Role of tensor force in light nuclei with tensor-optimized shell model

Takayuki MYO
Osaka Institute of Technology

In collaboration with Atsushi UMEYA (Nippon Inst. Tech.)
Hiroshi TOKI, Kaori HORII (RCNP)
Kiyomi IKEDA (RIKEN)

Outline

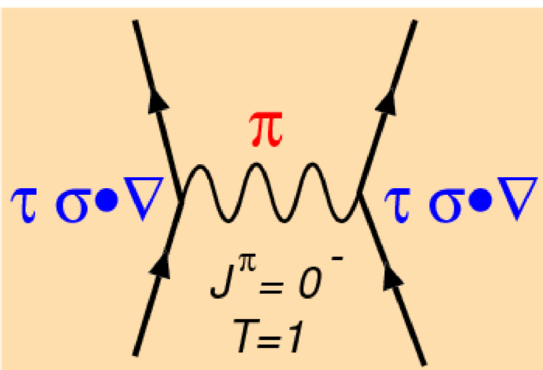
- **Role of V_{tensor}** in light nuclei
  - He, Li, Be isotopes
- Tensor Optimized Shell Model (**TOSM**) to describe tensor correlation.
- Unitary Correlation Operator Method (**UCOM**) to describe short-range correlation.

TM, A. Umeya, H. Toki, K. Ikeda PRC84 (2011) 034315
TM, A. Umeya, H. Toki, K. Ikeda PRC86 (2012) 024318
TM, A. Umeya, K. Horii, H. Toki, K. Ikeda PTEP (2014) 033D01

Pion exchange interaction vs. V_{tensor}

$$3(\vec{\sigma}_1\cdot\hat{q})(\vec{\sigma}_2\cdot\hat{q})\frac{q^2}{m^2+q^2}=(\vec{\sigma}_1\cdot\vec{\sigma}_2)\frac{q^2}{m^2+q^2}+S_{12}\frac{q^2}{m^2+q^2}$$

$$=(\vec{\sigma}_1\cdot\vec{\sigma}_2)\left[\frac{m^2+q^2}{m^2+q^2}-\frac{m^2}{m^2+q^2}\right]+S_{12}\frac{q^2}{m^2+q^2}$$



δ interaction

Yukawa interaction

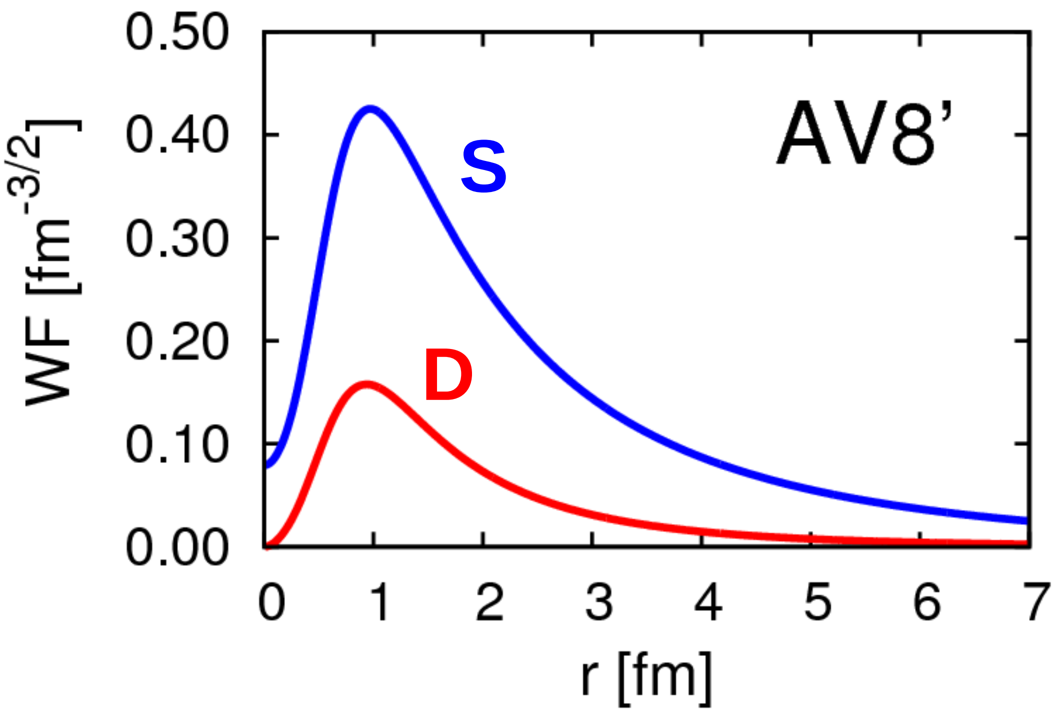
involve large momentum

Tensor operator

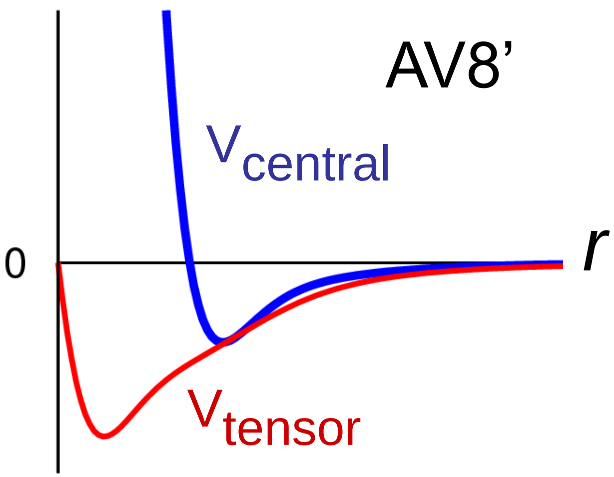
$$S_{12}=3(\vec{\sigma}_1\cdot\hat{q})(\vec{\sigma}_2\cdot\hat{q})-(\vec{\sigma}_1\cdot\vec{\sigma}_2)$$

- V_{tensor} produces the high momentum component.
- ***Horii*** (4P-2B) treats N→Δ transition in light nuclei.

Deuteron properties & tensor force



Energy	-2.24 MeV
Kinetic	19.88
Central	-4.46
Tensor	**-16.64**
LS	-1.02
P(L=2)	5.77%
Radius	1.96 fm



$R_m(s)$=2.00 fm

$R_m(d)$=1.22 fm

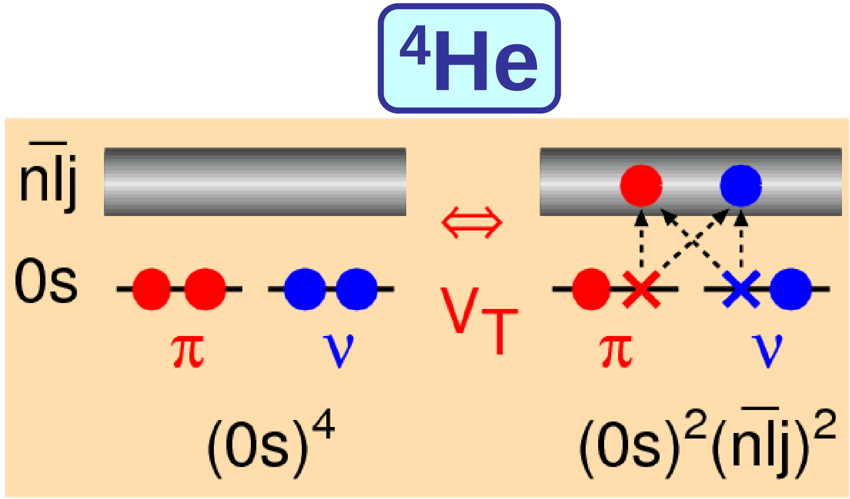
d-wave is "**spatially compact**" (high momentum)

Tensor-optimized shell model (TOSM)

TM, Sugimoto, Kato, Toki, Ikeda PTP117(2007)257

- 2p2h excitations with high-L orbits.
- V_{tensor} is **NOT** treated as residual interactions

  cf. $\dfrac{V_\pi}{V_{NN}}$ ~ 80% in GFMC



- Length parameters such as b_{0s}, b_{0p}, … are optimized independently, or superposed by many Gaussian bases.
  - spatial shrinkage of ***D*-wave** as seen in deuteron.
    HF (Sugimoto, NPA740) , RMF (Ogawa, PRC73), AMD (Dote et al., PTP115)
- Satisfy few-body results with Minnesota central force (^{4,6}He)

Hamiltonian and variational equations in TOSM

$$H = \sum_{i=1}^{A} t_i - T_G + \sum_{i<j}^{A} v_{ij},$$

(0p0h+1p1h+2p2h)

$$\Phi(A) = \sum_k C_k \cdot \psi_k(A)$$

Shell model type configuration with mass number A

Particle state : Gaussian expansion for each orbit

$$\varphi_{lj}^{n'}(\mathbf{r}) = \sum_{n=1}^{N} C_{lj,n}^{n'} \cdot \phi_{lj,n}(\mathbf{r})$$

$$\phi_{lj,n}(\mathbf{r}) \propto r^l \exp\left[-\frac{1}{2}\left(\frac{r}{b_{lj,n}}\right)^2\right]\left[Y_l(\hat{\mathbf{r}}), \chi_{1/2}^{\sigma}\right]_j$$

$$\left\langle \varphi_{lj}^{n'} \middle| \varphi_{lj}^{n''} \right\rangle = \delta_{n',n''}$$

