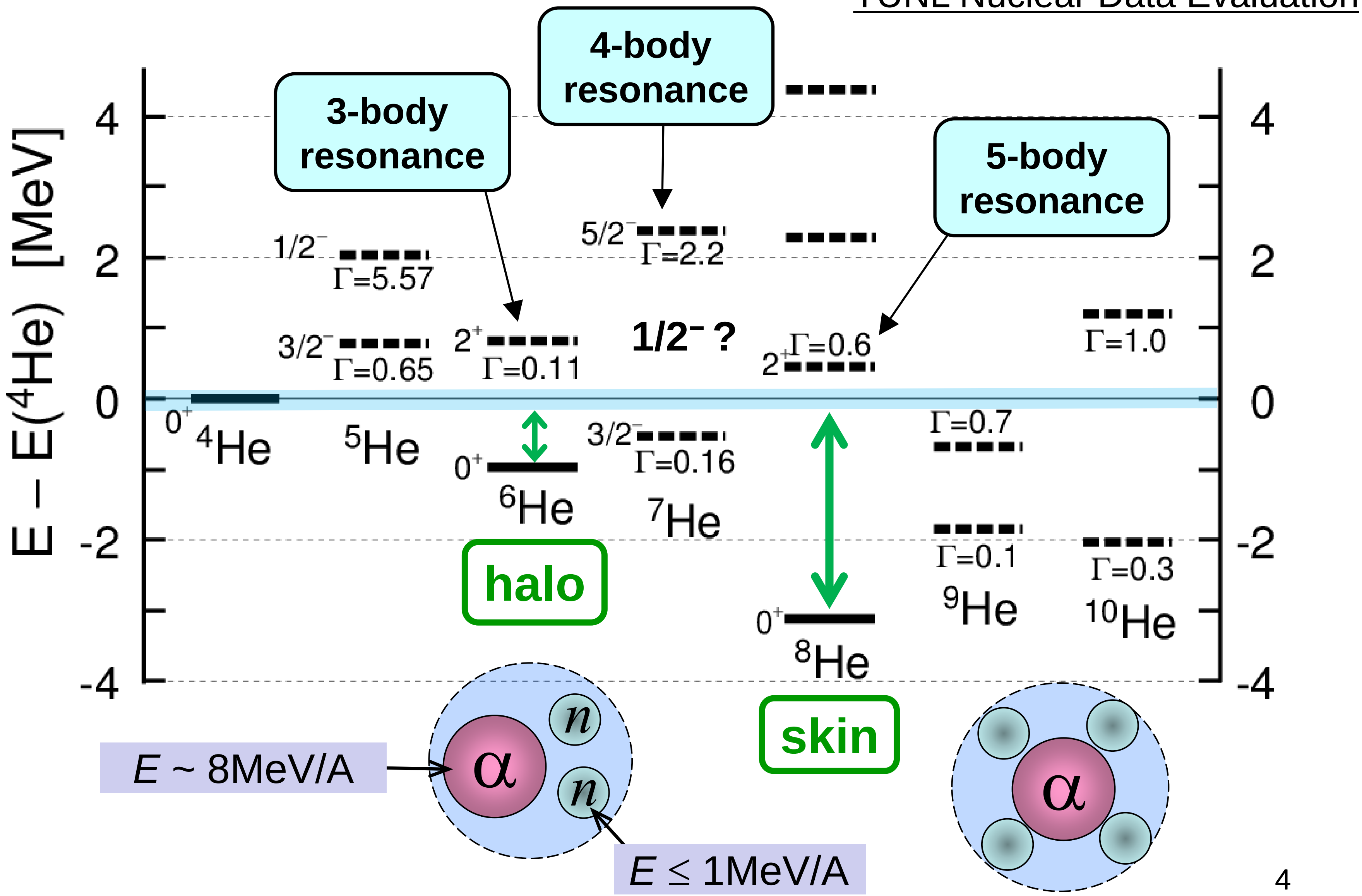


Neutron-rich
PRC 80 (2009) 014315
PLB 691 (2010) 150

Mirror symmetry between **proton-rich** & **neuron-rich**
(with Coulomb)

Neutron-rich He isotopes : experiment

TUNL Nuclear Data Evaluation



Proton-rich ^{7}B & ^{8}C

- Proton-rich unbound nucleus
  - ^{4}He-^{5}Li-^{6}Be-**^{7}B**-**^{8}C**, decay into $\alpha+p+p+p(+p)$ systems
- Experiments
  - Only the ground states are observed.
    - **^{7}B:** L. R. McGrath & J. Cerny, Phys. Rev. Lett. **19**, 1442 (1967).
    - **^{8}C:** R. G. H. Robertson, S. Martin, W. R. Falk, D. Ingham, A. Djaloeis, Phys. Rev. Lett. **32**, 1207 (1974). **^{8}C & ^{20}Mg**
  - R. J. Charity et al., Phys. Rev. C 84, 014320 (2011).
    ^{9}C beam: ^{7}B, ^{8}B*, ^{8}C, ... @MSU
- **NO theory** describes resonances of ^{7}B & ^{8}C, so far.
- Mirror symmetry of ***p***-rich & ***n***-rich unstable nuclei
  - ^{7}B-^{7}He, ^{8}C-^{8}He : energies levels, configurations

Method

- Cluster Orbital Shell Model (**COSM**)
  - Include open channel effects.

    ^{8}He : ^{7}He+n, ^{6}He+n+n, ^{5}He+n+n+n, ...

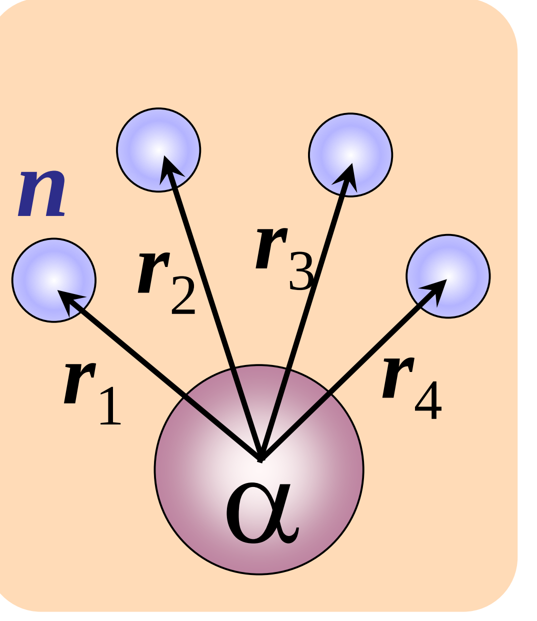
- Complex Scaling Method

$$\mathbf{r} \to \mathbf{r}e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

  - Obtain resonance w.f. with correct boundary condition as **Gamow states**

    $E=E_r-i\Gamma/2$

  - Give the continuum level density, ΔE
    - resonance+continuum, Green's function
    - strength function, Lippmann-Schwinger Eq., T-matrix



A.T. Kruppa, R.G. Lovas, B. Gyarmati, PRC37(1988) 383 (**^{8}Be as 2α**)
S. Aoyama, TM, K. Kato, K. Ikeda, PTP116(2006) 1 (**CSM review**)
C. Kurokawa , K. Kato, PRC71 (2005) 021301 (**^{12}C as 3α**)

Kikuchi (**LS eq.**)
Matsumoto (**CDCC**)

Cluster Orbital Shell Model (n-rich)

- System is obtained based on RGM equation

$$H({}^A\mathrm{He}) = H({}^4\mathrm{He}) + H_{\mathrm{rel}}(N_V n) \qquad \Phi({}^A\mathrm{He}) = \mathcal{A}\left\{\psi({}^4\mathrm{He}) \cdot \sum_{i=1}^{N} C_i \cdot \chi_i(N_V n)\right\}$$

valence neutron number

i : configuration

$\psi({}^4\mathrm{He})$: $(0\mathrm{s})^4$ ← No explicit tensor correlation

$\chi_i(N_V n) = \mathcal{A}\{\varphi_{i1}\varphi_{i2}\varphi_{i3}\cdots\}$ $\quad \varphi_i : L \le 2$ few-body method with Gaussian expansion

- Orthogonarity Condition Model (OCM) is applied.

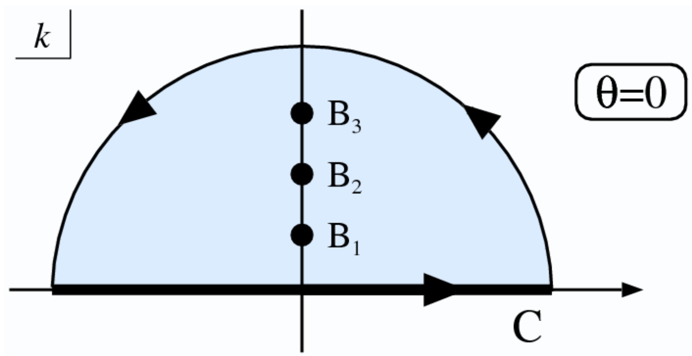
$$\sum_{i=1}^{N} \left\langle \chi_j \left| \sum_{k=1}^{N_V} \left(T_k + V_k^{cn}\right) + \sum_{k<l}^{N_V} \left(V_{kl}^{nn} + \frac{\vec{p}_i \cdot \vec{p}_j}{A_c m} \right) \right| \chi_i \right\rangle C_i = (E - E_{4\mathrm{He}})\, C_j$$

$\langle \varphi_i | \phi_{\mathrm{PF}} \rangle = 0$ Remove Pauli Forbidden states (PF)

Y. Suzuki, K. Ikeda, PRC38(1988)410, H. Masui, K. Kato, K. Ikeda, PRC73(2006)034318

Complex Scaling for 2-body case

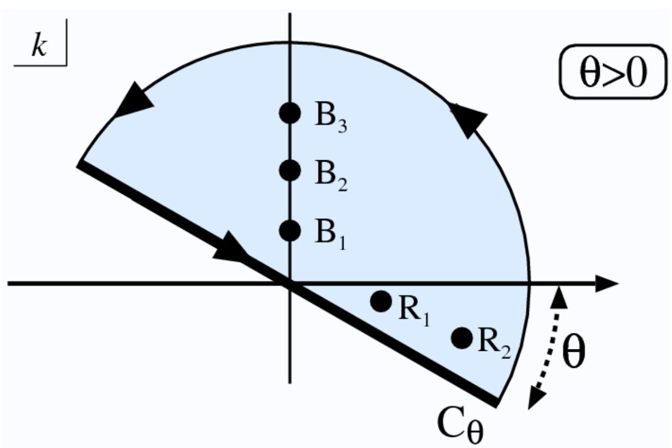
$$U(\theta)\ :\ \mathbf{r}\to\mathbf{r}\cdot\exp(i\theta)\ ,\quad \mathbf{k}\to\mathbf{k}\cdot\exp(-i\theta)\ ,\quad \theta\in\mathbb{R}$$



Completeness relation

$$1=\sum_B|\varphi_B\rangle\langle\tilde{\varphi}_B|+\int_C dk|\varphi_k\rangle\langle\tilde{\varphi}_k|$$

T. Berggren, NPA109('68)265.



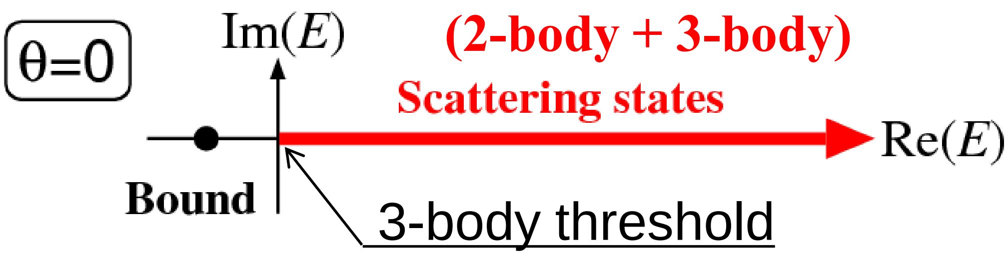
$$1=\sum_B|\varphi_B\rangle\langle\tilde{\varphi}_B| + \sum_R|\varphi_R\rangle\langle\tilde{\varphi}_R| + \int_{C_\theta} dk_\theta\left|\varphi_{k_\theta}\right\rangle\left\langle\tilde{\varphi}_{k_\theta}\right|$$

J.Aguilar and J.M.Combes, Commun. Math. Phys.,22('71)269.
E.Balslev and J.M.Combes, Commun. Math. Phys.,22('71)280.

