

# Role of tensor force in light nuclei with tensor-optimized shell model

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理研

# Outline

- **Role of  $V_{\text{tensor}}$**  in light nuclei
  - He, Li, Be isotopes with  $V_{\text{bare}}$
- Tensor Optimized Shell Model (**TOSM**)  
to describe tensor correlation.
- Unitary Correlation Operator Method (**UCOM**)  
to describe short-range correlation.

TM, A. Umeya, H. Toki, K. Ikeda PRC84 (2011) 034315

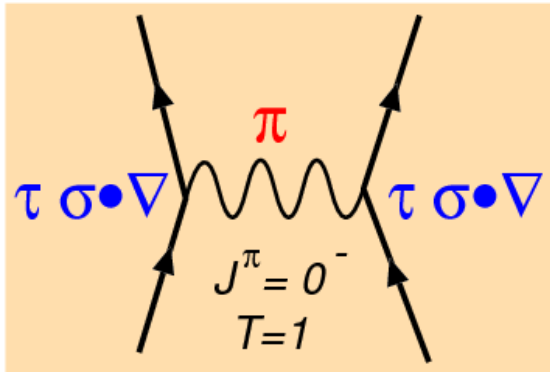
TM, A. Umeya, H. Toki, K. Ikeda PRC86 (2012) 024318

TM, A. Umeya, K. Horii, H. Toki, K. Ikeda PTEP (2014) 033B02

# Pion exchange interaction vs. $V_{\text{tensor}}$

$$3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) \frac{q^2}{m^2 + q^2} = (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \frac{q^2}{m^2 + q^2} + S_{12} \frac{q^2}{m^2 + q^2}$$

$$= (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left[ \frac{m^2 + q^2}{m^2 + q^2} - \frac{m^2}{m^2 + q^2} \right] + S_{12} \frac{q^2}{m^2 + q^2}$$



Tensor operator

$\delta$  interaction

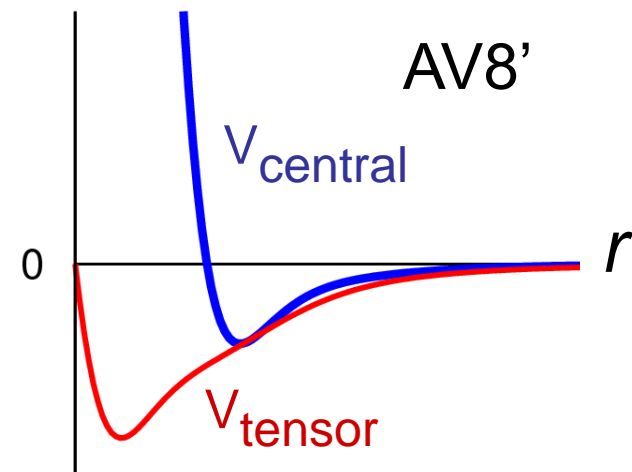
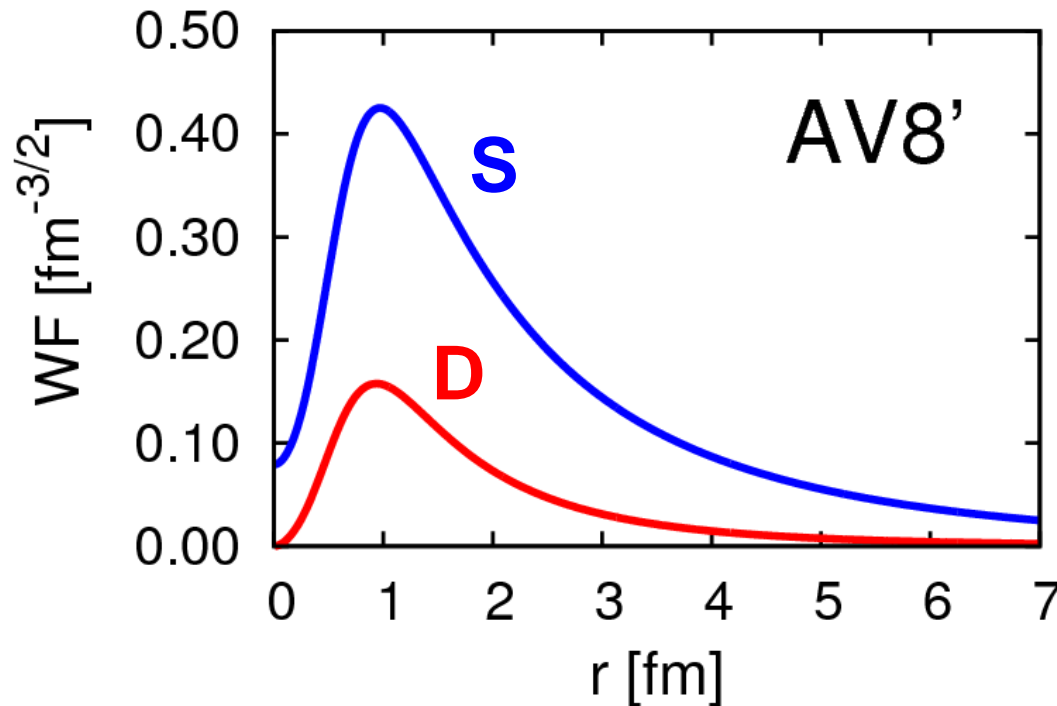
Yukawa interaction

Involve large momentum

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

-  $V_{\text{tensor}}$  produces the high momentum component.

# Deuteron properties & tensor force



$$R_m(s) = 2.00 \text{ fm}$$

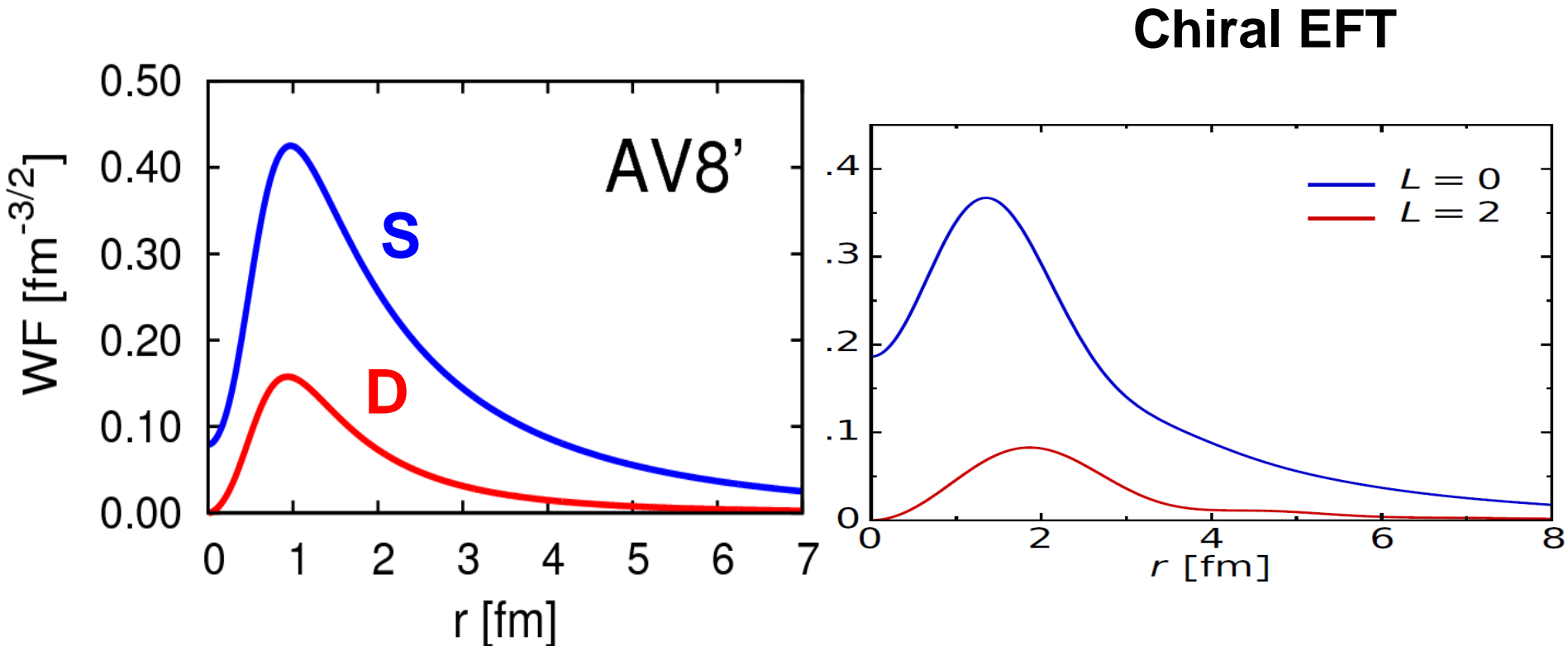
$$R_m(d) = 1.22 \text{ fm}$$

Energy	-2.24 MeV
Kinetic	19.88
Central	-4.46
<b>Tensor</b>	<b>-16.64</b>
LS	-1.02
P(L=2)	5.77%
Radius	1.96 fm

*d*-wave is  
**“spatially compact”**  
 (high momentum)

