Hyperon resonance Λ(1405) and the K^-pp three-body resonance

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    - *Feshbach projection on coupled-channel Complex Scaling Method (ccCSM+Feshbach method)*
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4. Summary and future plan

The 10th International Workshop on the Physics of Excited Nucleons (NSTAR2015)
25. May. '15 @ Icho Kaikan (Pharmaceutical Information Center), Suita Campus, Osaka Univ., Osaka, Japan

1. Introduction

1. Introduction

The 10th International Workshop on the Physics of Excited Nucleons

NSTAR 2015

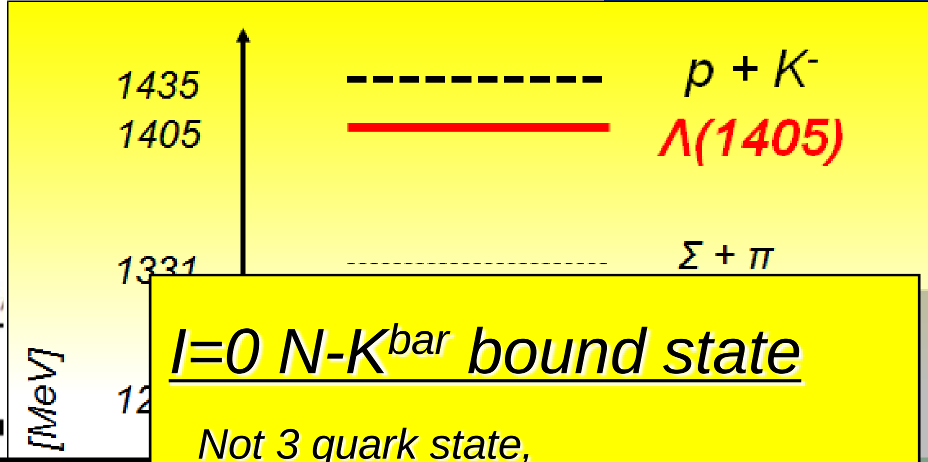
May 25(Mon)-28(Thu), 2015

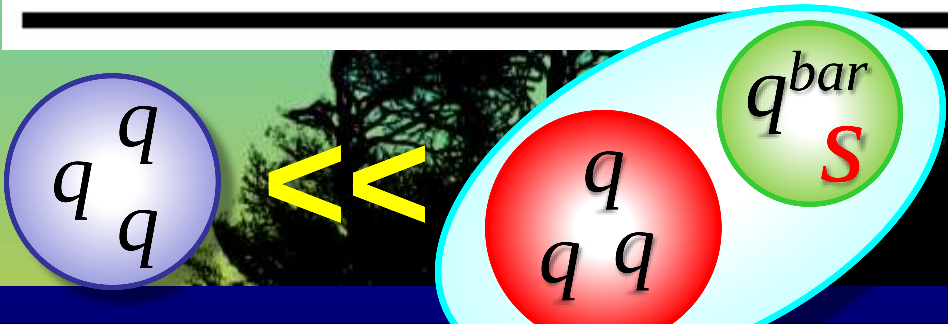
Y* is also interesting!

Λ(1405) 1/2⁻ $I(J^P) = 0(\frac{1}{2}^-)$

Mass $m = 1405.1^{+1.3}_{-1.0}$ MeV
Full width $\Gamma = 50 \pm 2$ MeV
Below $\overline{K}N$ threshold

Λ(1405) DECAY MODES	Fraction (Γ_i
$\Sigma\pi$	100 %

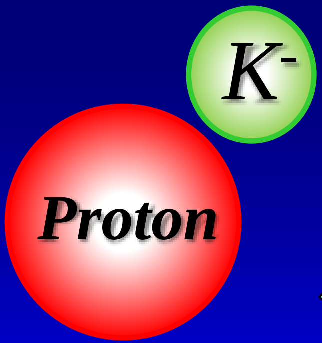




I=0 N-K^{bar} bound state

Not 3 quark state,
← A naive quark model fails.
N. Isgar and G. Karl, Phys. Rev. D18, 4187 (1978)

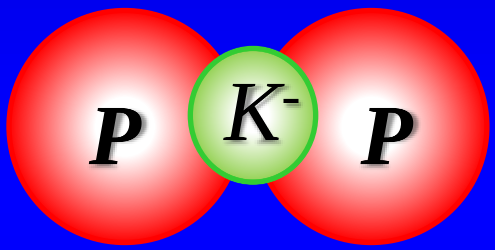
But rather a molecular state
T. Hyodo and D. Jido,
Prog. Part. Nucl. Phys. 67, 55 (2012)



Λ(1405)

… I=0 N-K^{bar} quasi-bound state

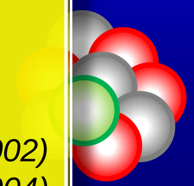
K^-pp



The simplest kaonic nucleus

"Prototype of kaonic nuclei"

Nuclear many-body system with K^-



$^3HeK^-$, $pppK^-$, $^4HeK^-$, $pppnK^-$, …, $^8BeK^-$,…

Kaonic nuclei = Exotic system??

- Doorway to dense matter
  → Chiral symmetry restoration in dense matter
- Interesting structure
- Neutron star…

Y. Akaishi and T. Yamazaki, PRC 52, 044005 (2002)
A. Dote, H. Horiuchi, Y. Akaishi and T. Yamazaki, PRC70, 044313 (2004)

2. Method

- ***Complex Scaling Method***
- ***Feshbach projection on coupled-channel Complex Scaling Method***

"ccCSM+Feshbach method"

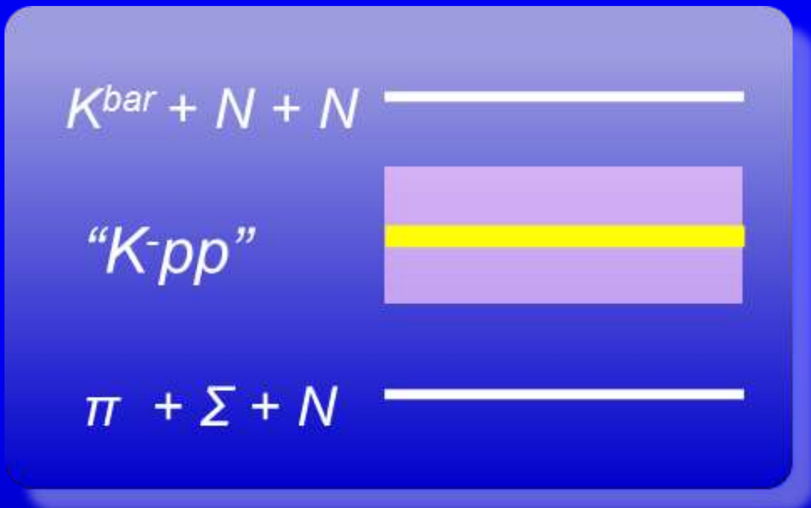
A. D., T. Inoue, T. Myo, PTEP 2015, 043D02 (2015)

- *Λ(1405) = Resonant state & $K^{bar}N$ coupled with πΣ*

- *"$K^{-}pp$" … Resonant state of*
  $K^{bar}NN$-πYN coupled-channel system

Doté, Hyodo, Weise, PRC79, 014003(2009). Akaishi, Yamazaki, PRC76, 045201(2007)
Ikeda, Sato, PRC76, 035203(2007). Shevchenko, Gal, Mares, PRC76, 044004(2007)
Barnea, Gal, Liverts, PLB712, 132(2012)

