

Dubna-Mainz-Taipei (DMT) dynamical model for pion scattering and EM meson production

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Outline

- Motivation
- DMT πN model
- DMT model for electromagnetic production of pion
- Results
 - threshold production
 - resonances properties
- Summary

Motivation

- To construct a meson-exchange model for πN scattering and e.m. production of pion to achieve a unified description for both reactions over a wide range of energies, i.e., from threshold to ~ 2 GeV.
- comparison with ChPT predictions near threshold to gain a glimpse of chiral symmetry
- consistent extraction of the resonance properties like, mass, width, and form factors, from both reactions.
 - Comparison with LQCD results requires reliable extraction. consistent extractions \rightarrow minimize model dependence ?
- The resonances studied are always assumed to be of the type which results from dressing of the quark core by meson cloud.
 - \rightarrow understand the underlying structure and dynamics

History of DMT model

- J. Phys. **G 11**, (1985) L205.
 - Dynamical approach for pion photoproduction reaction proposed
- J. Phys **G 20**, (1994) 1531; Phys. Rev **C 64**, (2001) 034309.
 - Taipei-Argonne meson-exchange πN model (in collab w/H.T-S. Lee)
- Phys. Rev. Lett. **83**, (1999) 4494.
 - DMT results for Δ deformation
- Phys. Rev. **C 64**, (2001) 032201; Phys. Lett. **B 522**, (2001) 27.
 - DMT results for form factors and threshold π production
- Nucl. Phys. **A 723**, (2003) 447; Phys. Rev. **C 76**, (2007) 035206.
 - DMT results for nucleon higher resonances

Taipei-Argonne πN model:

meson-exchange πN model below 400 MeV

Bethe-Salpeter equation

$$T_{\pi N} = B_{\pi N} + B_{\pi N} G_0 T_{\pi N},$$

where

$B_{\pi N}$ = sum of **all irreducible** two-particle Feynman amplitudes

G_0 = relativistic free pion-nucleon propagator

can be rewritten as \Rightarrow

$$T_{\pi N} = \bar{B}_{\pi N} + \bar{B}_{\pi N} \bar{G}_0 T_{\pi N},$$

with

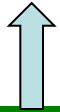
$$\bar{B}_{\pi N} = B_{\pi N} + B_{\pi N} (G_0 - \bar{G}_0) \bar{B}_{\pi N}.$$

Three-dimensional reduction

Choose a $\overline{G}_0(k, P)$ such that

1. $T_{\pi N} = \overline{B}_{\pi N} + \overline{B}_{\pi N} \overline{G}_0 T_{\pi N}$ becomes
three-dimensional

2. \overline{G}_0 can reproduce πN elastic cut



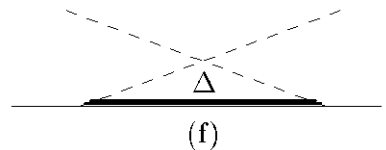
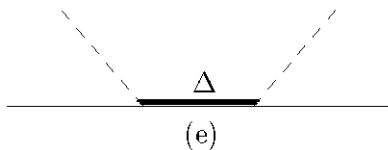
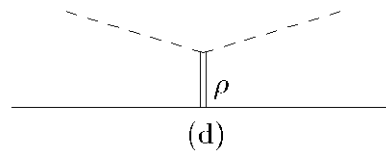
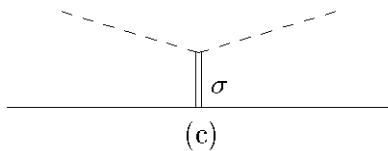
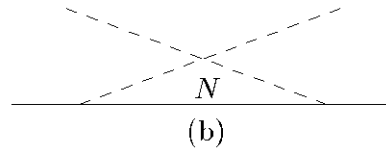
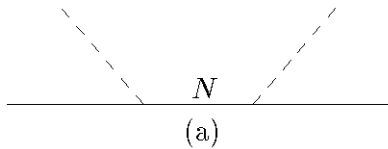
Cooper-Jennings reduction scheme

It satisfies both
the soft pion theorems and unitarity

Cooper, Jennings, NP A483, 601 (1988).

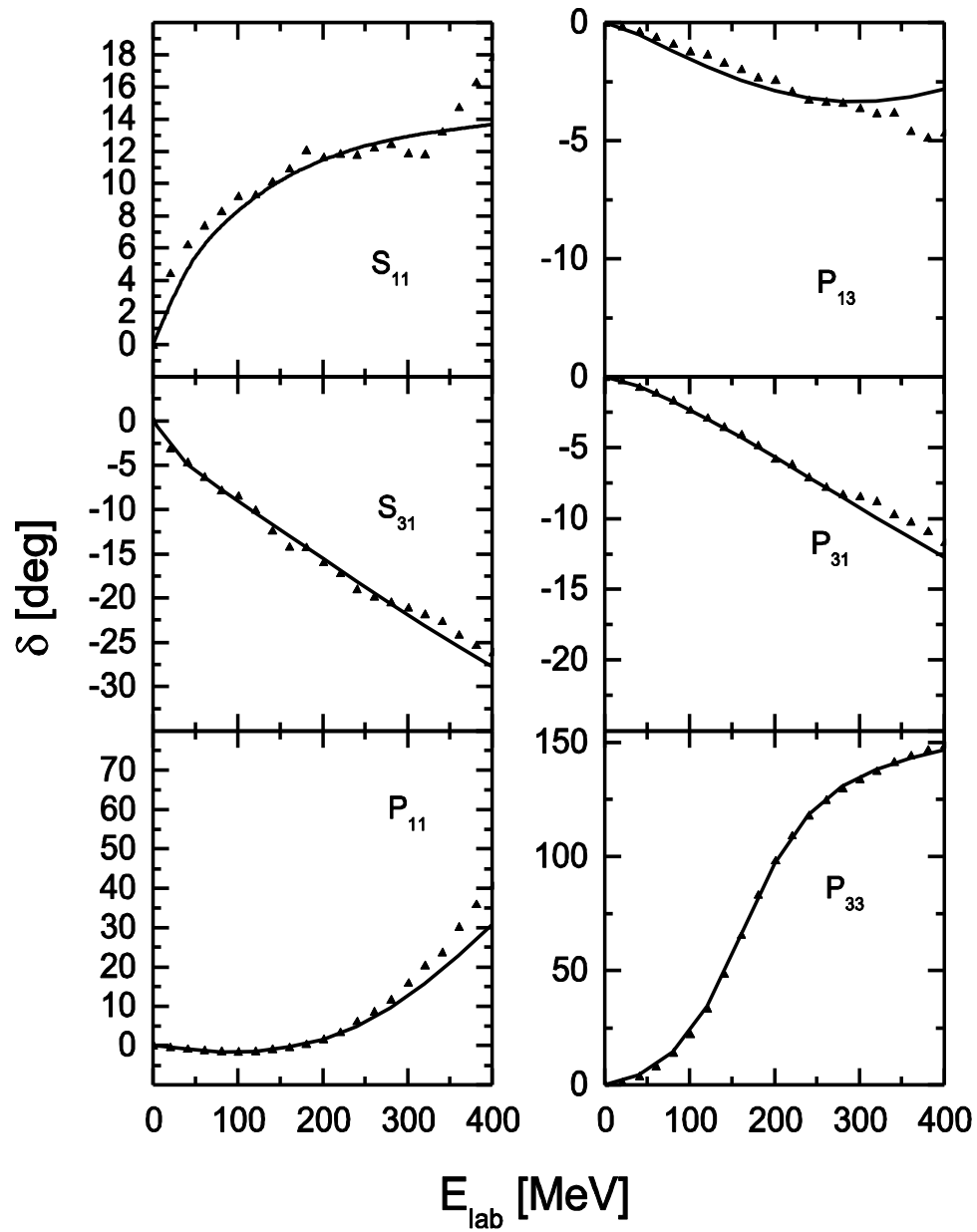
Approximate $\overline{B}_{\pi N}$ by

tree approximation of a **chiral** effective Lagrangian



$$F(p^2) = \left[\frac{n\Lambda^4}{n\Lambda^4 + (m^2 - p^2)^2} \right]^n,$$

$n = 10$



13 parameters

Dynamical model for $\gamma N \rightarrow \pi N$

To order e , the t -matrix for $\gamma N \rightarrow \pi N$ is written as

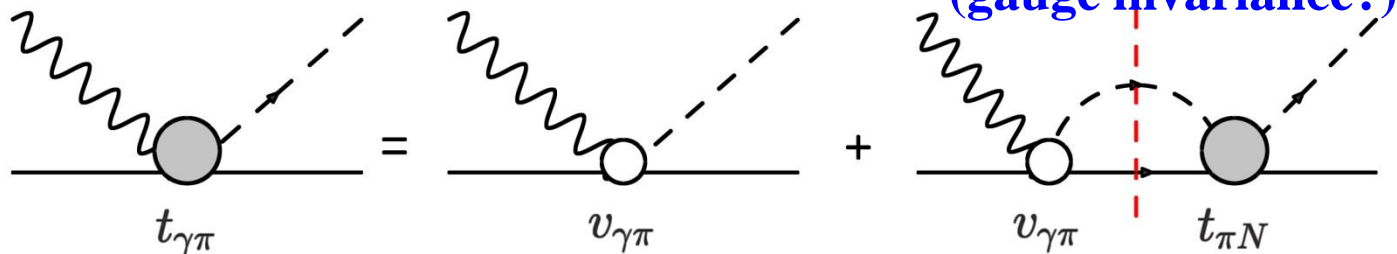
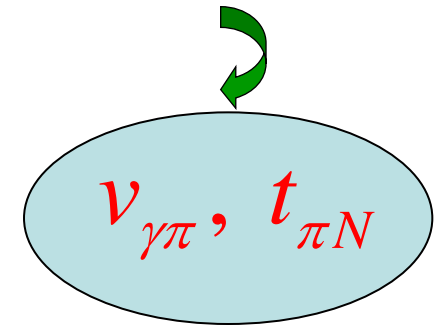
$$t_{\gamma\pi}(E) = v_{\gamma\pi} + v_{\gamma\pi} g_0(E) t_{\pi N}(E),$$