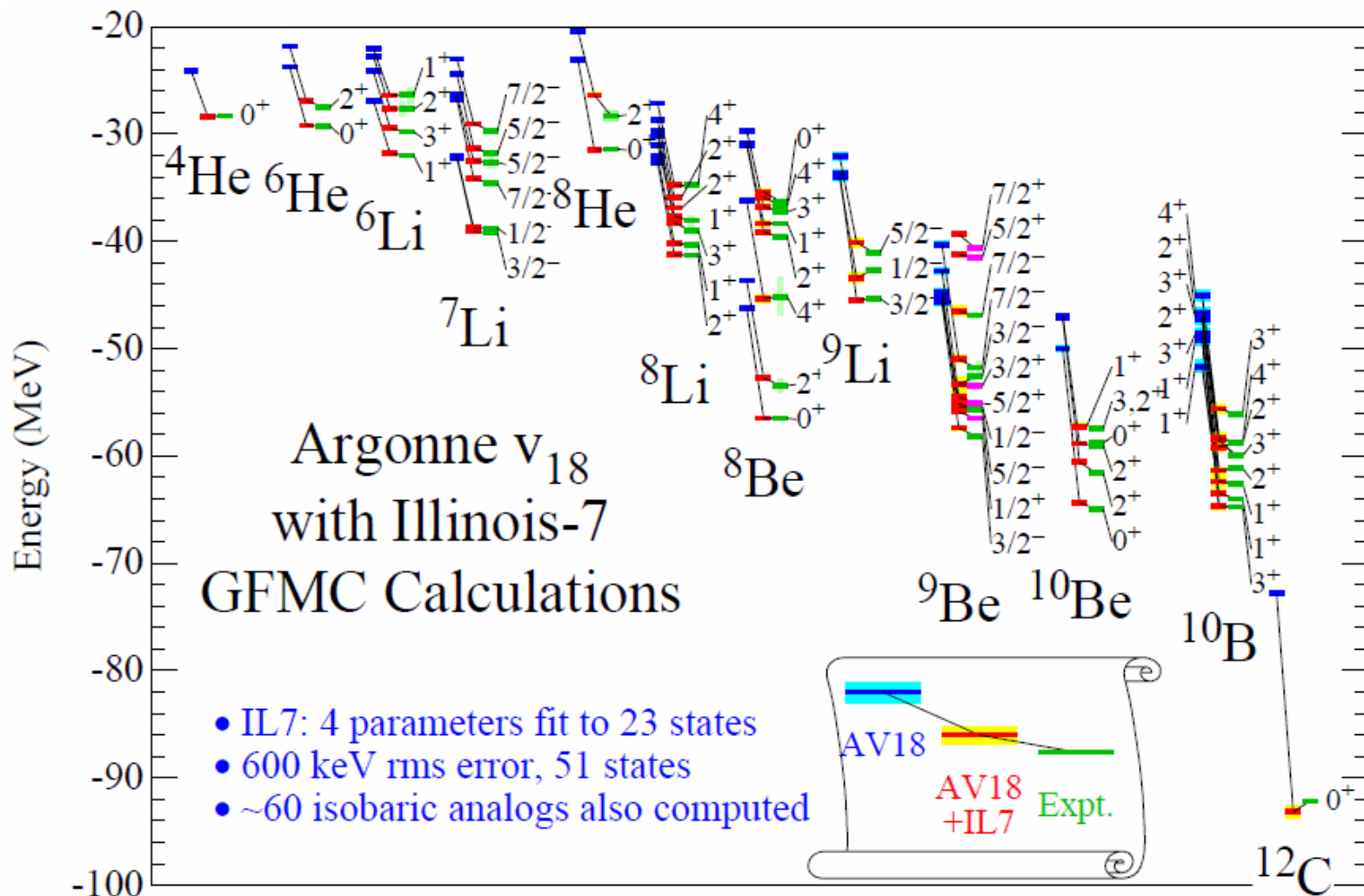
# Deuteron Wave Function

## - AV8' vs. Chiral EFT -



R. Roth @Bad Honnef 2012.3

# Argonne Group calculation 2012



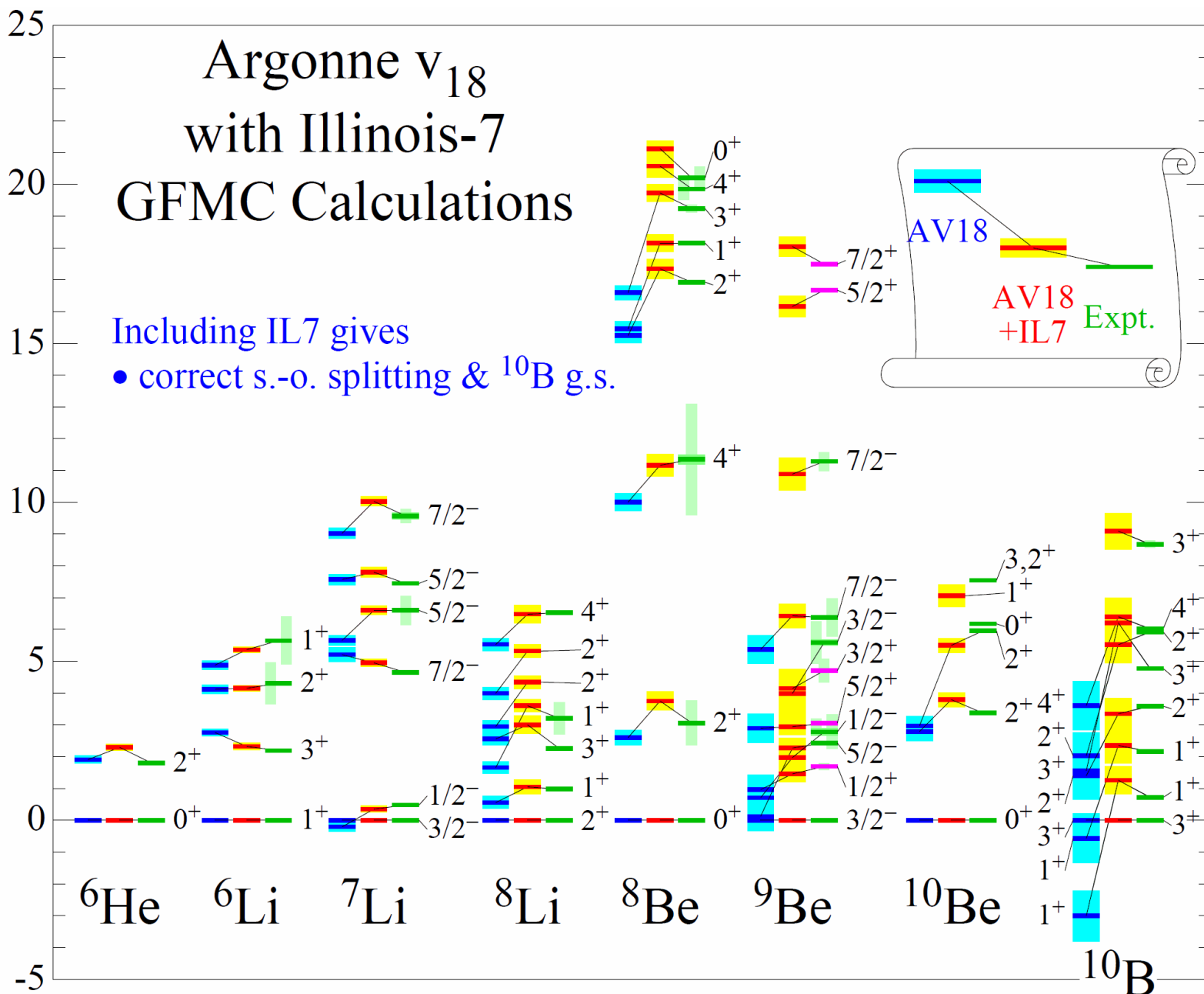
Talk by Wiringa  
@Bad Honnef, 2012.3

$$\frac{V_{\pi}}{V_{NN}} \sim 80\%$$

# Argonne $v_{18}$ with Illinois-7 GFMC Calculations

Excitation energy (MeV)

Including IL7 gives  
• correct s.-o. splitting &  $^{10}\text{B}$  g.s.

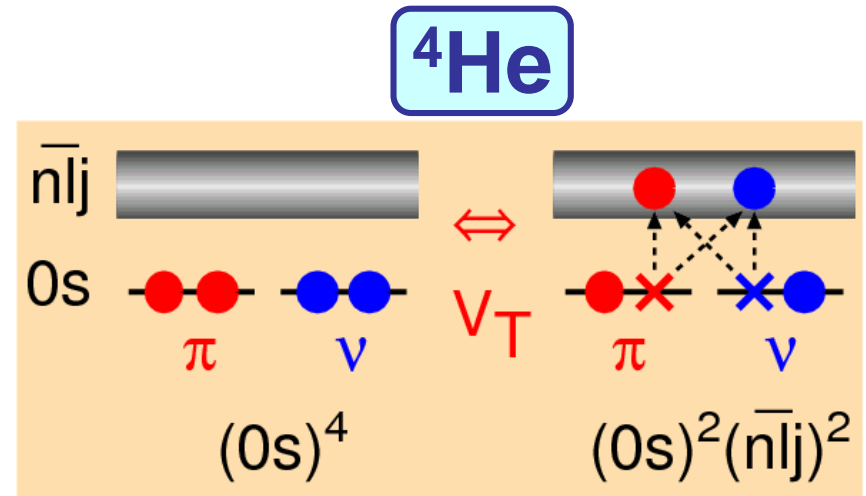


# Tensor-optimized shell model (TOSM)

TM, Sugimoto, Kato, Toki, Ikeda PTP117(2007)257

- **2p2h excitations**  
with high- $L$  orbits.
- $V_{\text{tensor}}$  is **NOT** treated as residual interactions

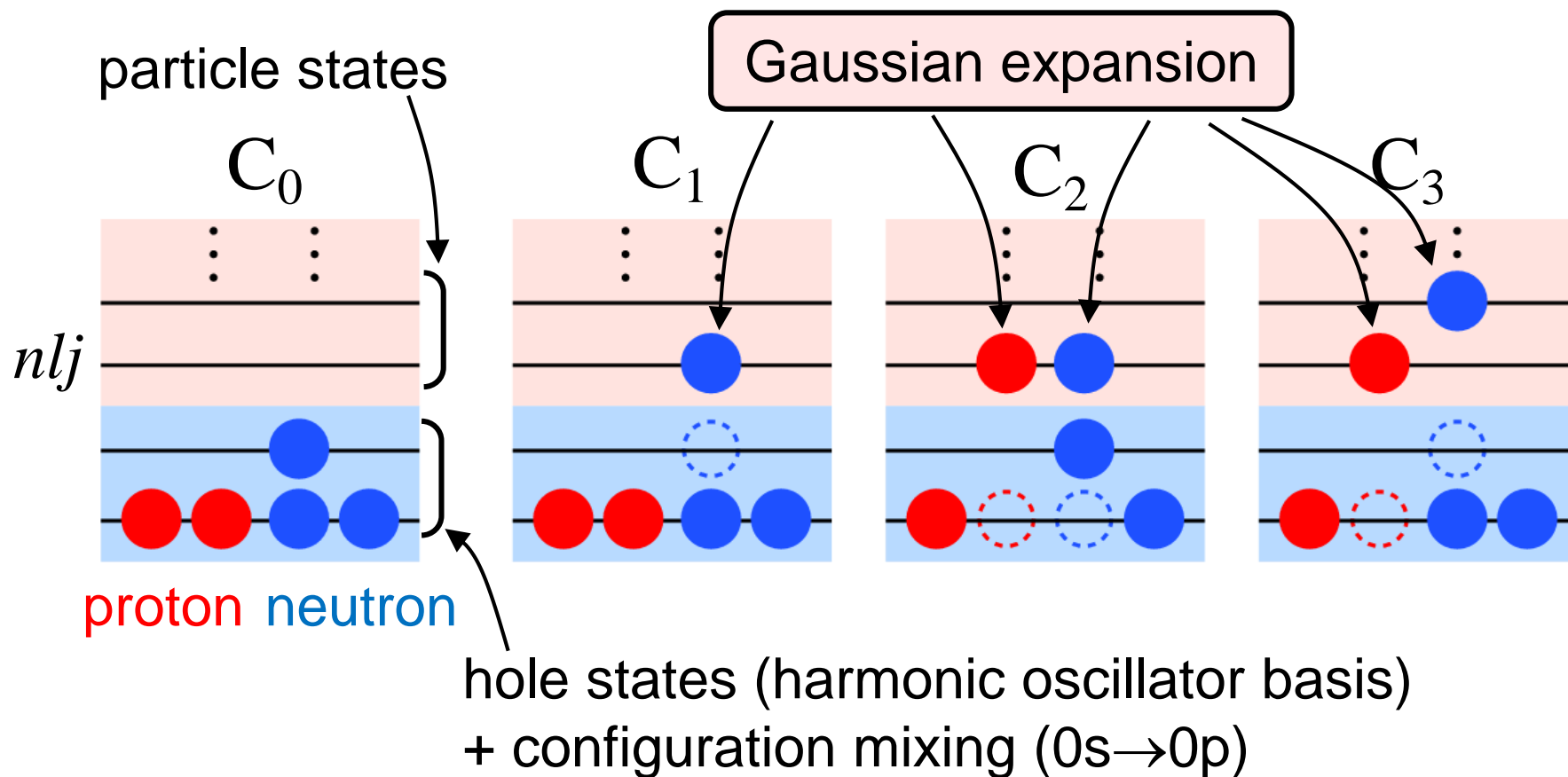
cf.  $\frac{V_{\pi}}{V_{NN}} \sim 80\%$  in GFMC



- Length parameters such as  $b_{0s}$ ,  $b_{0p}$ , ... are optimized **independently**, or **superposed by many Gaussian bases**.
  - **Spatial shrinkage** of **D-wave** as seen in deuteron.  
HF (Sugimoto, NPA740), RMF (Ogawa, PRC73), AMD (Dote et al., PTP115)
- Satisfy few-body results with Minnesota central force ( $^4, ^6\text{He}$ )

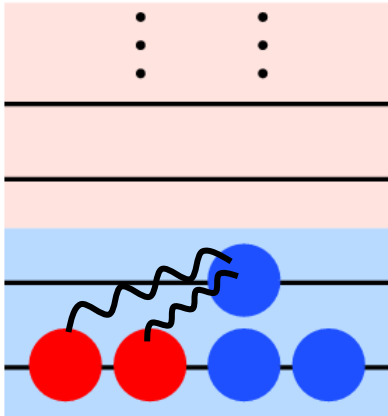


# Configurations in TOSM



Application to Hypernuclei to investigate  $\Lambda N$ - $\Sigma N$  coupling by **Umeya** (NIT), **Hiyama** (RIKEN)

# Tensor force matrix elements



$$b_D \sim b_S$$

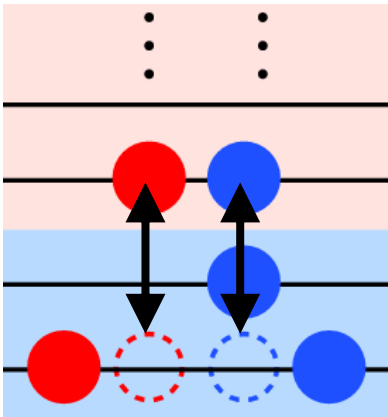
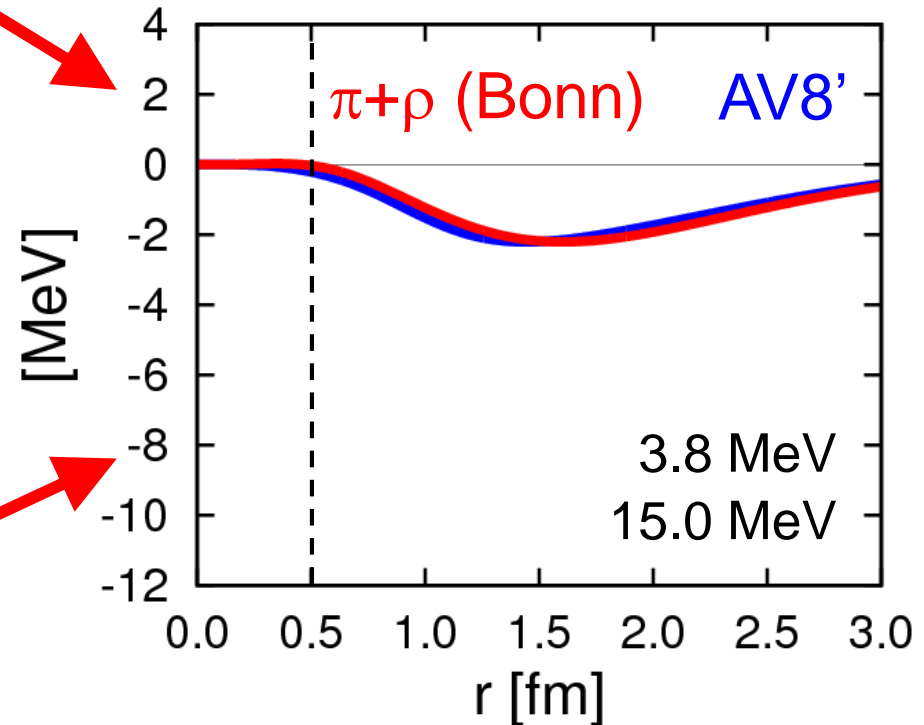
$$V_T = V_{\text{residual}}$$

