Electromagnetic form factors of the octet baryons from lattice QCD

Phiala Shanahan

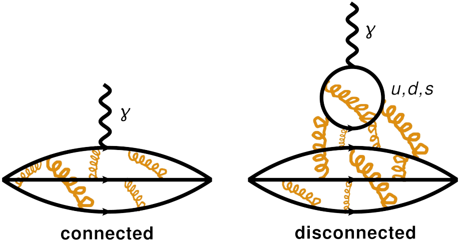
Collaborators: Anthony Thomas, Ross Young
James Zanotti and QCDSF/UKQCD lattice group

May 25th 2015

Electromagnetic form factors on the lattice

Strange quark nucleon form factors ⇒ ‘hidden flavour’ contributions

Strange quark observables in the proton are generated **entirely** by interactions with the vacuum



Key test of nonperturbative QCD

Electromagnetic form factors on the lattice

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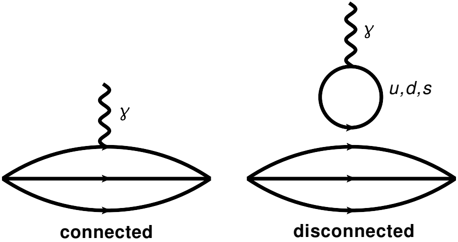

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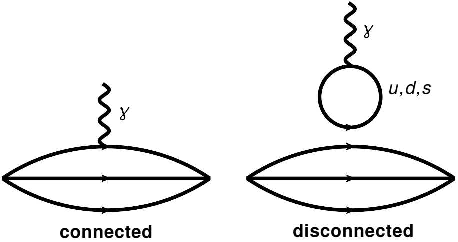


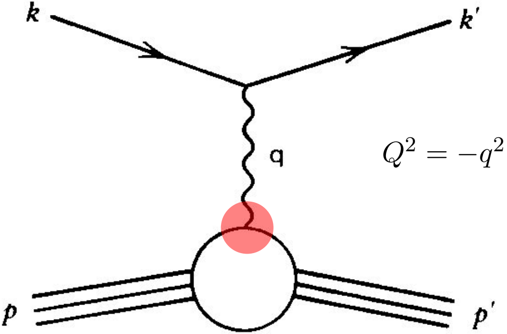
Key test of nonperturbative QCD

Nucleon form factors ⇒ c.f. experiment

Hyperon form factors ⇒ environment sensitivity of quark contributions

Electromagnetic form factors

Form factors characterize the extended nature of composite particles



$Q^2 = -q^2$

$$\langle P'|J^{\mu}_{\mathrm{EM}}|P\rangle = \overline{u}(p')\left[\gamma^{\mu} F_1(Q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2M} F_2(Q^2)\right] u(p)$$

Electromagnetic form factors

Sachs form factors:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$G_E(Q^2 = 0) = \mathcal{Q}$$

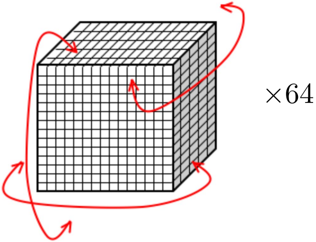
$$G_M(Q^2 = 0) = \mu$$

Lattice QCD (Ken Wilson 1974)

Numerical first-principles approach

Discretise space-time (4D box)

Lattice spacing a, volume $L^3 \times T$
order $32^3 \times 64 \approx 2 \times 10^6$ lattice sites



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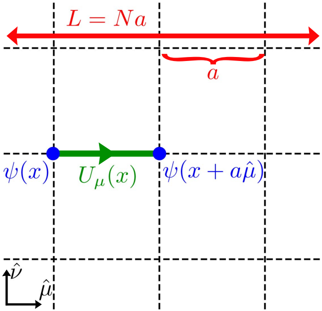


Discretise space-time (4D box)

Lattice spacing a, volume $L^3 \times T$
order $32^3 \times 64 \approx 2 \times 10^6$ lattice sites

Quark fields reside on sites: $\psi(x)$
Gauge fields on the links: $U_\mu = e^{-iagA_\mu(x)}$

Approximate the QCD path integral by Monte Carlo:

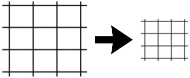
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{O}[A,\overline{\psi}\psi]e^{-S[A,\overline{\psi}\psi]} \quad \longrightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\text{conf}}} \sum_i^{N_{\text{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$.