where

$v_{\gamma\pi}$ = transition potential,

$t_{\pi N}$ = πN t -matrix,

two ingredients



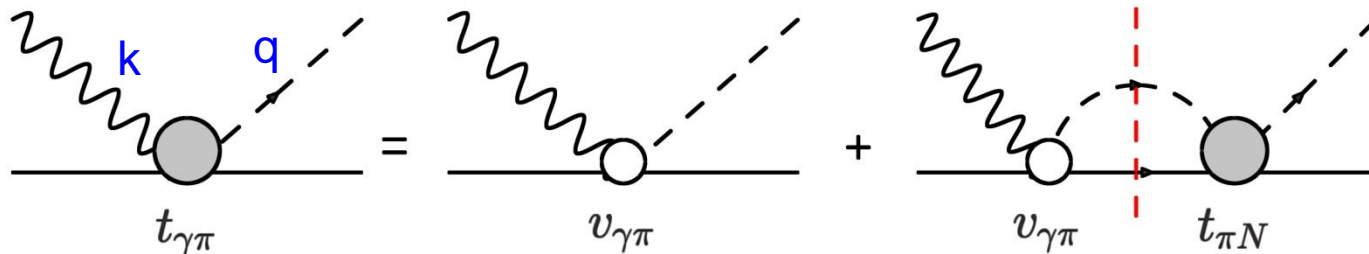
- **Multipole decomposition** of $t_{\gamma\pi}$ gives the physical amplitude in channel $\alpha=(\xi, l_\pi, j)$,

$$t_{\gamma\pi}^{(\alpha)}(q_E, k; E + i\varepsilon) = \exp(i\delta^{(\alpha)}) \cos \delta^{(\alpha)} \times \left[v_{\gamma\pi}^{(\alpha)}(q_E, k) + P \int_0^\infty dq' \frac{q'^2 R_{\pi N}^{(\alpha)}(q_E, q'; E) v_{\gamma\pi}^{(\alpha)}(q', k)}{E - E_{\pi N}(q')} \right]$$

where

- $\delta^{(\alpha)}$, $R_{\pi N}^{(\alpha)}$: πN scattering phase shift and reaction matrix in channel α
- $k=|k|$, q_E : photon and pion on-shell momentum

**Both on- & off-shell
(gauge invariance?)**



DMT Model $V_{\gamma\pi}^B$ obtained from tree diagrams of a **chiral** effective Lagrangian

$$V_{\gamma\pi}^B = \left\{ \begin{array}{l} \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \\ \text{Diagram 5} \\ \text{Diagram 6} \end{array} \right\}$$

Diagram 1: A horizontal solid line with an arrow pointing right. A wavy line enters from the top left and connects to the solid line. A dashed line with an arrow pointing up and right exits from the solid line.

Diagram 2: A horizontal solid line with an arrow pointing right. A wavy line enters from the top left and connects to the solid line. A dashed line with an arrow pointing up and right enters from the bottom left and connects to the solid line.

Diagram 3: A horizontal solid line with an arrow pointing right. A wavy line enters from the top left and connects to the solid line. A dashed line with an arrow pointing up and right enters from the bottom left and connects to the solid line.

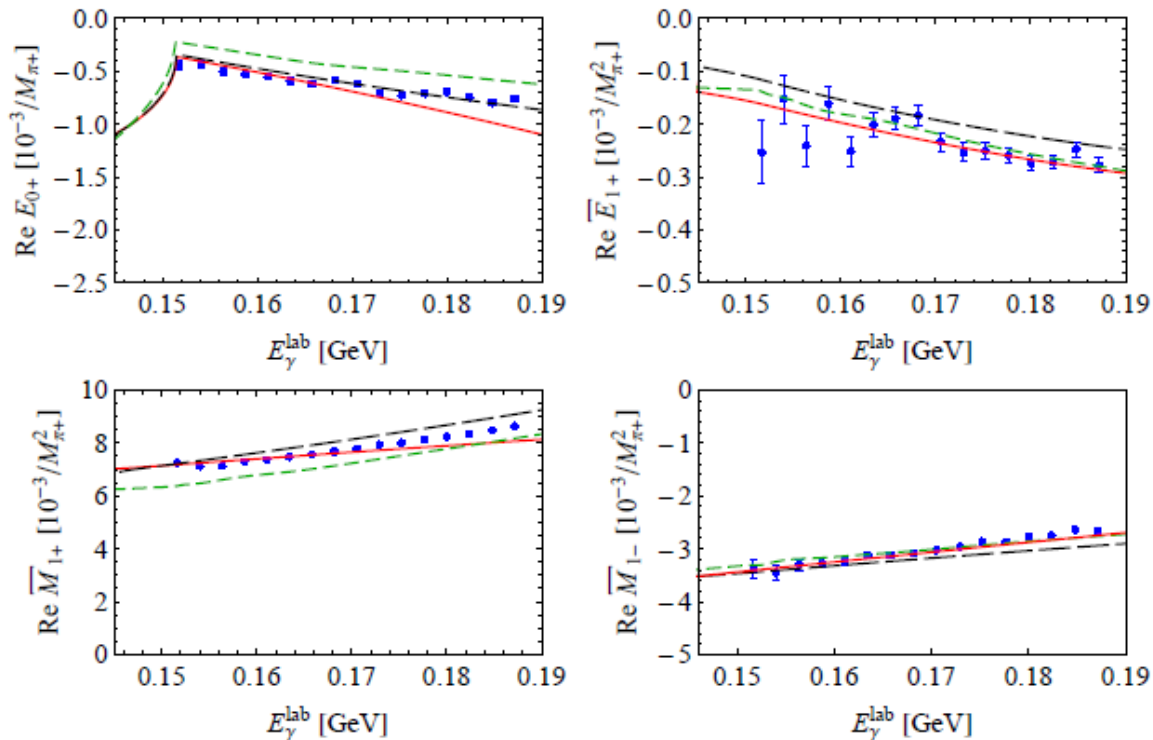
Diagram 4: A horizontal solid line with an arrow pointing right. A wavy line enters from the top left and connects to the solid line. A dashed line with an arrow pointing up and right enters from the bottom left and connects to the solid line.

Diagram 5: A horizontal solid line with an arrow pointing right. A wavy line enters from the top left and connects to a vertical solid line labeled ρ, ω . A dashed line with an arrow pointing up and right exits from the top of the vertical line.

Diagram 6: A horizontal solid line with an arrow pointing right. A wavy line enters from the top left and connects to the solid line. A dashed line with an arrow pointing up and right enters from the bottom left and connects to the solid line. An upward-pointing arrow is below the solid line, with the text $(PV \text{ only})$ below it.

Results of DMT model near threshold
(depends only on $v_{\lambda\pi}^B$ and $t_{\pi N}^B$)

$\vec{\gamma} + p \rightarrow \pi^0 + p$, Σ : $E_\gamma = 145 - 190$ MeV
 Hornidge et al., PRL 111, (2013) 062004
 A2 and CB-TAPS Collab. @ MAMI