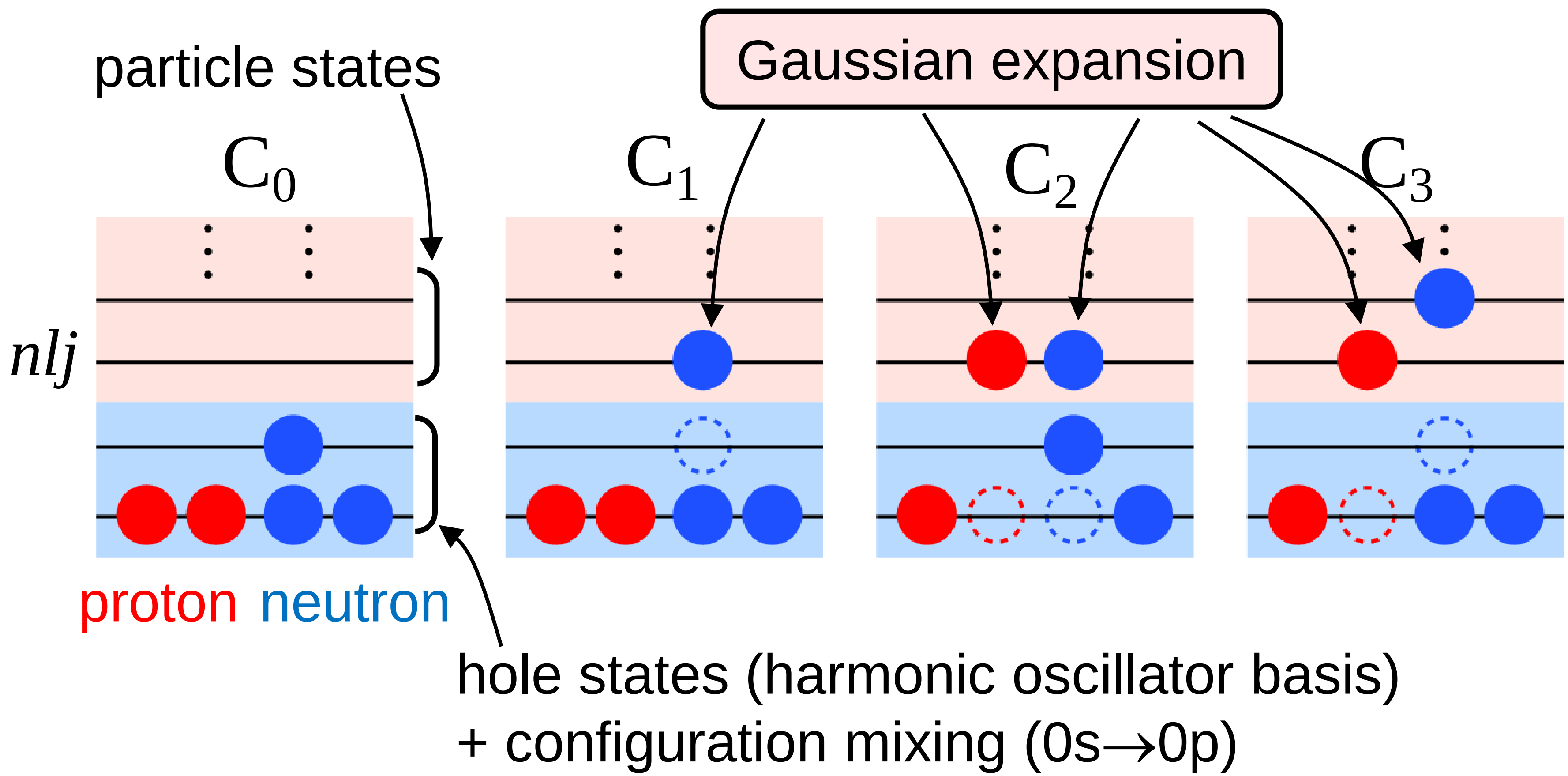
Gaussian basis function

Hiyama, Kino, Kamimura
PPNP51(2003)223

$$\frac{\partial \langle H-E \rangle}{\partial C_k} = 0, \quad \frac{\partial \langle H-E \rangle}{\partial b_{lj,n}} = 0$$

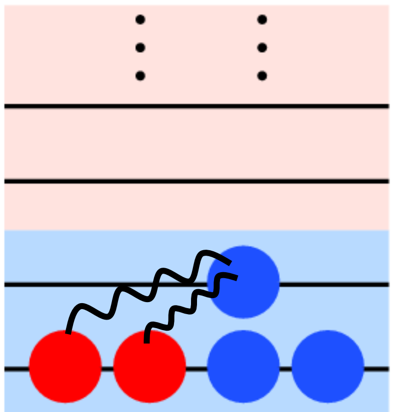
c.m. excitation is excluded by Lawson's method

Configurations in TOSM



Application to Hypernuclei to investigate ΛN-ΣN coupling by **Umeya** (NIT), **Hiyama** (RIKEN)

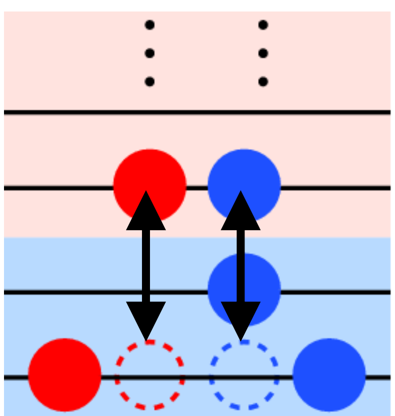
Tensor force matrix elements



$b_D \sim b_S$

$V_T = V_{residual}$

1st order



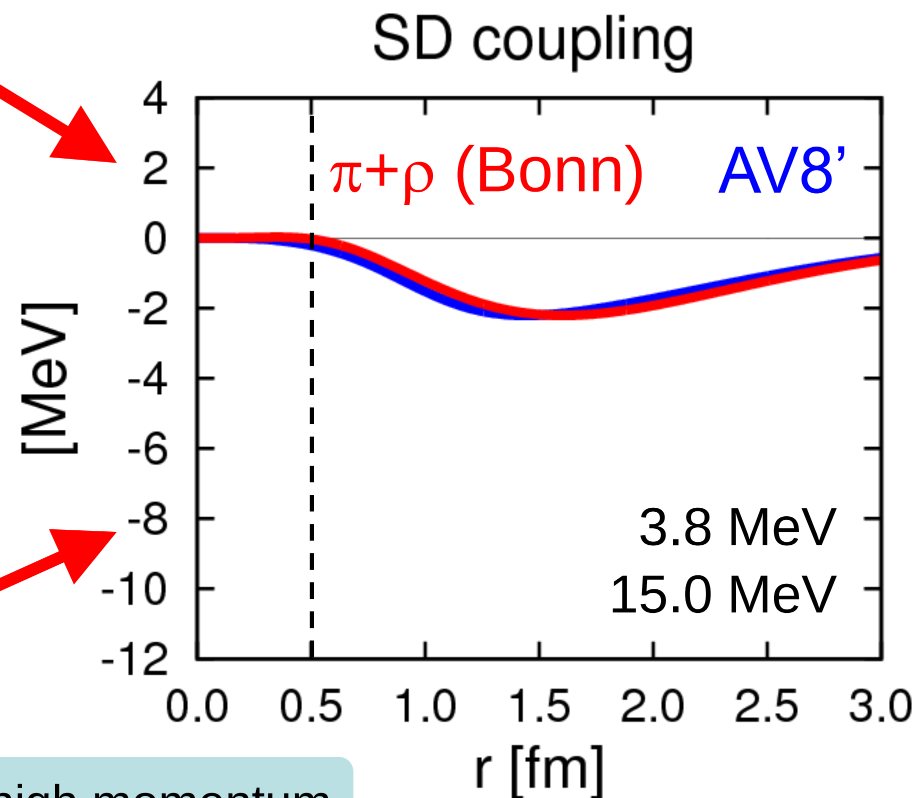
$b_D \sim b_S \times 0.5$

$V_T \neq V_{residual}$

0p0h-2p2h

$$M_{SD}(r) = r^2 \phi_S(r,b_S)\cdot V_T \cdot \phi_D(r,b_D)$$

Integrand of Tensor ME



high momentum

- Centrifugal potential (1GeV@0.5fm) pushes away *D*-wave.

Unitary Correlation Operator Method

(short-range part)

$$\Psi_{\text{corr.}} = C \cdot \Phi_{\text{uncorr.}}$$

TOSM

short-range correlator

$$C^{\dagger} = C^{-1} \quad \text{(Unitary trans.)}$$

$$H\Psi = E\Psi \quad \rightarrow \quad C^{\dagger} H C \Phi \equiv \hat{H}\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance

$$C = \exp(-i\sum_{i<j} g_{ij}), \qquad g_{ij} = \tfrac{1}{2}\left\{ p_r s(r_{ij}) + s(r_{ij}) p_r \right\} \qquad \vec{p} = \vec{p}_r + \vec{p}_\Omega$$

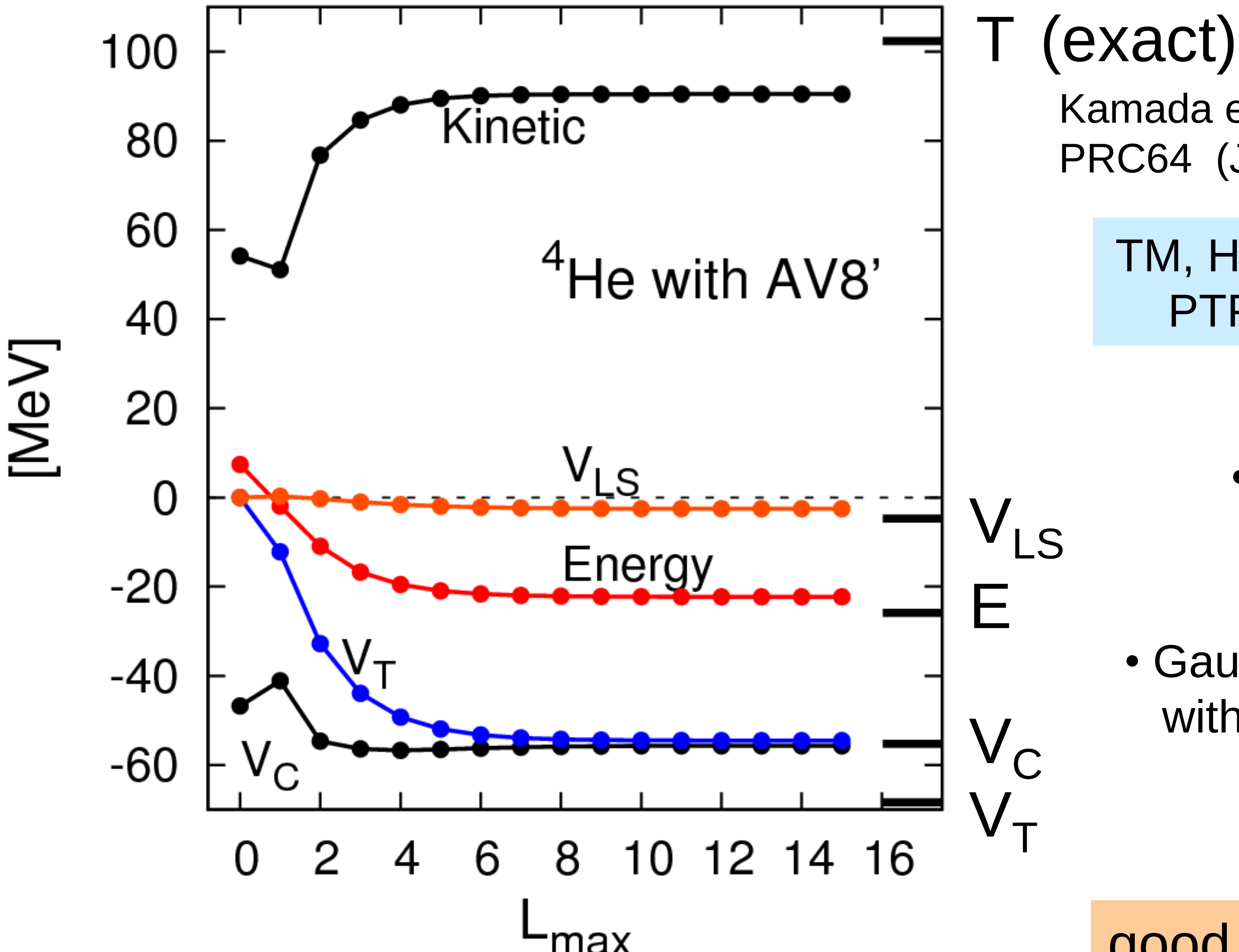
Amount of shift, variationally determined.