Want to compare
LATTICE
and
EXPERIMENT

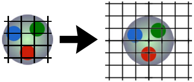
Lattice QCD - systematics and limitations

- **Finite lattice spacing** a
  discretisation artifacts
  Continuum extrapolation



$$a \to 0$$

- **Finite box size** L
  $\Rightarrow$ momentum quantized, finite-volume effects
  Finite-volume corrections

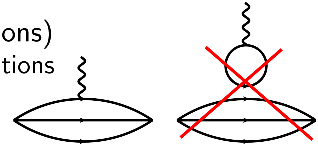


$$L \to \infty$$

- **Large pion mass** m_π
  Chiral extrapolation
  BUT: Can map out m_ϕ-dependence of observables

$$m_\pi \to 140\mathrm{MeV}$$

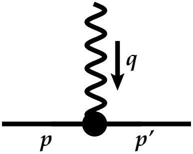
- **Omitted disconnected loops** (our simulations)
  BUT: can separate ‘valence’ and ‘sea’ contributions
  This is the key here



The lattice simulations

Have: lattice simulations for G_E and G_M

- Quark contributions to outer-ring octet baryons $G^{p,u}$, $G^{p,d}$, $G^{\Sigma,u}$, $G^{\Sigma,s}$, $G^{\Xi,s}$, $G^{\Xi,u}$
- Two lattice spacings and volumes a, L
- Nine sets of pseudoscalar masses (m_π, m_K)
- Thirteen values of the momentum transfer $q = p' - p$



CSSM/QCDSF/UKQCD Collaborations

PES *et al.* PRD89 074511, PRD90 034502 (2014), PRL 114 091802 (2015)

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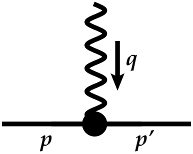

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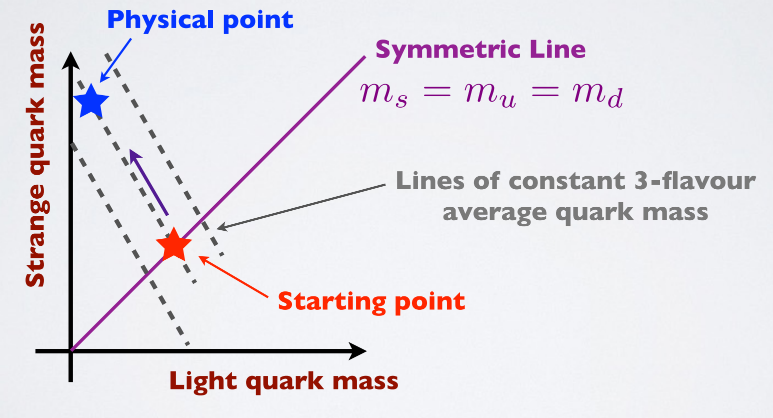
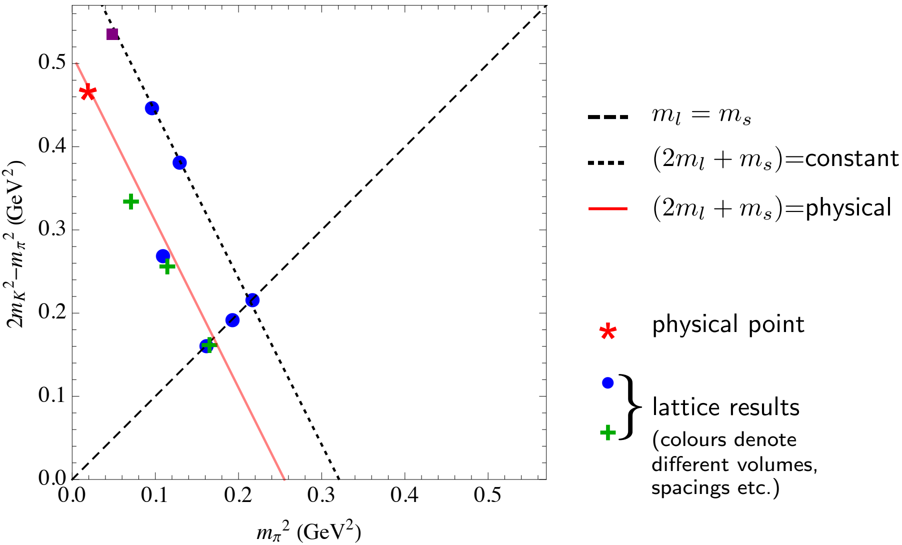
The lattice simulations: QCDSF-UKQCD lattices

Tune lattice parameters to have exact SU(3) symmetry at the physical average (light) quark mass

$$\overline{m}_q^{\text{latt.}} = (m_u + m_d + m_s)^{\text{phys.}}/3$$



The lattice simulations: $2m_l$–$2m_s$ plane



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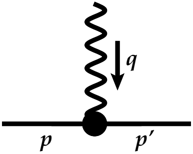

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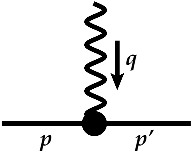