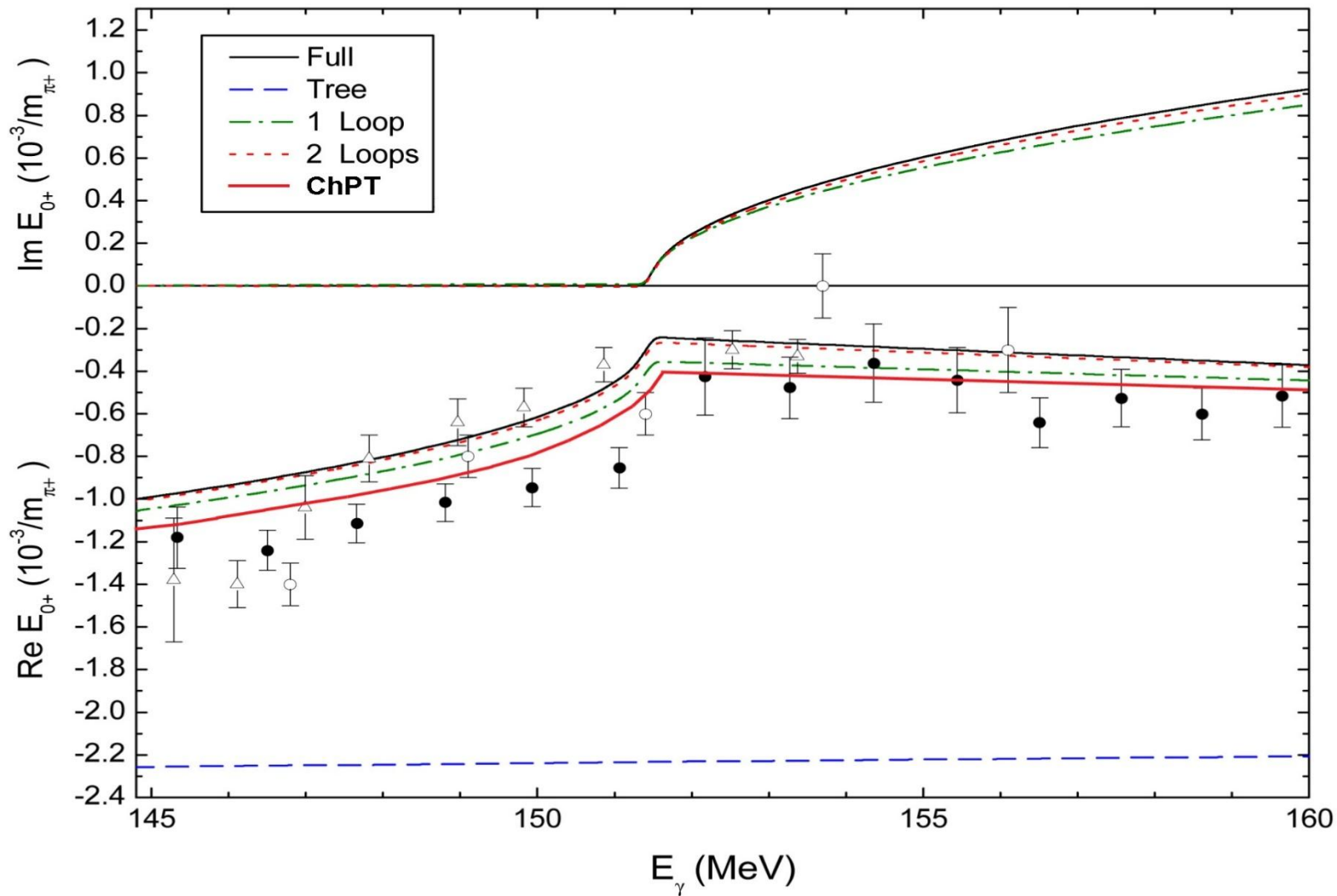


- - - - : DMT

- - - - : Gasparyan and Lutz, unitary EFT w/
 chiral Lagrangian

——— : RChPT (Hilt, Scherer & Tiator)

$$\gamma p \rightarrow \pi^0 p$$

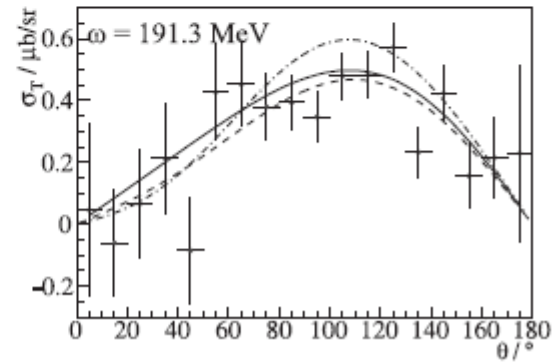
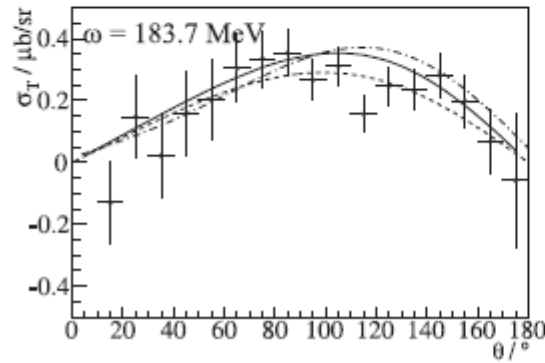
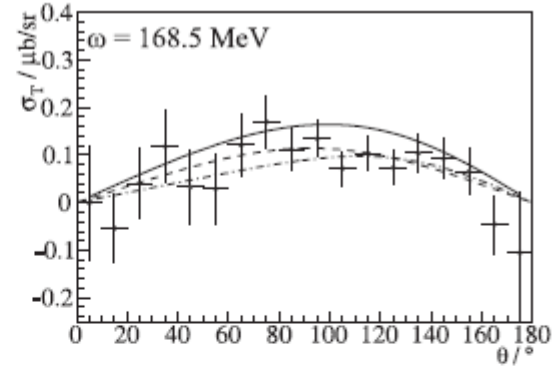
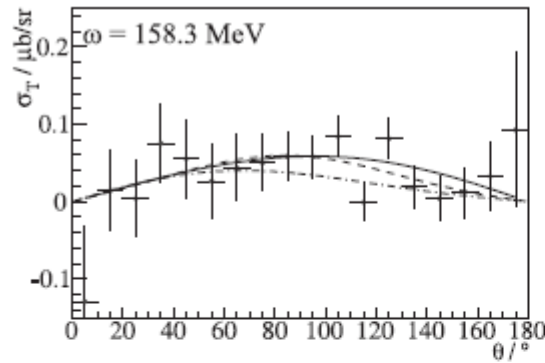


$\gamma + \vec{p} \rightarrow \pi^0 + p$, σ_T : $E_\gamma = 145 - 190$ MeV
 S. Schumann et al., A2 Collab. @ MAMI, submitted to PL B.

$$\sigma_T = \sigma_0 T,$$

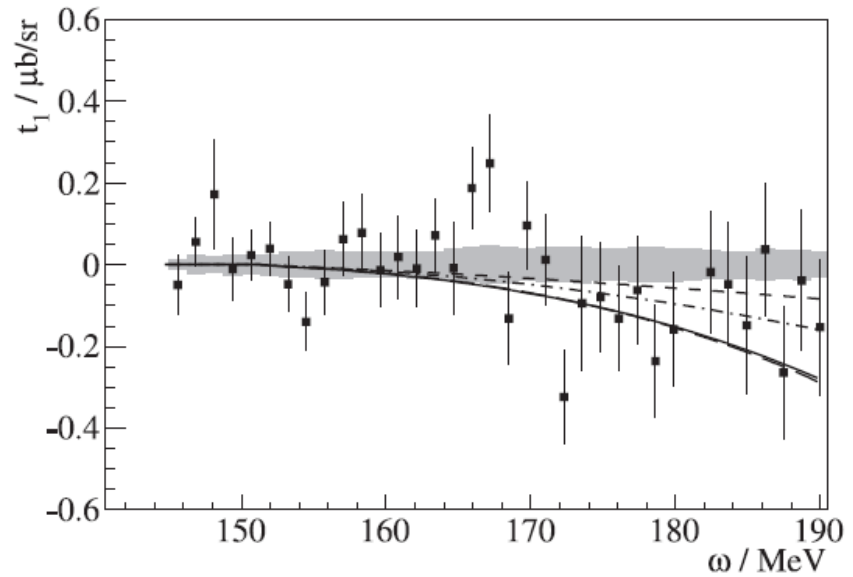
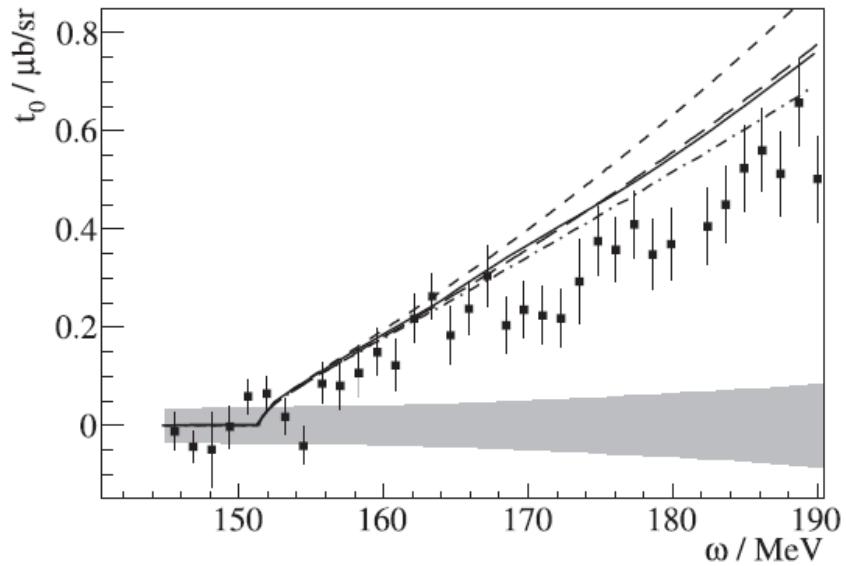
$$T = \frac{1}{P_T \sin \varphi} \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-},$$