B.G.Giraud, K.Kato, A.Ohnishi
J. Phys. A **37** ('04)11575

Complex Scaling for 3-body case

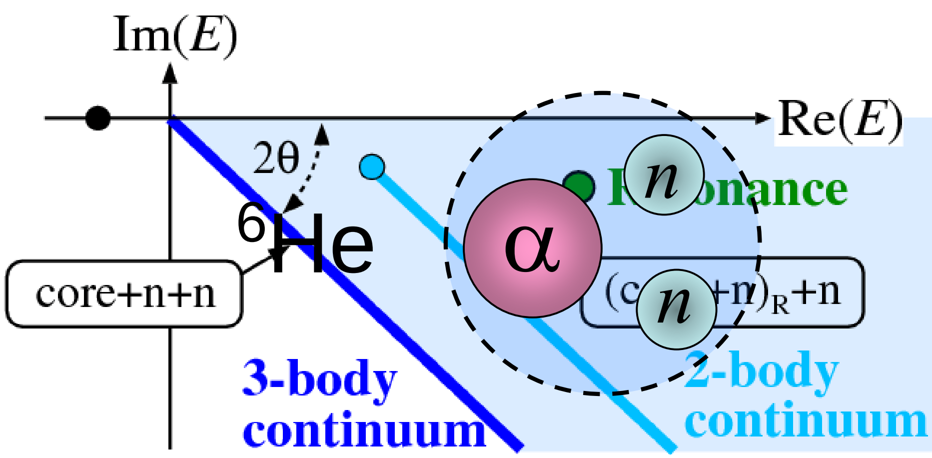
$$U(\theta)\ :\ \mathbf{r} \to \mathbf{r}\cdot\exp(i\theta)\ ,\qquad \mathbf{k} \to \mathbf{k}\cdot\exp(-i\theta)\ ,\qquad \theta \in \mathbb{R}$$



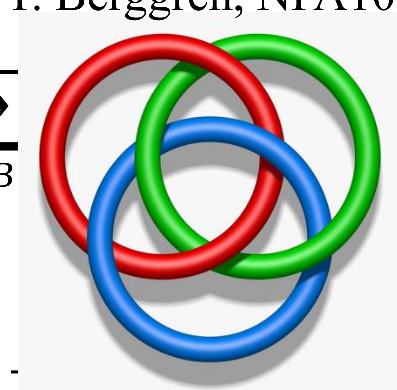
Completeness relation

$$1 = \sum_B |\varphi_B\rangle\langle\tilde{\varphi}_B| + \int_C dE\,|\varphi_E\rangle\langle\tilde{\varphi}_E|$$

T. Berggren, NPA109('68)265.



$$1 = \sum_B \ldots + \ldots \varphi_{E_\theta}\rangle\langle\tilde{\varphi}_{E_\theta}|$$

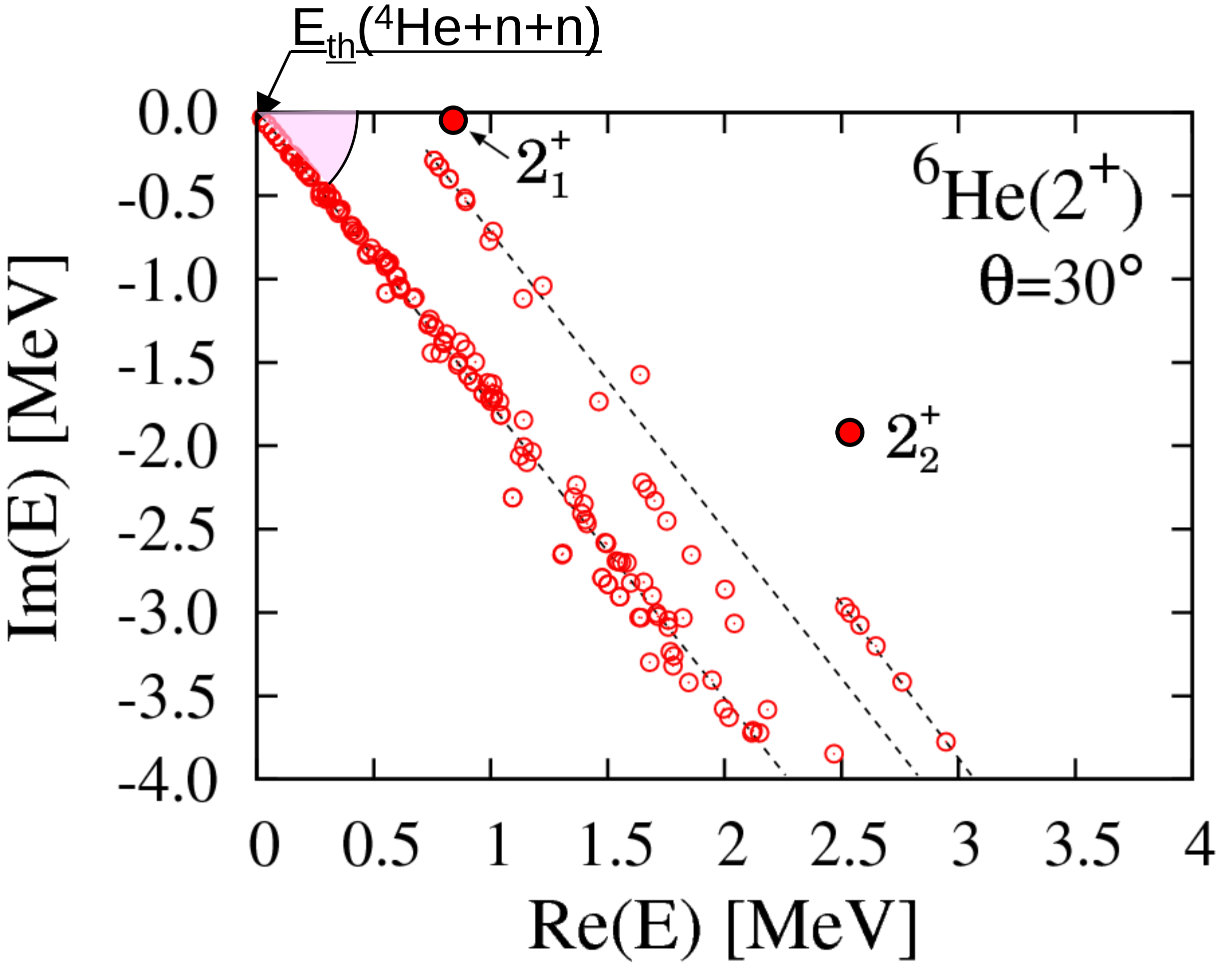


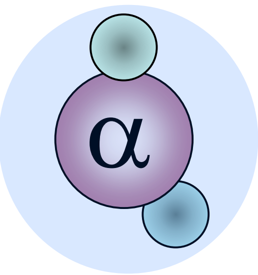
Borromean rings

Halo nuclei : "core+n+n" with Borromean condition

$^{6}He = {}^{4}He+n+n$, $^{11}Li = {}^{9}Li+n+n$, $^{14}Be = {}^{12}Be+n+n$, …

Spectrum of ^{6}He with ^{4}He+n+n model





^{6}He$^{(*)}$
^{5}He+n
^{4}He+n+n

Continuum states are discretized using **<u>Gaussian basis functions</u>** (Kamimura)

$$\phi_\ell(\mathbf{r}) = \sum_n C_n \cdot r^\ell e^{-(r/b_n)^2} Y_\ell(\hat{\mathbf{r}})$$

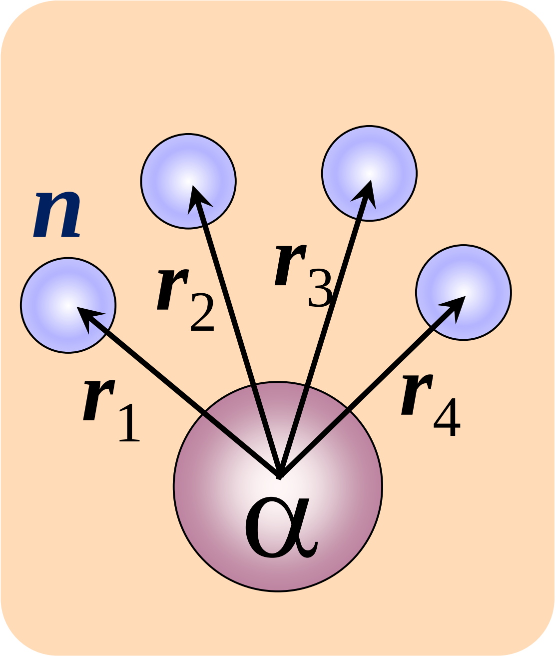
A. Csoto, PRC49 ('94) 3035,
S. Aoyama et al. PTP94('95)343, T. Myo et al. PRC63('01)054313

Hamiltonian

- $V_{\alpha\text{-n}}$: microscopic KKNN potential
  - s,p,d,f-waves of α-n scattering
- V_{nn} : Minnesota potential with slightly strengthened (+ Coulomb for *p*-rich nuclei)

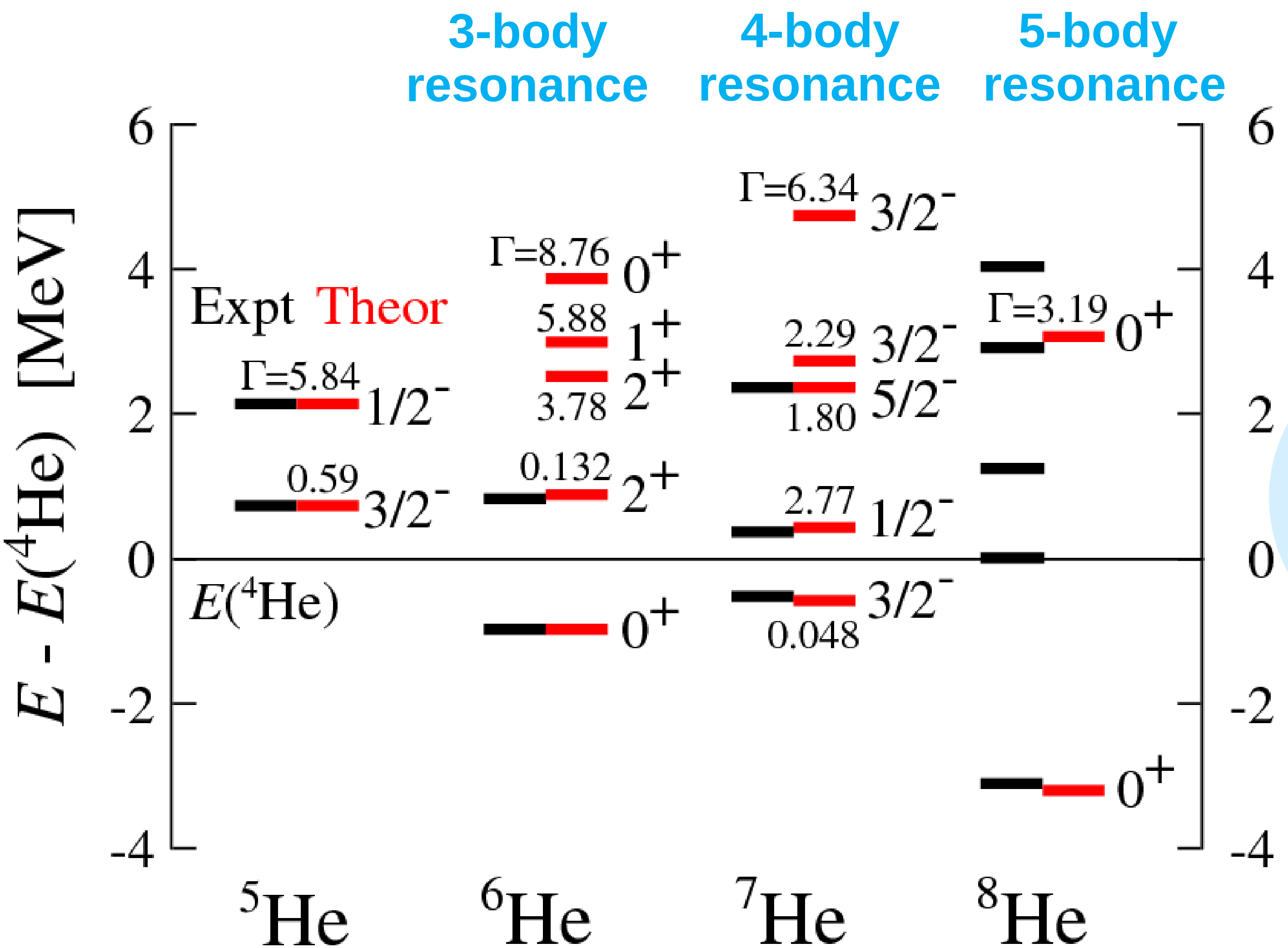
Fit energy of $^6He(0^+)$

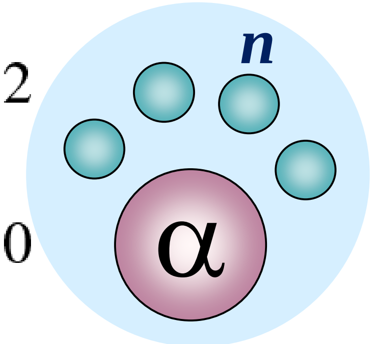
COSM



A. Csoto, PRC48(1993)165.
K. Arai, Y. Suzuki and R.G. Lovas, PRC59(1999)1432.
TM, S. Aoyama, K. Kato, K. Ikeda, PRC63(2001)054313.
TM et al. PTP113(2005)763.