- ➢ ***Resonant state***
- ➢ ***Coupled-channel system***



⇒ ***<u>"coupled-channel Complex Scaling Method"</u>***

Complex Scaling Method

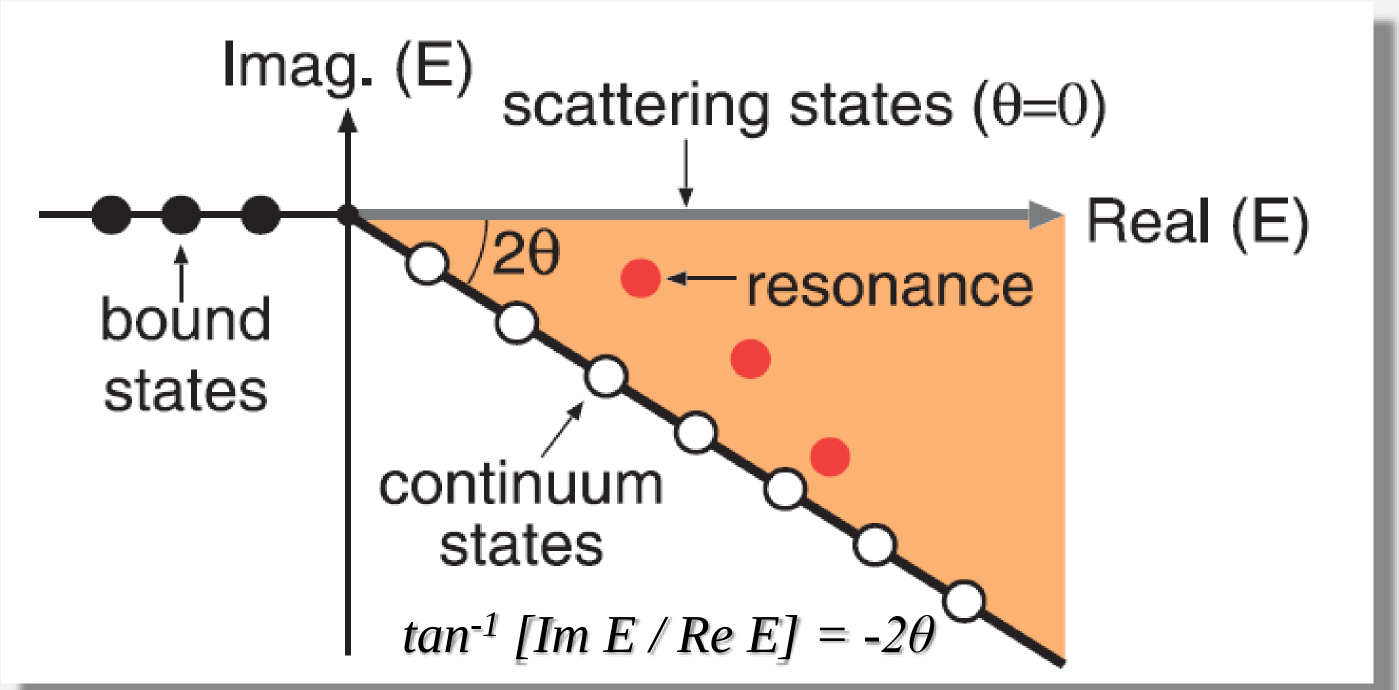
S. Aoyama, T. Myo, K. Kato, K. Ikeda, PTP116, 1 (2006)
T. Myo, Y. Kikuchi, H. Masui, K. Kato, PPNP79, 1 (2014)

… Powerful tool for resonance study of many-body system

Complex rotation (Complex scaling) of coordinate
Resonance wave function → L^2 integrable

$$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

Diagonalize $H_\theta = U(\theta)\, H\, U^{-1}(\theta)$ with Gaussian base,

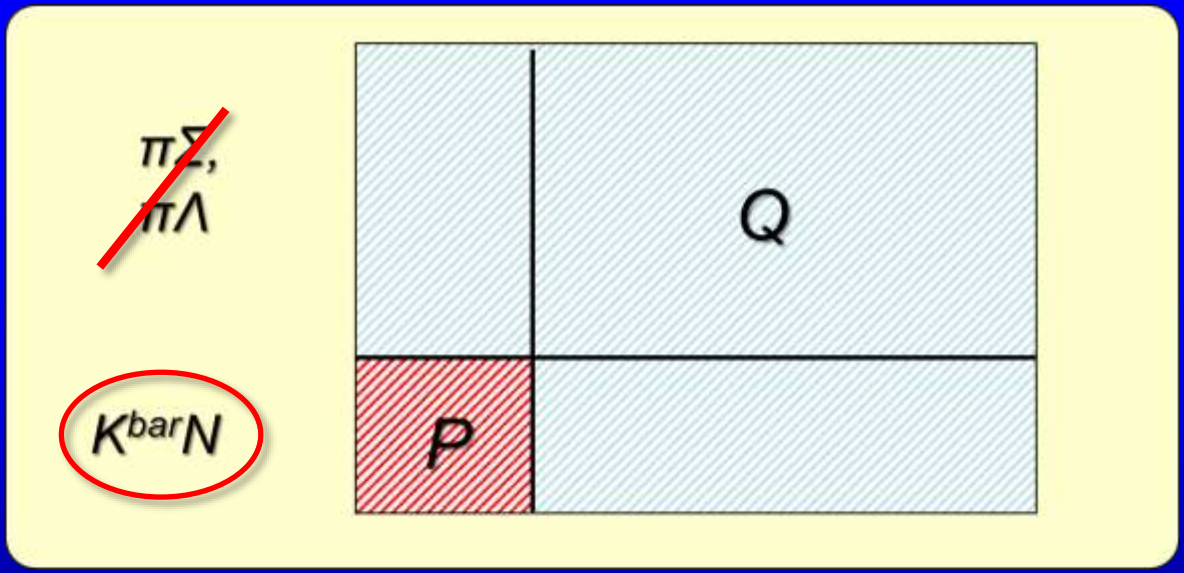


- *Continuum state appears on 2ϑ line.*
- *Resonance pole is off from 2ϑ line, and independent of ϑ. (ABC theorem)*

ccCSM+Feshbach method

- *Λ(1405) = two-body system of $K^{bar}N$-πΣ*
  → Explicitly treat coupled-channel problem

- *"$K^{-}pp$" = three-body system of $K^{bar}NN$-πYN*
  … High computational cost



For economical treatment of "$K^{-}pp$", we construct an effective $K^{bar}N$ single-channel potential by means of Feshbach projection on CSM.

Formalism of ccCSM + Feshbach method

Elimination of channels by Feshbash method

Schrödinger eq.
in model space "P" and out of model space "Q"

$$\begin{pmatrix} T_P + v_P & V_{PQ} \\ V_{QP} & T_Q + v_Q \end{pmatrix} \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix} = E \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix}$$

Schrödinger eq. in P-space :

$$\left(T_P + U_P^{Eff}(E) \right) \Phi_P = E \Phi_P$$

Effective potential for P-space

$$U_P^{Eff}(E) = v_P + V_{PQ} G_Q(E) V_{QP}$$

Q-space Green function:

$$G_Q(E) = \frac{1}{E - H_{QQ}}$$

Extended Closure Relation in Complex Scaling Method

$$H_{QQ}^\theta \left| \chi_n^\theta \right\rangle = \varepsilon_n^\theta \left| \chi_n^\theta \right\rangle$$

$$H_{QQ}^\theta = U(\theta) H_{QQ} U^{-1}(\theta)$$

$$\int_C \sum_{R+B} \left| \chi_n^\theta \right\rangle \left\langle \chi_n^\theta \right| = 1$$

Diagonalize H^θ_{QQ} with Gaussian base,

$$\sum_n \left| \chi_n^\theta \right\rangle \left\langle \chi_n^\theta \right| \approx 1$$

Well approximated

T. Myo, A. Ohnishi and K. Kato, PTP 99, 801 (1998)
R. Suzuki, T. Myo and K. Kato, PTP 113, 1273 (2005)

Express the $G_Q(E)$ with Gaussian base using ECR

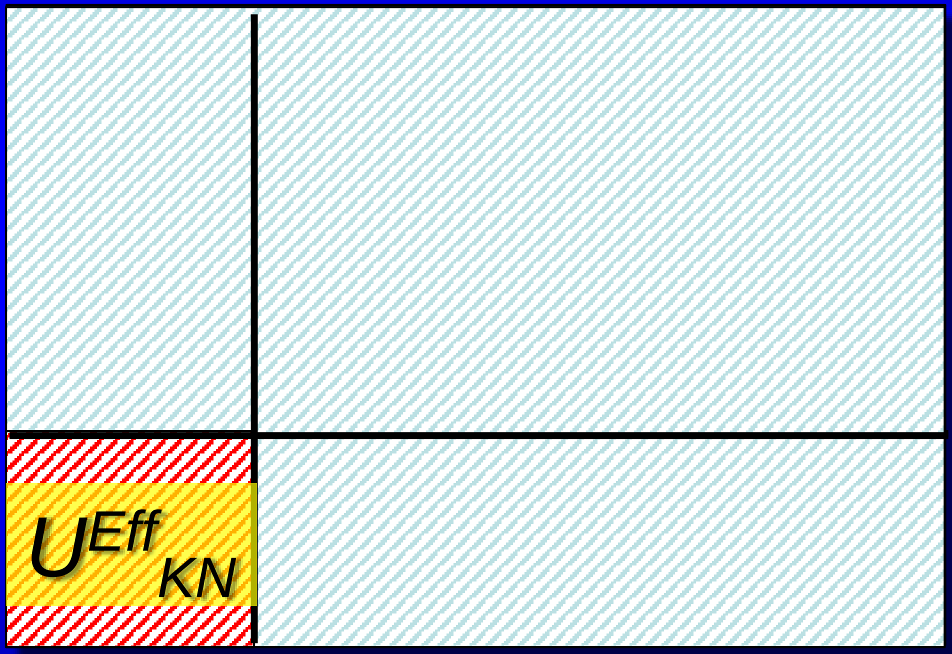
$$G_Q^\theta(E) = \frac{1}{E - H_{QQ}^\theta} \approx \sum_n \left| \chi_n^\theta \right\rangle \frac{1}{E - \varepsilon_n^\theta} \left\langle \chi_n^\theta \right|$$

$$U_P^{Eff}(E) = v_P + V_{PQ} \underbrace{U^{-1}(\theta) G_Q^\theta(E) U(\theta)}_{G_Q(E)} V_{QP}$$

$\left\{ \left| \chi_n^\theta \right\rangle \right\}$: expanded with Gaussian base.