1<sup>st</sup> order

$$M_{SD}(r) = r^2 \phi_S(r, b_S) \cdot V_T \cdot \phi_D(r, b_D)$$

Integrand of Tensor ME

SD coupling



$$b_D \sim b_S \times 0.5$$

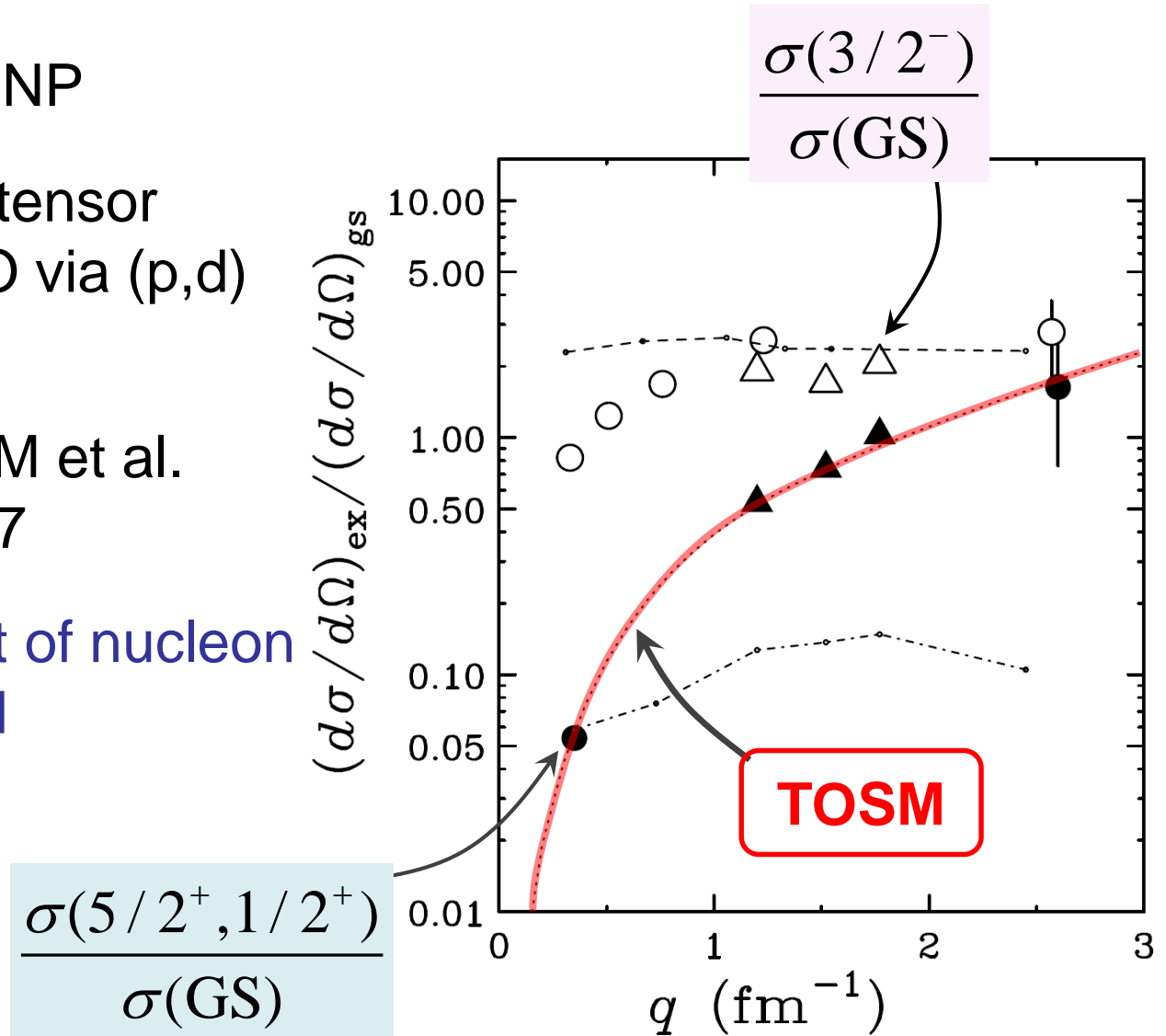
$$V_T \neq V_{\text{residual}}$$

0p0h-2p2h

- Centrifugal potential (1GeV@0.5fm) pushes away D-wave.

# High- $k$ component of nucleon in nucleus

- $^{16}\text{O}(p,d)^{15}\text{O}$  @RCNP
- Probing effect of tensor interactions in  $^{16}\text{O}$  via (p,d) reaction
- Ong, Tanihata, TM et al. PLB725(2013)277
- Compact  $SD$ -orbit of nucleon in  $^{16}\text{O}$  with TOSM



# Hamiltonian and variational equations in TOSM

$$H = \sum_{i=1}^A t_i - T_G + \sum_{i<j}^A v_{ij},$$

(0p0h+1p1h+2p2h)

$$\Phi(A) = \sum_k C_k \cdot \psi_k(A)$$

Shell model type configuration  
with mass number  $A$

**Particle state** : Gaussian expansion for each orbit

$$\phi_{lj}^{n'}(\mathbf{r}) = \sum_{n=1}^N C_{lj,n}^{n'} \cdot \phi_{lj,n}(\mathbf{r}) \quad \phi_{lj,n}(\mathbf{r}) \propto r^l \exp\left[-\frac{1}{2}\left(\frac{r}{b_{lj,n}}\right)^2\right] [Y_l(\hat{\mathbf{r}}), \chi_{1/2}^\sigma]_j$$

$$\langle \phi_{lj}^{n'} | \phi_{lj}^{n''} \rangle = \delta_{n',n''}$$

**Gaussian basis function**

Hiyama, Kino, Kamimura  
PPNP51(2003)223

$$\frac{\partial \langle H - E \rangle}{\partial C_k} = 0, \quad \frac{\partial \langle H - E \rangle}{\partial b_{lj,n}} = 0$$

c.m. excitation is excluded  
by Lawson's method

# Unitary Correlation Operator Method

(short-range part)

$$\Psi_{\text{corr.}} = \textcolor{red}{C} \cdot \Phi_{\text{uncorr.}}$$

TOSM

short-range correlator

$$C^\dagger = C^{-1} \quad (\text{Unitary trans.})$$

$$H\Psi = E\Psi \rightarrow C^\dagger H C \Phi \equiv \hat{H}\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance

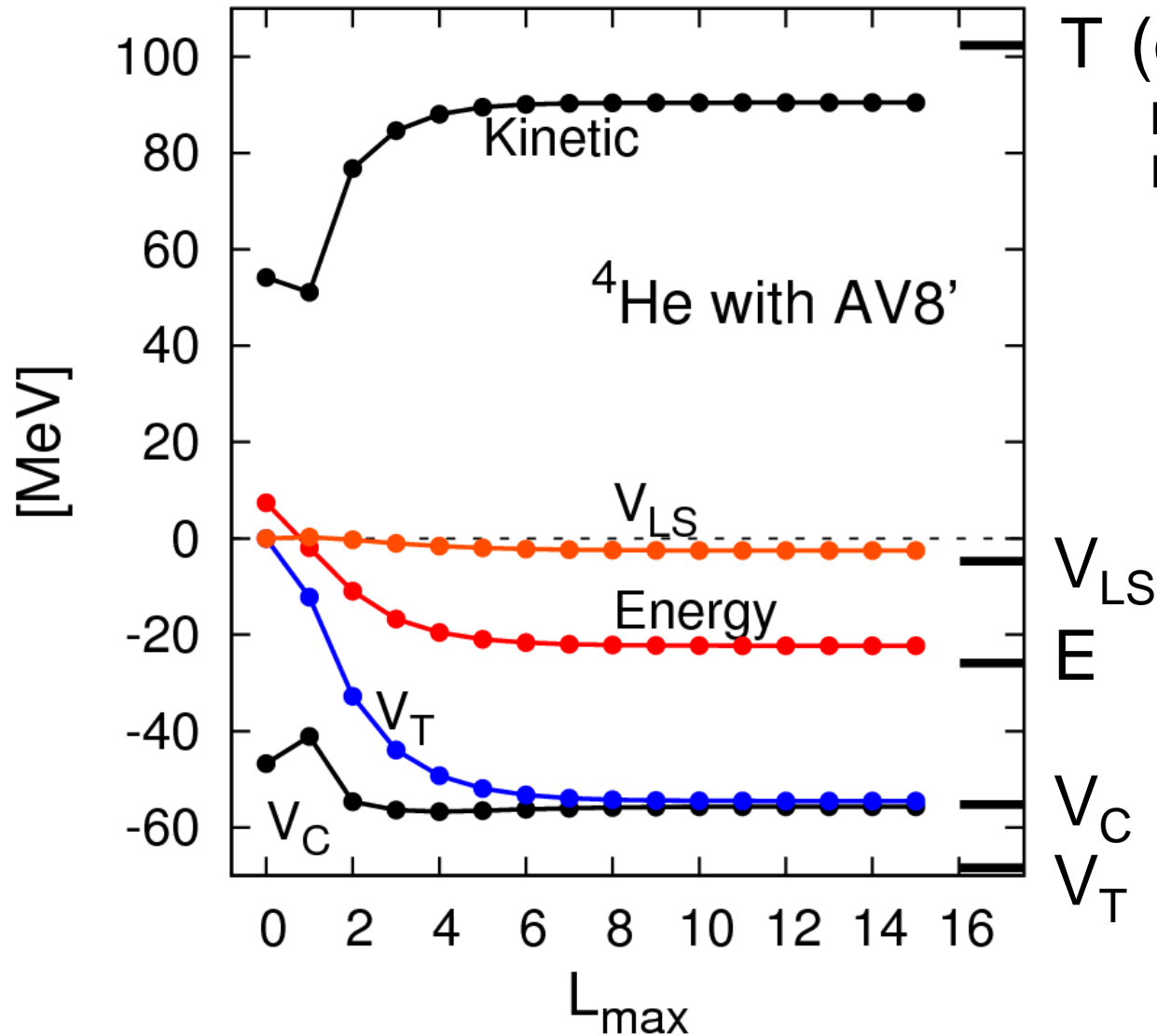
$$C = \exp(-i \sum_{i < j} g_{ij}), \quad g_{ij} = \frac{1}{2} \{ \underline{p_r s(r_{ij})} + \underline{s(r_{ij}) p_r} \} \quad \vec{p} = \vec{p}_r + \vec{p}_\Omega$$

Amount of shift, variationally determined.