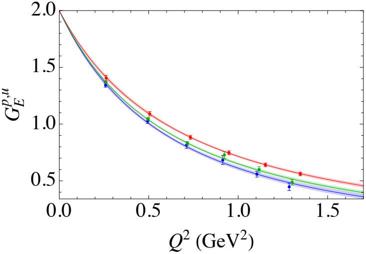

Colours:

different masses (m_π, m_K)

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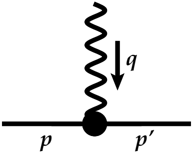

Want: G_E and G_M at the physical point

- Finite-volume corrections
- Chiral extrapolation - simultaneous fit to all baryons

CSSM/QCDSF/UKQCD Collaborations

PES *et al.* PRD89 074511, PRD90 034502 (2014), PRL 114 091802 (2015)

Chiral extrapolation

Idea: Write form factors as a function of quark mass: fit to lattice results

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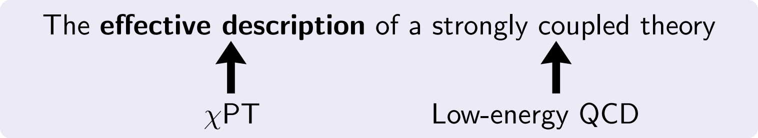
Approach: Use chiral perturbation theory (χPT)



The **effective description** of a strongly coupled theory

χPT

Low-energy QCD

Chiral extrapolation

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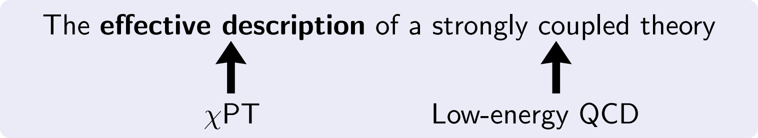

The **effective description** of a strongly coupled theory

χPT

Low-energy QCD

Goldstone bosons (pions) become the fundamental degrees of freedom

- Built on the symmetries of QCD
- Preserves non-analyticity of loops (correct chiral behavior of QCD)
- Same IR behaviour as underlying theory, different UV behaviour

Expansion in small momenta and light quark masses

Chiral extrapolation at *fixed* Q^2

q quark contribution to magnetic form factor of the baryon B

$$G_M^{B,q}(Q^2) = \text{terms analytic in } m_\phi^2$$

$$+ \text{chiral loop corrections}$$

Chiral extrapolation at *fixed* Q^2

q quark contribution to magnetic form factor of the baryon B

Linear in m_ϕ^2

$$\begin{aligned} G_M^{B,q}(Q^2) =& \alpha^{Bq} + \sum_{q'} \overline{\alpha}^{Bq(q')} \underline{\mathcal{B} m_{q'}} \\ &+ \frac{m_N}{16\pi^3 f^2} \sum_\phi \left(\beta_O^{Bq(\phi)} \underline{\mathcal{I}_O(m_\phi, Q^2)} + \beta_D^{Bq(\phi)} \underline{\mathcal{I}_D(m_\phi, Q^2)} \right) \end{aligned}$$

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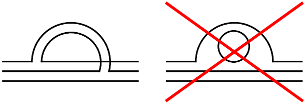
Chiral coefficients: choose **connected** contributions only



Finite-volume corrections

e.g., Hall, Leinweber, Young, PL B725, 101 (2013), PR D88, 014504 (2013)

- Use χPT formalism to estimate finite-volume effects
- Can check by comparing explicit results on different volumes

Shift all lattice results by the extrapolation expression for $G^{B,q}$ with loop integral contributions $\int \mathcal{I}$ replaced by the difference

$$\underbrace{\sum \mathcal{I}}_{\text{finite-volume sum}} - \underbrace{\int \mathcal{I}}_{\text{infinite-volume integral}}$$

After the chiral extrapolation

Have: Quark contributions to outer-ring octet baryons
$G^{p,u}$, $G^{p,d}$, $G^{\Sigma,u}$, $G^{\Sigma,s}$, $G^{\Xi,s}$, $G^{\Xi,u}$

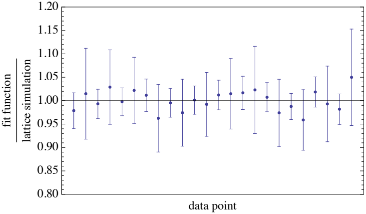
- Several different values of Q^2
- Infinite volume
- Any (m_π, m_K), e.g., physical point