Applying this tequnique to the *two-body* $K^{bar}N$-πY system,

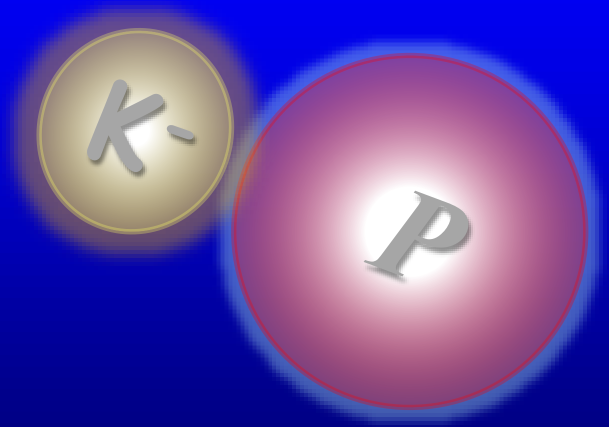
Effective single-channel $K^{bar}N$ potential is constructed.



Using the U^{Eff}_{KN} in "$K^{-}pp$" three-body calculation, the $K^{bar}NN$-πYN coupled-channel problem is reduced to the $K^{bar}NN$ single-channel problem.

3. Result

Hyperon resonance Λ (1405)



Chiral SU(3) potential with a Gaussian form

A. D., T. Inoue, T. Myo, Nucl. Phys. A 912, 66 (2013)

- **Anti-kaon = Nambu-Goldstone boson**

⇒ Chiral SU(3)-based $K^{bar}N$ potential

- Weinberg-Tomozawa term of effective chiral Lagrangian
- Gaussian form in r-space
- Semi-rela. / Non-rela.
- Based on Chiral SU(3) theory

→ **Energy dependence**

A non-relativistic potential (NRv2c)

$$V_{ij}^{(I=0,1)}(r) = -\frac{C_{ij}^{(I=0,1)}}{8 f_\pi^2}\left(\omega_i + \omega_j\right)\sqrt{\frac{1}{m_i m_j}}\, g_{ij}(r)$$

$$g_{ij}(r) = \frac{1}{\pi^{3/2} d_{ij}^3} \exp\left[-\left(r/d_{ij}\right)^2\right]$$: Gaussian form

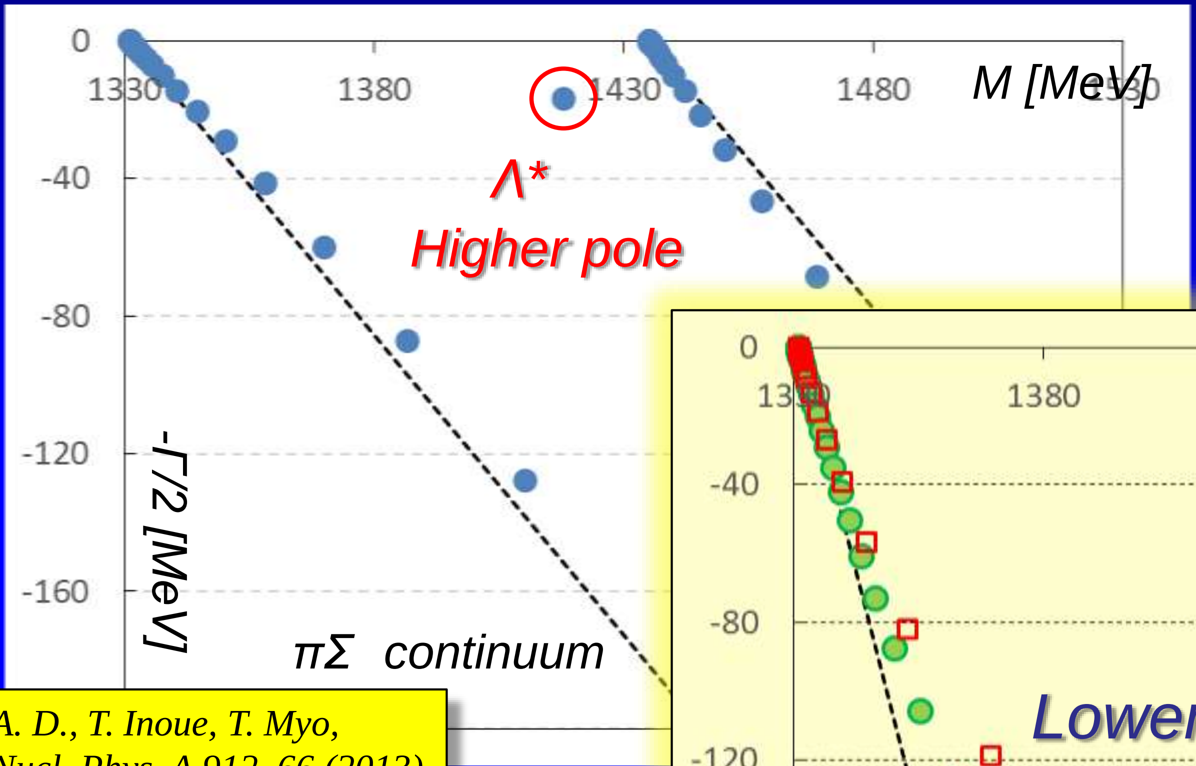
ω_i: meson energy

Constrained by $K^{bar}N$ scattering length

$a_{KN(I=0)}$ = -1.70+i0.67fm, $a_{KN(I=1)}$ = 0.37+i0.60fm

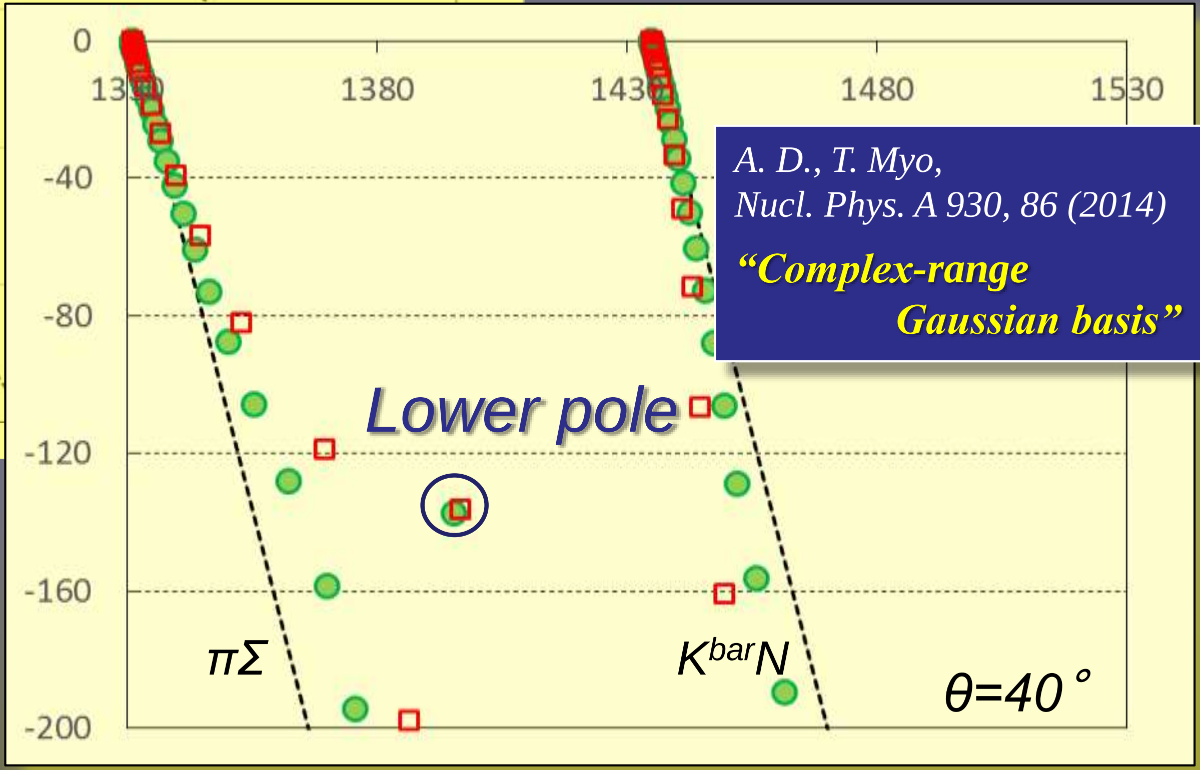
A. D. Martin, NPB179, 33(1979)

Poles of I=0 $K^{bar}N$-πΣ system found by ccCSM



$K^{bar}N$ potential:
a chiral SU(3) potential
(NRv2, f_π=110)