$$C^\dagger r C \simeq r + s(r) + \frac{1}{2} s(r) s'(r) \dots$$

2-body cluster expansion

# $^4\text{He}$ in TOSM + short-range UCOM

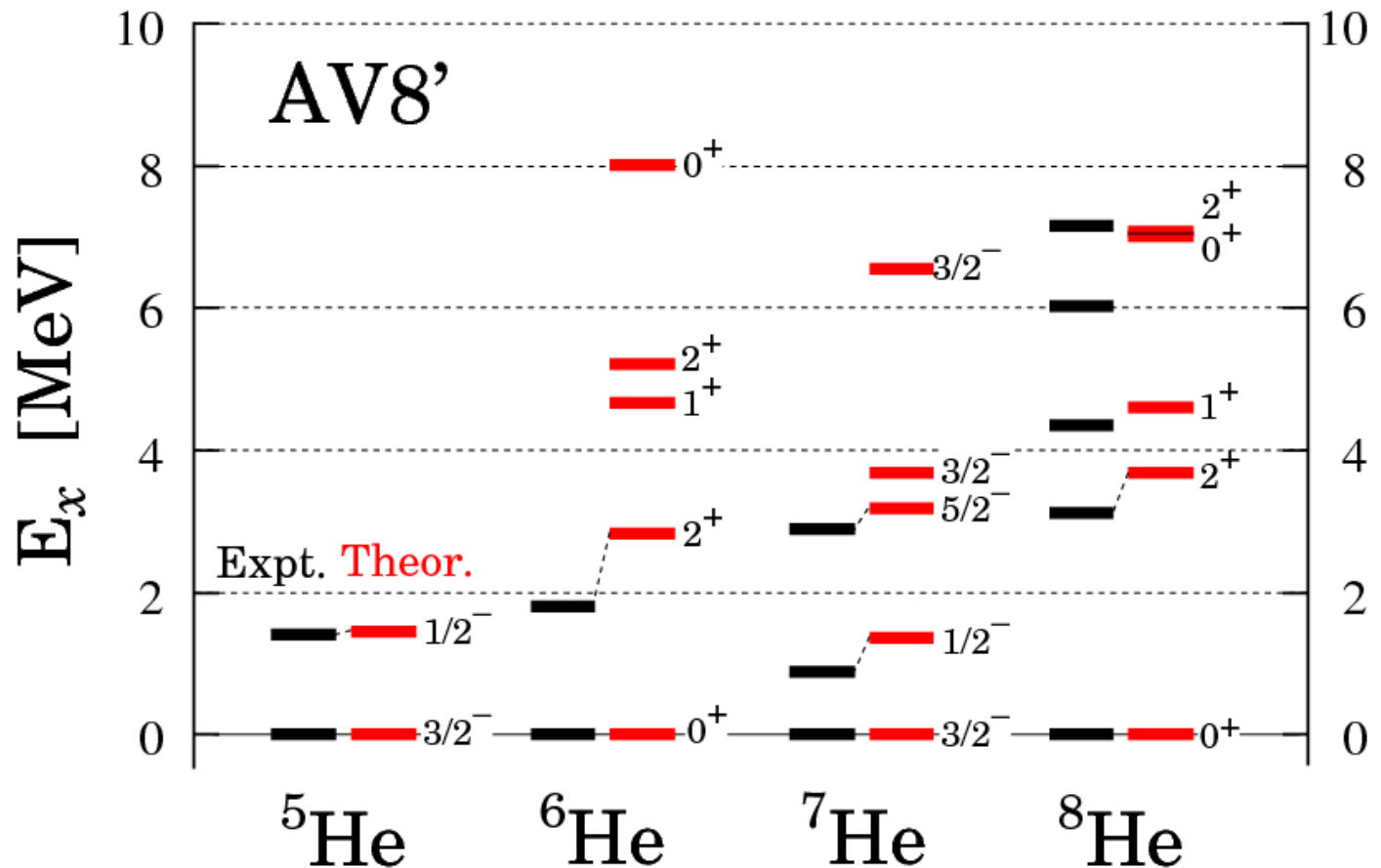


good convergence

# $5\text{-}^8\text{He}$ with TOSM+UCOM

TM, A. Umeya, H. Toki, K. Ikeda  
PRC84 (2011) 034315

- Excitation energies in MeV

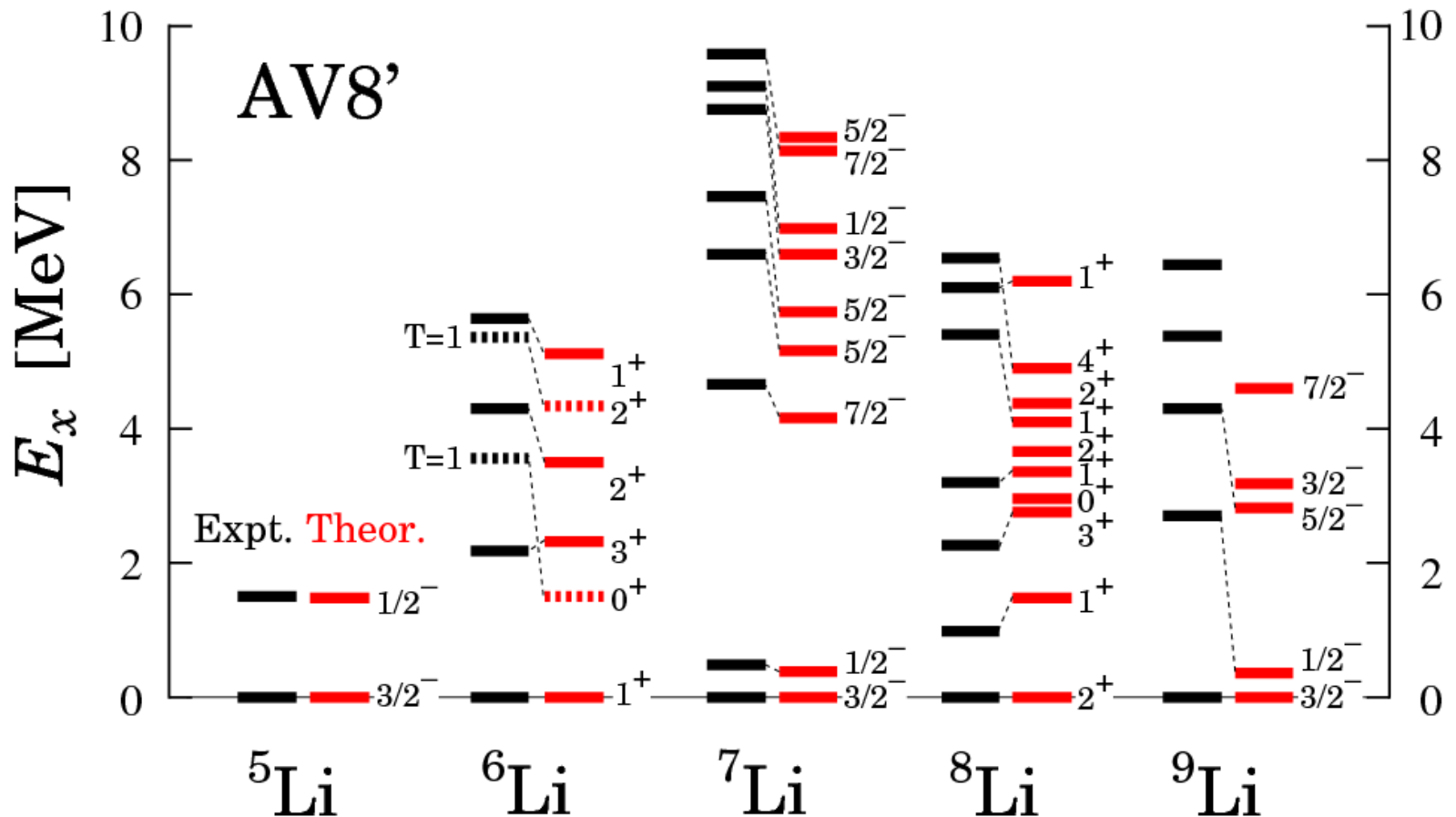


- Bound state app.
- No  $V_{\text{NNN}}$
- Excitation energy spectra are reproduced well

# ${}^5\text{-}{}^9\text{Li}$ with TOSM+UCOM

- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda  
PRC86(2012) 024318



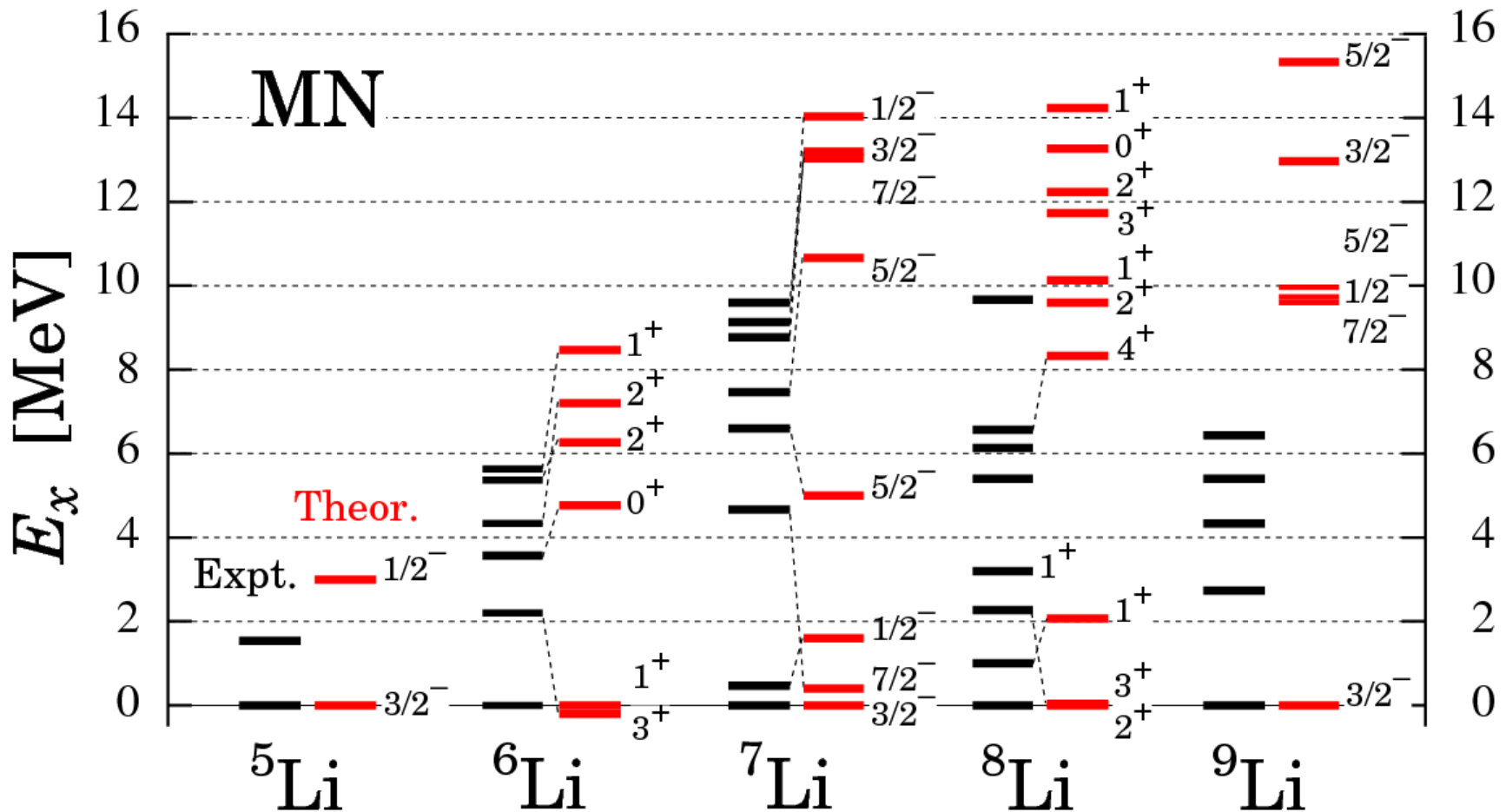
- Excitation energy spectra are reproduced well



# $^{5-9}\text{Li}$ with TOSM

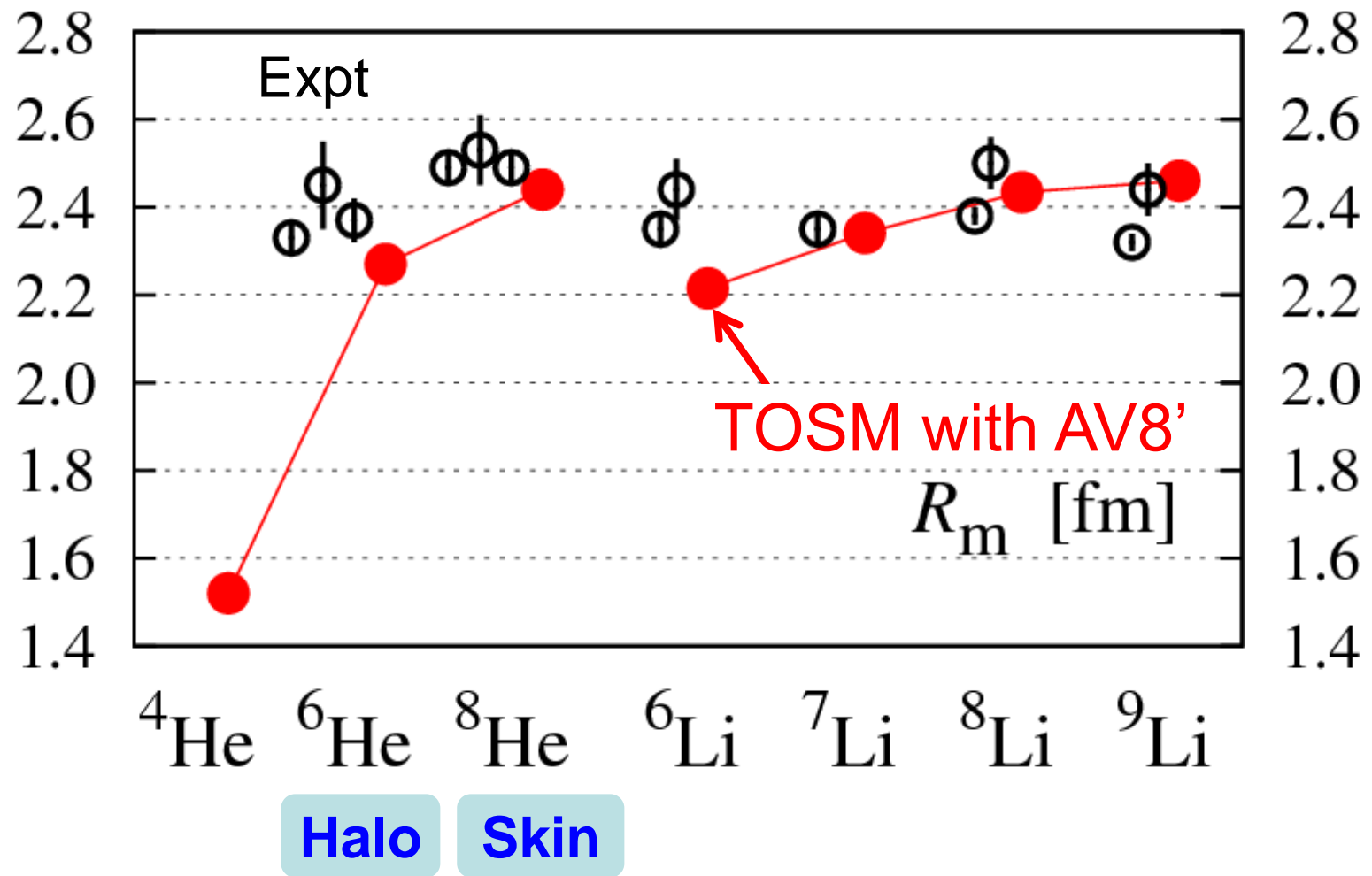
Minnesota force  
**NO tensor**

- Excitation energies in MeV



- Too large excitation energy

# Matter radius of He & Li isotopes



I. Tanihata et al., PLB289('92)261

O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215.