$$C^{\dagger} r C = r + s(r) + \tfrac{1}{2} s(r) s'(r) \cdots$$

2-body cluster expansion

H. Feldmeier, T. Neff, R. Roth, J. Schnack, NPA632(1998)61

^{4}He in TOSM + short-range UCOM



Kamada et al.
PRC64 (Jacobi)

TM, H. Toki, K. Ikeda
PTP121(2009)511

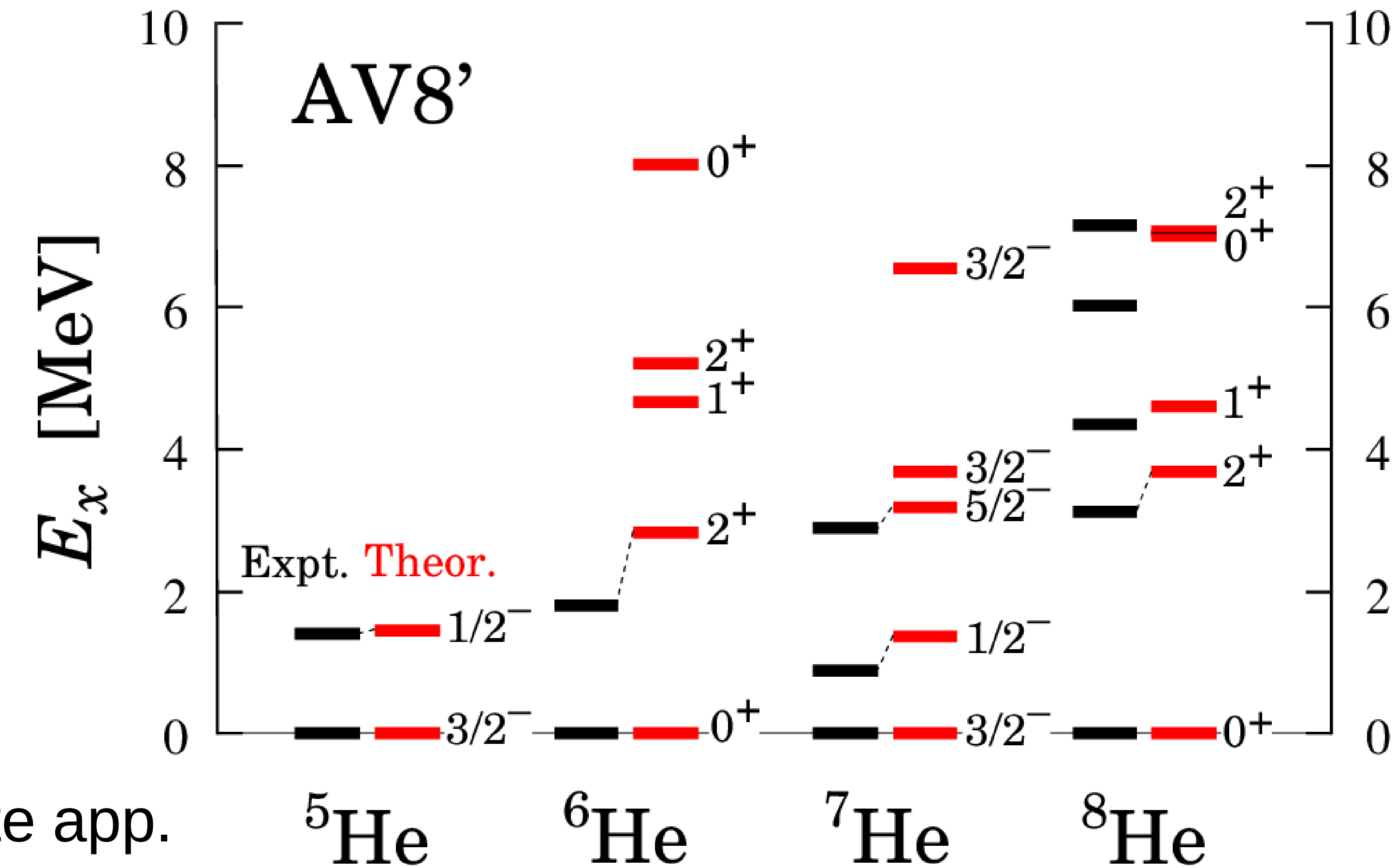
- variational calculation
- Gaussian expansion with 9 bases

good convergence

$^{5-8}$He with TOSM+UCOM

- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda
PRC84 (2011) 034315



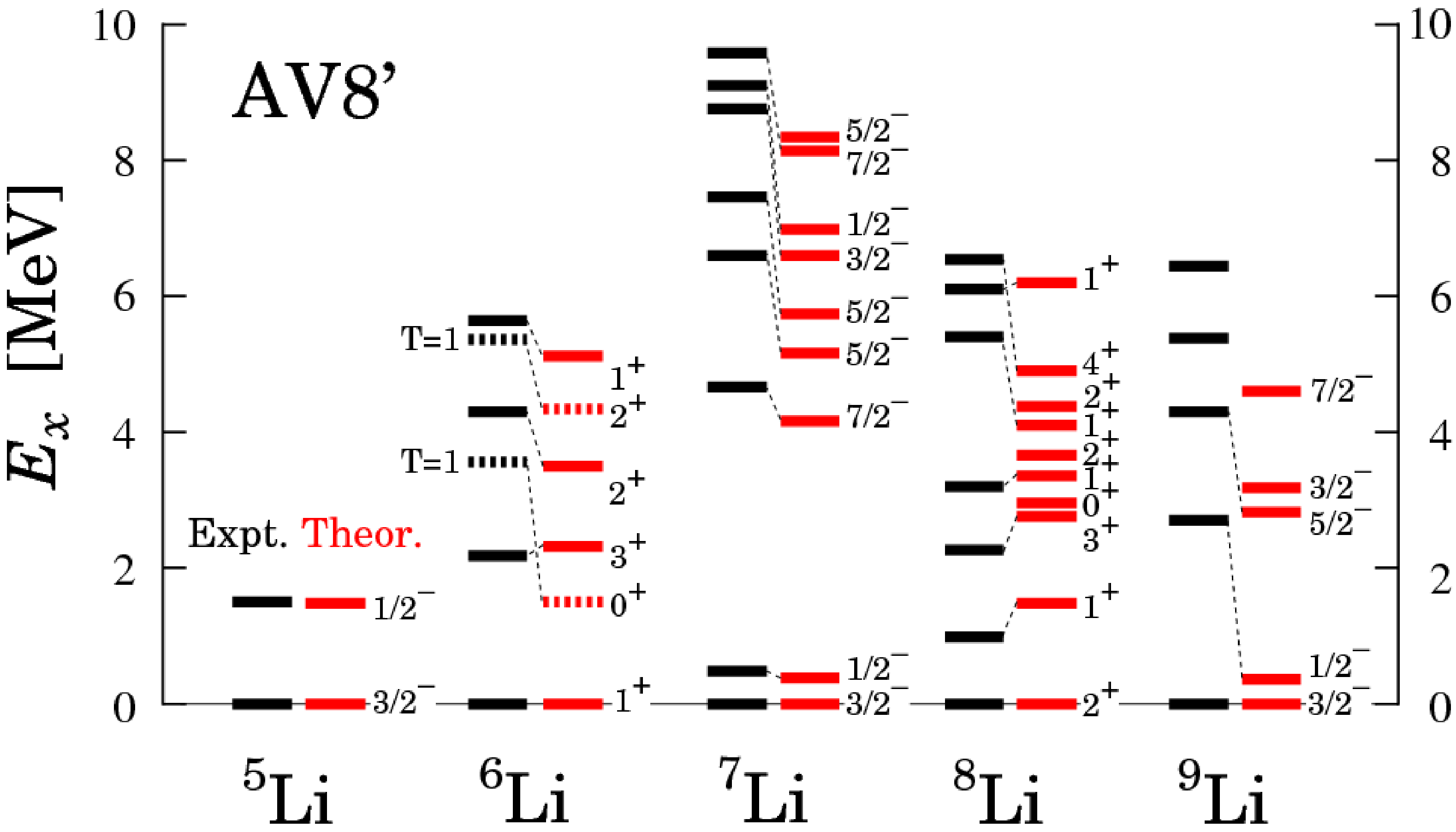
- Bound state app.
- No continuum
- No V_{NNN}