A. D., T. Inoue, T. Myo,
Nucl. Phys. A 912, 66 (2013)



A. D., T. Myo,
Nucl. Phys. A 930, 86 (2014)

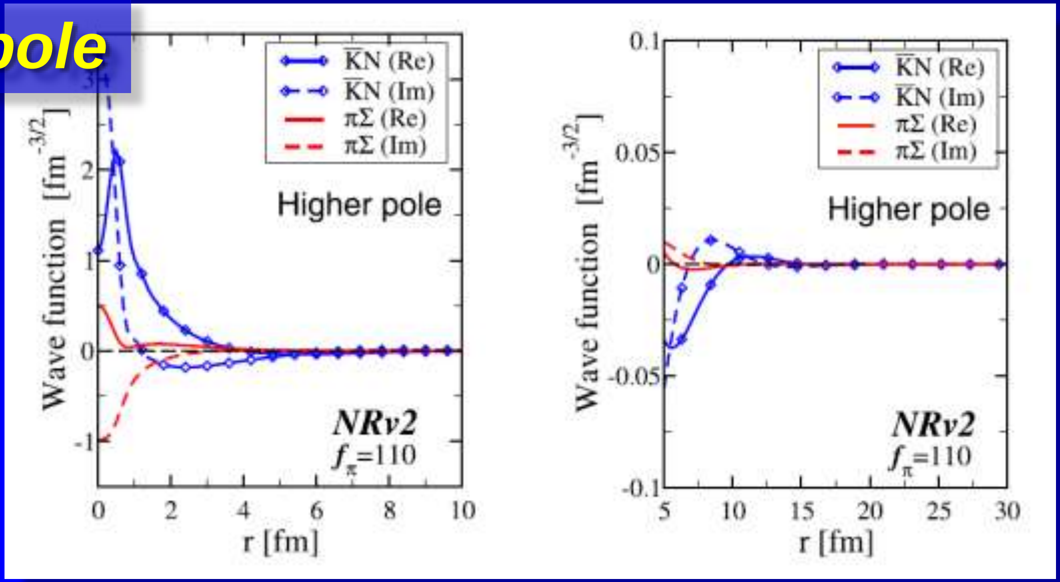
"Complex-range Gaussian basis"

Double-pole structure of Λ(1405) is confirmed!

ccCSM wfnc. of double pole

A. D., T. Myo,
Nucl. Phys. A 930, 86 (2014)

Higher pole

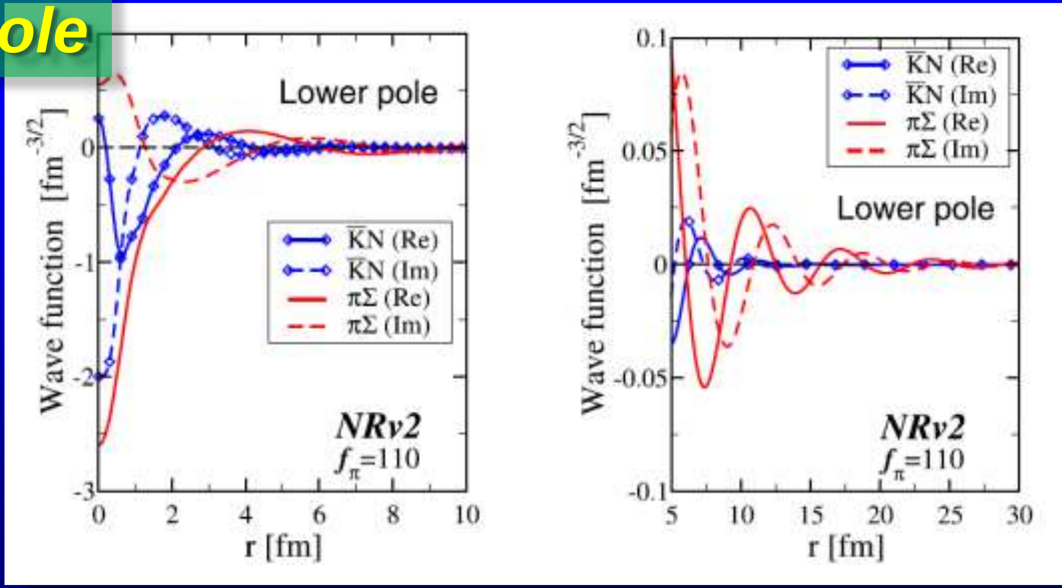


Norm ($K^{bar}N$)
1.115+0.098i

Norm (πΣ)
-0.115-0.098i

$K^{bar}N$ dominant

Lower pole



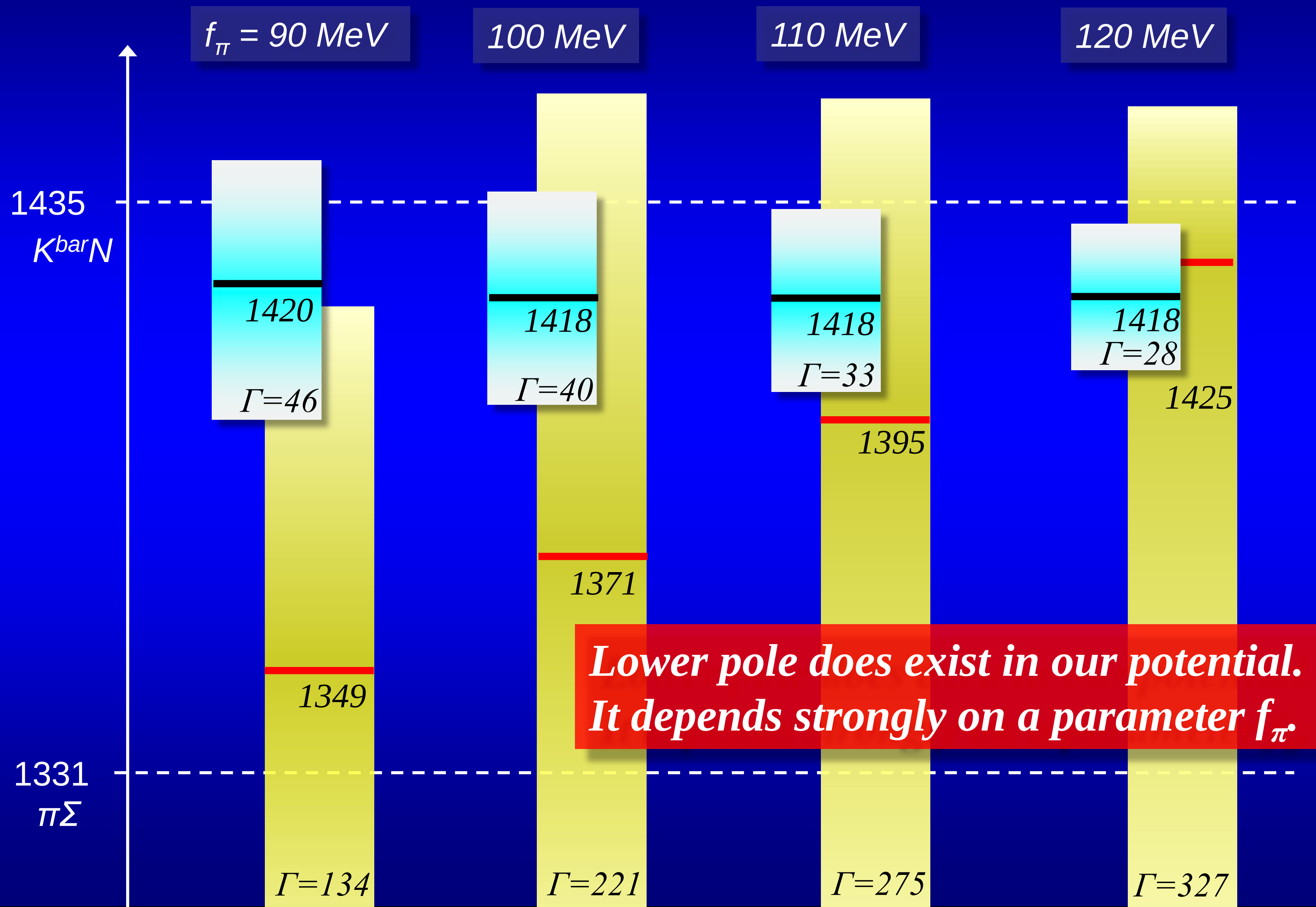
Norm ($K^{bar}N$)
0.097+0.154i

Norm (πΣ)
0.903-0.154i

πΣ dominant

Double-pole structure of Λ(1405)

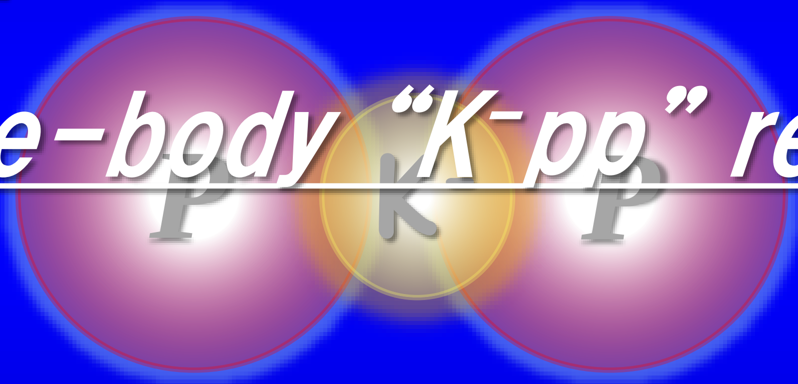
A. D., T. Myo,
Nucl. Phys. A 930, 86 (2014)



Lower pole does exist in our potential.
It depends strongly on a parameter f_π.