He isotopes : Expt vs. Complex Scaling

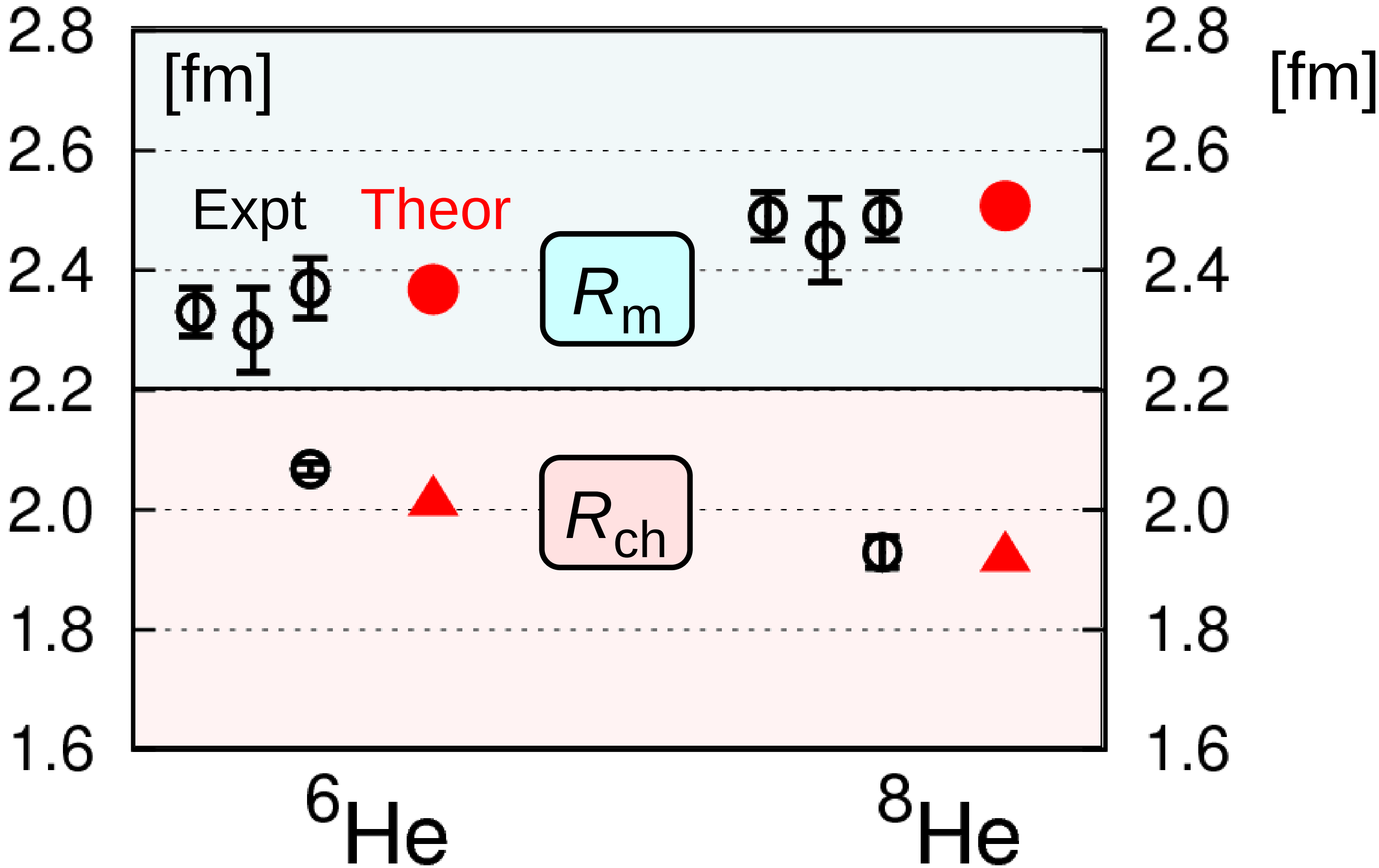




TM, K.Kato, K.Ikeda PRC76('07)054309
TM, R.Ando, K.Kato PRC80('09)014315

TM, R.Ando, K.Kato, PLB691('10)150
TUNL Nuclear Data Evaluation

Matter & Charge radii of $^{6,8}He$



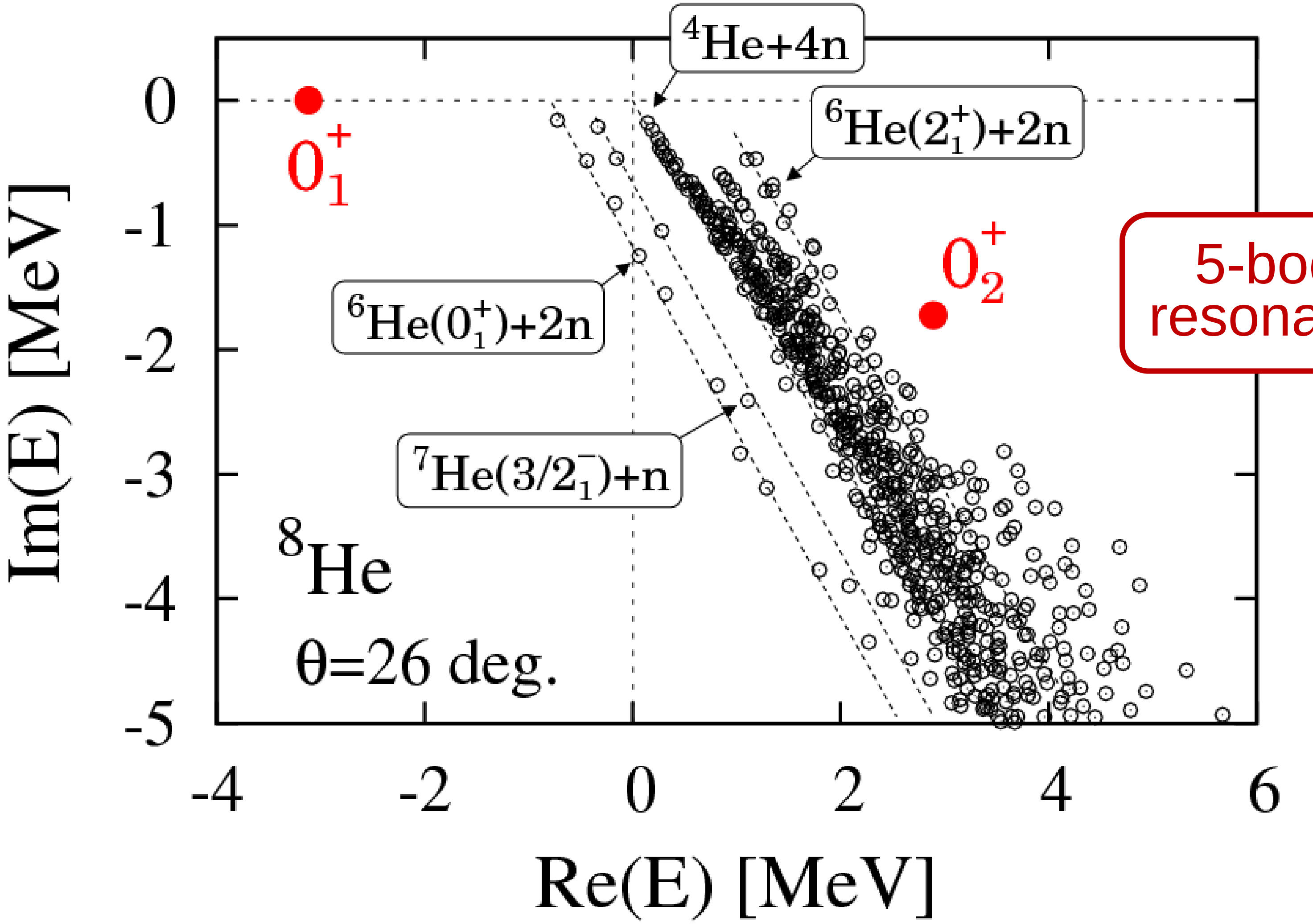
I. Tanihata et al., PLB289('92)261

O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215.

G. D. Alkhazov et al., PRL78('97)2313

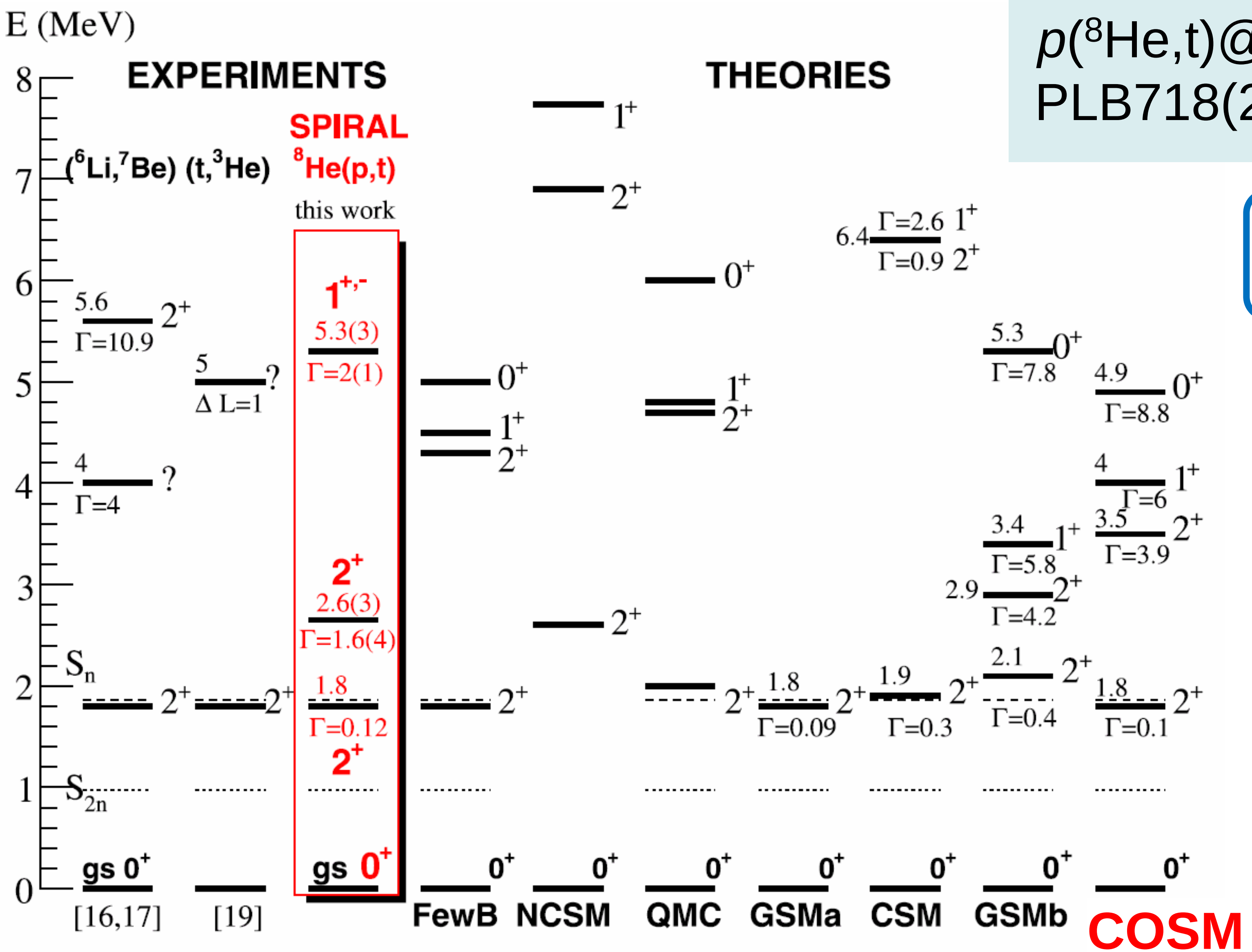
P. Mueller et al., PRL99(2007)252501

Energy of ^{8}He with complex scaling



5-body resonance

Eigenvalue problem with 32,000 dim.
Full diagonalization of complex matrix @ SX8R of NEC