- Excitation energy spectra are reproduced well

$^{5\text{-}9}$Li with TOSM+UCOM

- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda
PRC86(2012) 024318

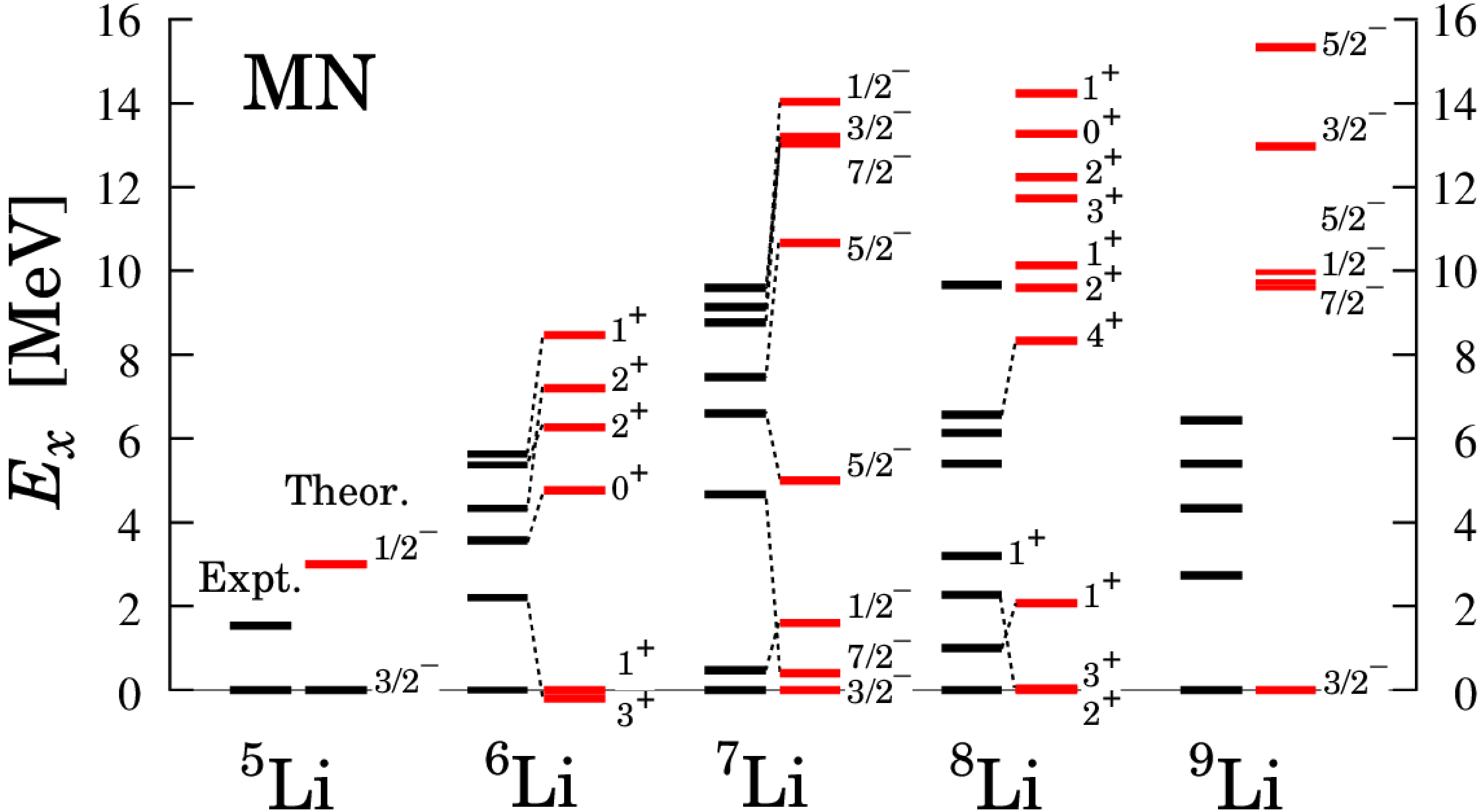


- Excitation energy spectra are reproduced well

$^{5\text{-}9}$Li with TOSM

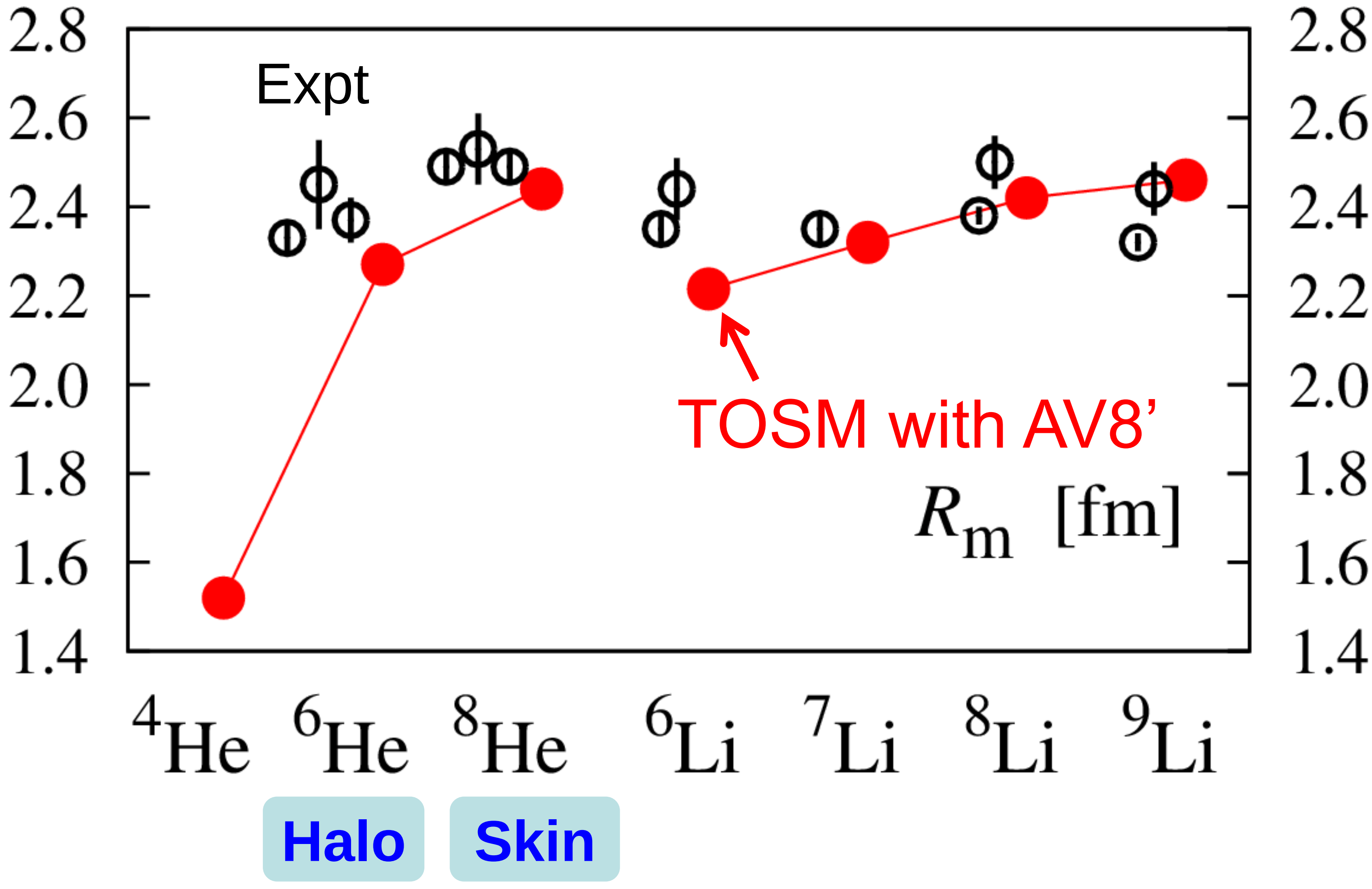
Minnesota force
NO tensor

- Excitation energies in MeV



- Too large excitation energy

Radius of He & Li isotopes



I. Tanihata et al., PLB289('92)261
O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215.

A. Dobrovolsky, NPA 766(2006)1
G. D. Alkhazov et al., PRL78('97)2313
P. Mueller et al., PRL99(2007)252501

Configurations of ^{4}He with AV8'

TM, H. Toki, K. Ikeda
PTP121(2009)511

$(0s_{1/2})^4$	83.0 %
$(0s_{1/2})^{-2}{}_{JT}(p_{1/2})^2{}_{JT}$ ***JT*=10**	2.6
JT=01	0.1
$(0s_{1/2})^{-2}{}_{10}(1s_{1/2})(d_{3/2})_{10}$	2.3
$(0s_{1/2})^{-2}{}_{10}(p_{3/2})(f_{5/2})_{10}$	1.9
Radius [fm]	1.54

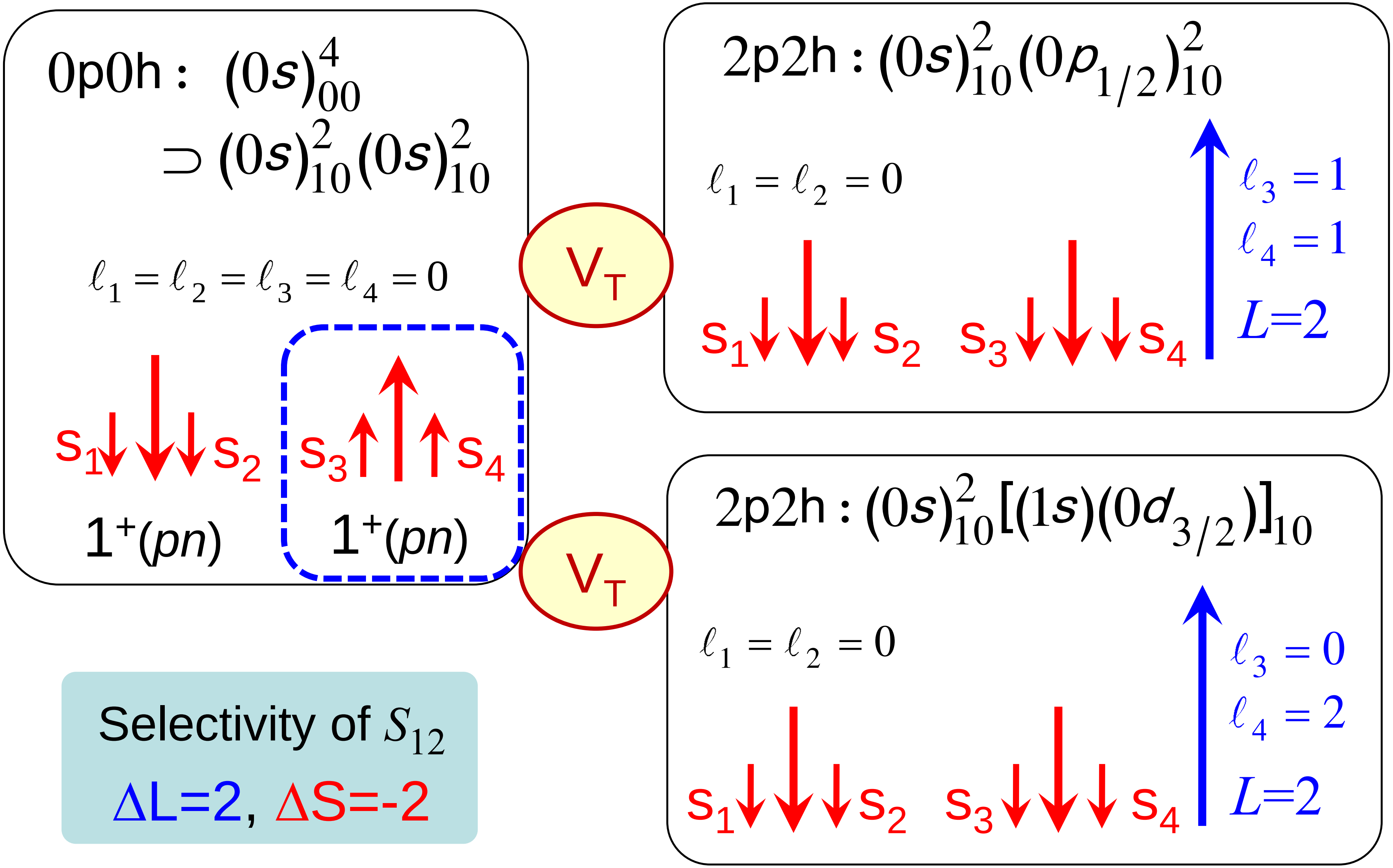
- deuteron correlation with $(J,T)=(1,0)$