P_T : target pol.
 φ : π^0 azimuthal angle



———— : DMT, - - - - , - - - - - fits

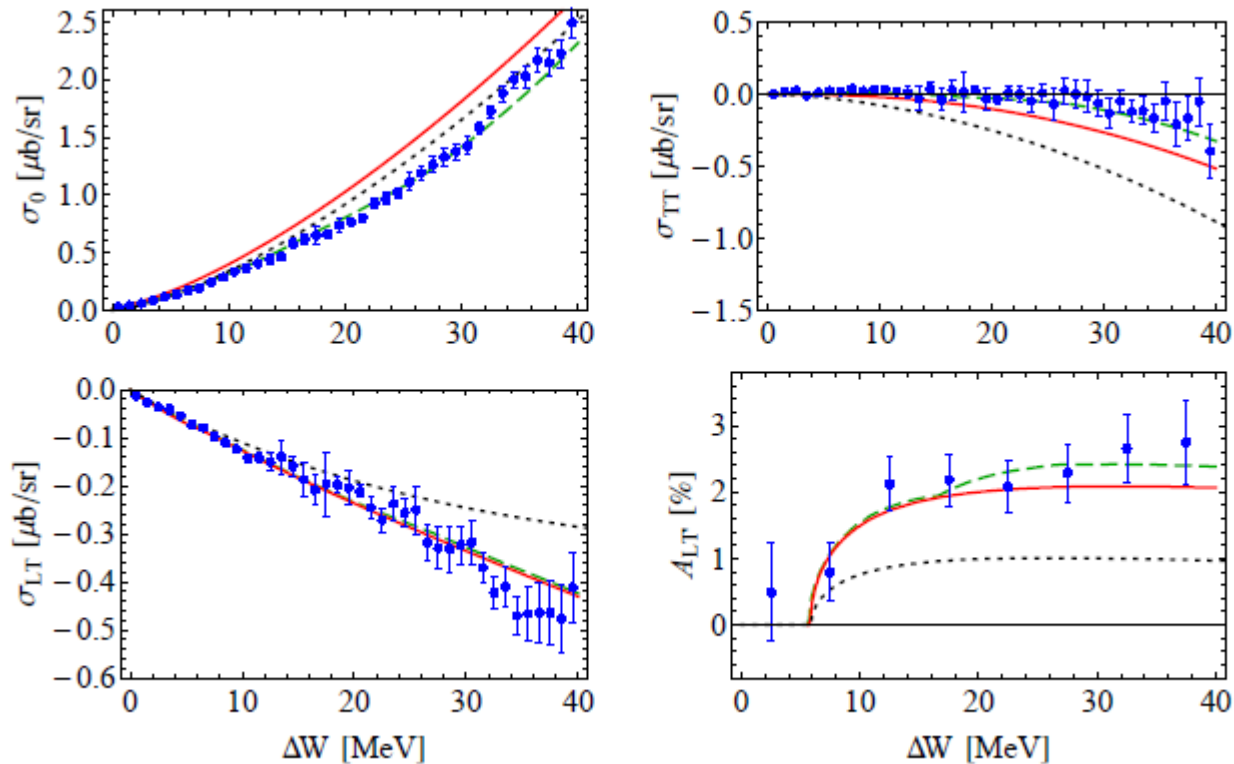
$$\sigma_T = \frac{q}{k} \sin \theta [t_0 P_0(\cos \theta) + t_1 P_1(\cos \theta)]$$



- : DMT
- : Gasparyan - Lutz
- · - · - · : RChPT
- - - - - : fit of Hornidge et al.

$$\frac{d\sigma_v}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \epsilon \frac{d\sigma_L}{d\Omega_\pi} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{d\Omega_\pi} \cos \Phi_\pi + \epsilon \frac{d\sigma_{TT}}{d\Omega_\pi} \cos 2\Phi_\pi + h \sqrt{2\epsilon(1-\epsilon)} \frac{d\sigma_{LT'}}{d\Omega_\pi} \sin \Phi_\pi$$

$$Q^2 = 0.05 \text{ GeV}^2, \Theta_\pi = 90^\circ, \Phi_\pi = 90^\circ, \text{ and } \epsilon = 0.93$$



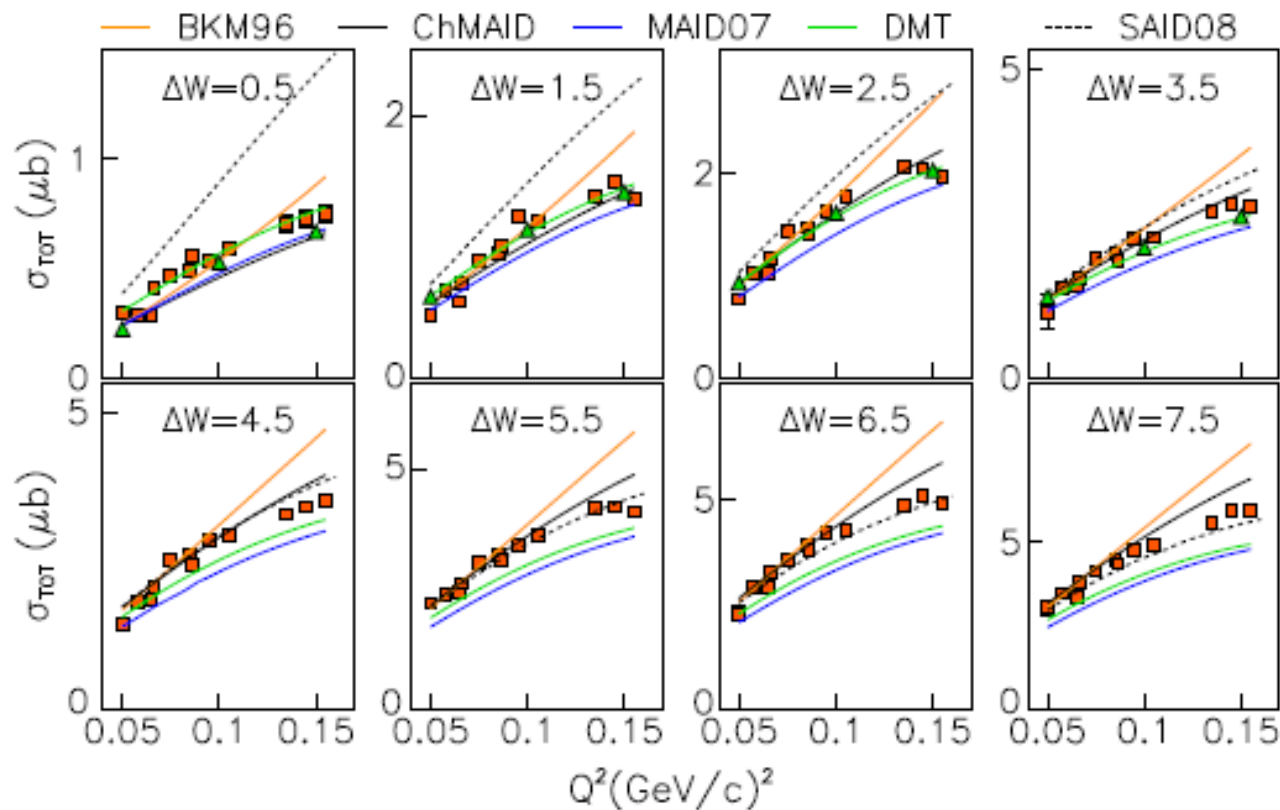
data: *M. Weis et al., Eur. Phys. J. A 38 (2008) 27*

--- : **DMT**

..... : HBChPT [Bernard et al., PLB 383, 116 (1996)]

— : RChPT (Hilt, Scherer & Tiator, 2013)

Chirapatpimol et al., PRL 114, 192503 (2015) , Hall A Collab.
 $p(e,e'p)\pi^0$ with $E_e = 1192$ MeV



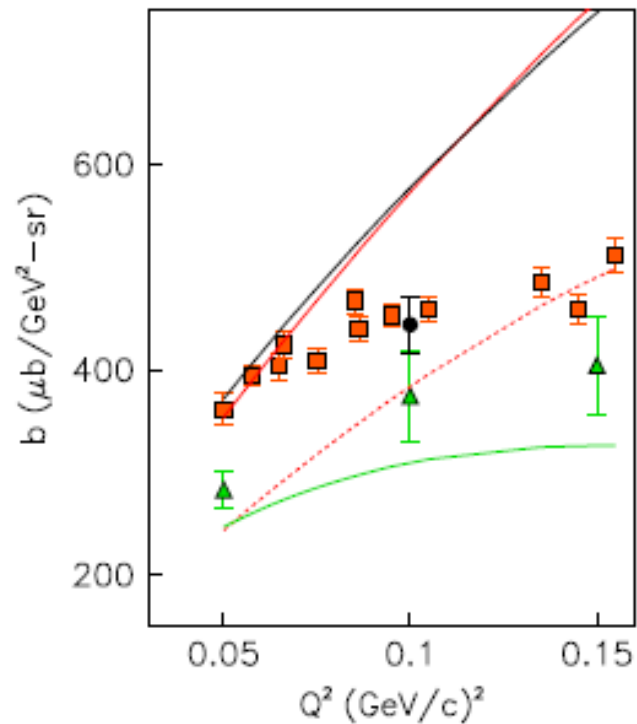
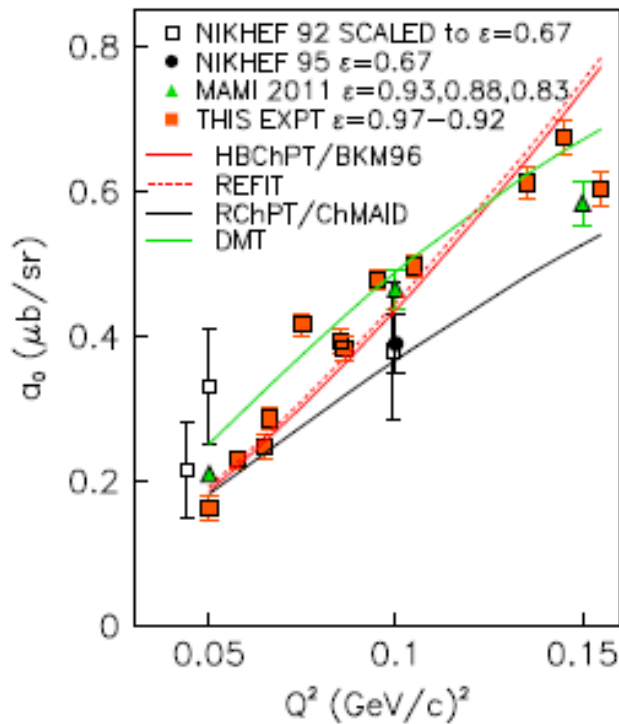
$$\frac{d\sigma}{d\Omega_\pi^*} = \frac{p_\pi^*}{k_\gamma^*} (R_T + \epsilon_L R_L + \epsilon R_{TT} \cos 2\phi_\pi^* + \sqrt{2\epsilon_L(\epsilon + 1)} R_{LT} \cos \phi_\pi^*).$$