The fits are

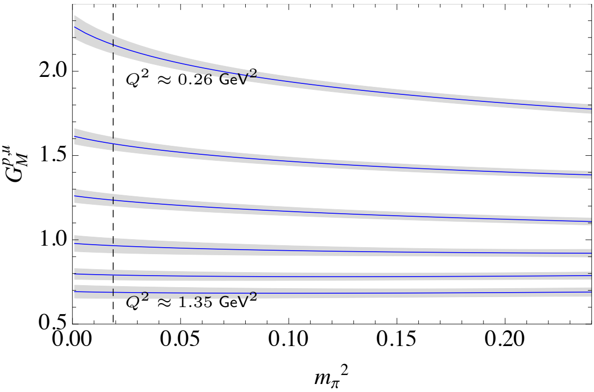
- Independent at each Q^2
- Simultaneous for the different baryons

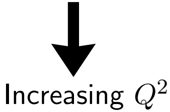
24 data points, 8 fit parameters
at each Q^2

G_M, lowest Q^2



Fit to lattice results: chiral extrapolation

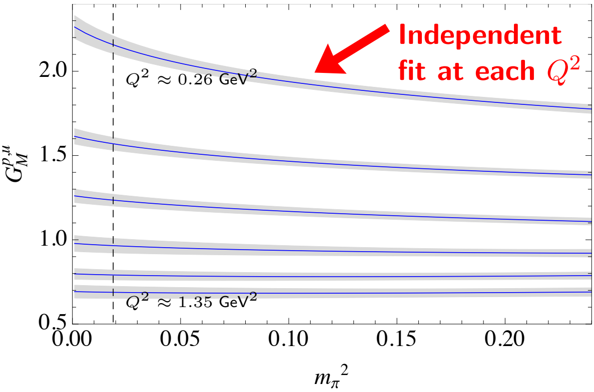




Trajectory: Singlet pseudoscalar mass $(m_K^2 + m_\pi^2/2)$ fixed to its physical value

Recall: Simultaneous fit with other baryon FFs ($G_M^{\Sigma,u}$) etc.

Fit to lattice results: chiral extrapolation



Independent fit at each Q^2

Increasing Q^2

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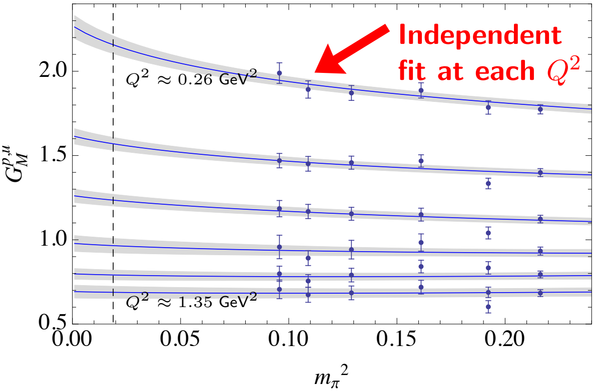

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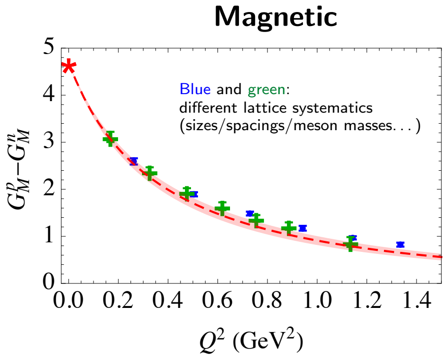
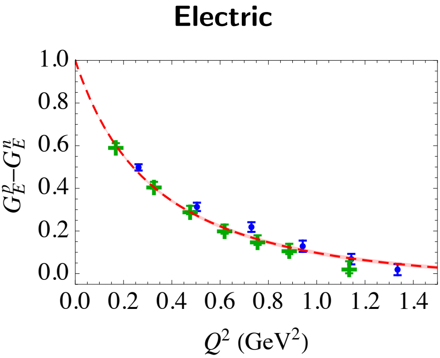
Isovector nucleon form factors

Systematics under control ⇒ Isovector nucleon FFs agree with experiment

$$(p-n)_{\text{total}} = (p_{\text{connected}} + O_N) - (n_{\text{connected}} + O_N)$$
$$= (p-n)_{\text{connected}}$$