Cf. R.Schiavilla et al. (VMC) PRL98(2007)132501
R. Subedi et al. (JLab) Science320(2008)1476
^{12}C(e,e'*p*N)

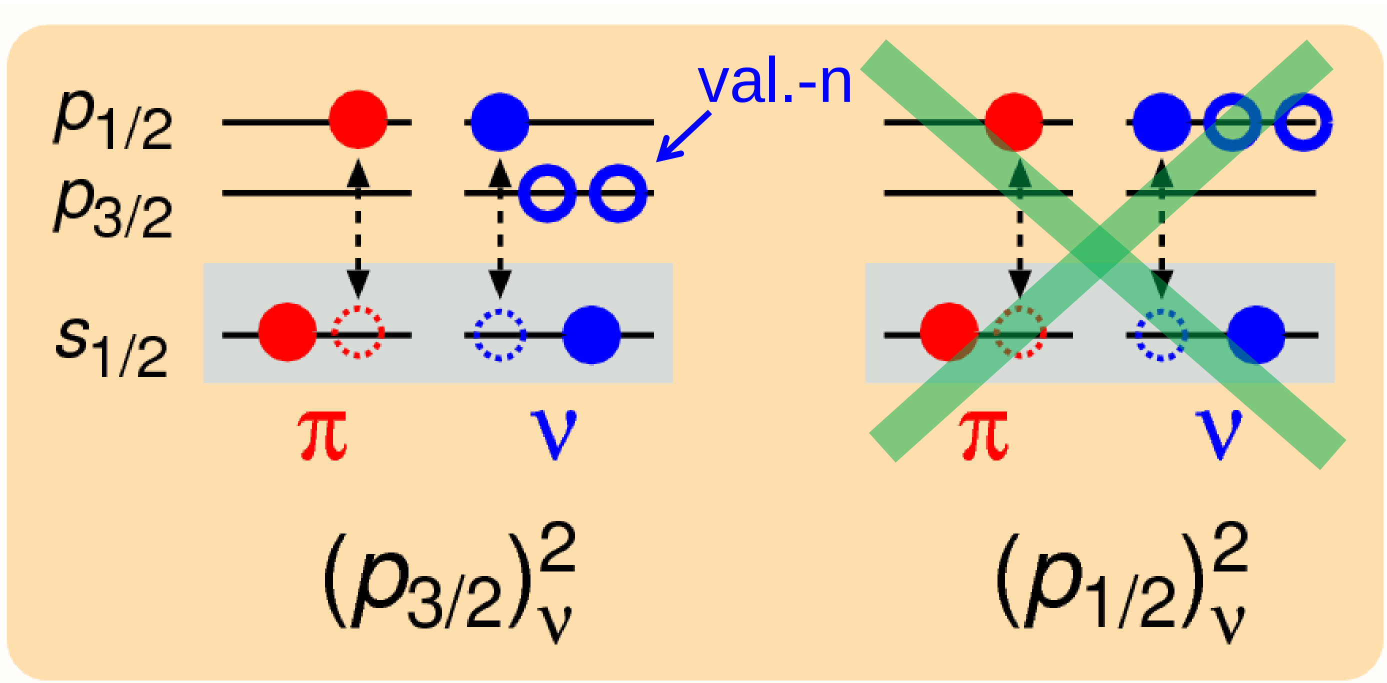
S.C.Simpson, J.A.Tostevin PRC83(2011)014605
$^{12}C \rightarrow {}^{10}B + pn$

- ^{4}He contains ***$p_{1/2}$*** of "***pn*-pair**"
  – Same feature in ^{5}He-^{8}He ground state

Selectivity of the tensor coupling in ^{4}He



Tensor correlation in ^{6}He



Ground state

Excited state

halo state (0^+)

Tensor correlation is **suppressed** due to Pauli-Blocking

TM, K. Kato, K. Ikeda, J. Phys. G31 (2005) S1681

^{6}He : Hamiltonian components

- Difference from ^{4}He in MeV

^{6}**He**	0^+_1	0^+_2
n^2 **config**	$(p_{3/2})^2$	$(p_{1/2})^2$
ΔKin.		
ΔCentral		
ΔTensor		
ΔLS		

b_{hole}=1.5 fm

$\hbar\omega$=18.4 MeV

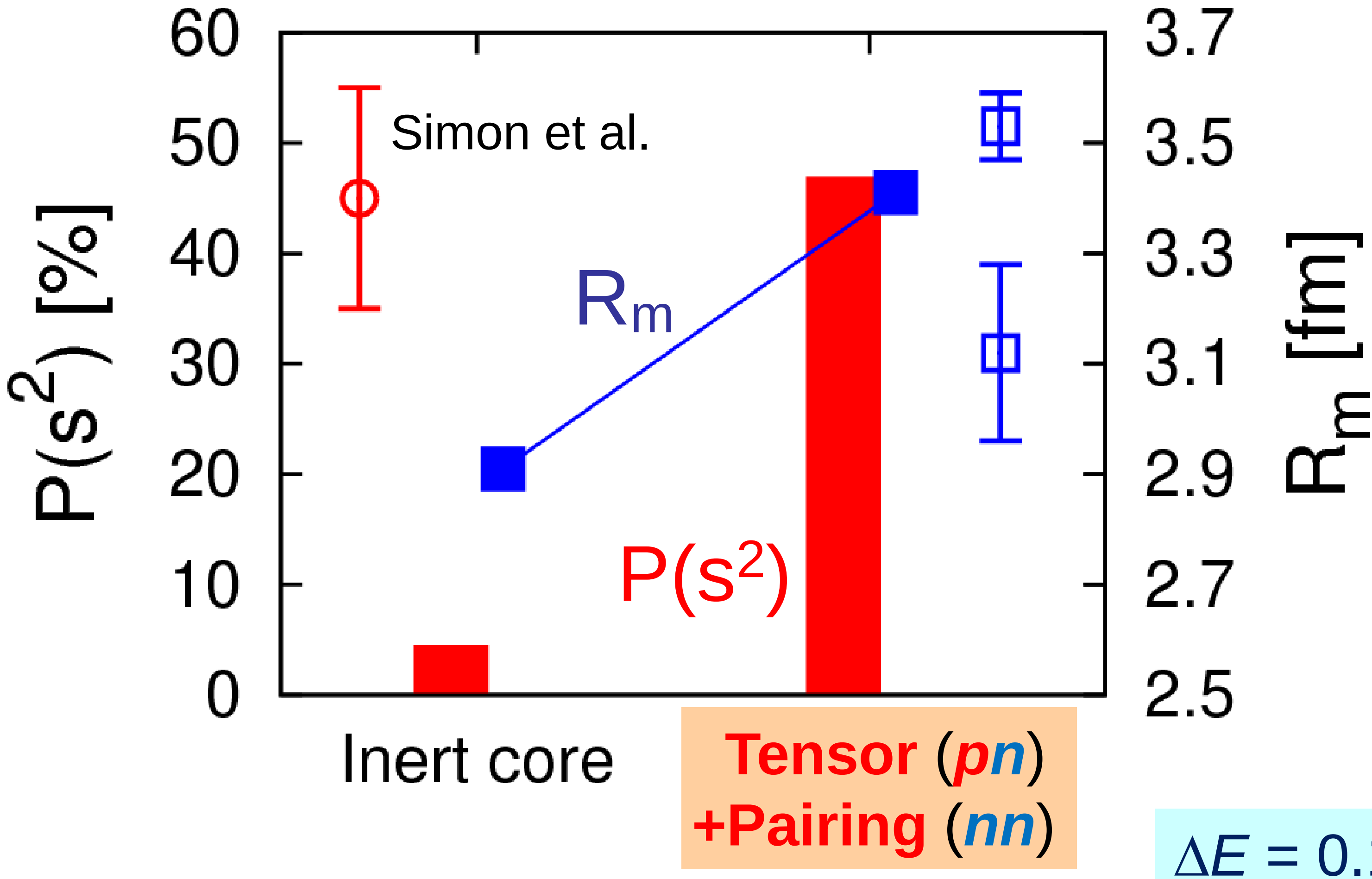
LS splitting energy in ^{6}He

same trend in $^{5-8}$He, ^{10,11}Li

Halo formation with large *s*-wave in ^{11}Li

Ikeda, Myo, Kato, Toki
Lecture Notes in Physics 818 (2010) “Clusters in Nuclei” Vol.1