3. Result

Three-body "K^-pp" resonance



"K^-pp" =

$K^{bar}NN - \pi\Sigma N - \pi\Lambda N$ ($J^{\pi} = 0^-$, $T=1/2$)

Apply ccCSM + Feshbach method to K^-pp

"K^-pp" … $K^{bar}NN$ - $\pi\Sigma N$ - $\pi\Lambda N$ ($J^\pi=0^-$, $T=1/2$)

For the two-body system, P = $K^{bar}N$, Q = πY

$$V\left(K^{bar}N-\pi Y; I=0,1\right)$$
$$V\left(\pi Y-\pi Y' \quad ; I=0,1\right)$$

Feshbach + ccCSM →

$$U^{Eff}_{K^{bar}N(I=0,1)}\left(E\right)$$

- Schrödinger eq. for $K^{bar}NN$ channel :

$$\left(T_{K^{bar}NN}+V_{NN}+\sum_{i=1,2}U^{Eff}_{K^{bar}N_i(I)}\left(E_{K^{bar}N}\right)\right)\Phi_{K^{bar}NN} = E\,\Phi_{K^{bar}NN}$$

- Trial wave function

$$\left|"K^-pp"\right\rangle = \sum_a C_a^{(KNN,1)}\left\{G_a^{(KNN,1)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)+G_a^{(KNN,1)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_1\right]_{T=1/2}\right\rangle$$

Ch. 1: $K^{bar}NN$, NN: ^1E

$$+\sum_a C_a^{(KNN,2)}\left\{G_a^{(KNN,2)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)-G_a^{(KNN,2)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_0\right]_{T=1/2}\right\rangle$$

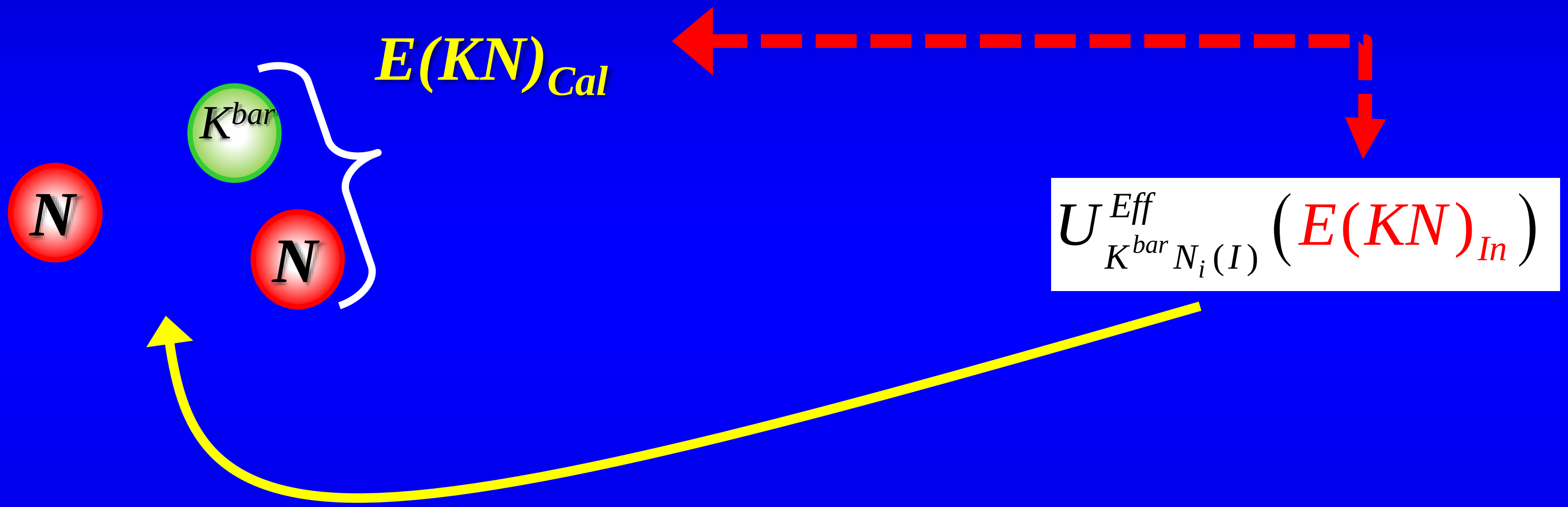
Ch. 2: $K^{bar}NN$, NN: ^1O

- Basis function = Correlated Gaussian
…including 3-types Jacobi-coordinates

$$G_a^{(KNN,i)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)=N_a^{(KNN,i)}\exp\left[-\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)A_a^{(KNN,i)}\begin{pmatrix}\mathbf{x}_1^{(3)}\\ \mathbf{x}_2^{(3)}\end{pmatrix}\right]$$

Self-consistency for complex $K^{bar}N$ energy

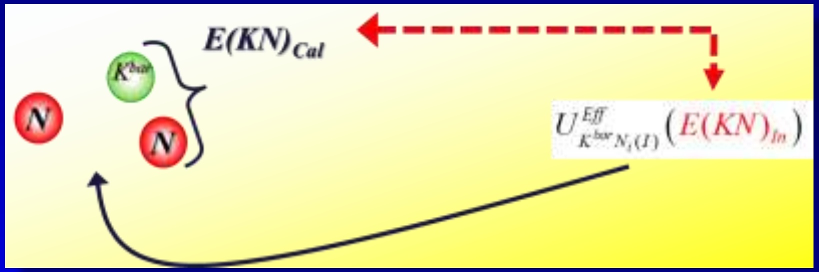
Effective $K^{bar}N$ potential has energy dependence…



- $E(KN)_{In}$: assumed in the $K^{bar}N$ potential
- $E(KN)_{Cal}$: calculated with the obtained $K^{-}pp$

When $E(KN)_{In} = E(KN)_{Cal}$, a self-consistent solution is obtained.

Self-consistency for complex $K^{bar}N$ energy



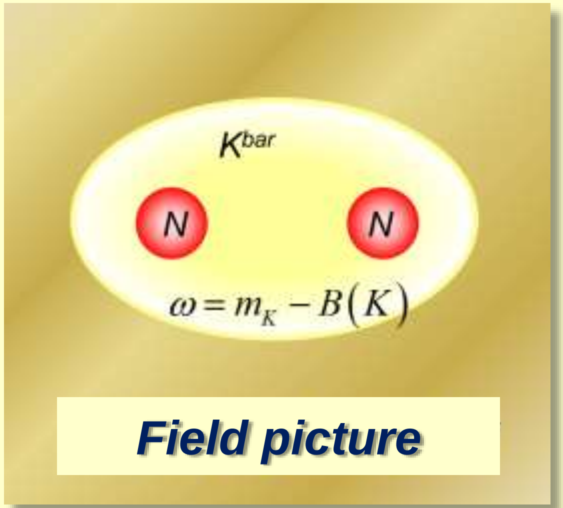
How to determine the two-body energy in the three-body system?

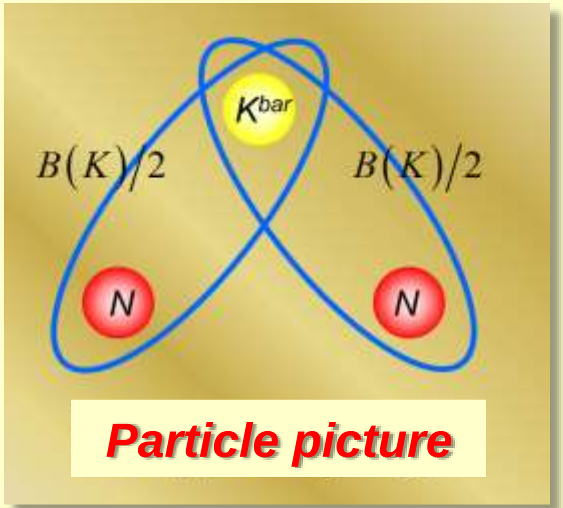
A. D., T. Hyodo, W. Weise, PRC79, 014003 (2009)

1. *Kaon's binding energy:* $B(K) \equiv -\left\{ \langle H \rangle - \langle H_{NN} \rangle \right\}$ $\quad H_{NN}$ *: Hamiltonian of two nucleons*