A. Dobrovolsky, NPA 766(2006)1

G. D. Alkhazov et al., PRL78('97)2313

P. Mueller et al., PRL99(2007)252501

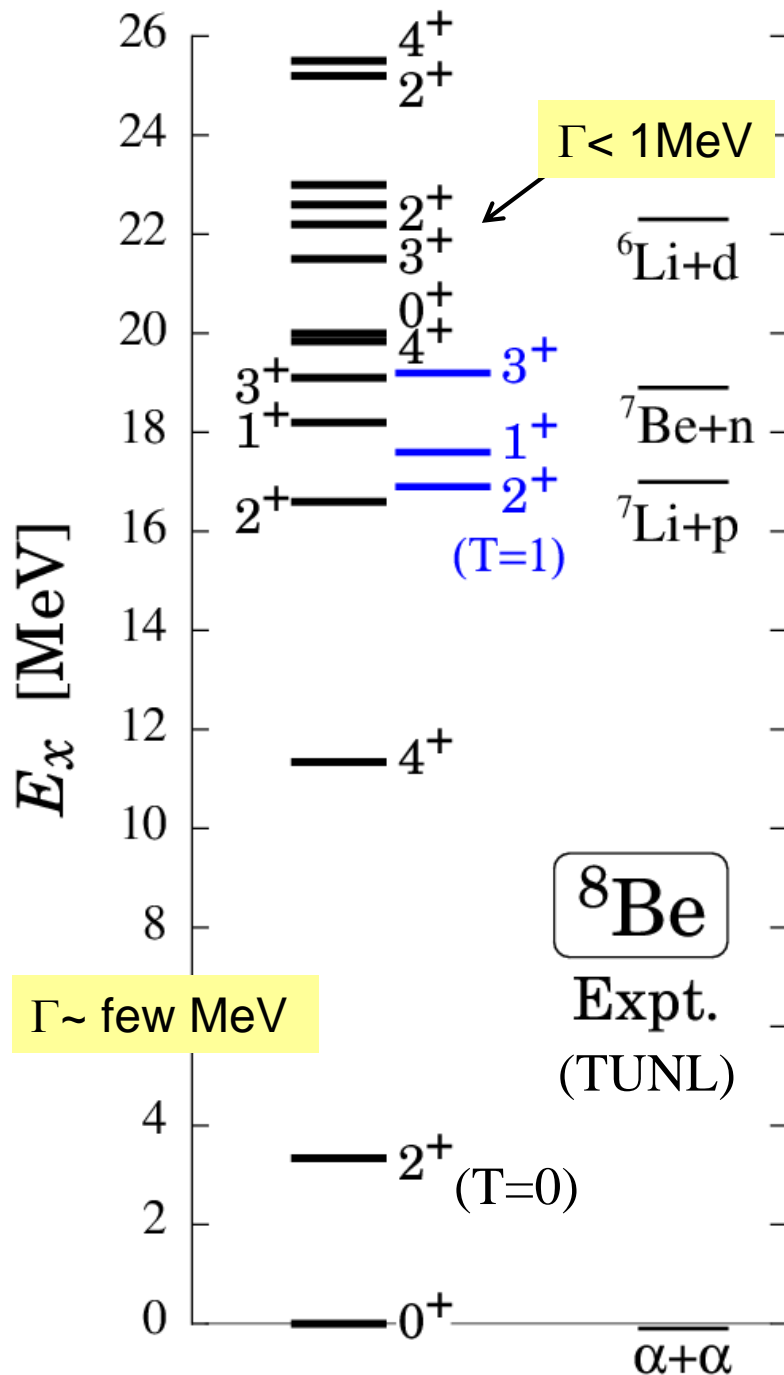
# Be isotopes

$^8\text{Be}$

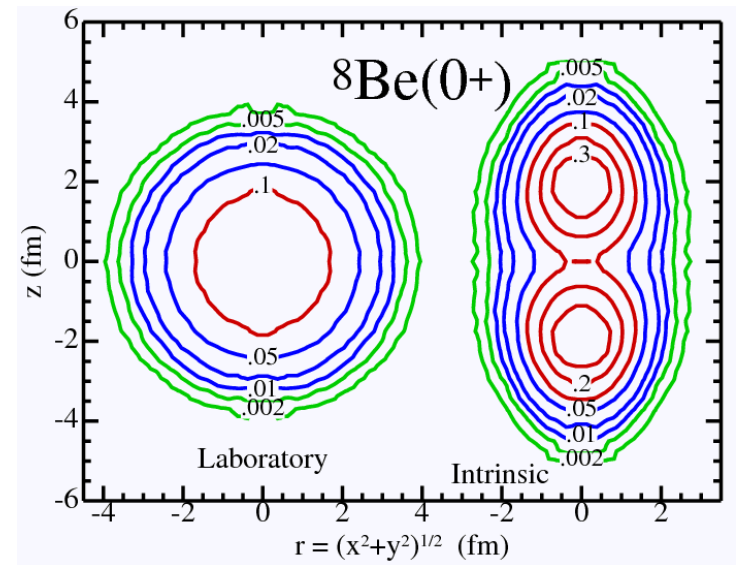
TM, A. Umeya, K. Horii, H. Toki, K. Ikeda  
PTEP (2014), in press

$^9\text{Be}, ^{10}\text{Be}$  JPS meeting

# $^8\text{Be}$ spectrum



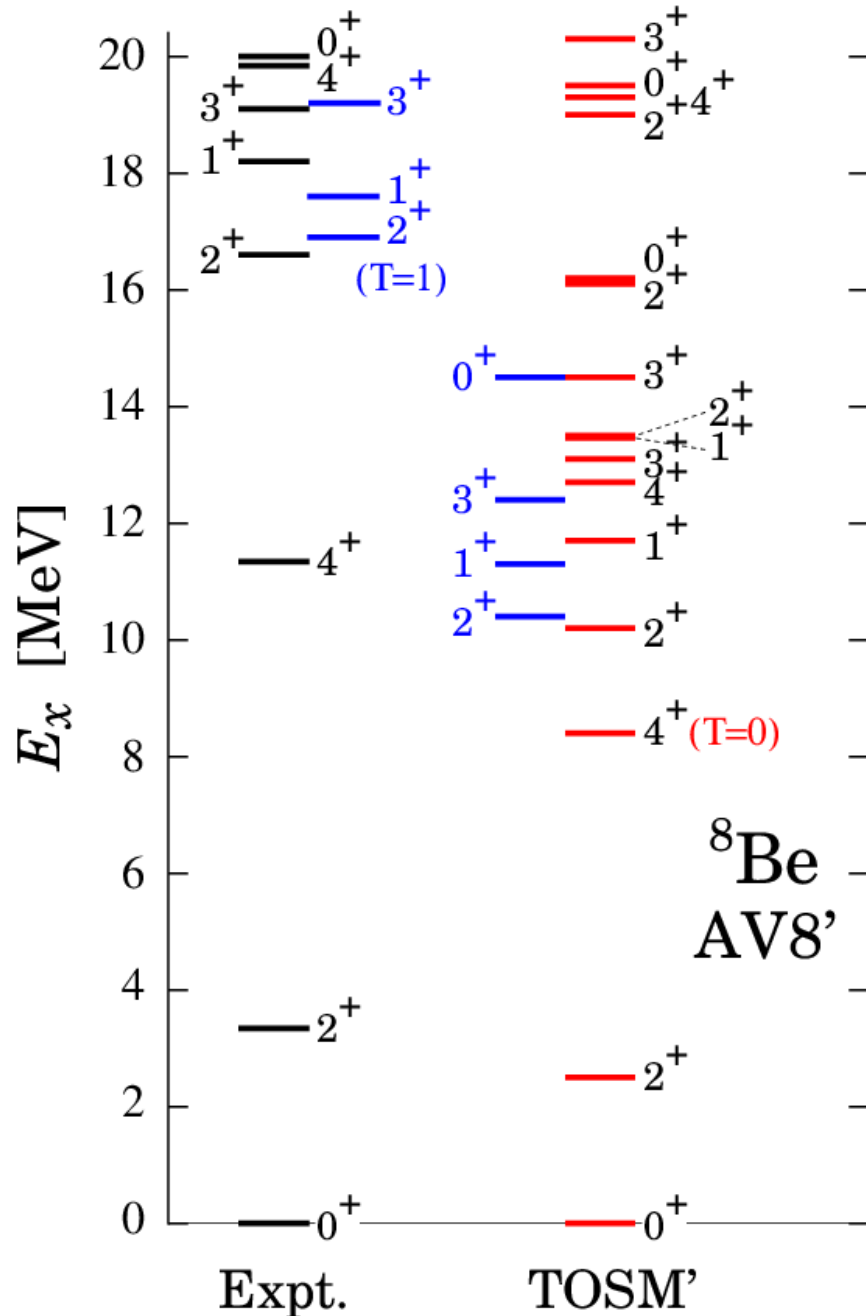
- Argonne Group
  - Green's function Monte Carlo  
C. Pieper, R.B. Wiringa,  
Annu. Rev. Nucl. Part. Sci. 51 (2001)



$\alpha$ - $\alpha$  structure

# $^8\text{Be}$ in TOSM

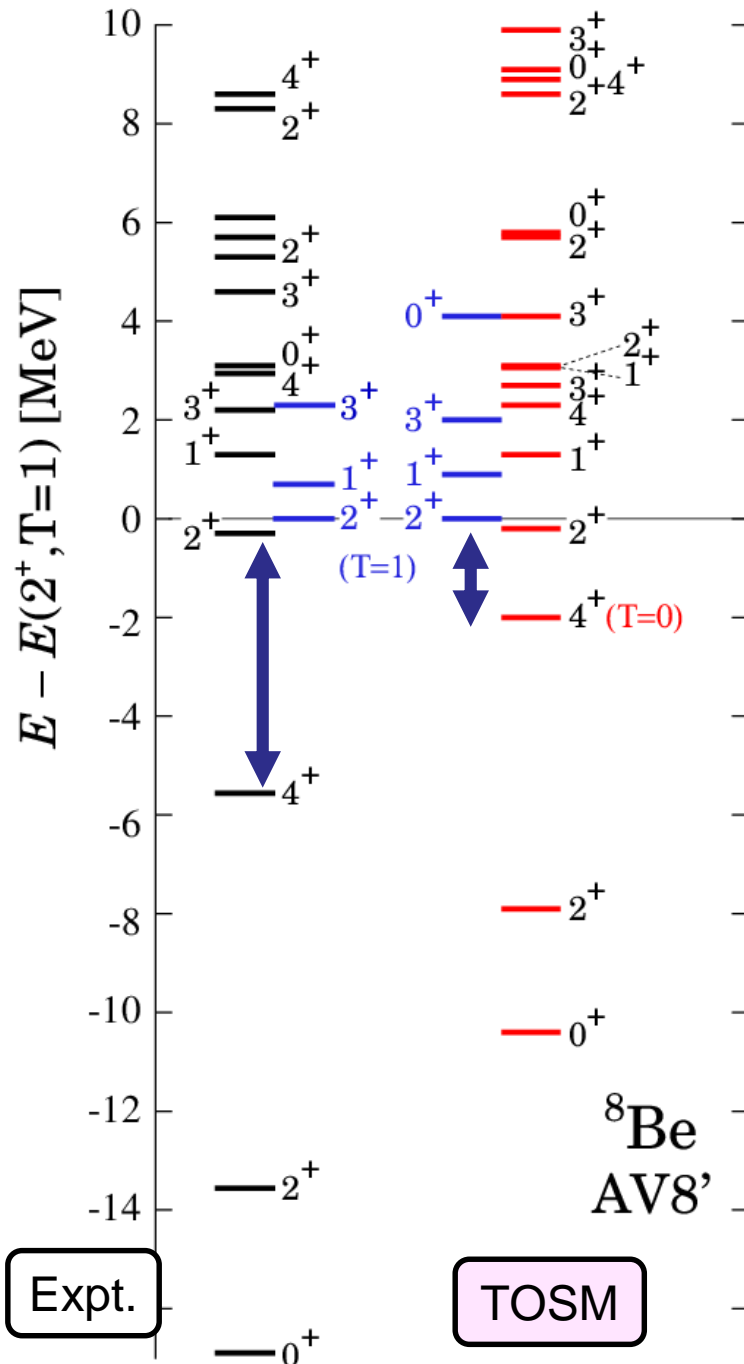
## – AV8' –



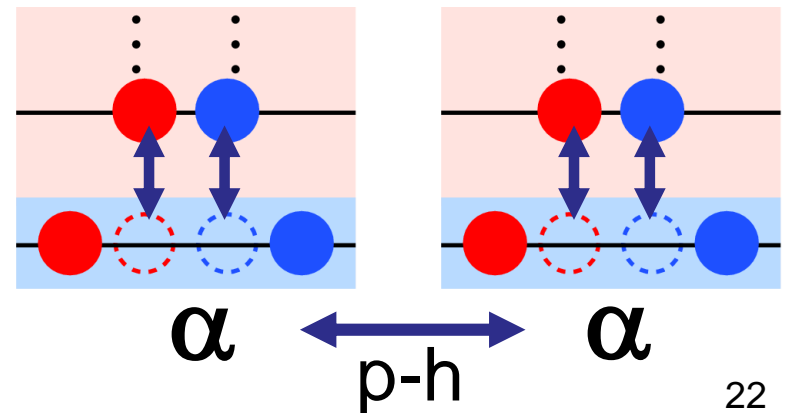
- $V_T \times 1.1, V_{LS} \times 1.4$ 
  - simulate  $^4\text{He}$  benchmark (Kamada et al., PRC64)
- ground band
- highly excited states
  - small  $E_x$
  - correct level order (T=0,1)
- $R_m(^8\text{Be})=2.21$  fm
  - Brink  $2\alpha$  model: 2.48 fm
  - $^4\text{He}$  :1.52 fm
  - $^{12}\text{C}$  :2.35 fm  $\sim R_m(\text{exp})$