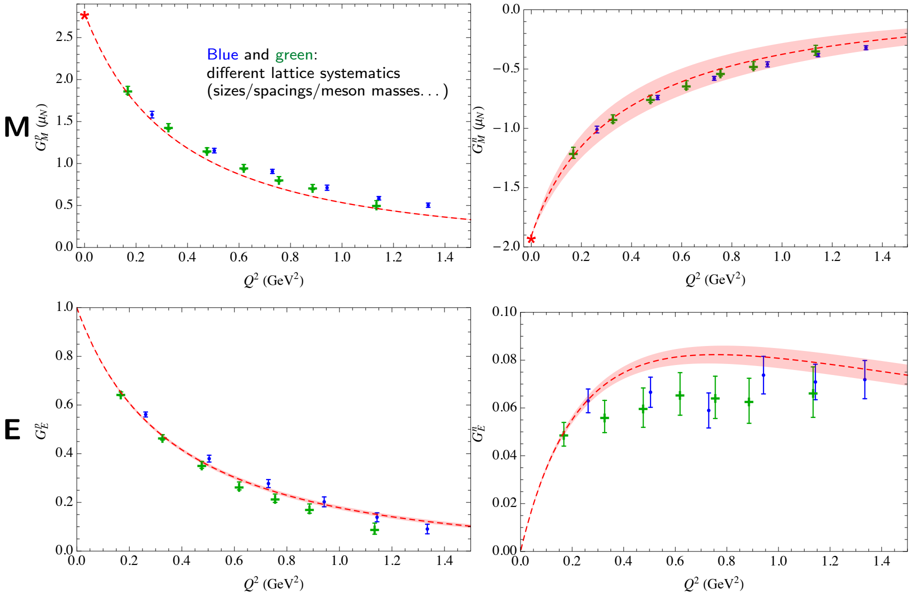




Kelly experimental parameterization: J.J. Kelly, PR C70, 068202 (2004)

Proton

Neutron

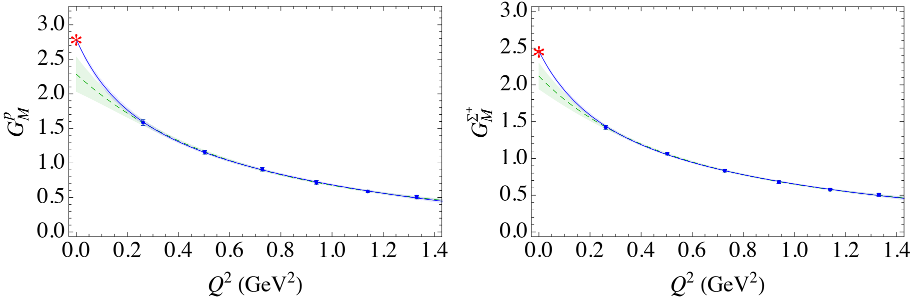


Blue and green:
different lattice systematics
(sizes/spacings/meson masses. . .)

Kelly experimental parameterization: J.J. Kelly, PR C70, 068202 (2004)

Predict hyperon magnetic radii: dipole-like fits

$$G_M^{\mathrm{fit}}(Q^2) = \frac{\mu}{1 + d_1 Q^2 + d_2 Q^4}$$



	$\langle r_M^2 \rangle^B$ (fm^2)					
	p	n	Σ^+	Σ^-	Ξ^0	Ξ^-
Extrapolated	$0.71(8)$	$0.86(9)$	$0.66(5)$	$1.05(9)$	$0.53(5)$	$0.44(5)$
Experimental	$0.777(16)$	$0.862(9)$				

Many other results

**PRD89 074511,
PRD90 034502 (2014)**

- Hyperon electromagnetic FFs against Q^2
- Ratio of electric and magnetic FFs - for nucleon and hyperons
- Electric radii
- Magnetic moments
- Environment sensitivity of contribution from quarks

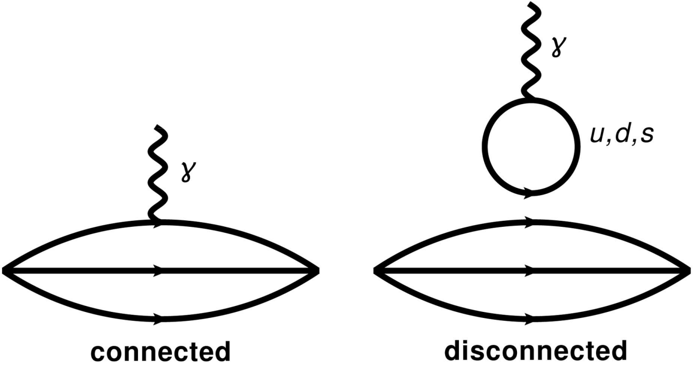
Here we focus on

Strange nucleon form factors

PRL 114 091802 (2015)

Method

Use lattice results to **deduce** the strange form factors of the nucleon.