$$R_T + \epsilon_L R_L = A_0^{T+L} + A_1^{T+L} P_1(x) + A_2^{T+L} P_2(x),$$

$$R_{TT} = A_0^{TT} (1 - x^2),$$

$$R_{LT} = (A_0^{LT} + A_1^{LT} P_1(x)) (1 - x^2)^{1/2}.$$

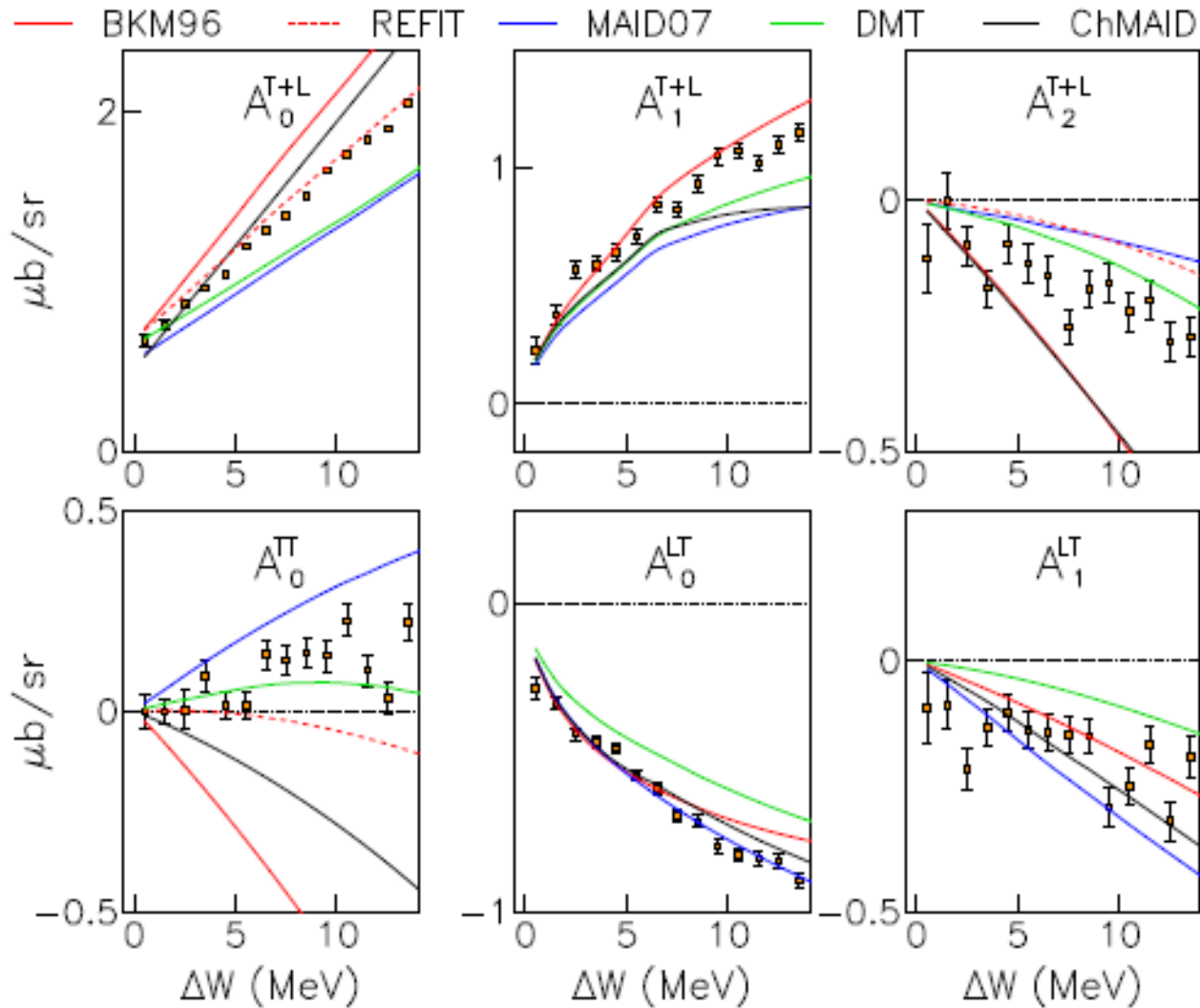
$$A_0^{T+L} = a_0 + b |q_\pi^*|^2$$



$$R_T + \epsilon_L R_L = A_0^{T+L} + A_1^{T+L} P_1(x) + A_2^{T+L} P_2(x),$$

$$R_{TT} = A_0^{TT} (1 - x^2),$$

$$R_{LT} = (A_0^{LT} + A_1^{LT} P_1(x)) (1 - x^2)^{1/2}.$$



	DMT	HBChPT/ RChPT
chiral symmetry	yes	yes
crossing symmetry	no	yes
unitarity	yes	no
counting	loop	chiral power

If a resonance appears,
then transition potential $v_{\gamma\pi}$ consists of two terms,

$$v_{\gamma\pi}(E) = v_{\gamma\pi}^B(E) + v_{\gamma\pi}^R(E),$$

where

$v_{\gamma\pi}^B$ = background transition potential

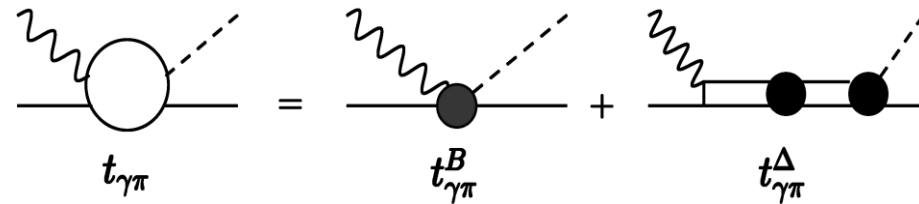
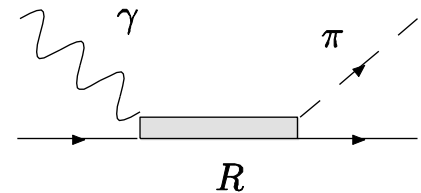
$v_{\gamma\pi}^R$ = contribution of a bare resonance R

then one obtains

$$t_{\gamma\pi}(E) = t_{\gamma\pi}^B(E) + t_{\gamma\pi}^R(E),$$

with

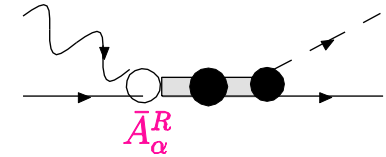
$$\begin{cases} t_{\gamma\pi}^B(E) = v_{\gamma\pi}^B + v_{\gamma\pi}^B g_0(E) t_{\pi N}(E) \\ t_{\gamma\pi}^R(E) = v_{\gamma\pi}^R + v_{\gamma\pi}^R g_0(E) t_{\pi N}(E) \end{cases}$$



both t^B and t^R satisfy
Fermi-Watson theorem,
respectively.

In DMT, we approximate the resonance contribution $A_\alpha^R(W, Q^2)$ by the following Breit-Wigner form

$$A_\alpha^R(W, Q^2) = \bar{A}_\alpha^R(Q^2) \frac{f_{\gamma R}(W) \Gamma_R M_R f_{\pi R}(W)}{M_R^2 - W^2 - iM_R \Gamma_R} e^{i\phi},$$