# $^8\text{Be}$ in TOSM

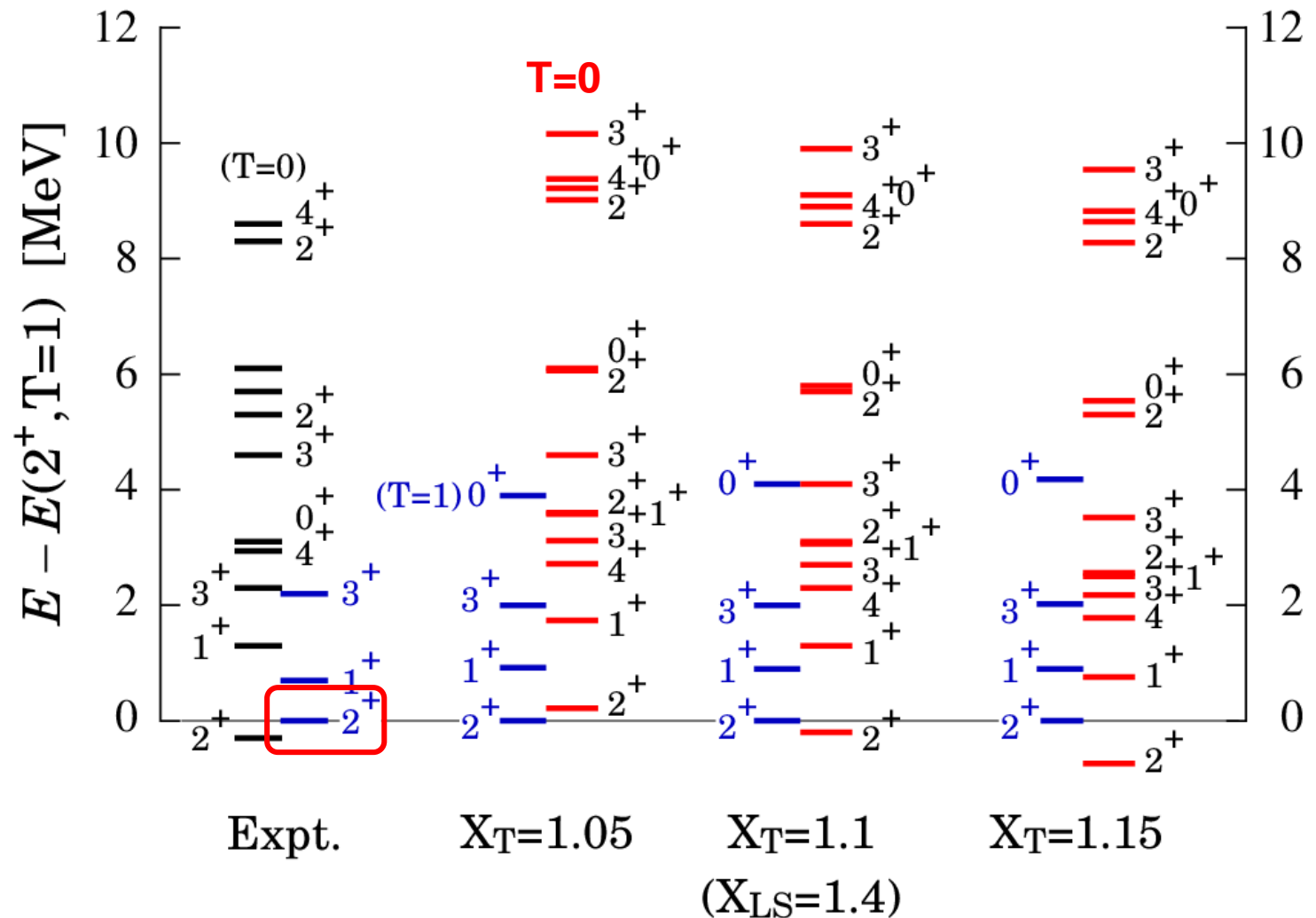
## – AV8' –



- $V_T \times 1.1, V_{LS} \times 1.4$ 
  - simulate  $^4\text{He}$  benchmark (Kamada et al., PRC64)
- correct level order ( $T=0,1$ )
- $\alpha$  :  $0p0h+2p2h$  with high- $k$ 
  - naively  $2\alpha$  needs  $4p4h$ .



# $V_{\text{tensor}}$ dependence of ${}^8\text{Be}$



- S-wave UCOM can be simulated with  $X_T \sim 1.1$  (PTP121)
- Stronger tensor correlation in **T=0 states** than T=1 states.

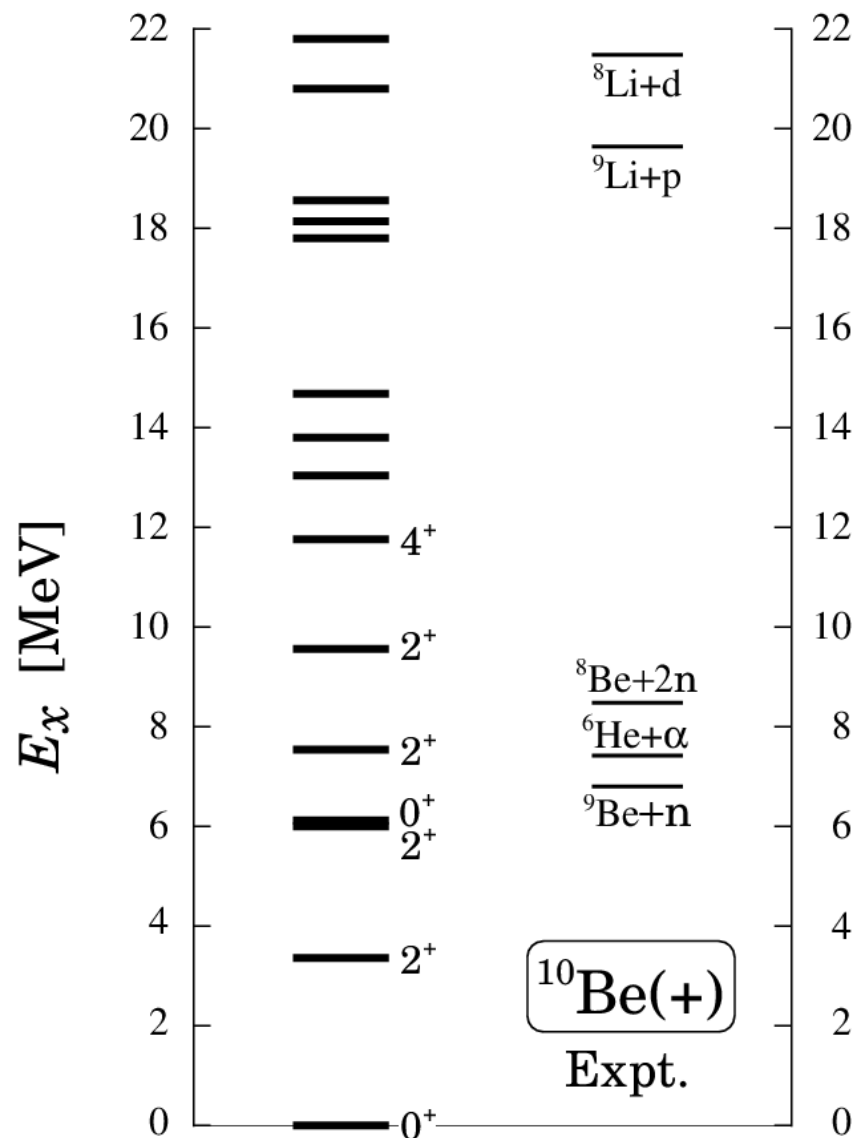
# Hamiltonian components in $^8\text{Be}$

State		Kinetic	Central	Tensor
$^4\text{He}$		95	-56	-62
$^8\text{Be}$	$0^+_1$	192	-115	-97
	$2^+_1$	191	-112	-95
	$2^+_2$	185	-98	-92
	$2^+_{T=1}$	168	-94	-82

- Grand state
  - Kinetic & Central  
~ twice of  $^4\text{He}$
  - Tensor ~ 1.6 of  $^4\text{He}$
  - larger  $\langle H \rangle$  components than highly excited states.
- Kinetic & Tensor
  - $T=0$  states >  $T=1$  states