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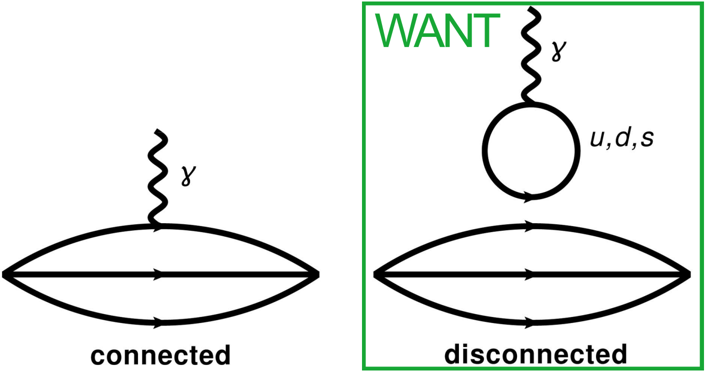

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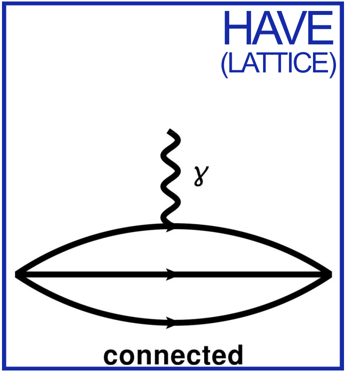

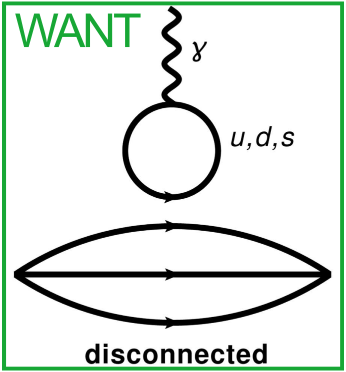

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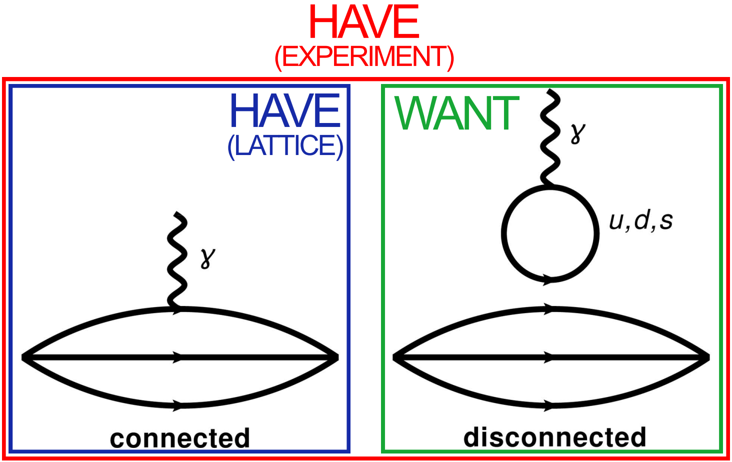

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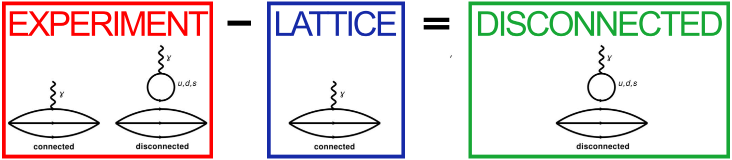

Difference between lattice and experiment gives disconnected terms

Method

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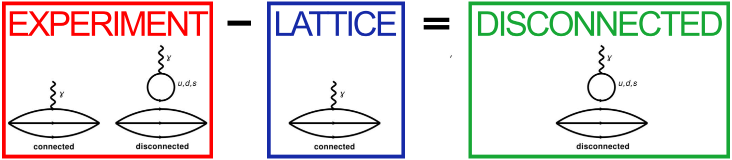


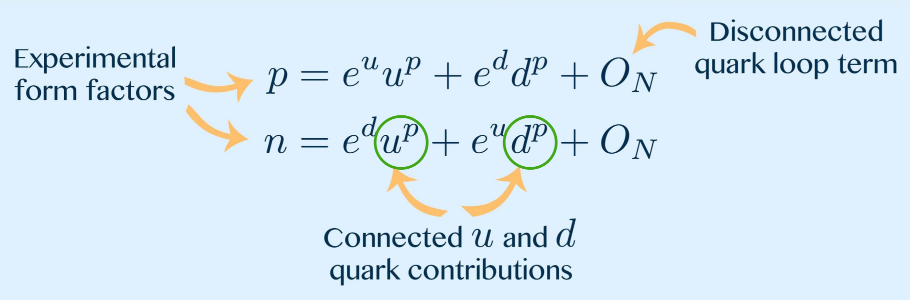
EXPERIMENT − LATTICE = DISCONNECTED

Difference between lattice and experiment gives disconnected terms

IF all systematics are under control . . .