2. *Define a K^{bar}N-bond energy in two ways*

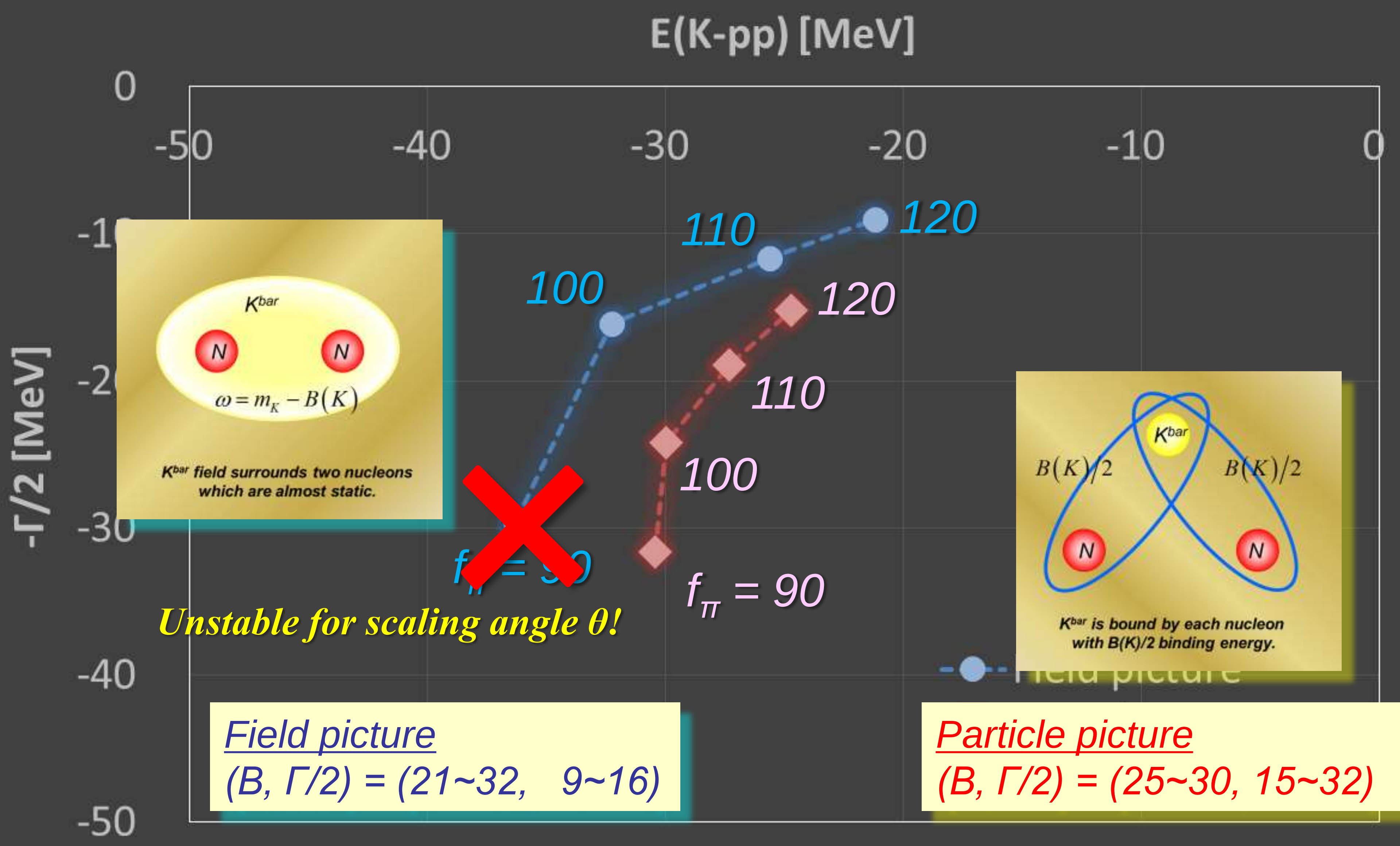
$$E_{KN} = M_N + \omega = \begin{cases} M_N + m_K - B(K) & \text{: Field picture} \\ M_N + m_K - B(K)/2 & \text{: Particle picture} \end{cases}$$





Self-consistent results
f_π=90~120MeV

NN pot. : Av18 (Central)
K^{bar}N pot. : NRv2c potential
(f_π=90 - 120MeV)



Field picture
(B, Γ/2) = (21~32, 9~16)

Particle picture
(B, Γ/2) = (25~30, 15~32)

NN correlation density

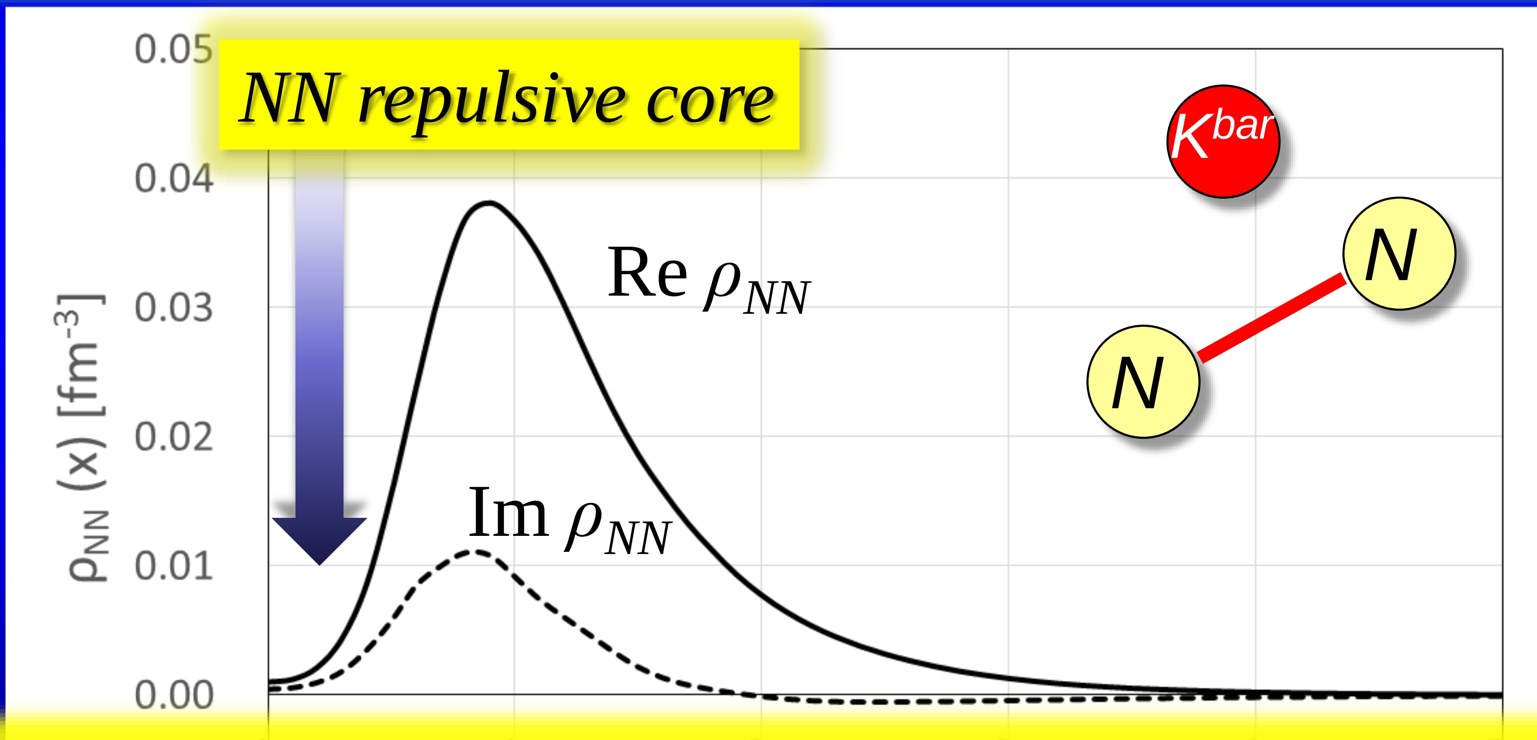
NN pot. : Av18 (Central)
$K^{bar}N$ pot. : NRv2c potential
f_{π}=110, Particle pict.

Correlation density in Complex Scaling Method

$$\rho_{NN,\theta}(\mathbf{x}) = \delta^3\left(\hat{\mathbf{r}}_{NN,\theta} - \mathbf{x}\right)$$

$$\hat{\mathbf{r}}_{XN,\theta} = \hat{\mathbf{r}}_{XN}\, e^{i\theta}$$

$$\rho_{NN}(\mathbf{x}) \equiv \left\langle \Phi_\theta \middle| \rho_{XN,\theta}(\mathbf{x}) \middle| \Phi_\theta \right\rangle = e^{-3i\theta}\int d^3\mathbf{R}\, \Phi_\theta^2\left(\mathbf{x}e^{-i\theta}, \mathbf{R}\right)$$