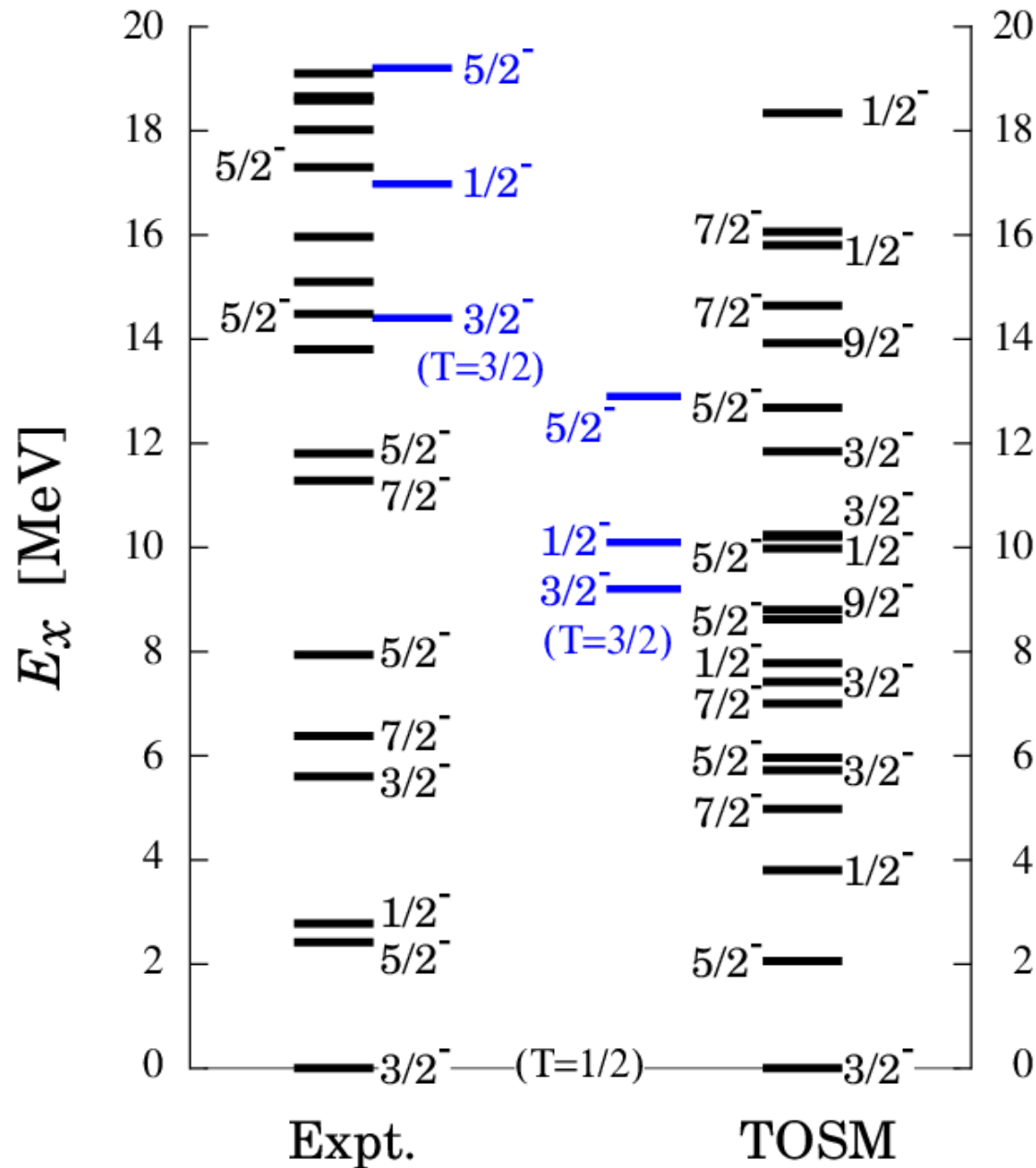


# TUNL

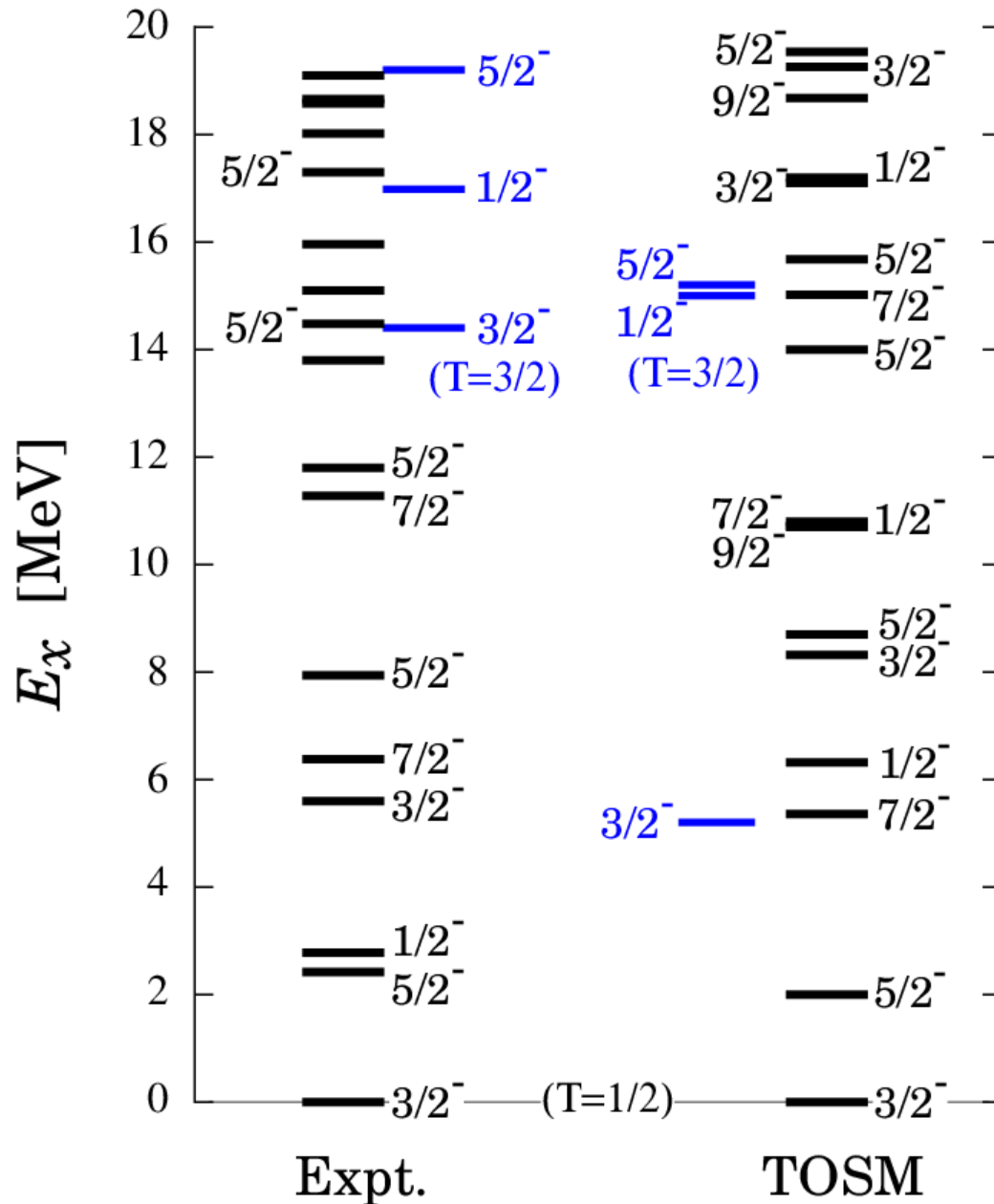


# <sup>9</sup>Be in TOSM – AV8' –

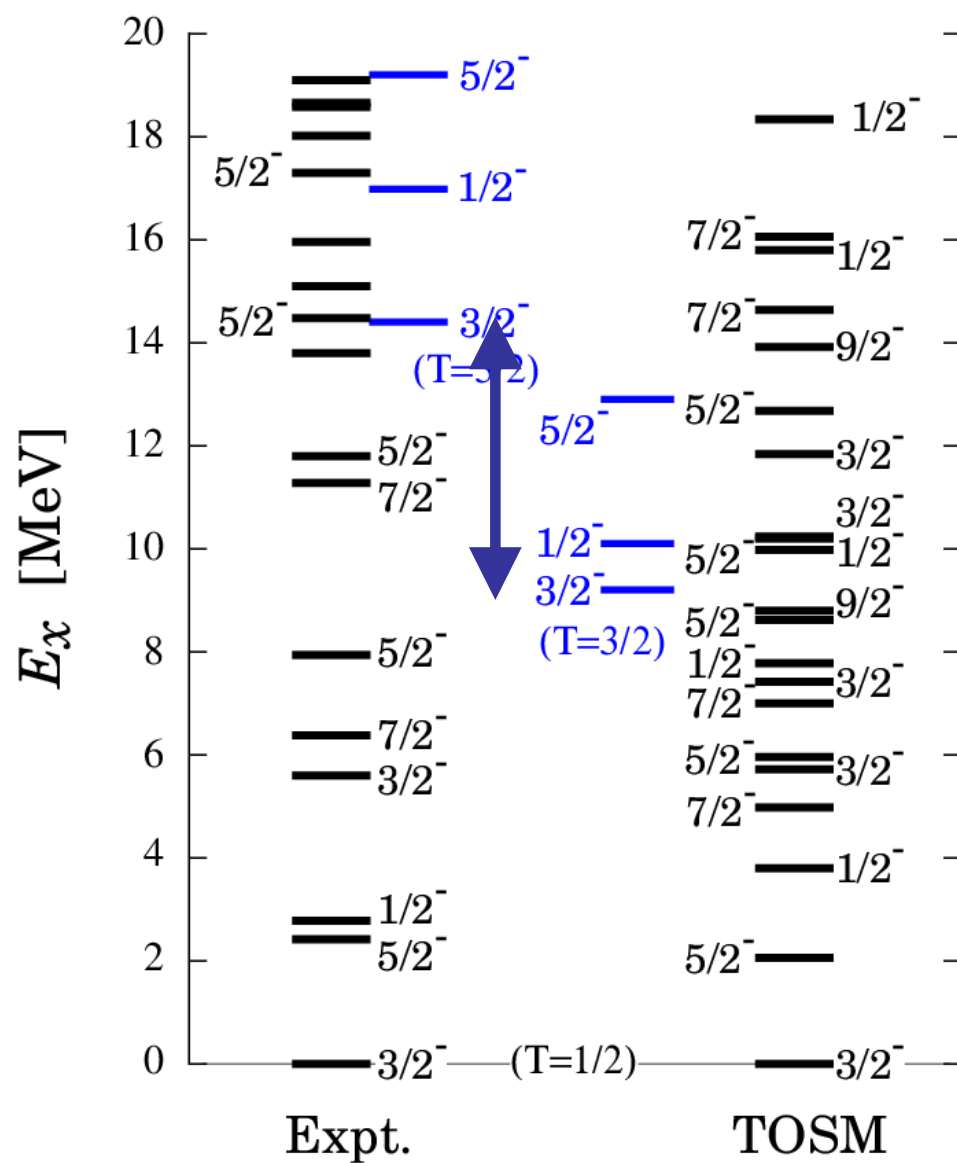
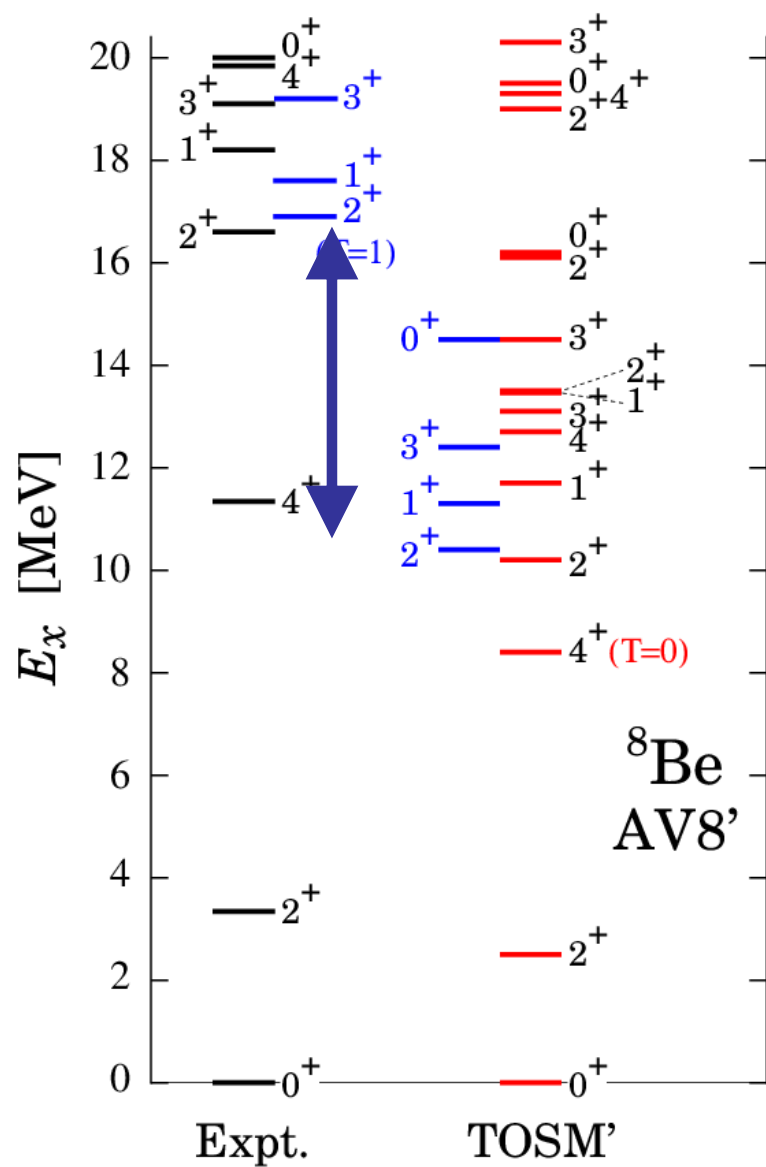
*Preliminary*



# $^9\text{Be}$ in TOSM – Minnesota –



# $^8\text{Be}$ & $^9\text{Be}$



# Preliminary



# Toward the description of cluster states

- **Tensor Optimized Shell Model (TOSM)**
  - TM, A.Umeya, H. Horii, H. Toki, K. Ikeda
  - He, Li, Be isotopes
- **Tensor Optimized Few-body Model (TOFM)**
  - K. Horii, H.Toki, TM, K. Ikeda, PTP127(2012)1019
- **Tensor Optimized AMD (Tensor-AMD)**
  - Clustering & tensor force

# Formulation of Tensor-AMD

$$|\Phi_{\text{T-AMD}}\rangle = C_0 |\Phi_{\text{AMD}}\rangle + \sum_{i < j}^A \sum_{S,T} F_{ij}^{ST}(\vec{r}_{ij}) |\Phi'_{\text{AMD}}\rangle$$

$$F^{ST}(\vec{r}) = r^2 S_{12} \sum_n C_n^{ST} \exp(-\rho_n^{ST} r^2)$$

- Variational parameters
  - $\nu, \mathbf{Z}_i$  ( $i=1, \dots, A$ ), spin-direction (up/down)
  - $C_0, C_n^{ST}, \rho_n^{ST}$  (Gaussian expansion)
  - Tensor-type correlation for **relative motion**
  - Decided by using cooling equation + parity projection.

# Tensor matrix elements

$$|\Phi_{\text{AMD}}\rangle = \frac{1}{\sqrt{A!}} \det \{ \varphi_1, \dots, \varphi_A \}$$

$$|\varphi\rangle = |\mathbf{Z}\rangle |\chi^{\sigma\tau}\rangle$$

$$\langle \mathbf{r} | \mathbf{Z} \rangle \propto \exp \left[ -\nu \left( \mathbf{r} - \frac{\mathbf{Z}}{\sqrt{\nu}} \right)^2 \right]$$

Matrix elements

$$\langle \varphi_i \varphi_j \dots | \hat{O} | \varphi_i' \varphi_j' \dots \rangle_A$$

Corr. func.(bra)