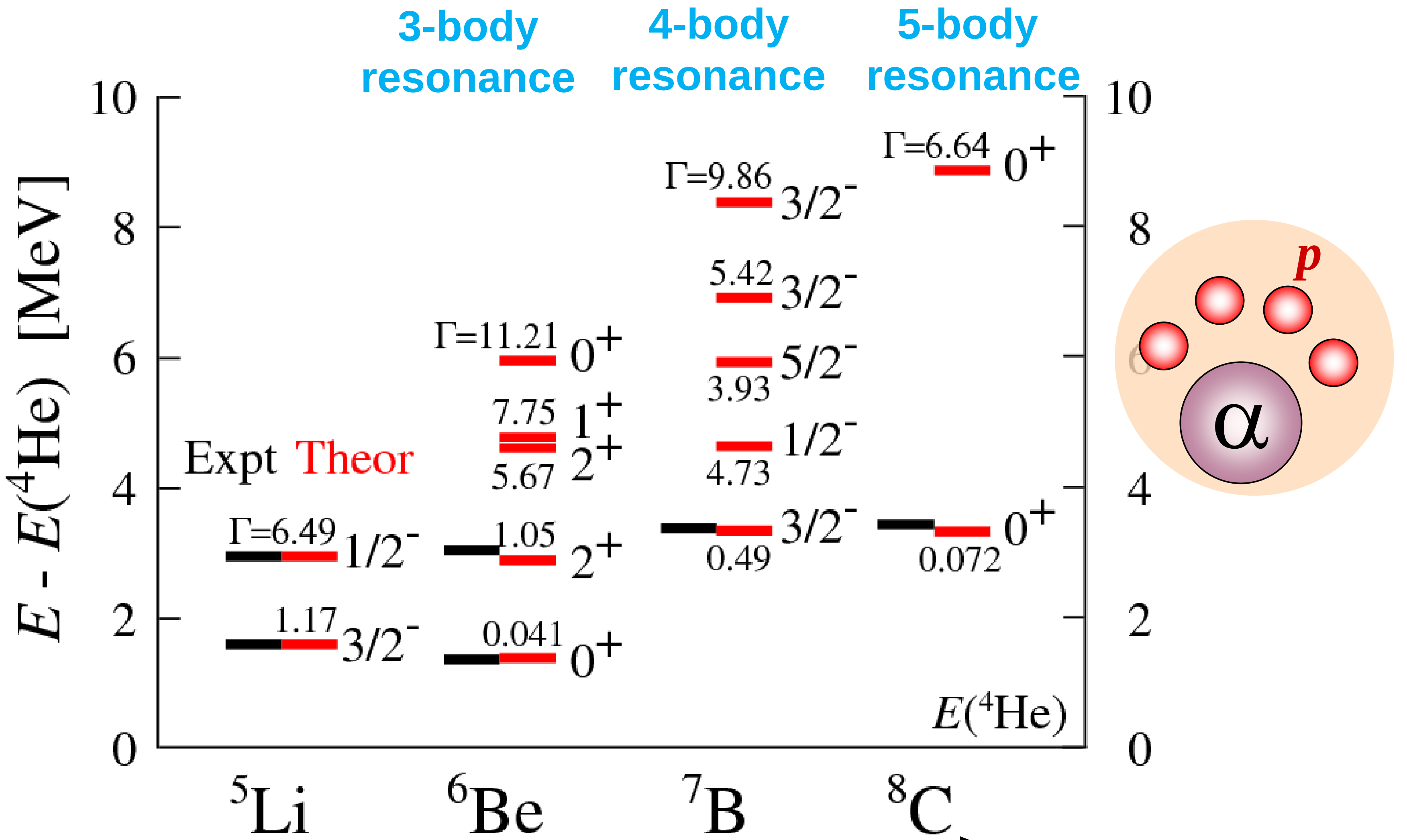


p(^{8}He,t)@SPIRAL
PLB718(2012)441

^{6}He

Fig. 6. Spectroscopy of ^{6}He: comparison between our new results with the previous experiments and with several theories, few-body model (FewB) [14], QMC [4], NCSM [15], CSM [10], GSMa [9], GSMb [8], and the COSM [11].

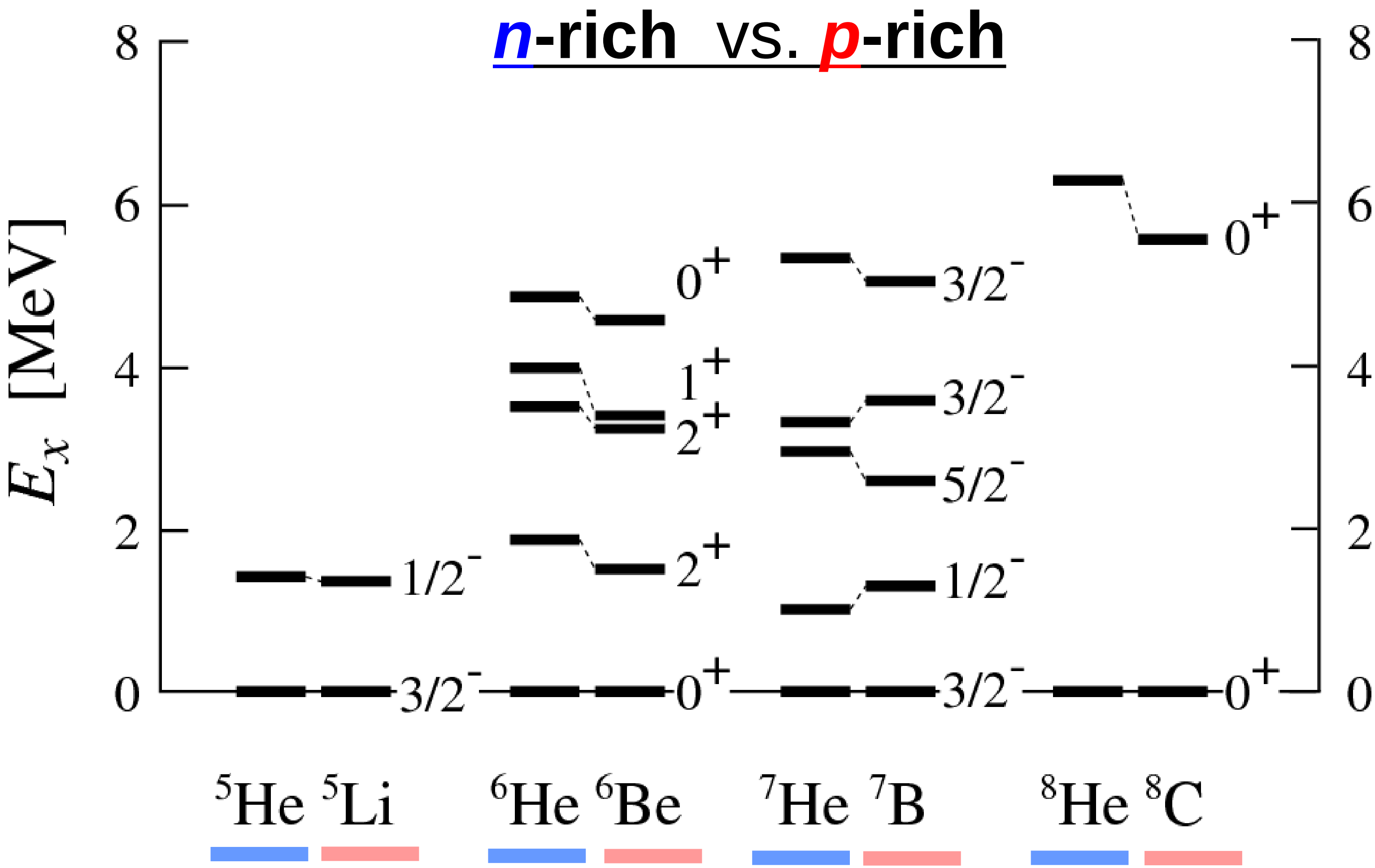
Proton-rich side : ^{4}He+4*p*



Γ(exp) ~ 0.130(50) MeV

Charity et al. PRC84(2011)014320.
TUNL Nuclear Data Evaluation

Mirror symmetry in resonances



TM, Kikuchi, Kato PRC84 (2011) 064306
PRC85 (2012) 034338

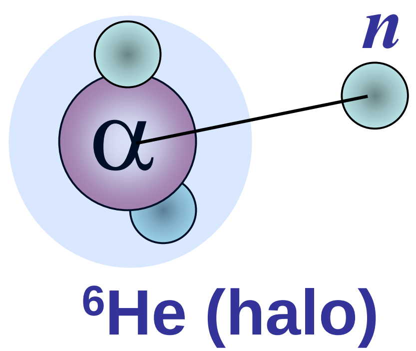
- Good symmetry

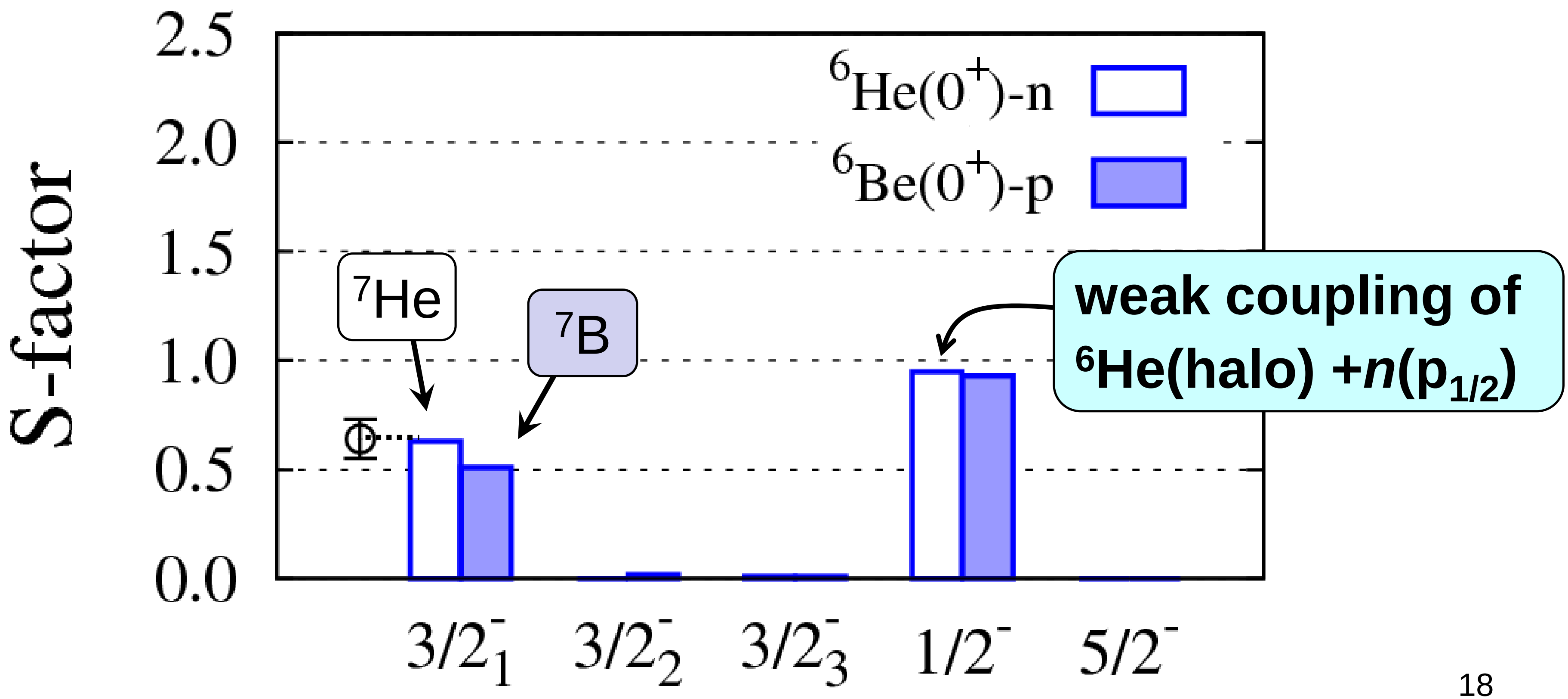
S-factors of ^{7}He & ^{7}B

neutron removal

$$S_{J',J} = \sum_{nlj} \left\langle {}^{6}\mathrm{He}(0^{+}) \left| a_{nlj}(n) \right| {}^{7}\mathrm{He}(J^{\pi}) \right\rangle^{2}$$

$$S_{J',J} = \sum_{nlj} \left\langle {}^{6}\mathrm{Be}(0^{+}) \left| a_{nlj}(p) \right| {}^{7}\mathrm{B}(J^{\pi}) \right\rangle^{2}$$