with

$f_{\pi R}$ = Breit-Wigner factor describing the decay of the resonance R

$\Gamma_R(W)$ = total width

M_R = physical mass

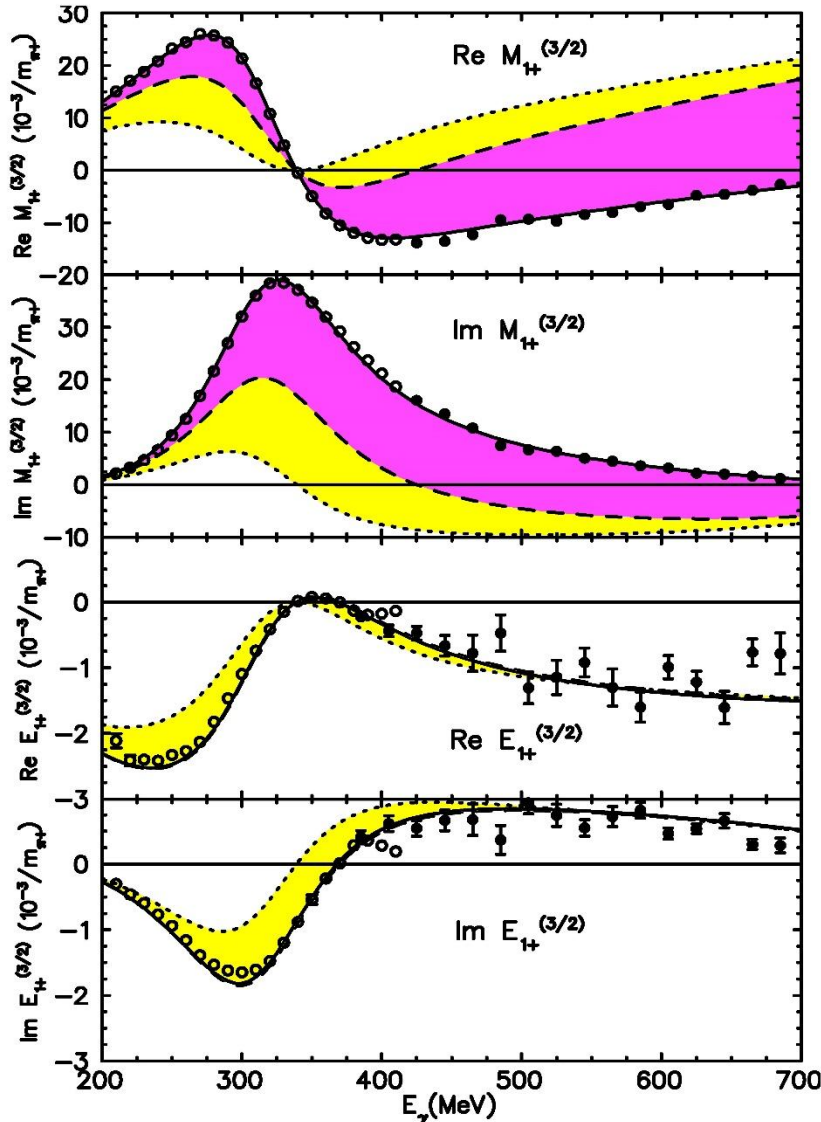
$\phi(W)$ = to adjust the phase of the total multipole to be equal to the corresponding πN phase shift $\delta^{(\alpha)}$.

Note that $\bar{A}_\alpha^R(Q^2)$ refers to a bare vertex

- Δ deformation

△ deformation (w/ two parameters GM1 & GE2)

(hyperfine qq interaction → D-state component in the △)



Pion Cloud
 Bare Δ

$$R_{EM} = -2.4\%$$

Dashed (dotted) curves are results for $t_{\gamma\pi}^B$ including (excluding) the principal value integral contribution.

$$t_{\gamma\pi}^{B,\alpha} = \exp(i\delta^{(\alpha)}) \cos \delta^{(\alpha)} \left\{ v_{\gamma\pi}^{B,\alpha}(W, Q^2) + P \int_0^\infty dq' \frac{q'^2 R_{\pi N}^{(\alpha)}(q_E, q'; E) v_{\gamma\pi}^{B,\alpha}(q', k)}{E - E_{\pi N}(q')} \right\}$$

	$A_{1/2}$ ($10^{-3}GeV^{-1/2}$)	$A_{3/2}$	$Q_{N \rightarrow \Delta}$ (fm^2)	$\mu_{N \rightarrow \Delta}$
PDG	-135	-255	-0.072	3.512
LEGS	-135	-267	-0.108	3.642
MAINZ	-131	-251	-0.0846	3.46
DMT	-134 (-80)	-256 (-136)	-0.081 (0.009)	3.516 (1.922)
SL	-121 (-90)	-226 (-155)	-0.051 (0.001)	3.132 (2.188)

Comparison of our predictions for the helicity amplitudes, $Q_{N \rightarrow \Delta}$ and $\mu_{N \rightarrow \Delta}$ with experiments and Sato-Lee's prediction. The numbers within the parenthesis in red correspond to the bare values. Small bare value of $Q_{N \rightarrow \Delta}$ indicate that bare Delta is almost spherical.

DMT πN model:

extension of Taipei-Argonne model
to energies ≤ 2 GeV

- Inclusion of ηN channel and effects of $\pi\pi N$ channel in S_{11}
- Introducing higher resonances as indicated by the data

G.Y. Chen et al., Phys. Rev. C 76 (2007) 035206.

decomposition of bkg and reson.

(in the case of only one resonance)

$$t_{\pi N}(E) = t_{\pi N}^B(E) + t_{\pi N}^R(E),$$

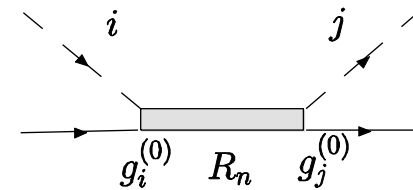
where

$$\left\{ \begin{array}{l} t_{\pi N}^B(E) = v_{\pi N}^B(E) + v_{\pi N}^B g_0(E) t_{\pi N}(E), \\ t_{\pi N}^R(E) = v_{\pi N}^R(E) + v_{\pi N}^R g_0(E) t_{\pi N}(E). \end{array} \right.$$

Introduction of higher resonances

If there are n resonances, then

$$\nu_{ij}^R(q, q'; E) = \sum_{n=1}^N \nu_{ij}^{R_n}(q, q'; E)$$

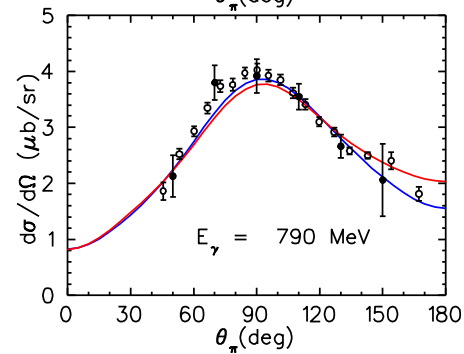
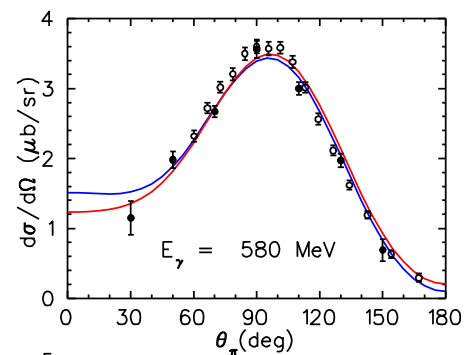
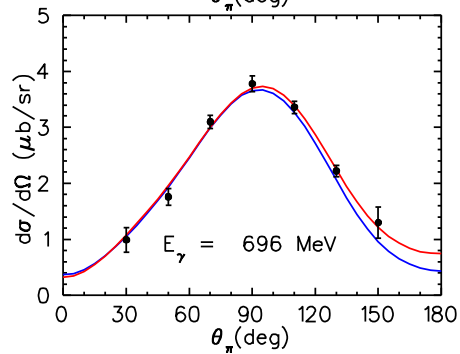
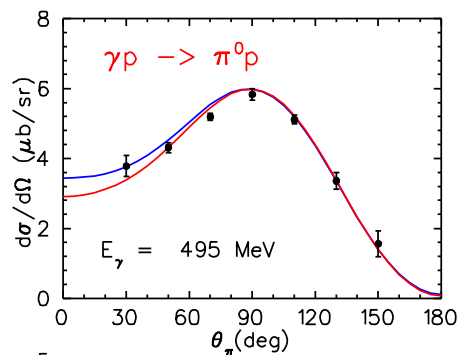
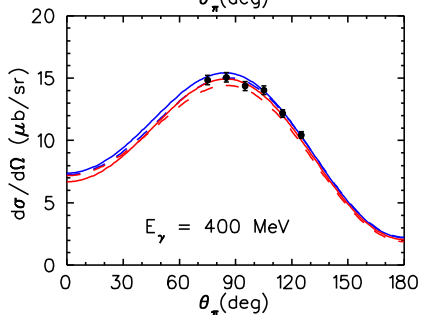
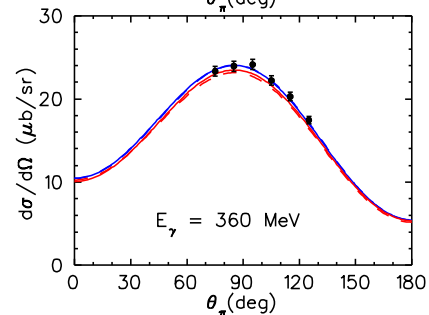
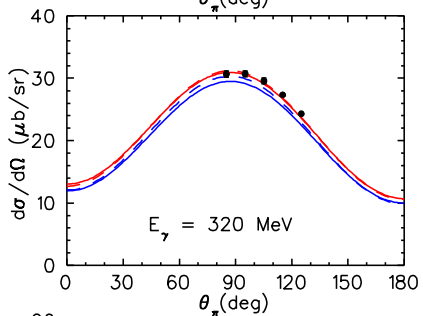
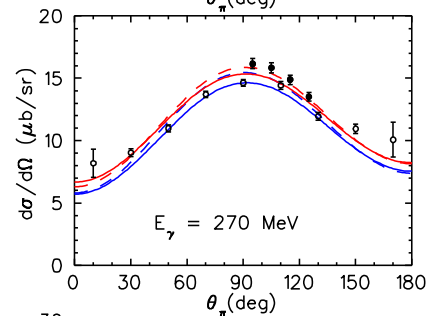
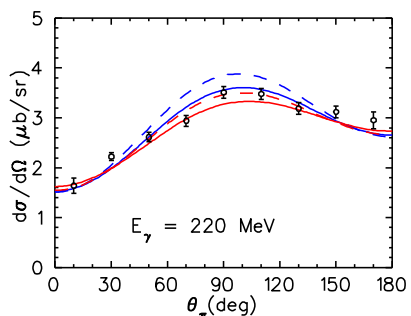
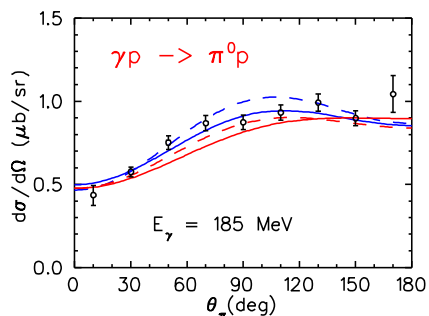