Hamiltonian

$$\hat{O} = S_{12} \cdot S_{12} \cdot S_{12}$$

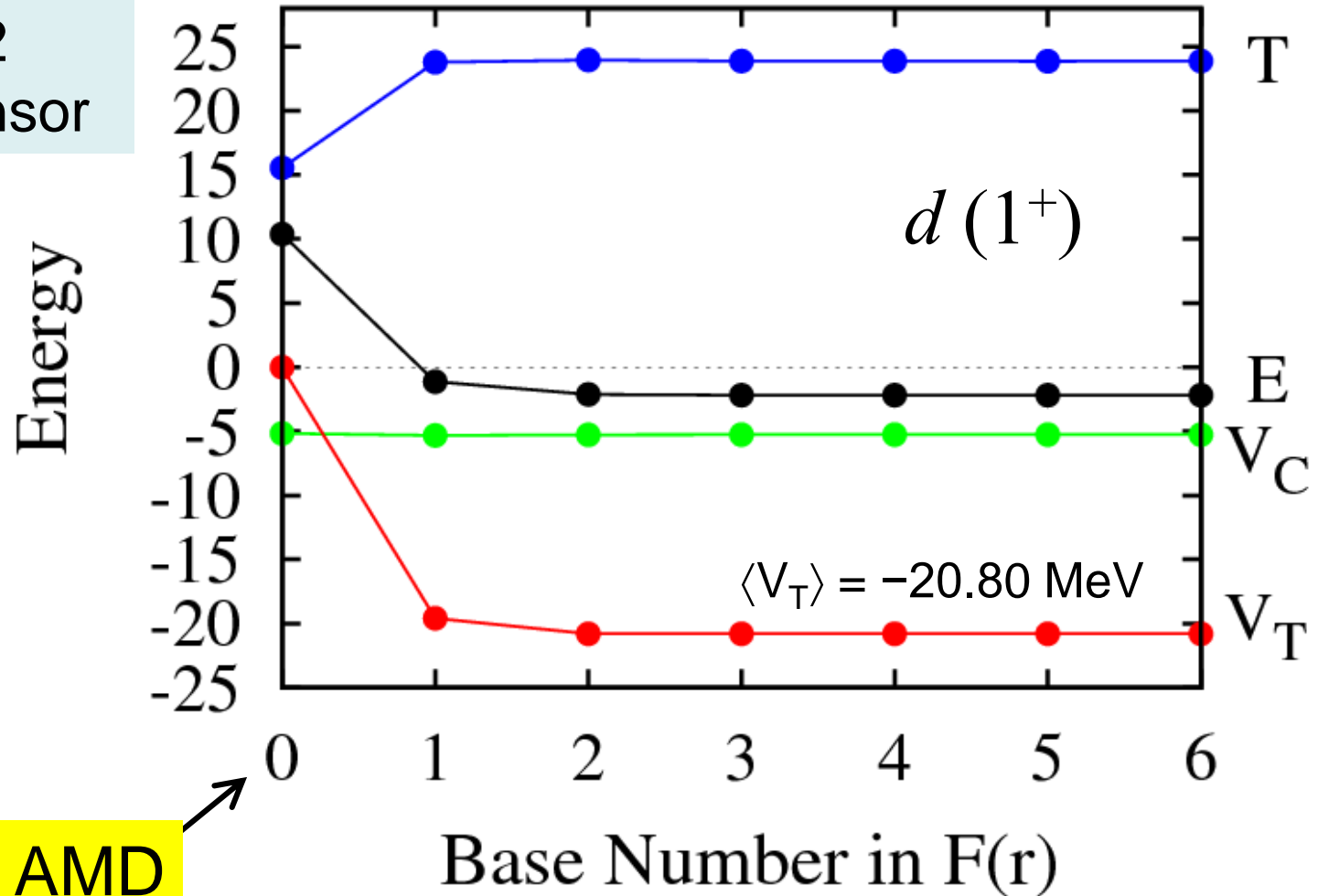
Corr. func.(ket)

- 6-body matrix elements within 2-body Hamiltonian.
- At most, 4-body matrix elements to be evaluated.
  - 6-body ME : {2-body ME} × {2-body ME} × {2-body ME}
  - 5-body ME : {3-body ME} × {2-body ME}



# Deuteron in Tensor-AMD

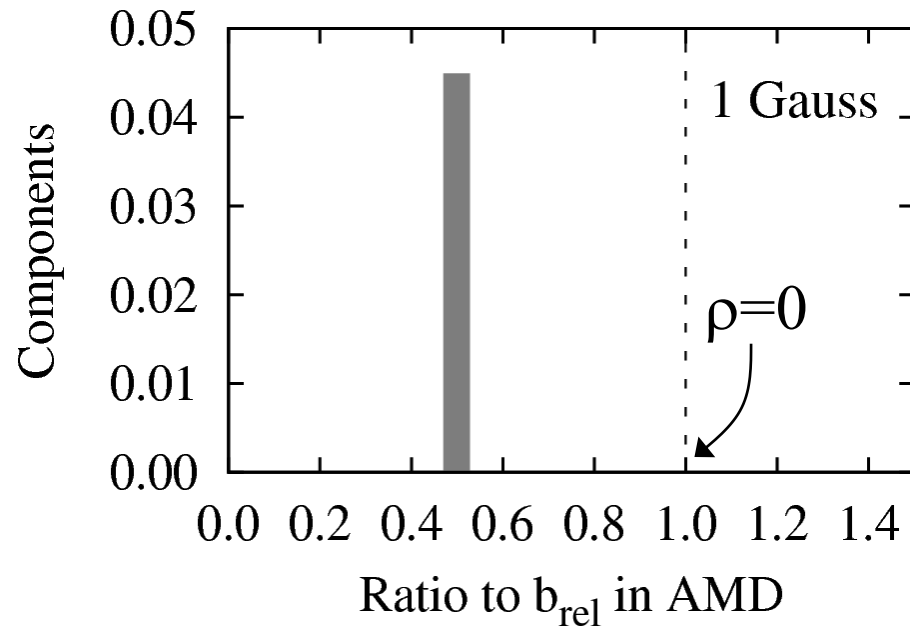
- Volkov No.2
- Furutani tensor



- Good convergence

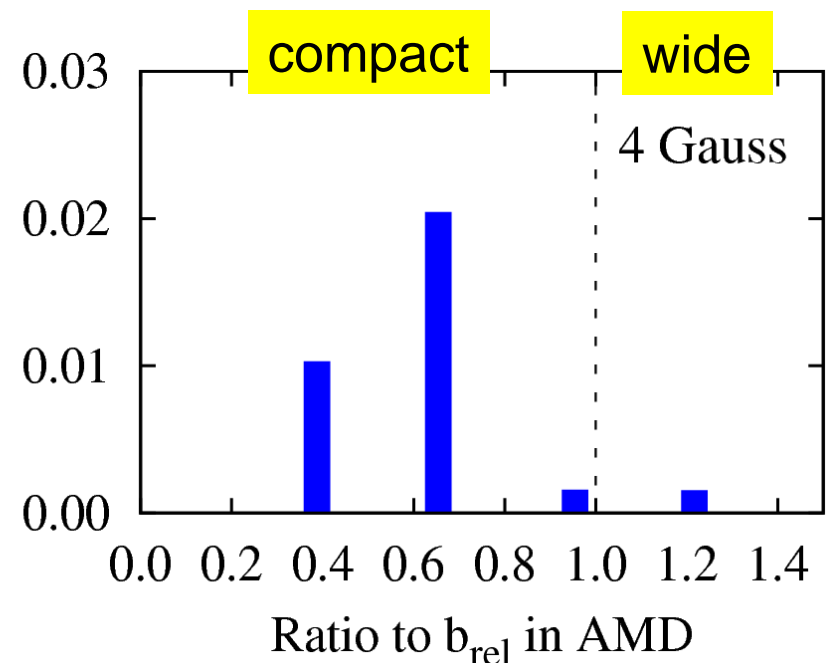
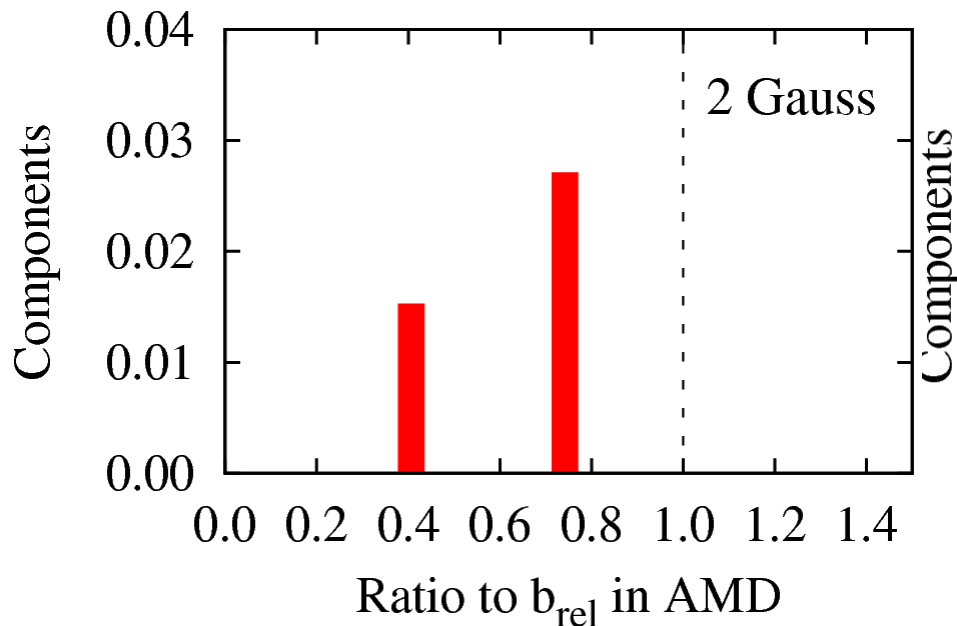
$$F(r) = r^2 S_{12} \sum_n C_n \exp(-\rho_n r^2)$$

# Gaussian expansion in $F(r)$



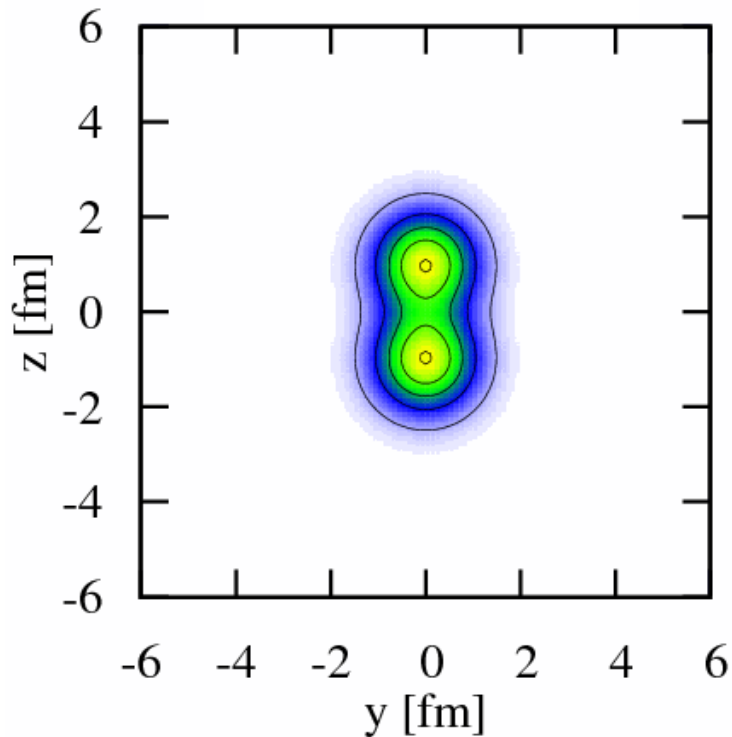
$$F(r) = r^2 S_{12} \sum_n C_n \exp(-\rho_n r^2)$$

- Ratio to the Gaussian length (fm) of AMD basis
- Compact component is dominant in relative  $D$ -state



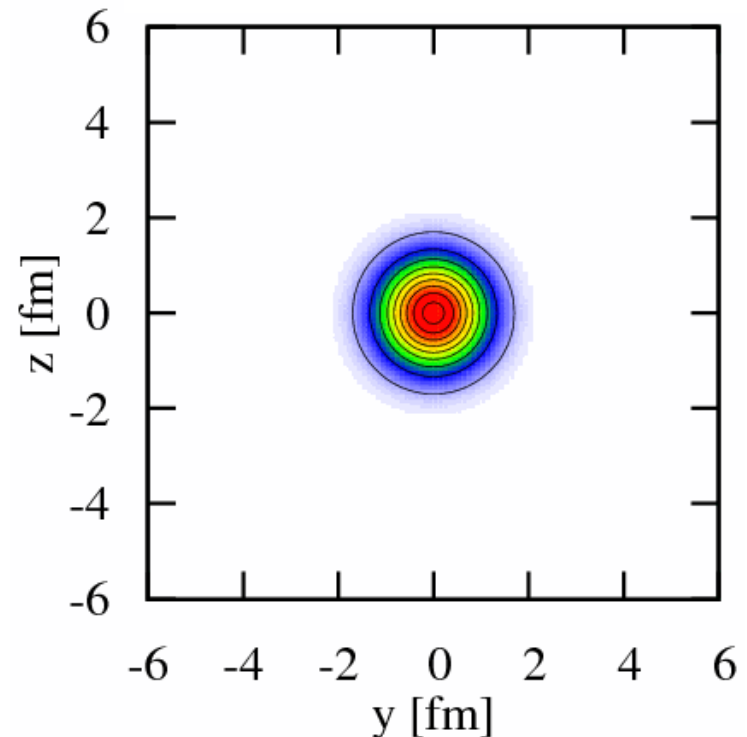
# Intrinsic density of deuteron

AMD



$$\langle V_T \rangle_{\text{AMD}} = -0.63 \text{ MeV}$$

AMD part  
in Tensor-AMD



Describe S-wave in  $d$

$$\langle V_T \rangle_{\text{T-AMD}} = -20.80 \text{ MeV}$$

# Summary

- **TOSM+UCOM** using  $V_{\text{bare}}$ .
  - strong tensor correlation from 0p0h-2p2h.
- He & Li isotopes
  - energy spectra, radius
  - $^4\text{He}$  contains “ **$pn$ -pair of  $p_{1/2}$** ” due to  $V_T$ .
- Be isotopes
  - $^8\text{Be}$ : grand band & highly excited states ( $T=0$  &  $T=1$ ). similar tendency in  $^9\text{Be}$ .
  - more configurations of such as 4p4h to describe  $2\alpha$  structure in the grand band.