Method - Details

Use **charge symmetry**:



Experimental form factors

Disconnected quark loop term

$$p = e^u u^p + e^d d^p + O_N$$

$$n = e^d u^p + e^u d^p + O_N$$

Connected u and d quark contributions

$$O_N = \frac{2}{3} {}^{\ell}G^u - \frac{1}{3} {}^{\ell}G^d - \frac{1}{3} {}^{\ell}G^s$$

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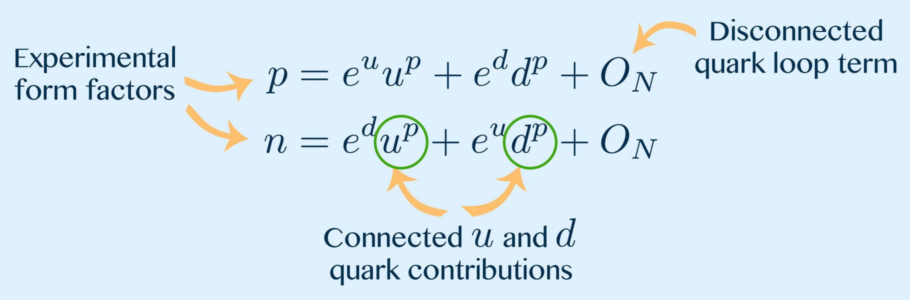

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Connected u and d quark contributions

$$O_N = \frac{2}{3}{}^{\ell}G^u - \frac{1}{3}{}^{\ell}G^d - \frac{1}{3}{}^{\ell}G^s$$

Method - Details

Rearrange:

$$^{\ell}G^{s} = \left(\frac{^{\ell}R_{d}^{s}}{1 - {^{\ell}R_{d}^{s}}} \right) \left[\frac{3}{2}(p+n)_{\mathrm{Exp.}} - \frac{1}{2}(u^{p}+d^{p})_{\mathrm{Latt.}} \right].$$

1. The p and n form factors **from experiment**.
2. The connected u and d contributions to the proton form factor (u^{p}, d^{p}) **from lattice QCD**.
3. The ratio of strange to light disconnected contributions ${^{\ell}R_{d}^{s}} = \frac{^{\ell}G^{s}}{^{\ell}G^{d}}$ **from a model based on chiral perturbation theory**.

Method - Details

Rearrange:

$$ {}^{\ell}G^{s} = \left(\frac{{}^{\ell}R^{s}_{d}}{1 - {}^{\ell}R^{s}_{d}} \right) \left[\frac{3}{2}(p+n)_{\text{Exp.}} - \frac{1}{2}(u^{p} + d^{p})_{\text{Latt.}} \right]. $$

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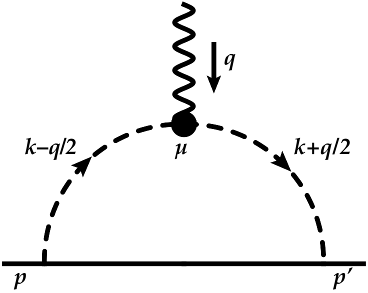
Approximate ${}^{\ell}R^s_d = \frac{{}^{\ell}G^s}{{}^{\ell}G^d}$ by the ratio of the strange to light **disconnected** contributions to the loops:



Loop contribution $\mathcal{I}(m_\phi, Q^2)$ for meson ϕ in loop

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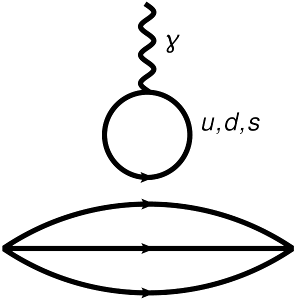

disconnected

Disconnected contribution depends only on m_q for quark q in the loop

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$$ {}^{\ell}R^s_d(Q^2) = \frac{{}^{\ell}G^s}{{}^{\ell}G^d} = \frac{\mathcal{I}(m_K, Q^2)}{\mathcal{I}(m_\pi, Q^2)} $$

Uncertainties:

- Model-dependence (range of regulator masses Λ in FRR scheme)
- Higher-order terms (use decuplet intermediate state loops to estimate)

③ The ratio of strange to light disconnected loops ${}^{\ell}R^s_d = \frac{{}^{\ell}G^s}{{}^{\ell}G^d}$ **from a model based on chiral perturbation theory.**

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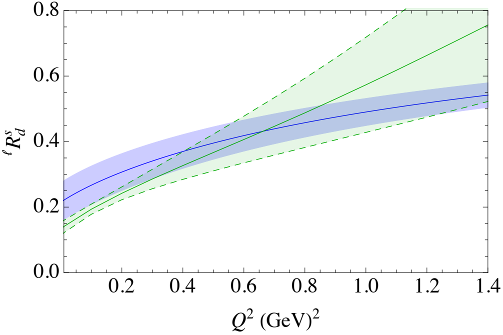

Magnetic (solid)
Electric (dashed)