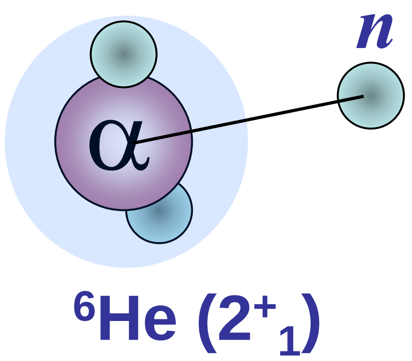


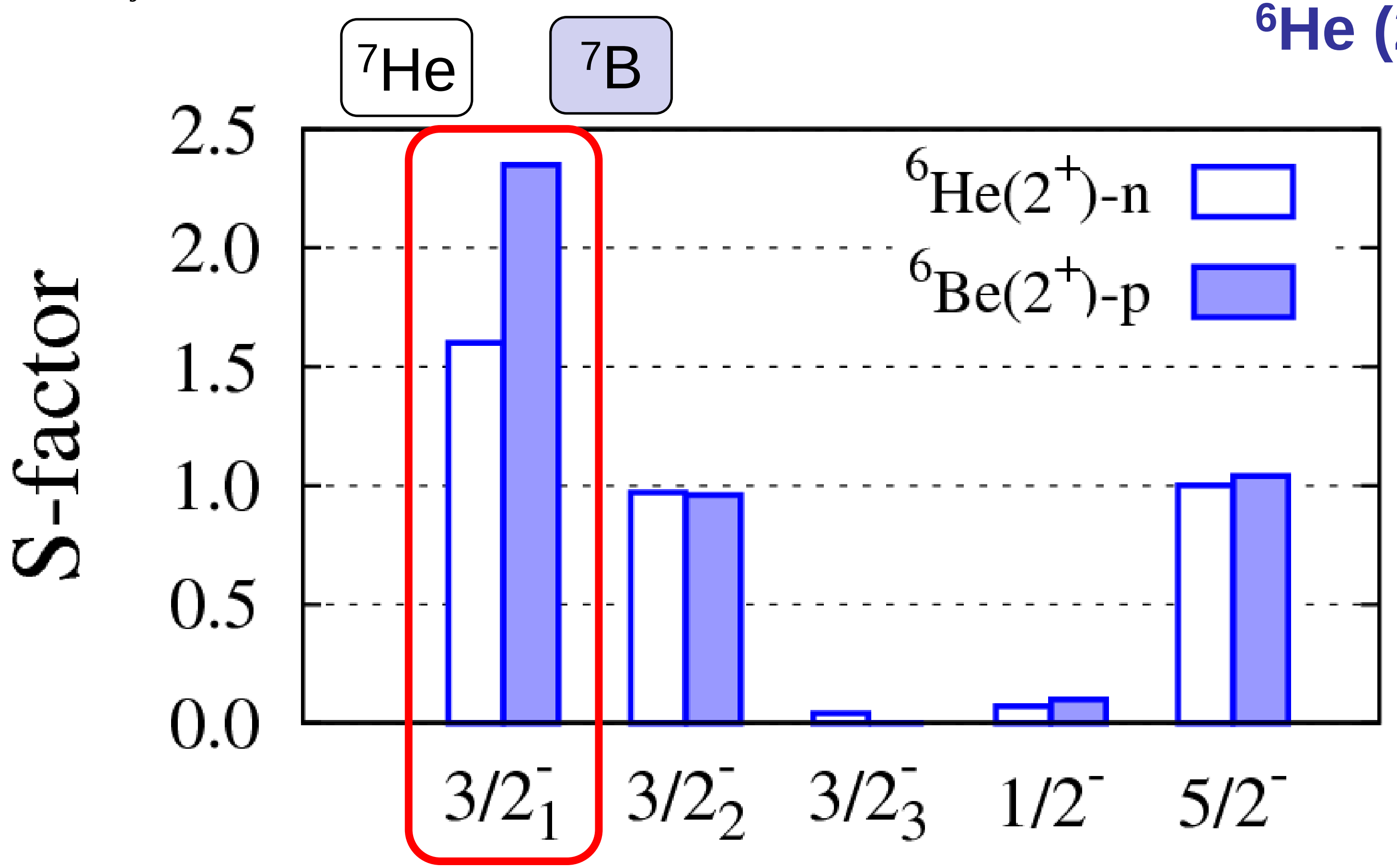
Expt. of ^{7}He : F. Beck et al., Phys. Lett. B 645 (2007) 128

S-factors of ^{7}He & ^{7}B

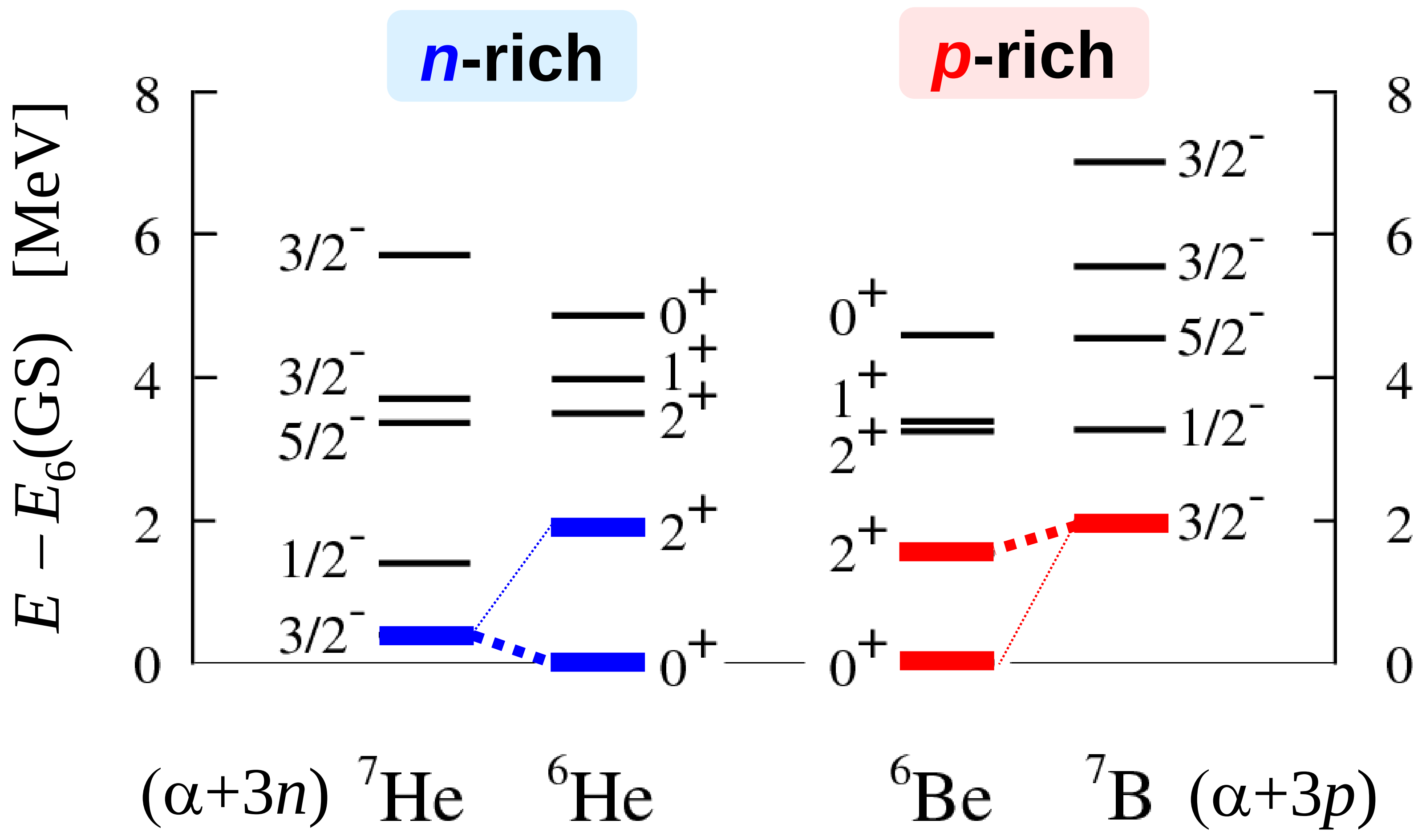
$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{He}(2^+) \middle| a_{nlj}(n) \middle| {}^7\mathrm{He}(J^\pi) \right\rangle^2$$

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{Be}(2^+) \middle| a_{nlj}(p) \middle| {}^7\mathrm{B}(J^\pi) \right\rangle^2$$





Thresholds of $[A=6]+N$ system



Mirror symmetry breaking due to the channel coupling effect caused by Coulomb force

Configuration weights of ^{8}C, ^{8}He

G.S.

0p0h

	^{8}C (4p)	^{8}He (4n)
$(p_{3/2})^4$	**0.88**	**0.86**
$(p_{3/2})^2(p_{1/2})^2$	0.06	0.07
$(p_{3/2})^2(d_{5/2})^2$	0.04	0.04

0^+_2

2p2h

	^{8}C (4p)	^{8}He (4n)
$(p_{3/2})^4$	0.04	0.02
$(p_{3/2})^2(p_{1/2})^2$	**0.93**	**0.97**
$(p_{3/2})^2(d_{3/2})^2$	0.02	0.02

- Good symmetry between ^{8}C & ^{8}He

Continuum effect in ^{8}He (r_n< 6 fm)

G.S.

0p0h

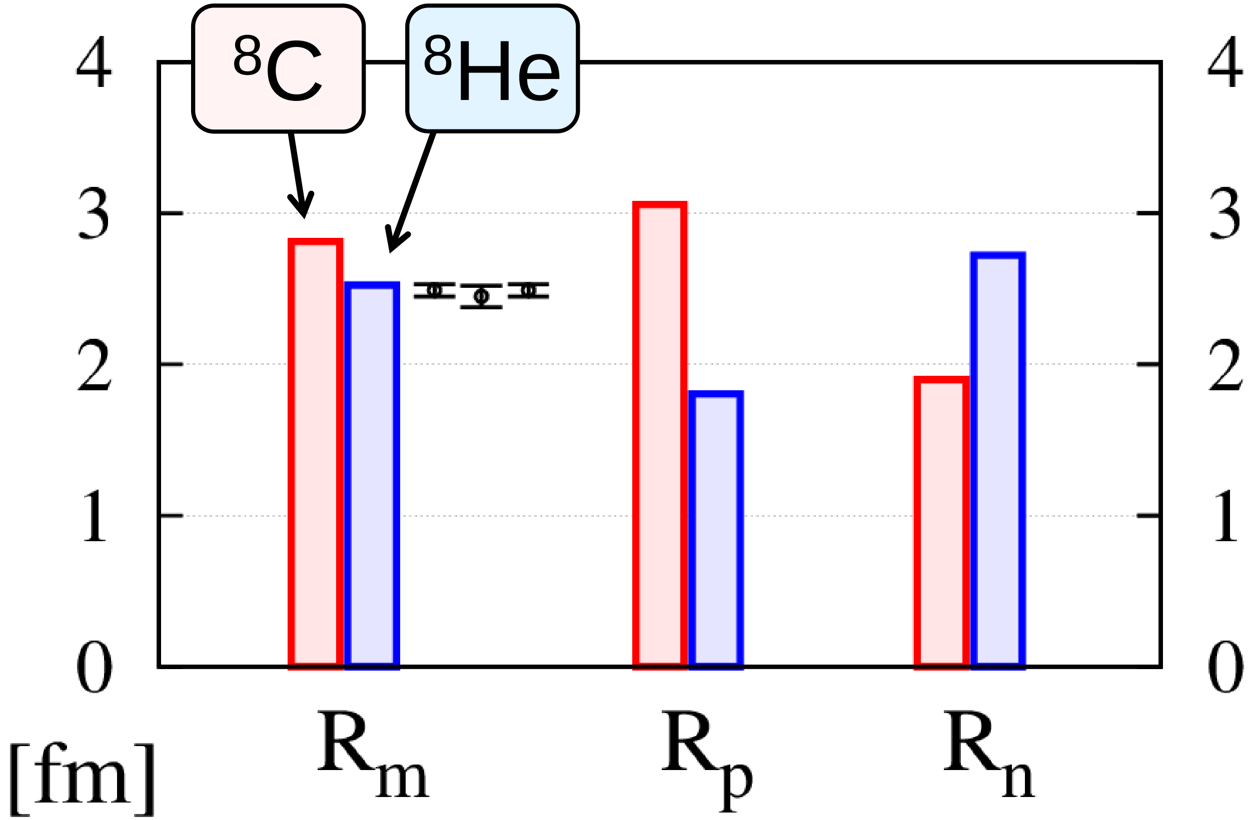
	Full	**No continuum**
$(p_{3/2})^4$	**0.86**	**0.86**
$(p_{3/2})^2(p_{1/2})^2$	0.07	0.07
$(p_{3/2})^2(d_{5/2})^2$	0.04	0.04