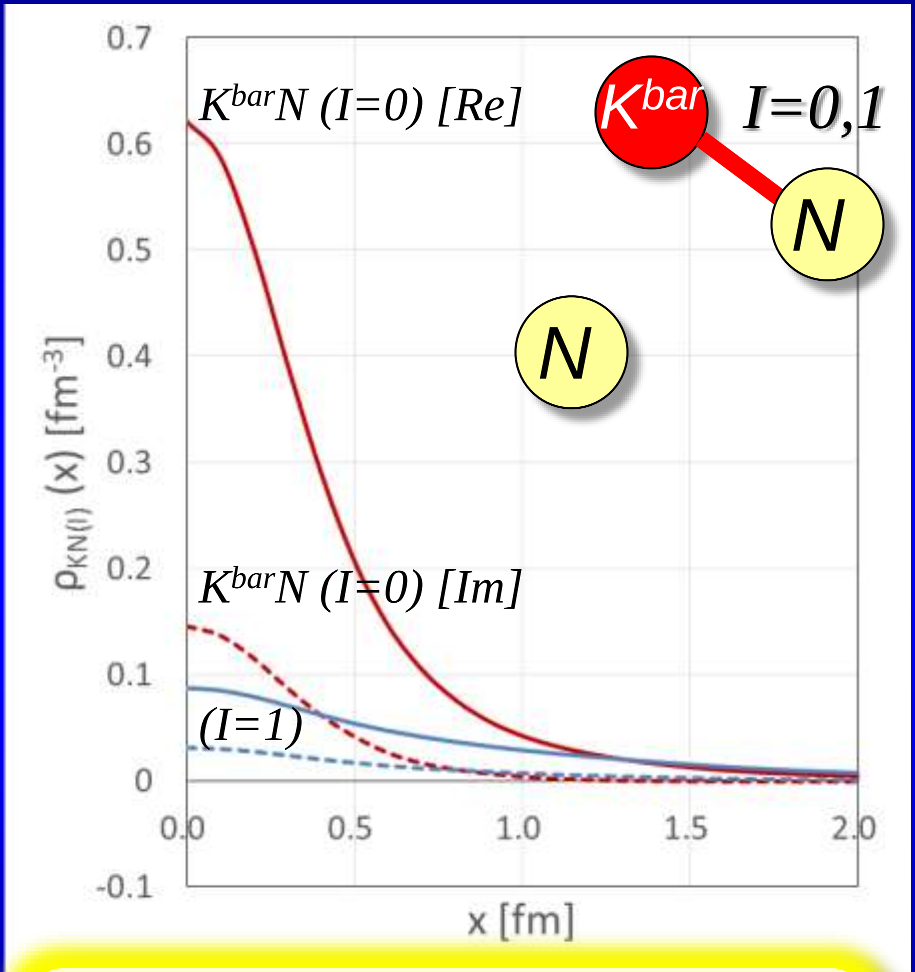


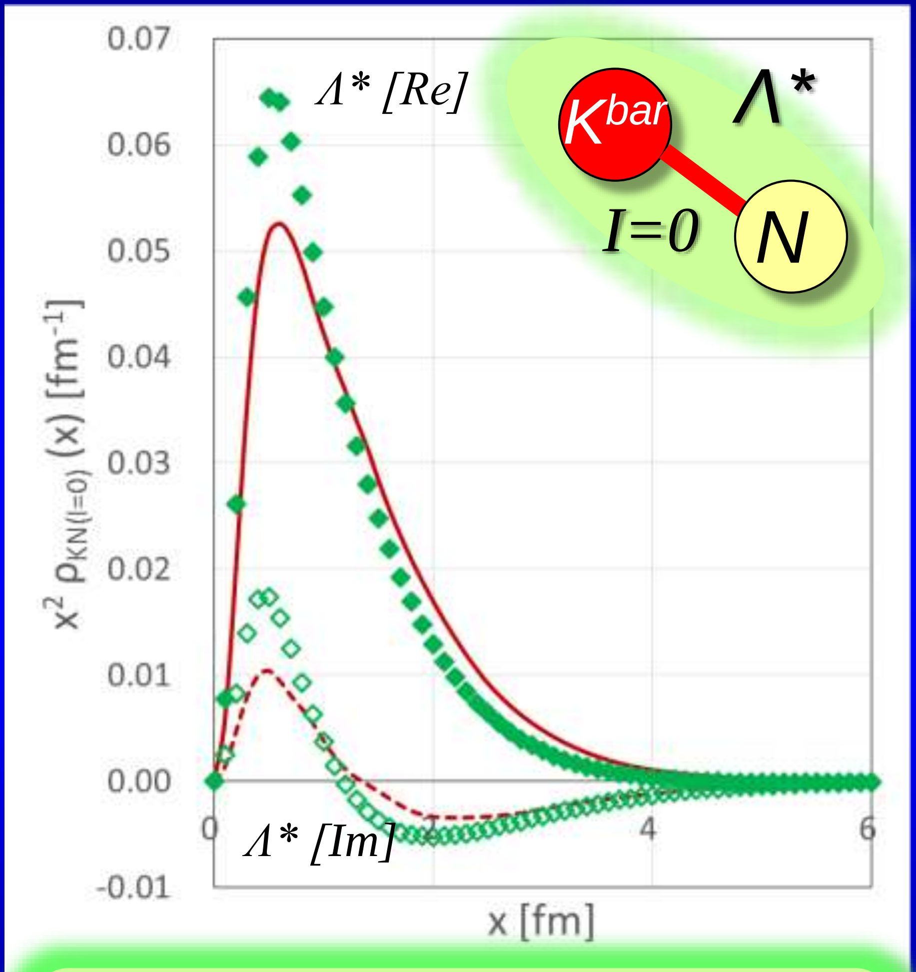
NN distance = 2.1 - i 0.3 fm

~ Mean distance of 2N in nuclear matter at normal density!

$K^{bar}N$ correlation density

NN pot. : Av18 (Central)
$K^{bar}N$ pot. : NRv2c potential
f_π=110, Particle pict.





I=0 $K^{bar}N$ compacter than I=1 one

Strong $K^{bar}N$ attraction in I=0

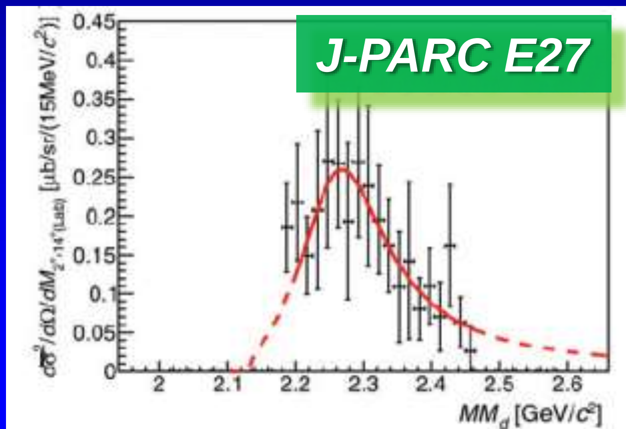
I=0 $K^{bar}N$ seems similar to Λ*

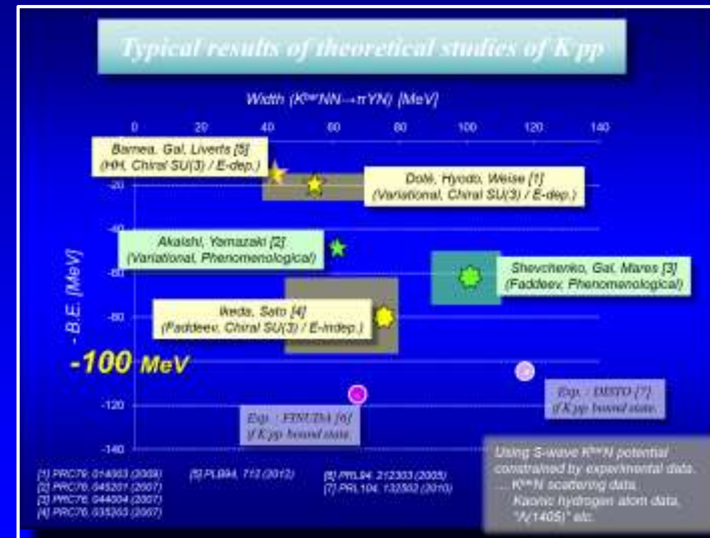
Λ* survives in $K^{-}pp$

How to understand experimental results?

FINUDA

DISTO





Signal at ~100 MeV below K^bar NN threshold

FINUDA: PRL 94, 212303 (2005)

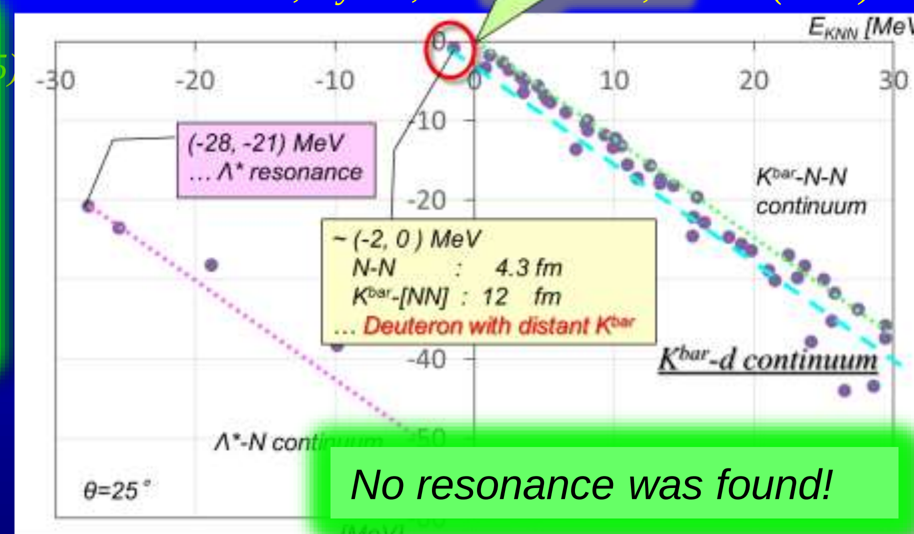
DISTO: PRL 104, 132502 (2010)

B(K⁻pp) < 100 MeV

Doté, Hyodo, We... ...C79, 014003(2009)

J-PARC E27: d(π⁺, K⁺) reaction
to search for K⁻pp

⇒ The observed state may be "K⁻pp"
with $J^\pi=1^-$ and I=1/2,
because target deuteron has spin 1.