^{11}Li with ^{9}Li(TOSM)+n+n S_{2n}=0.31 MeV



ΔE = 0.1MeV

Pairing correlation between halo neutrons couples $(0p)^2$ and $(1s)^2$

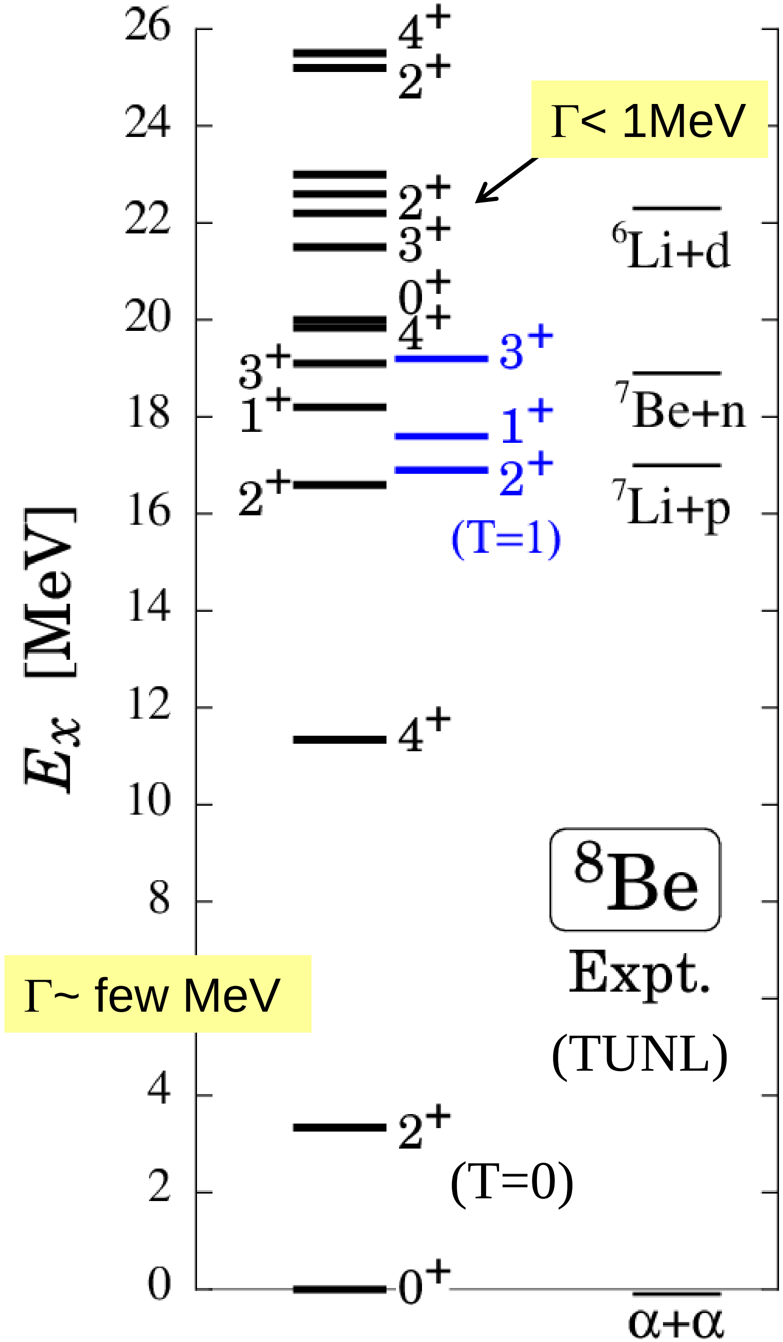
TM, K.Kato, H.Toki, K.Ikeda, PRC76(2007)024305
TM, Y.Kikuchi, K.Kato, H.Toki, K.Ikeda, PTP119(2008)561

Li isotopes: ground state configurations

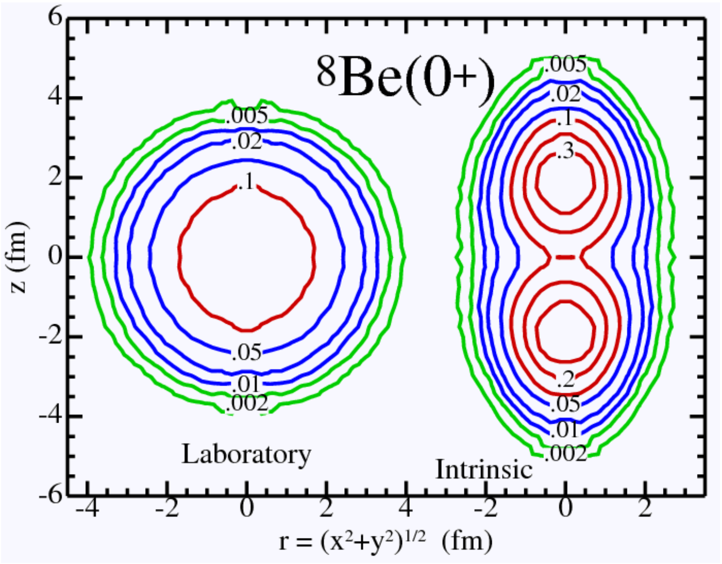
		p-shell config.	weight	
LS	^{6}Li (1^+,T=0)	$(0p_{1/2})(0p_{3/2})$ $\Delta V_T = -12$ MeV	43%	S=1
jj	^{6}Li (0^+,T=1)	$(0p_{3/2})^2$ $\Delta V_T = -4$ MeV	72%	IAS of ^{6}He
jj	^{7}Li ($3/2^-$)	$(0p_{3/2})^3$	48%	
jj	^{8}Li (2^+)	$(0p_{3/2})^4$	41%	
jj	^{9}Li ($3/2^-$)	$(0p_{3/2})^5$	46%	

- $^6\mathrm{Li}_{gs}$ … *LS* coupling → Indication of { $\alpha+d$ clustering / deformation }

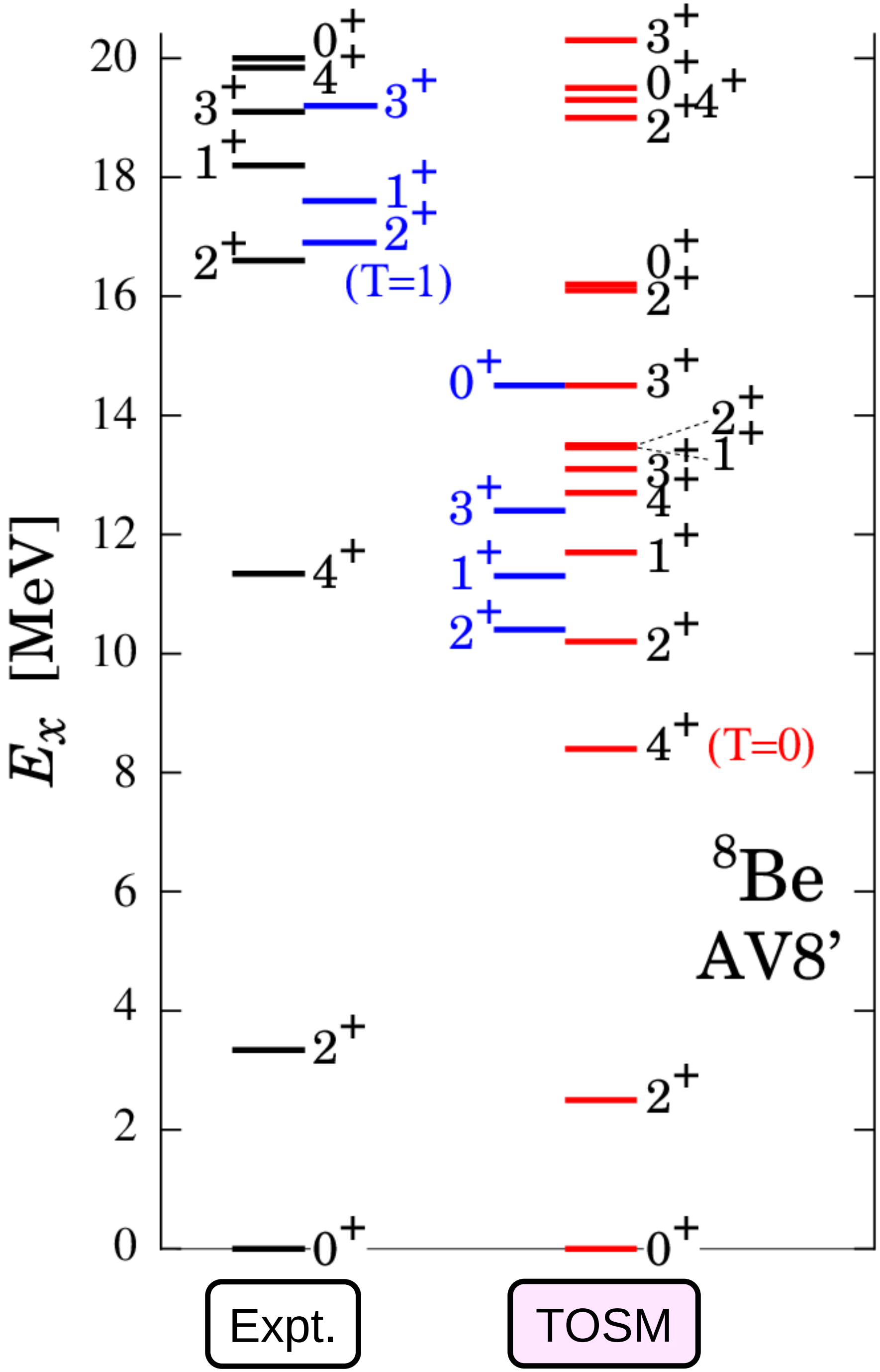
^{8}Be spectrum



– Green's function Monte Carlo
C. Pieper, R.B. Wiringa,
Annu.Rev.Nucl.Part.Sci.51 (2001)