0^+_2

2p2h

	Full	**No continuum**
$(p_{3/2})^4$	0.02	0.07
$(p_{3/2})^2(p_{1/2})^2$	**0.97**	**0.81**
$(p_{3/2})^2(1s_{1/2})^2$	–0.01	0.04
$(p_{3/2})^2(d_{3/2})^2$	0.02	0.02
$(p_{3/2})^2(d_{5/2})^2$	0.00	0.01

Radial properties of ^{8}C, ^{8}He – **G.S.** –

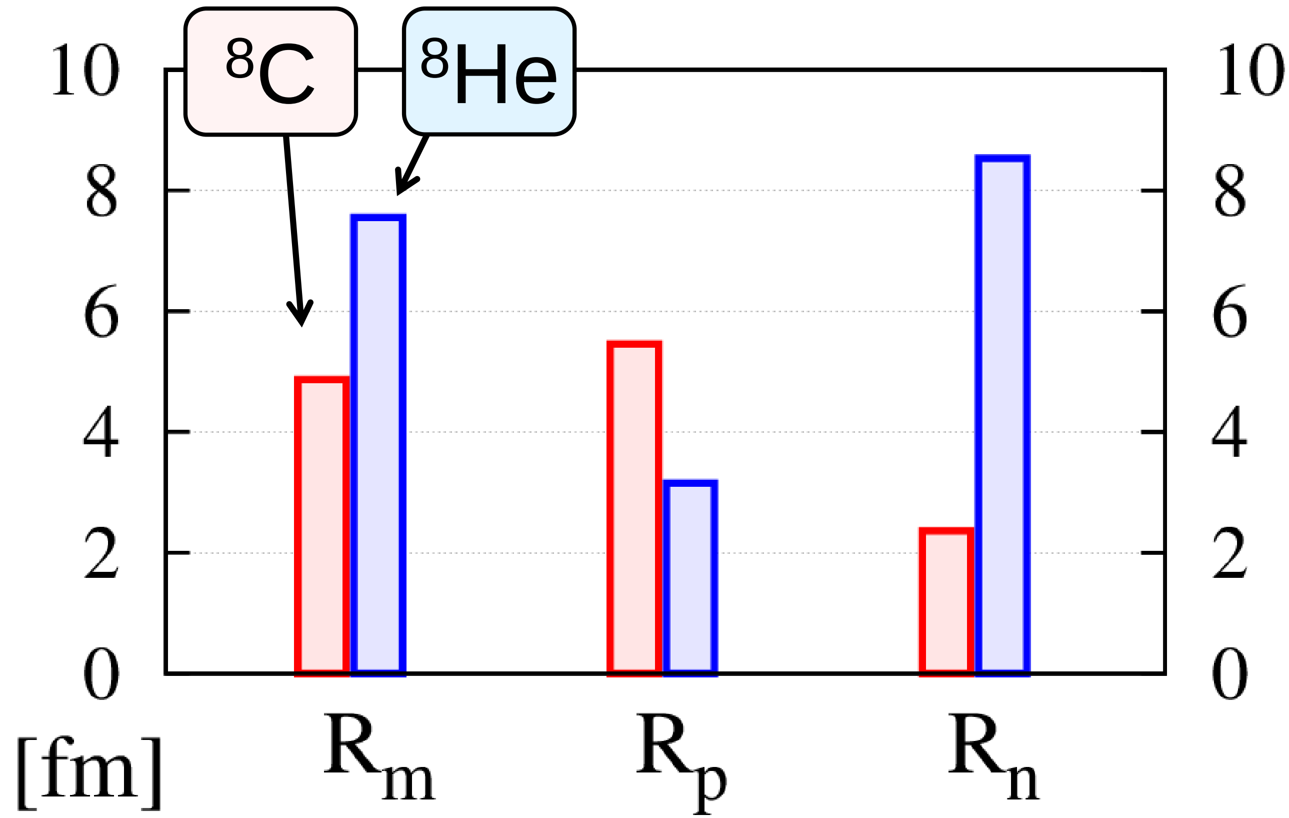


<u>10%-15% increase</u> due to Coulomb repulsion

cf. ^{6}Be-^{6}He, 20% increase
(2p) (2n)

I. Tanihata et al., PLB289(‘92)261
G. D. Alkhazov et al., PRL78(‘97)2313
O. A. Kiselev et al., EPJA 25, Suppl. 1(‘05)215

Radial properties of ^{8}C, ^{8}He – $\mathbf{0^+_2}$ –



<u>30% decrease</u> due to Coulomb barrier

$\mathbf{0^+_2}$

- ^{8}C (E_r, Γ) = (8.9, 6.4) (MeV)
- ^{8}He (E_r, Γ) = (3.1, 3.2) comparable

Continuum Level Density (CLD) in CSM

$$\Delta E = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G(E) - G_0(E)\right]\right], \qquad G_{(0)} = \frac{1}{E - H_{(0)}},$$

$$\Delta E = \frac{1}{2i\pi}\mathrm{Tr}\left[S(E)^{\dagger}\frac{d}{dE}S(E)\right] \quad \rightarrow \quad \frac{1}{\pi}\frac{d\delta_{\ell}}{dE} \quad \text{(single channel case)}$$

S. Shlomo, NPA539('92)17

K. Arai and A. Kruppa, PRC60('99)064315

R. Suzuki, T. Myo and K. Kato, PTP113('05)1273.

CLD in CSM

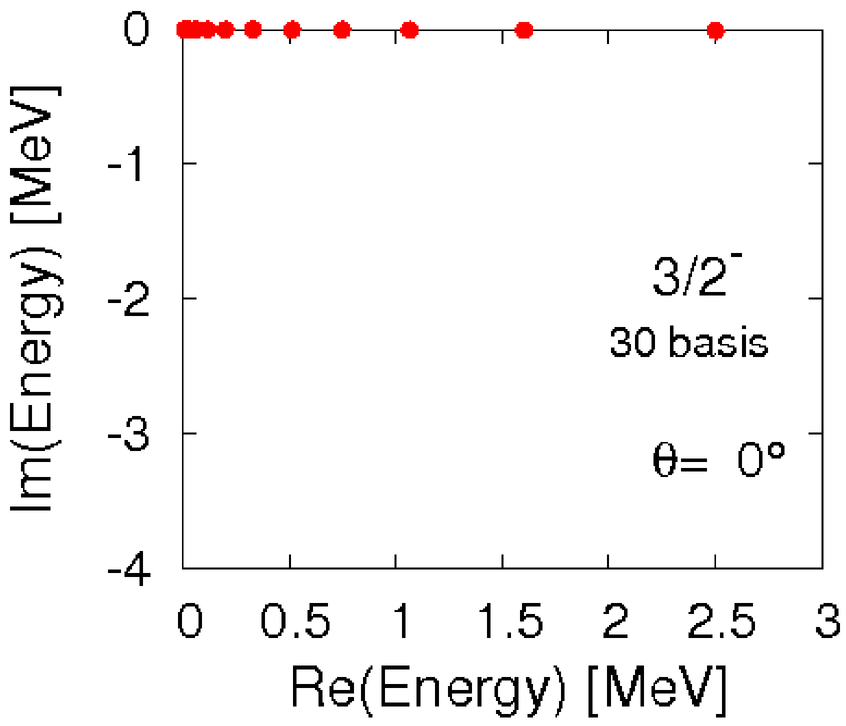
$$\Delta E = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G^{\theta}(E) - G_0^{\theta}(E)\right]\right]$$

$$G = \frac{1}{E - H^{\theta}}$$

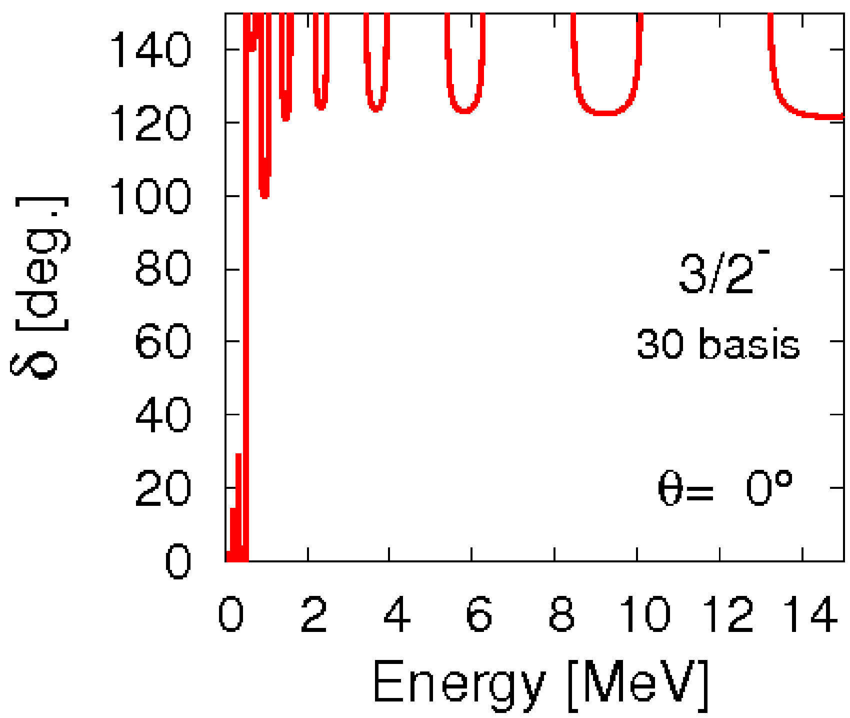
$$G_0 = \frac{1}{E - H_0^{\theta}} \quad \text{(asymptotic)}$$

α+n scattering with complex scaling using discretized continuum states

energy eigenvalues



$P_{3/2}$ scattering phase shift



30 Gaussian basis functions

Strength function $S(E)$ in CSM

- Strength function and response function

Bi-orthogonal relation

$$S(E) = \sum_i \langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_i \rangle \langle \tilde{\varphi}_i | \hat{O} | \Phi_0 \rangle \cdot \delta(E - E_i)$$

$$= -\frac{1}{\pi} \mathrm{Im}\left[R(E) \right]$$

initial state

$$R(E) = \sum_i \frac{\langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_i \rangle \langle \tilde{\varphi}_i | \hat{O} | \Phi_0 \rangle}{E - E_i}$$

Response function

- Complex-scaled Green's function

complete set in CSM

$$G^\theta(E) = \frac{1}{E - H_\theta} = \sum_i \frac{|\varphi_i^\theta\rangle\langle\tilde{\varphi}_i^\theta|}{E - E_i^\theta}$$

Bound+Resonance+Continuum

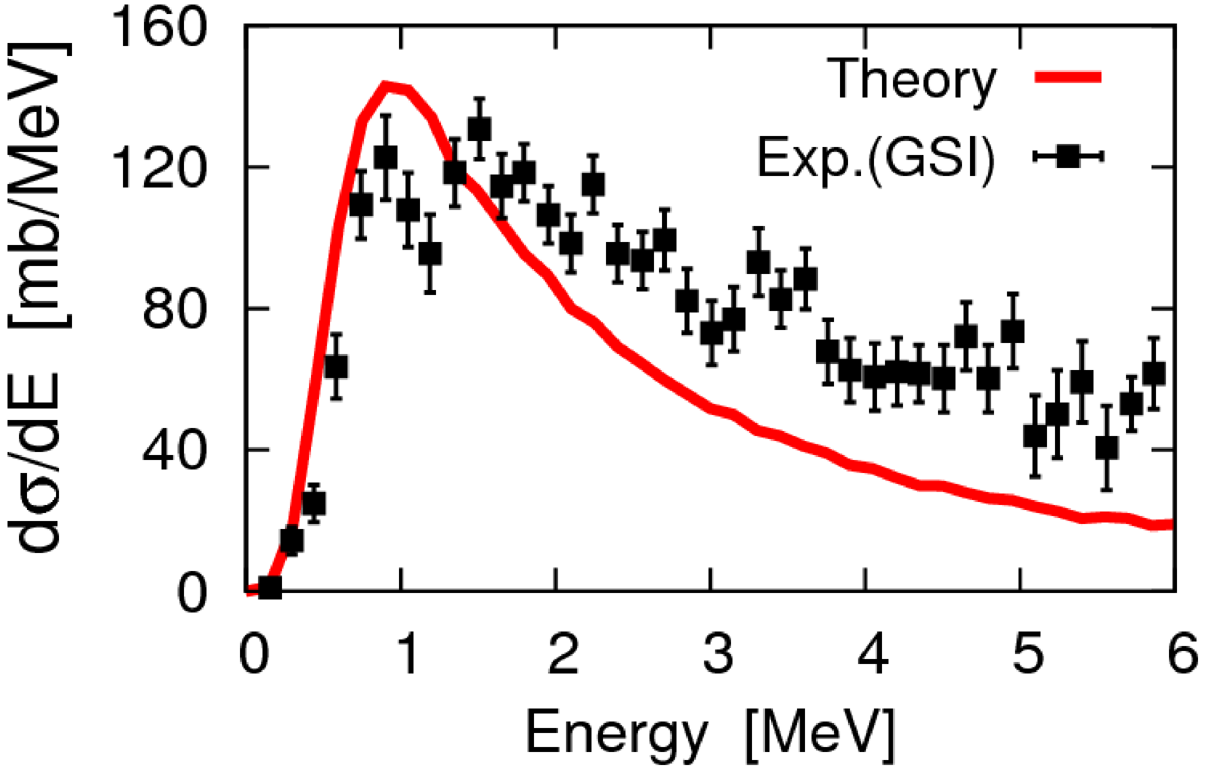
Reaction theory

- LS-eq. (Kikuchi)
- CDCC (Matsumoto)
- Scatt. Amp. (Kruppa, Dote($K^{bar}N$))