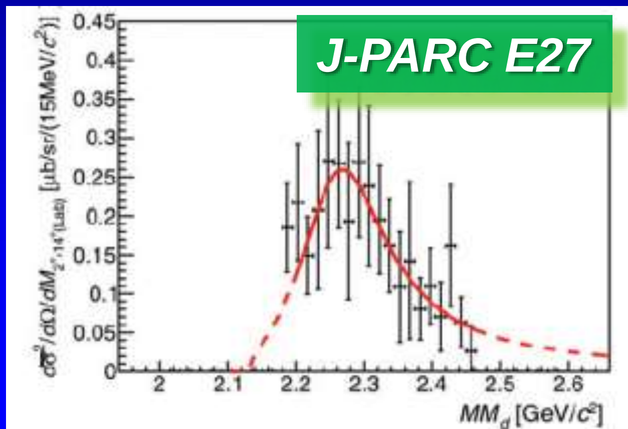


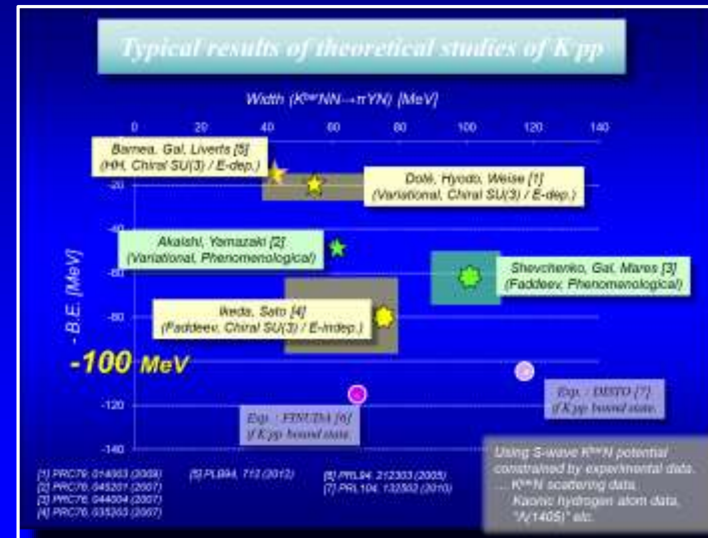
No resonance was found!

How to understand experimental results?

FINUDA

DISTO





Signal at ~100 MeV below $K^{bar}NN$ threshold

B(K^-pp) < 100 MeV

FINUDA: PRL 94, 212303 (2005)

Doté, Hyodo, Weise, PRC79, 014003(2009)
, 045201(2007)
3(2007).
RC76, 044004(2007)
712, 132(2012)

1. Partial restoration of chiral symmetry
… $K^{bar}N$ potential is enhanced by 17%.
S. Maeda, Y. Akaishi, T. Yamazaki, Proc. Jpn. Acad., Ser. B 89, 418 (2013)

2. Double-pole structute of "K^-pp" ($J^\pi=0^-$, $I=1/2$)
… "K^-pp" has two poles similarly to Λ(1405). The lower pole appears.
Y. Ikeda, H. Kamano, T. Sato, PTP124, 533 (2010)
A. Dote, T. Inoue, T Myo, PTEP 2015, 043D02 (2015)

3. Pion assisted dibaryon "Y = πΣN-πΛN ($J^\pi=2^+$, $I=3/2$)"
A. Gal, arXiv:1412.0198 (Proceeding of EXA2014)

4. Summary and future plans

4. Summary and future plans

A prototype of K^{bar} nuclei "K^-pp" = Resonance state of $K^{bar}NN$-πYN coupled system

"coupled-channel Complex Scaling Method + Feshbach projection"

… *Represent the Q-space Green function with the Extended Complete Set well approximated by Gaussian base*

⇒ *Eliminate πY channels to reduce the problem to a $K^{bar}NN$ single channel problem.*

K^-pp studied with ccCSM+Feshbch method

- *Used a Chiral SU(3)-based potential (Gaussian form in r-space)*
- *Self-consistency for kaon's complex energy*
- *Correlation density in CSM shows effect of NN repulsive core and Λ^* survival in K^-pp resonance.*
- *$J^\pi=1^-$ state ("Deuteron+K^-"-like channel) seems not to exist as a resonance state.*

K^-pp ($J^\pi=0^-$, $T=1/2$) --- NRv2c potential case

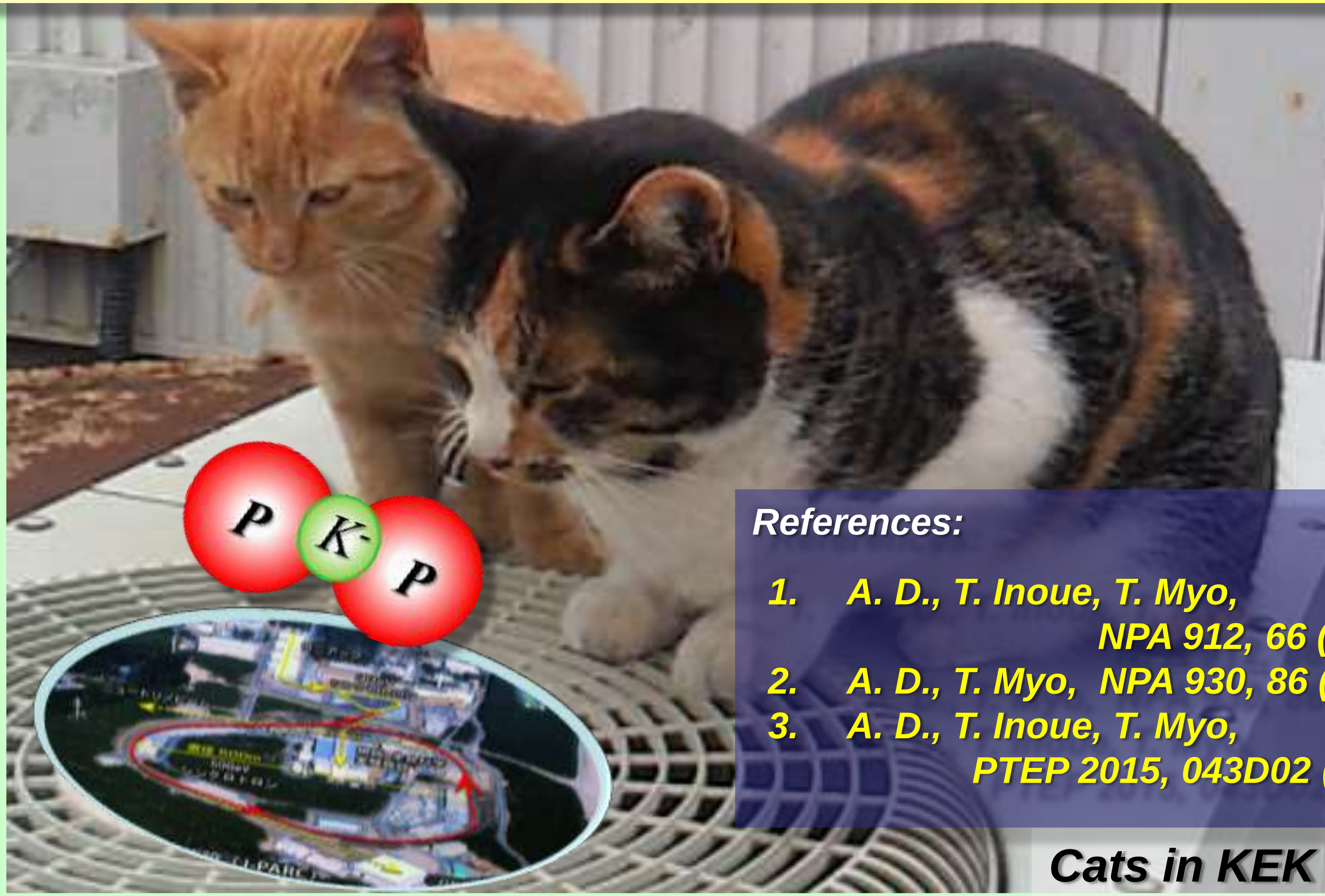
(B, $\Gamma/2$) = (21~31, 9~16) MeV : "Field picture"
(25~30, 15~32) MeV : "Particle pict."

Mean NN distance ~ 2.2 fm → Normal density

Future plans

- Full-coupled channel calculation of K^-pp
  … *Deailed study for the double pole structure of K^-pp*
- Application to resonances of other hadronic systems

Thank you for your attention!



Cats in KEK

References:

1. A. D., T. Inoue, T. Myo, NPA 912, 66 (2013)
2. A. D., T. Myo, NPA 930, 86 (2014)
3. A. D., T. Inoue, T. Myo, PTEP 2015, 043D02 (2015)