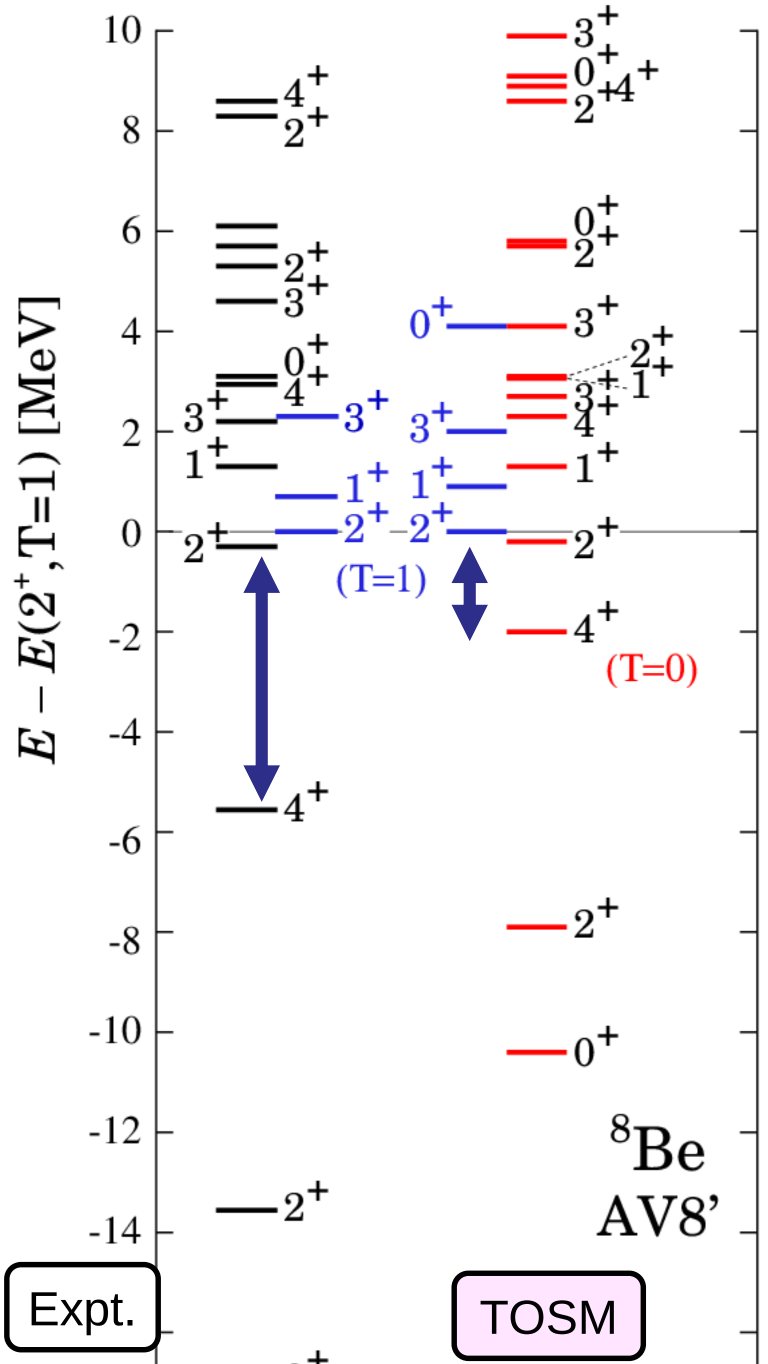


α-α structure



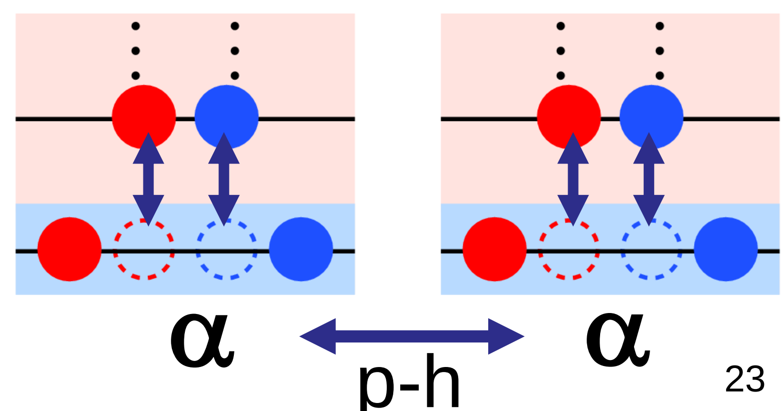
^{8}Be in TOSM – AV8' –

- $V_T \times 1.1$, $V_{LS} \times 1.4$
  - simulate ^{4}He benchmark (Kamada et al., PRC64)
- ground band
- highly excited states
  - small E_x
  - correct level order (T=0,1)
- $R_m(^8\text{Be})$=2.21 fm
  - 2α THSR : 2.8 fm
  - ^{4}He :1.52 fm
  - ^{12}C :2.35 fm ~ R_m(exp)

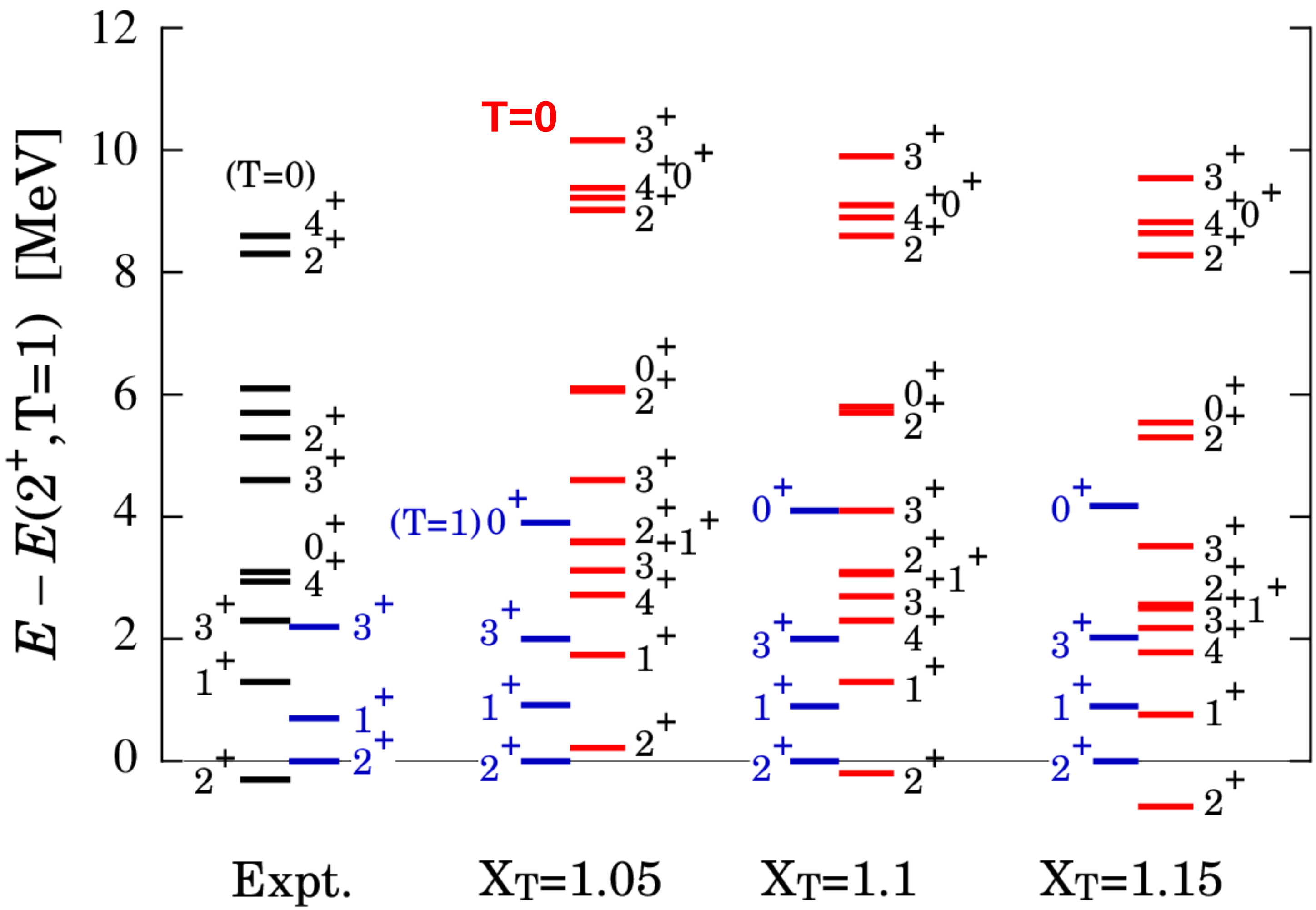


^{8}Be in TOSM – AV8' –

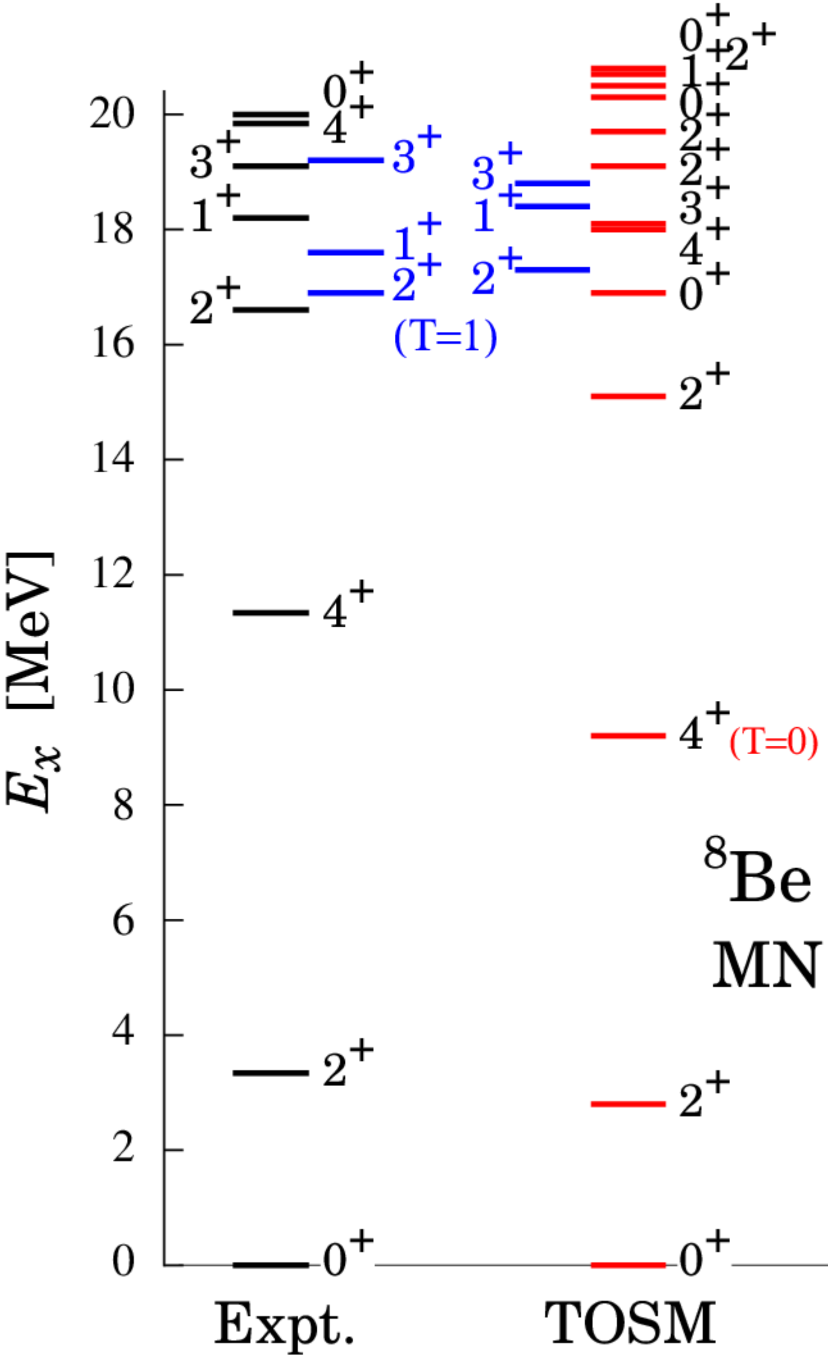
- $V_T \times 1.1$, $V_{LS} \times 1.4$
  - simulate ^{4}He benchmark (Kamada et al., PRC64)
- correct level order (T=0,1)
- α : 0p0h+2p2h with high-*k*
  - 2α needs 4p4h.
  - spatial asymptotic form of 2α