T. Berggren, NPA109('68)265, TM, A. Ohnishi and K. Kato, PTP99('98)801

Coulomb breakup strength of ^{6}He

$$^{6}\mathrm{He\,(G.S.)} \rightarrow {}^{4}\mathrm{He} + n + n$$



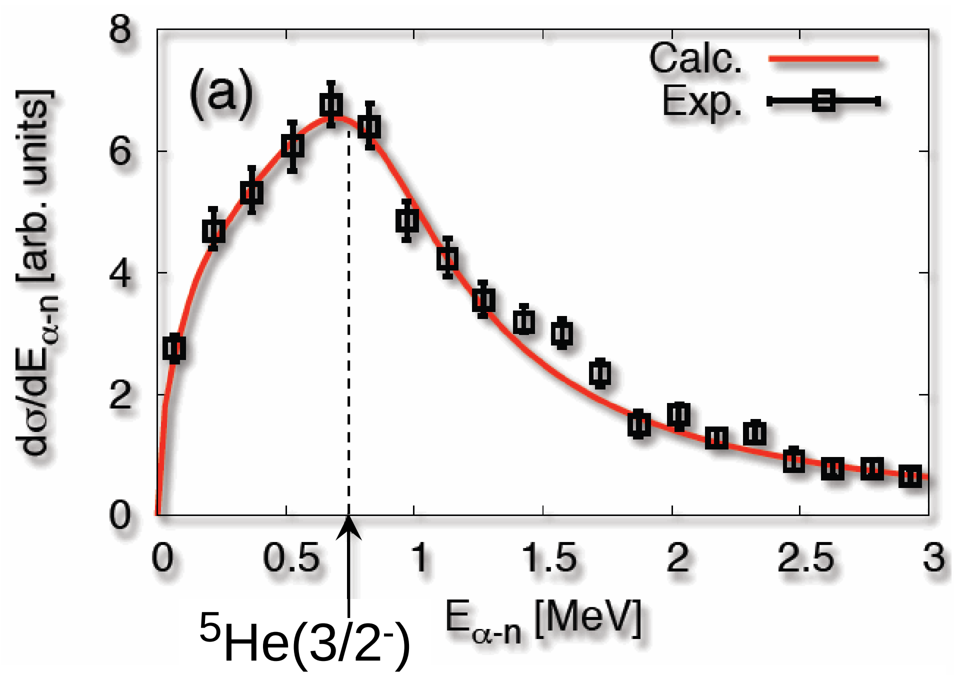
E1+E2 (complex scaling)
Equivalent photon method

TM, K. Kato, S. Aoyama and K. Ikeda PRC63(2001)054313.

Kikuchi, TM, Takashina, Kato, Ikeda
PTP122(2009)499
PRC81(2010)044308.
(invariant mass of α-n & n-n)

^{6}He : 240MeV/A, Pb Target (T. Aumann et.al, PRC59(1999)1252)

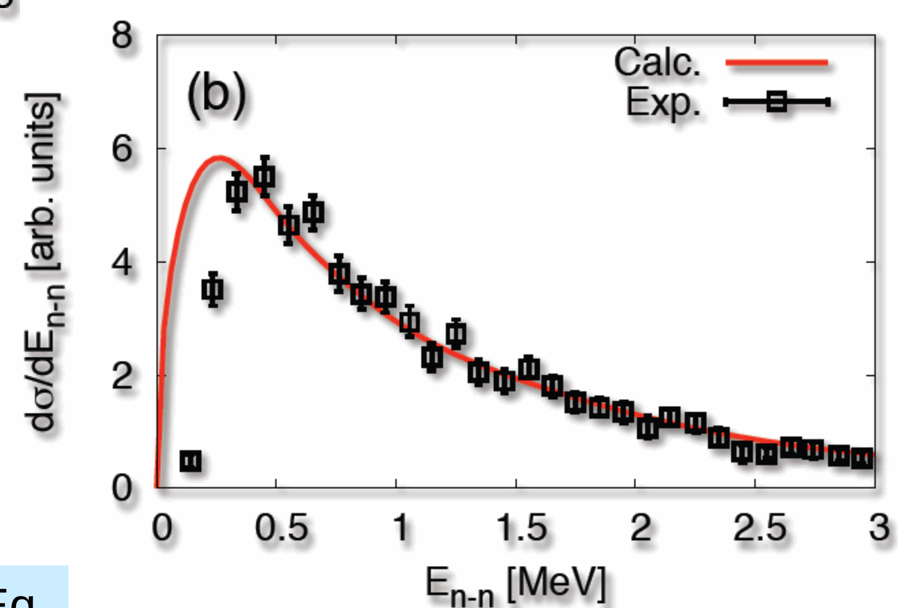
Invariant mass spectra of ^{6}He breakup



$$^6\text{He (G.S.)} \rightarrow {}^4\text{He} + n + n$$

α-*n* system

n-*n* system



Kikuchi, TM, Takashina, Kato, Ikeda

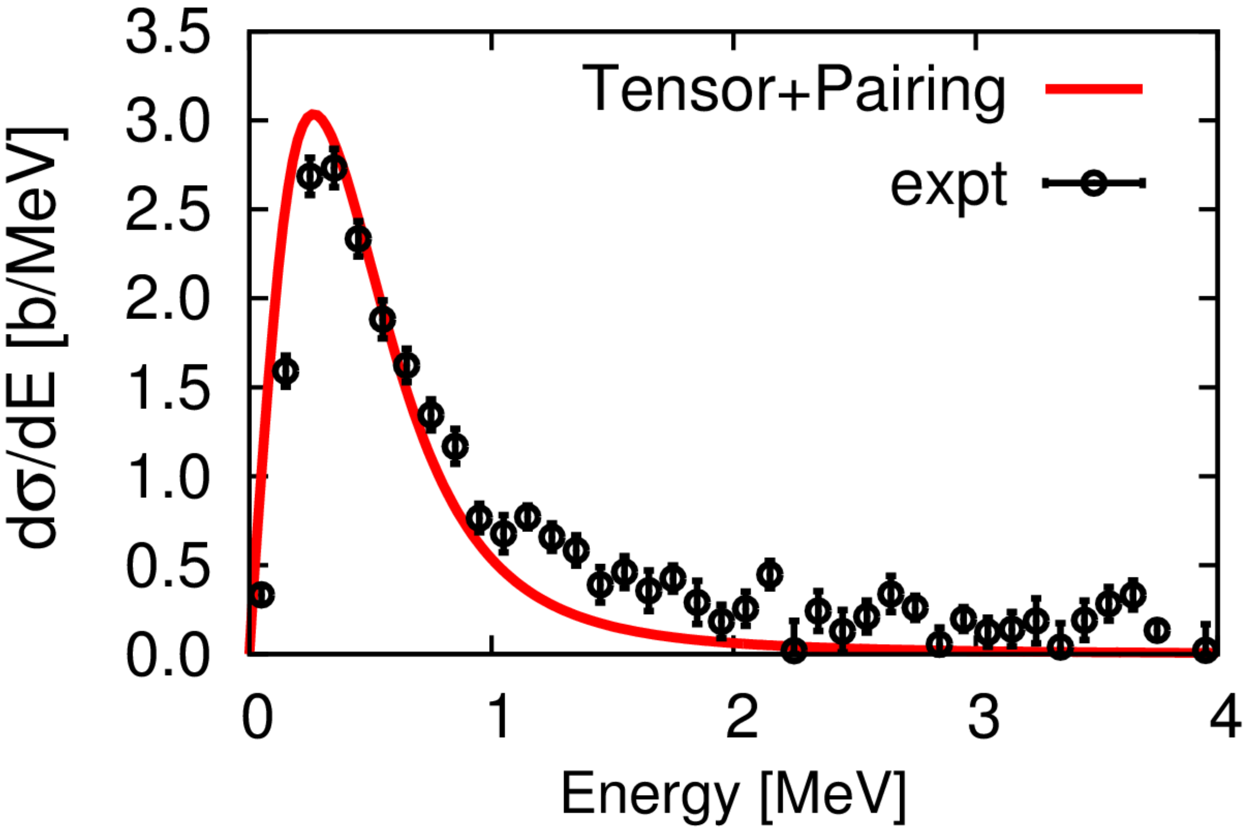
PRC81(2010)044308.

Complex Scaling + Lippmann-Schwinger Eq.

Coulomb breakup strength of ^{11}Li

T.Myo, K.Kato, H.Toki, K.Ikeda
PRC76(2007)024305

$^{11}Li \rightarrow {}^{9}Li+n+n$



No three-body resonance

E1 strength
+ **Complex scaling**
+Equivalent photon method

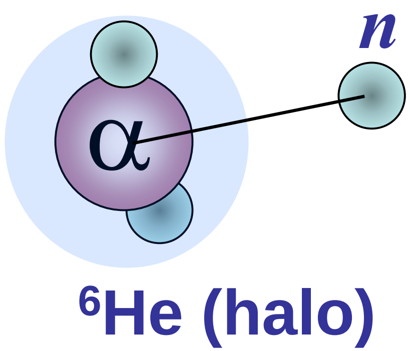
- Expt: T. Nakamura et al. , PRL96,252502(2006)
- Energy resolution with $\sqrt{E}$ =0.17 MeV.