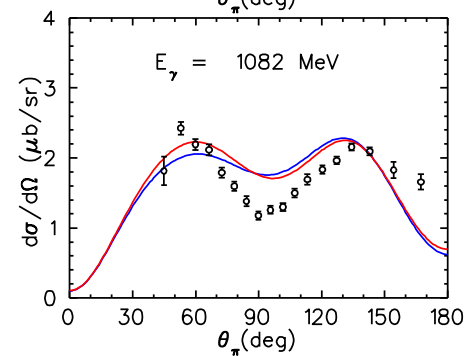
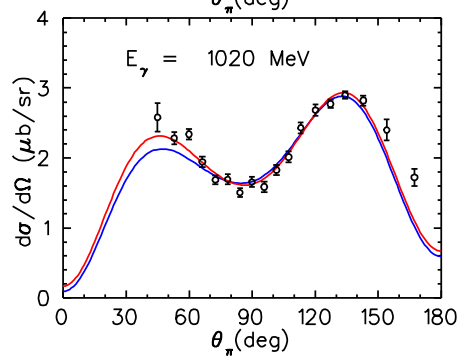
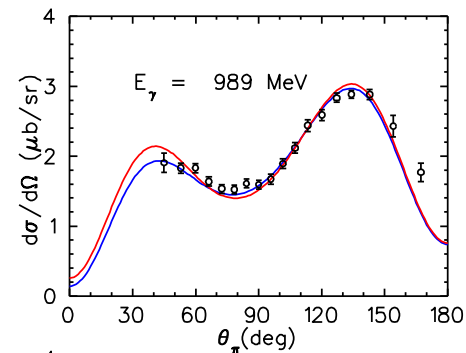
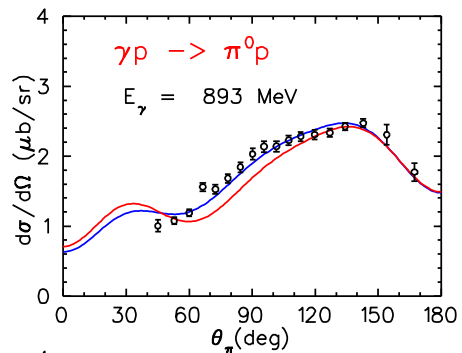
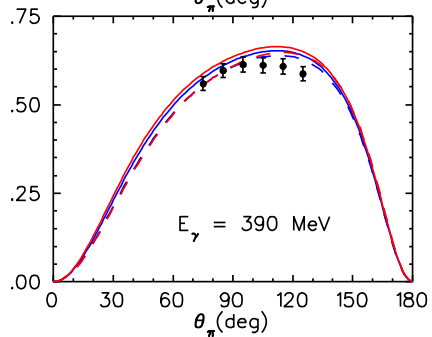
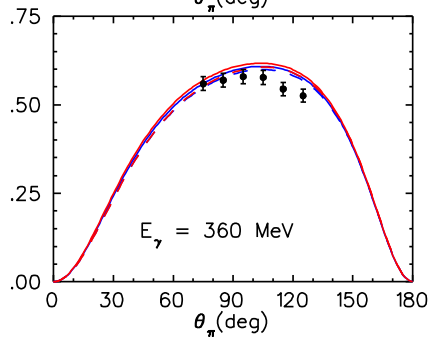
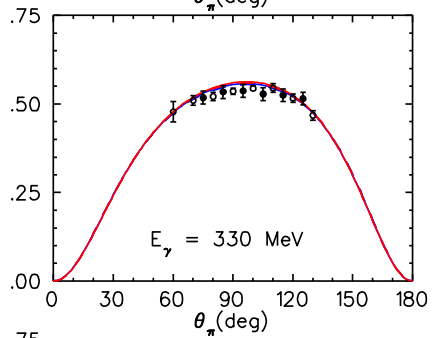
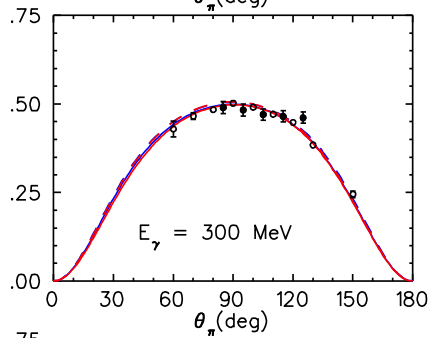
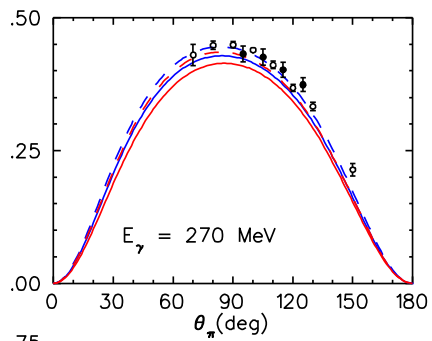
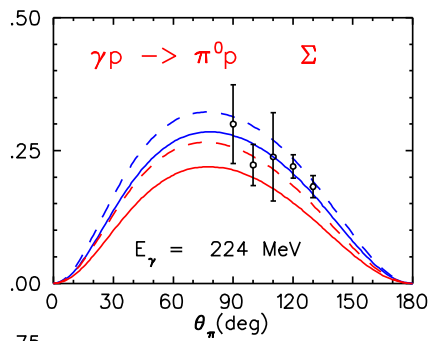
$$\nu_{ij}(E) = \nu_{ij}^B(E) + \nu_{ij}^R(E)$$

$$t_{ij}(E) = \nu_{ij}(E) + \sum_k \nu_{ik}(E) g_k(E) t_{kj}(E),$$

Coupled-channels equations can be solved

— MAID — DMT

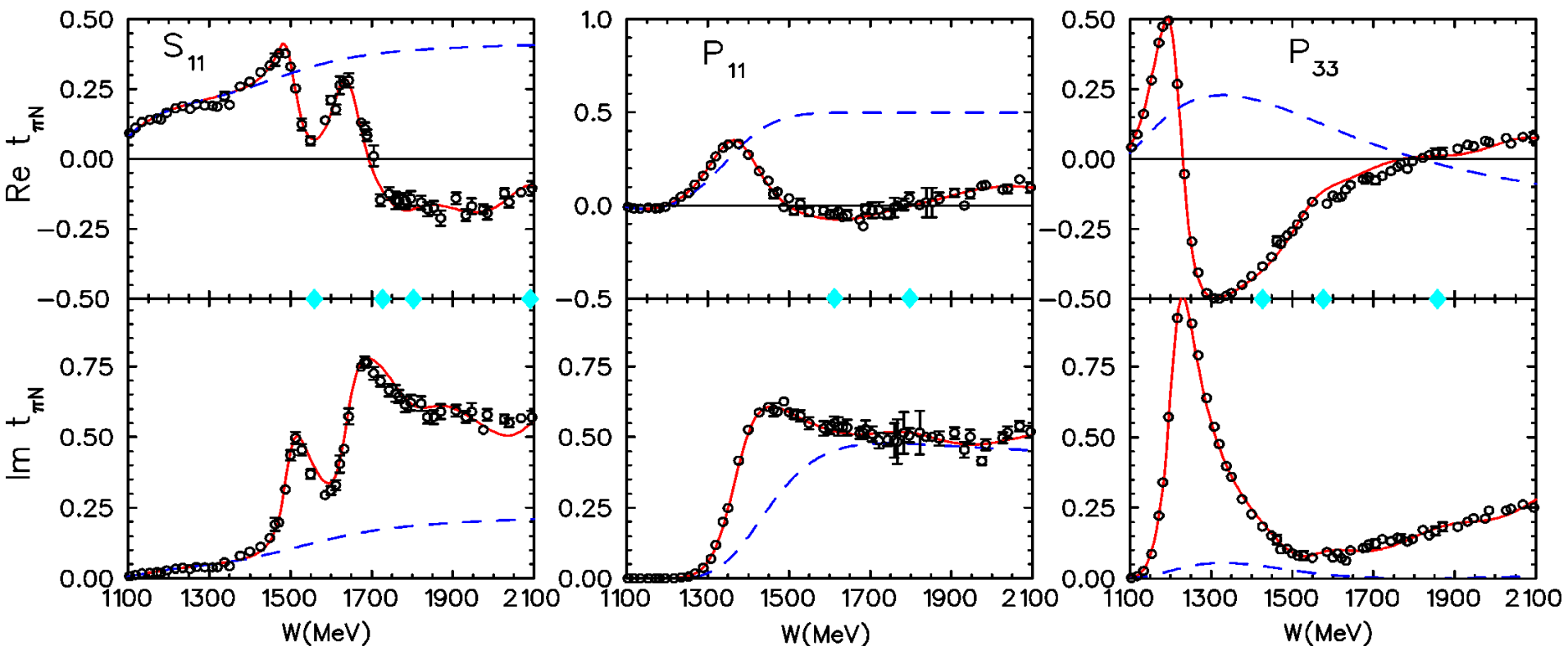




results of our fits to the SAID s.e. partial waves

◆ bare resonances

----- nonresonant background



requires 4 resonances in S_{11}

single-energy pw analysis from SAID

Resonance masses, widths,
and pole positions

bare and physical resonance masses, total widths,
 πN branching ratios and background phases
 for N^* resonances ($I=1/2$)