V_{tensor} dependence of ^{8}Be

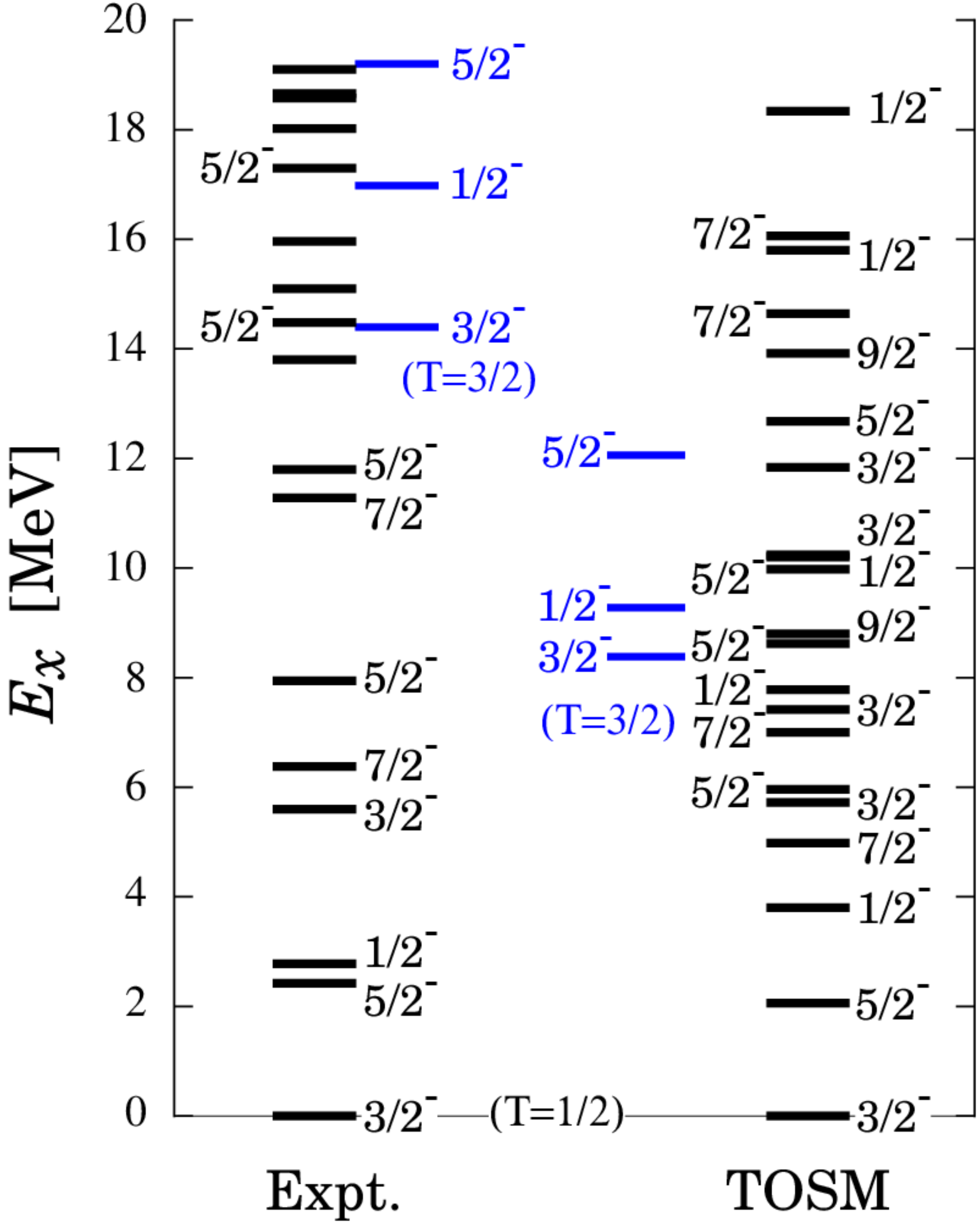


- *S*-wave UCOM can be simulated with X_T~1.1 (PTP121)
- Stronger tensor correlation in **T=0 states** than T=1 states.



^{8}Be in TOSM – Minnesota –

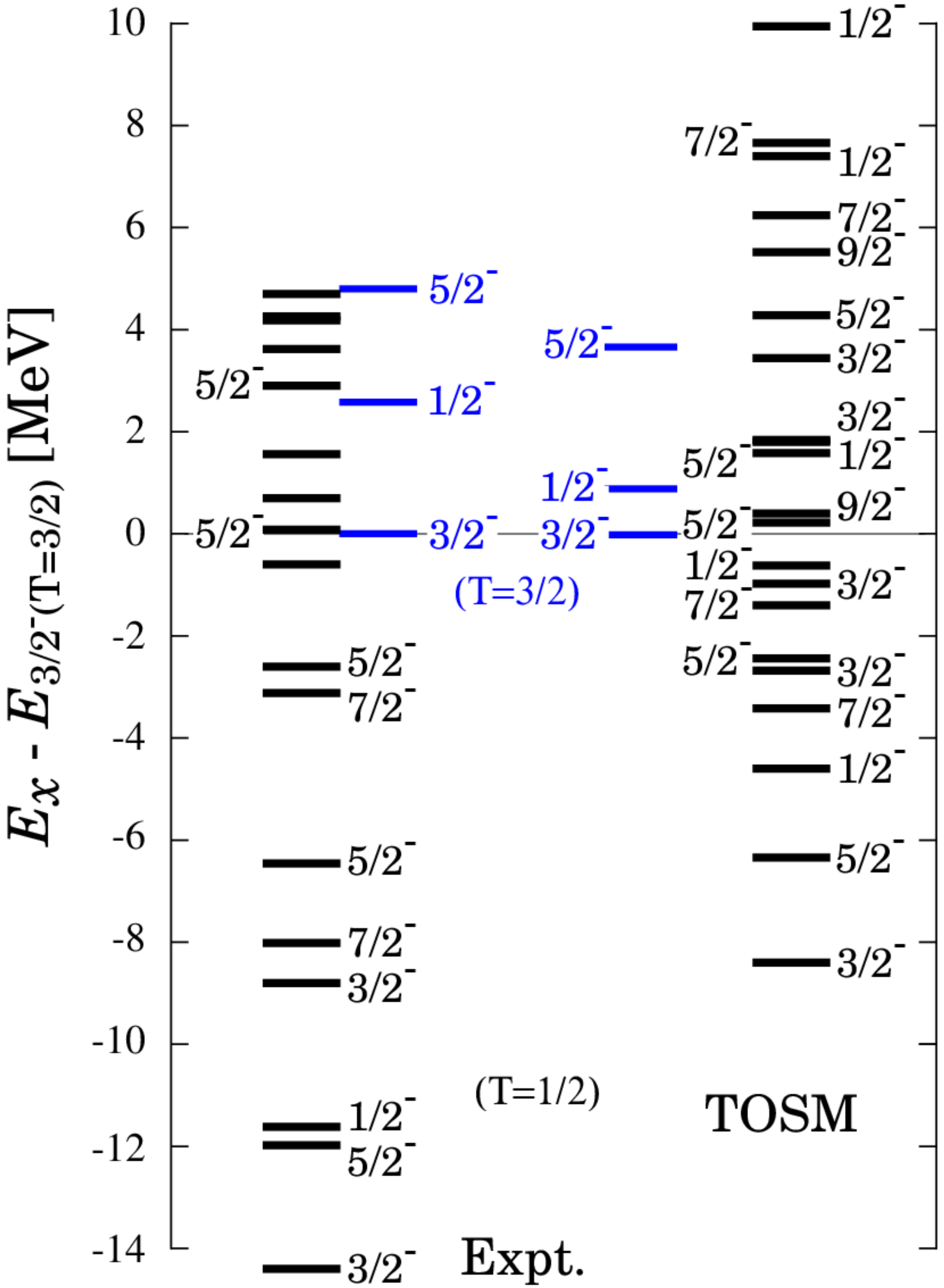
- ground band & highly excited states
  - good E_X
  - incorrect level order ($0^+, 1^+, 3^+$)
  - E_X(T=0) < E_X(T=1)
- Radius (R_m) is small
  - ^{4}He 1.39 fm
  - ^{8}Be 1.89 fm
  - ^{12}C 1.85 fm



^{9}Be in TOSM

- low-lying states
  - correct level order
- highly excited states
  - small E_X with T=1/2,3/2
- $R_m(^9Be)$=2.32 fm
  - exp: 2.45(1) fm
  - ^{4}He:1.52 fm
  - ^{8}Be: 2.21 fm < 2.8 fm (2α THSR, Funaki)
  - ^{12}C : 2.35 fm ~ R_m(exp)

^{9}Be in TOSM



- low-lying states
  - correct level order
- highly excited states
  - small E_X with T=1/2,3/2
- $R_m(^9Be)$=2.32 fm
  - exp: 2.45(1) fm
  - ^{4}He:1.52 fm
  - ^{8}Be: 2.21 fm < 2.8 fm (2α THSR, Funaki)
  - ^{12}C : 2.35 fm ~ R_m(exp)

Summary

- **TOSM+UCOM** using V_{bare}.
  - Strong tensor correlation from 0p0h-2p2h.
- He, Li, & Be isotopes
  - Energy spectra, Radius
  - ^{4}He contains **"*pn*-pair of $p_{1/2}$"** owing to S_{12} of V_{T}, which affect the spectrum of *n*-rich nuclei.
- ^{8}Be, ^{9}Be
  - Two aspects : grand state region & highly excited states.
  - For Be isotopes, indication of more configurations to describe 2α structure states.