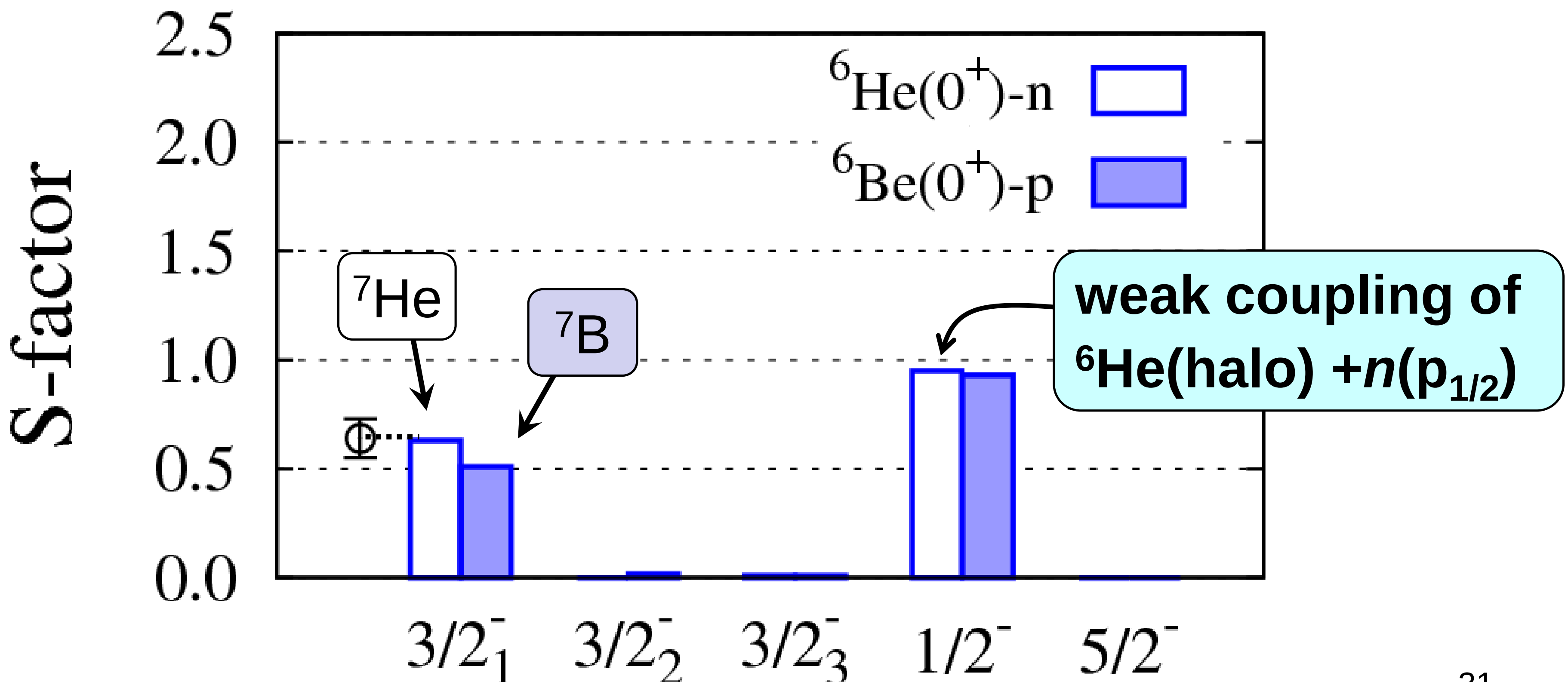
S-factors of ^{7}He & ^{7}B

neutron removal

$$S_{J',J}=\sum_{nlj}\left\langle {}^6\mathrm{He}(0^+)\middle|a_{nlj}(n)\middle|{}^7\mathrm{He}(J^\pi)\right\rangle^2$$

$$S_{J',J}=\sum_{nlj}\left\langle {}^6\mathrm{Be}(0^+)\middle|a_{nlj}(p)\middle|{}^7\mathrm{B}(J^\pi)\right\rangle^2$$





Expt. of ^{7}He : F. Beck et al., Phys. Lett. B 645 (2007) 128

One-neutron removal strength of $^7He_{GS}$

$$S_{J',J}(E) = \sum_{nlj} \left| \left\langle {}^6\mathrm{He}^{J'}(E) \middle| a_{nlj}(n) \middle| {}^7\mathrm{He}^J \right\rangle \right|^2$$

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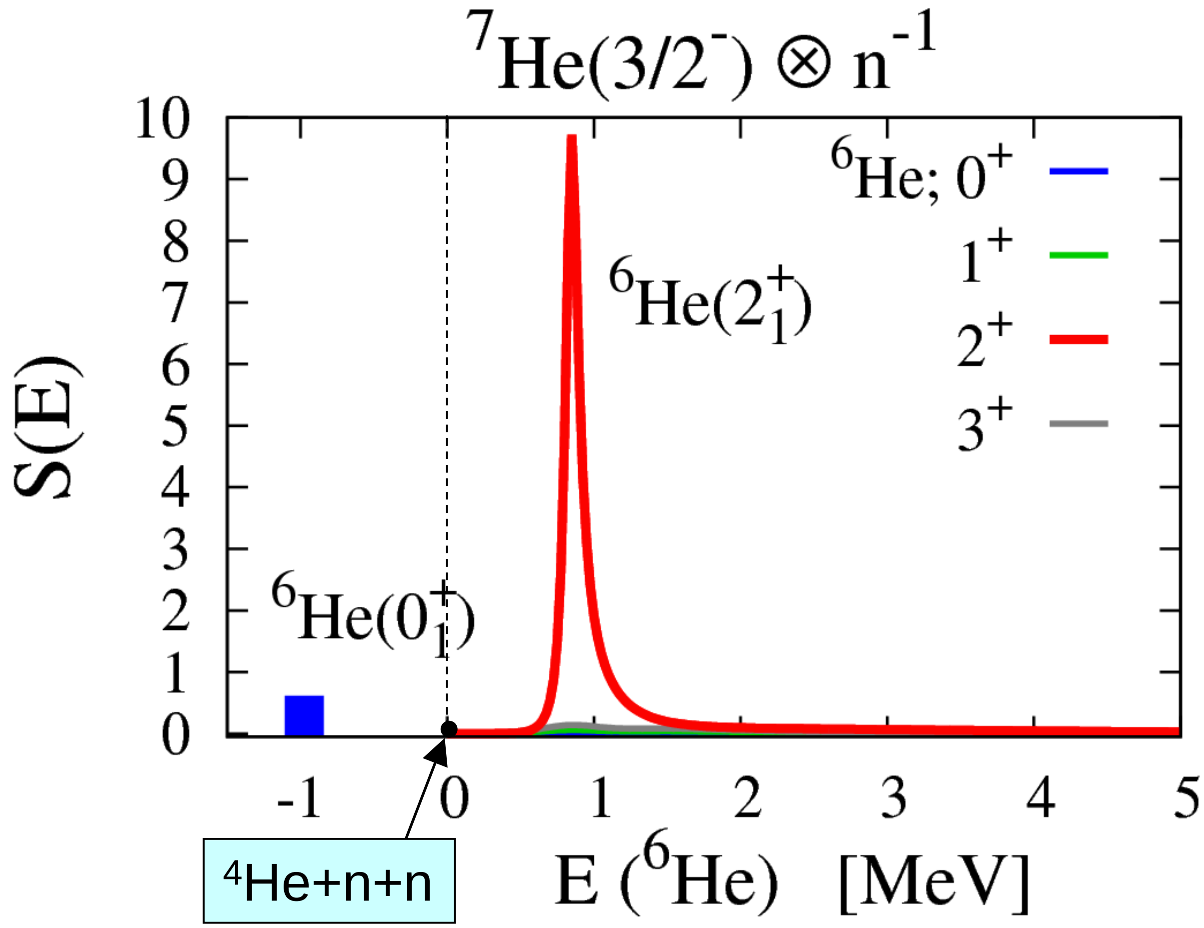
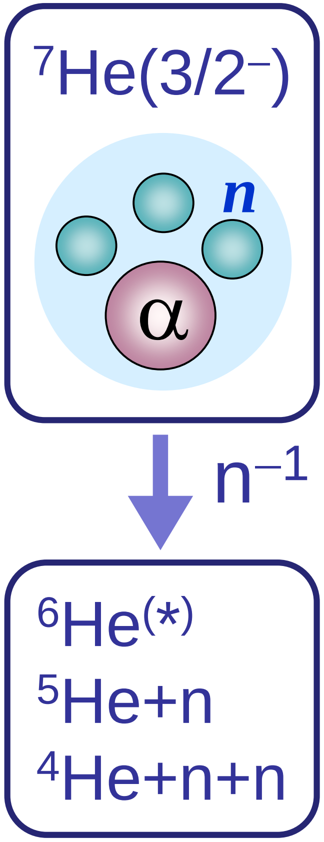
" ^{4}He+n+n" complete set with CSM





One-neutron removal strength of $^7\mathrm{He}_{GS}$

$$S_{J',J}(E) = \sum_{nlj} \left\langle {}^6\mathrm{He}^{J'}(E) \middle| a_{nlj}(n) \middle| {}^7\mathrm{He}^{J} \right\rangle^2$$

TM, Ando, Kato
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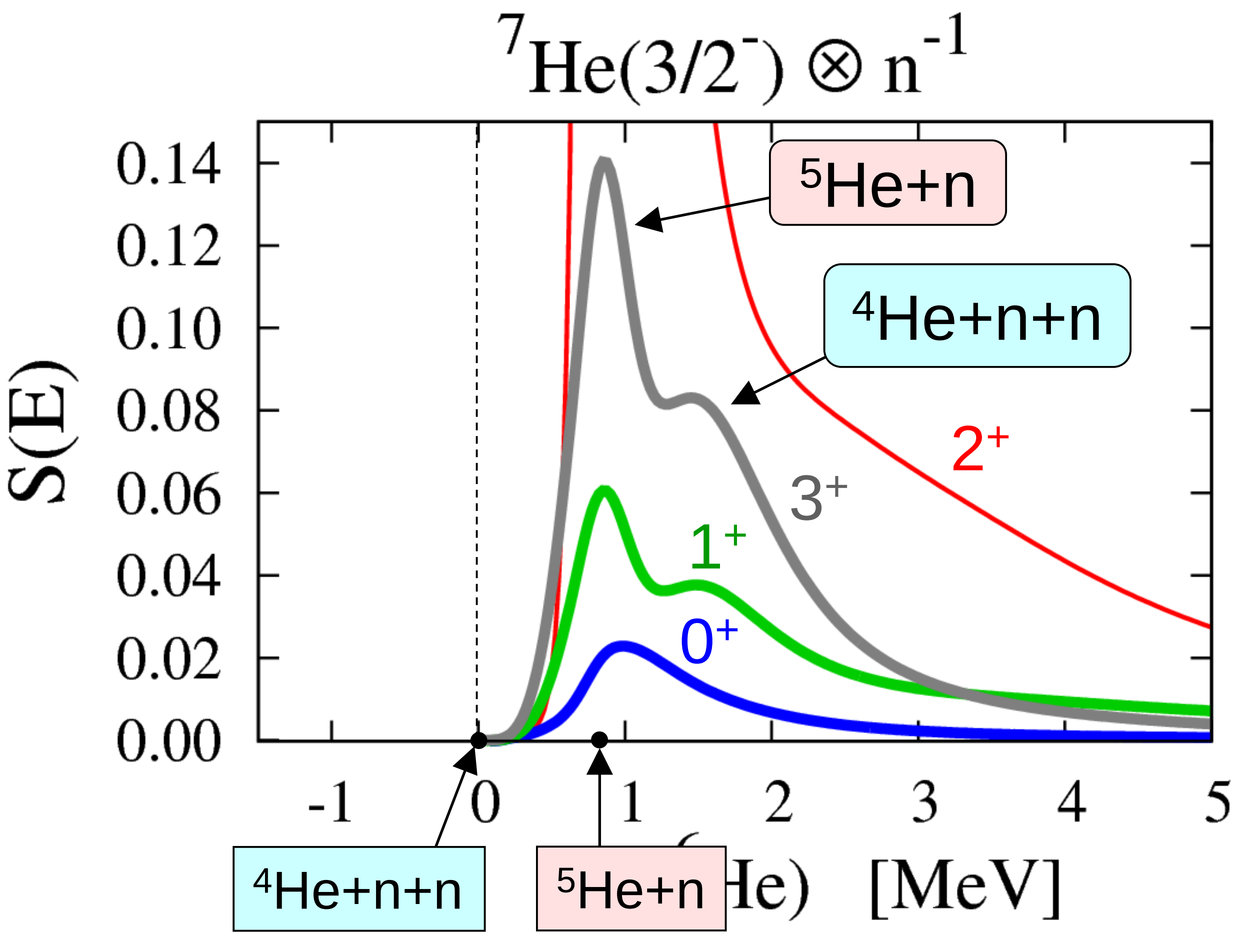
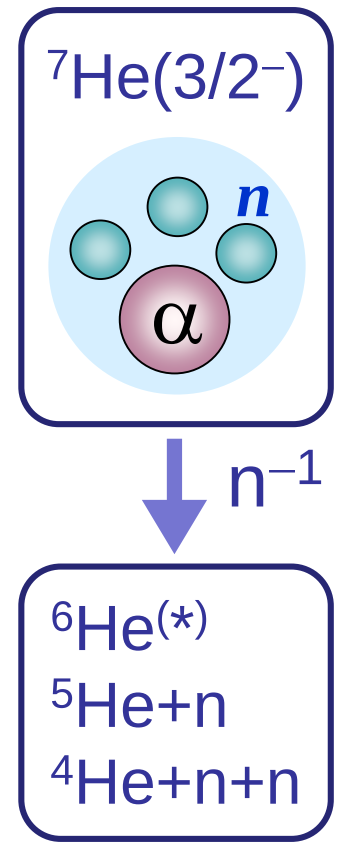
" ^{4}He+n+n" complete set using CSM





Summary

- **Light Unstable Nuclei**
  - He isotopes (***n*-rich**) & Mirror nuclei (***p*-rich**)
  - Mirror symmetry due to $\underline{V_{\mathrm{Coulomb}}}$
    - Channel coupling (threshold), Radius
- **Complex Scaling**
  - Many-body resonance spectroscopy
  - Continuum level density ΔE (resonance+continuum)
  - Strength functions using Green's function
    - Coulomb breakups, nucleon removal, ...
    - Application to reaction theory (LS eq., CDCC, …)