bare phys

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi} (\%)$	ϕ_R (deg)
$P_{11}(1440)$ ****	1612	1418 1430±20	436 350±100	44 65±10	32
$D_{13}(1520)$ ****	1590	1520 1515±5	94 115±15	62 60±5	1.2
$S_{11}(1535)$ ****	1559	1520 1535±10	130 150±25	43 45±10	20
$S_{11}(1650)$ ****	1727	1678 1655±10	200 140±30	73 70±20	24
$D_{15}(1675)$ ****	1710	1670 1675±5	154 147±17	18 40±5	49
$F_{15}(1680)$ ****	1748	1687 1685±5	156 130±10	67 67±2	7.9
$D_{13}(1700)$ ***	1753	1747 1700±50	156 175±75	5 12±5	-1
$P_{11}(1710)$ *****	1798	1803 1710±30	508 150±100	32 12±8	40

our analysis
 PDG
 star * in red,
 upgraded in
 2014

bare mass **physical mass**

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi} (\%)$	ϕ_R (deg)
$P_{13} (1720)$ ****	1725	1711 1725±25	278 225±75	13 11±3	0
$P_{13} (1900)$ ****	1922	1861 ~1900	1000 ~250	18 ~5	-3.5
$F_{15} (2000)$ **	1928	1926 2150±50	58 500±200	4 15±5	18
$D_{13} (2080)$ **	1972	1946 1804±55	494 450±185	15 ~4	5
$S_{11} (xxx)$	1803	1878	508	41	-5
$S_{11} (2090)$ *	2090	2124 2180±80	388 350±100	37 18±8	-18
$P_{11} (2100)$ *	2196	2247 2125±75	1020 260±100	42 12±2	32
$D_{13} (xxx)$	2162	2152	292	14	7
$P_{13} (xxx)$	2220	2204	406	15	-4
$D_{15} (2200)$ **	2300	2286 2180±80	532 400±100	16 10±3	8

our analysis
PDG

additional res.

additional res.
additional res.

resonance parameters for Δ resonances ($I=3/2$)

bare physical
mass mass

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi}$ (%)	ϕ_R (deg)
P_{33} (1232) ****	1425	1233 1232±1	132 117±3	100 100	12
P_{33} (1600) *****	1575	1562 1600±100	216 320±100	6 17±7	-9
S_{31} (1620) ****	1654	1616 1630±30	160 140±10	32 25±5	-41
D_{33} (1700) ****	1690	1650 1710±40	260 300±100	15 15±5	-5
P_{31} (1750) *	1765	1746 1744±36	554 300±120	4 8±3	-24
S_{31} (1890) **	1796	1770 1885±25	430 280±60	8 22±7	-44
F_{35} (1930) ****	1891	1854 1950±50	534 360±120	11 12±3	-12

our analysis
PDG

resonance parameters for Δ resonances ($I=3/2$)

bare physical
mass mass

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi} (\%)$	ϕ_R (deg)
P_{31} (1910) ****	1953	1937 1895±25	226 230±40	14 22±7	-21
P_{33} (1920) ***	1856	1827 1935±35	834 240±60	12 12±7	3
D_{35} (1930) ***	2100	2068 1950±50	426 360±140	15 10±5	-20
D_{33} (1940) *	2100	2092 2057±110	310 460±320	6 18±12	-10
F_{37} (1950) ****	1974	1916 1932±17	338 285±50	47 40±5	13
F_{35} (2000) **	2277	2260 2200±125	356 400±125	11 16±5	-26
P_{31} (xxx)	2160	2100	492	35	-25
S_{31} (2150) *	2118	1942 2150±100	416 200±100	70 8±2	-44

our analysis
PDG

additional res.

Comparison with ANL-Osaka results

Features of ANL-Osaka vs. DMT

1. 8 coupled-channels

(γN , πN , ηN , σN , ρN , $\pi \Delta$, $K\Lambda$, $K\Sigma$)

2. different treatments in

a. derivation of background potential

b. prescription in maintaining
gauge invariance

N* poles from ANL-Osaka DCC vs. DMT

$L_{21 2J}$	PDG (MeV)	DMT	ANL-Osaka (MeV)
$S_{11}(1535)$ (1650)	(1490 ~ 1530) - (45 ~ 125) <i>i</i>	1449 - 34 <i>i</i>	1482 - 98 <i>i</i>
	(1640 ~ 1670) - (50 ~ 85) <i>i</i>	1642 - 49 <i>i</i>	1656 - 85 <i>i</i>
$S_{31}(1620)$ (1900)	(1590 ~ 1610) - (60 ~ 70) <i>i</i>	1598 - 68 <i>i</i>	1592 - 68 <i>i</i>
	(1830 ~ 1910) - (65 ~ 115) <i>i</i>	1775 - 18 <i>i</i>	1746 - 177 <i>i</i>
$P_{11}(1440)$ (1710)	(1350 ~ 1380) - (80 ~ 110) <i>i</i>	1366 - 90 <i>i</i>	1374 - 76 <i>i</i>
	(1670 ~ 1770) - (40 ~ 190) <i>i</i>	1721 - 93 <i>i</i>	1746 - 177 <i>i</i>
$P_{13}(1720)$ (1900)	(1660 ~ 1690) - (75 ~ 200) <i>i</i>	1683 - 120 <i>i</i>	1703 - 70 <i>i</i>
	(1870 ~ 1930) - (70 ~ 150) <i>i</i>	1846 - 90 <i>i</i>	1763 - 159 <i>i</i>
$P_{31}(1910)$	(1830 ~ 1880) - (100 ~ 250) <i>i</i>	1896 - 65 <i>i</i>	1854 - 184 <i>i</i>
$P_{33}(1232)$ (1600)	(1209 ~ 1211) - (49 ~ 51) <i>i</i>	1218 - 45 <i>i</i>	1211 - 51 <i>i</i>
	(1460 ~ 1560) - (100 ~ 175) <i>i</i>	1509 - 118 <i>i</i>	1734 - 176 <i>i</i>
$D_{13}(1520)$ (1700)	(1505 ~ 1515) - (52 ~ 60) <i>i</i>	1516 - 62 <i>i</i>	1501 - 39 <i>i</i>
	(1650 ~ 1750) - (50 ~ 150) <i>i</i>	??	1702 - 141 <i>i</i>
$D_{15}(1675)$	(1655 ~ 1665) - (62 ~ 75) <i>i</i>	1657 - 66 <i>i</i>	1650 - 75 <i>i</i>
$D_{33}(1700)$ (1940)	(1620 ~ 1680) - (80 ~ 150) <i>i</i>	1609 - 67 <i>i</i>	1592 - 122 <i>i</i>
	(1800 ~ 2000) - (70 ~ 130) <i>i</i>	2070 - 134 <i>i</i>	1707 - 170 <i>i</i>
$F_{15}(1680)$	(1665 ~ 1680) - (55 ~ 68) <i>i</i>	1663 - 58 <i>i</i>	1665 - 49 <i>i</i>
$F_{35}(1905)$	(1805 ~ 1835) - (132 ~ 150) <i>i</i>	1771 - 95 <i>i</i>	1765 - 94 <i>i</i>
$F_{37}(1950)$	(1870 ~ 1890) - (110 ~ 130) <i>i</i>	1860 - 100 <i>i</i>	1872 - 103 <i>i</i>

Summary

- The DMT coupled-channel dynamical model gives excellent description of the pion scattering and pion photoproduction data from threshold up to $W \leq 2 \text{ GeV}$
 - Excellent agreement with π^0 threshold production data. Two-loop contributions small. DMT has become a **benchmark/guide** experiments and calculations for π^0 threshold production.
cf. talk by Scherer
 - DMT predicts $\mu_{N \rightarrow \Delta} = 3.514 \mu_N$, $Q_{N \rightarrow \Delta} = -0.081 \text{ fm}^2$, and $R_{EM} = -2.4\%$, all in close agreement with the experiments.
 \Rightarrow dressed Δ is **oblate**
 - Bare Δ is almost spherical. The oblate deformation of the dressed Δ arises almost exclusively from pion cloud

- The resonance masses, width, and pole positions extracted with DMT agree, in general, with PDG and ANL-Osaka numbers, except we get 4 resonances not yet seen in PDG table:

S11(1878), D13(2152), P13(2204), P31(2100)

DMT vs. ANL-Osaka

little dog vs. big tiger

but little could be effective

(小而美)

- Connection with LQCD
cf. talks by and Edwards

END