Method - Details

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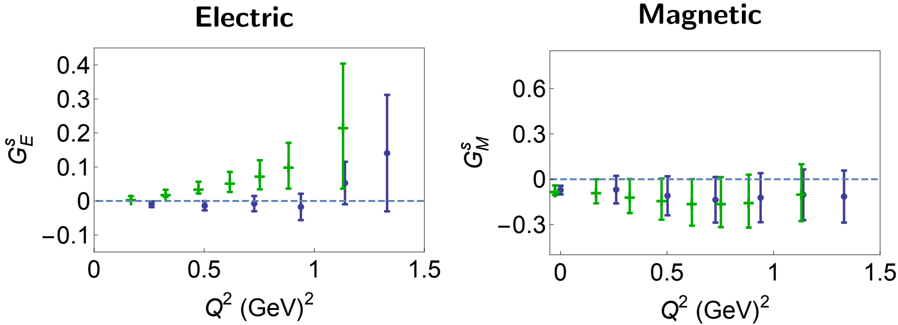
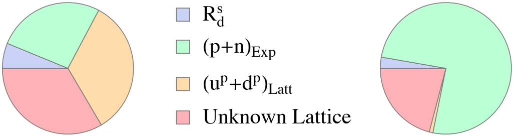
Results - no charge factor ($\times -\frac{1}{3}$ for physical units)



Uncertainties:



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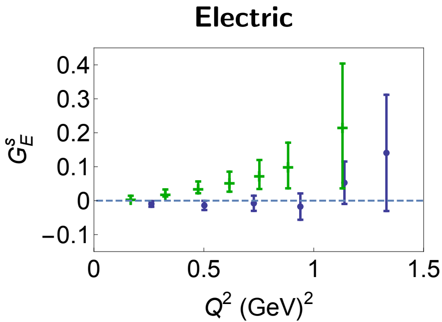
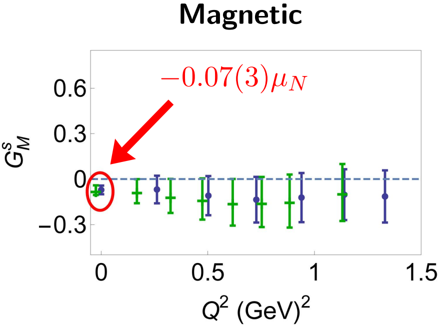

Electric


Magnetic

$-0.07(3)\mu_N$

Strange quarks contribute $0.8(3)\%$ to the proton magnetic moment.

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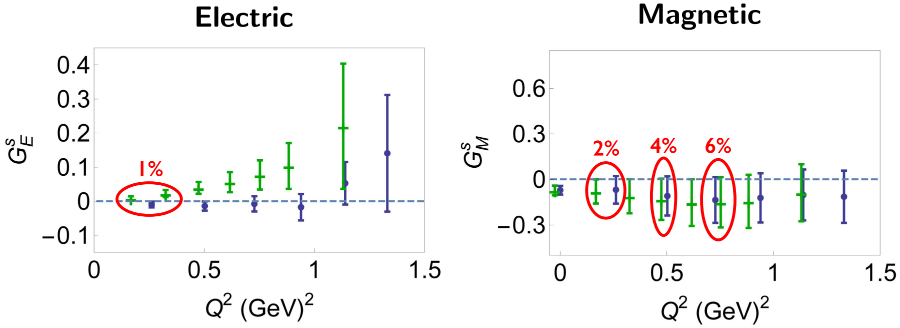

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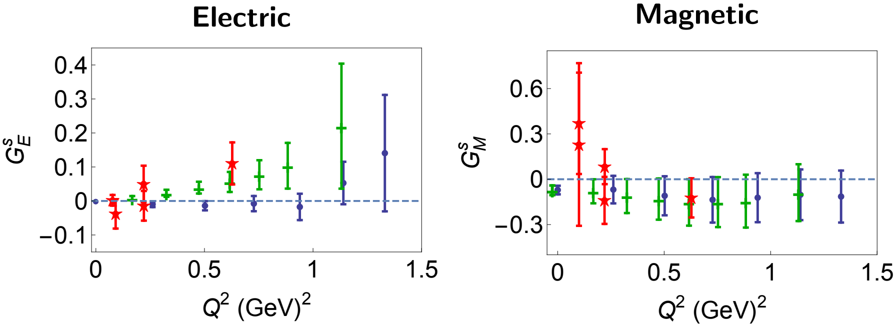

Red stars: G0, SAMPLE, HAPPEX, A4.

Summary

Lattice calculation of the electric and magnetic form factors of the octet baryons

- Chiral extrapolation at *fixed* values of Q^2
- Finite-volume corrections

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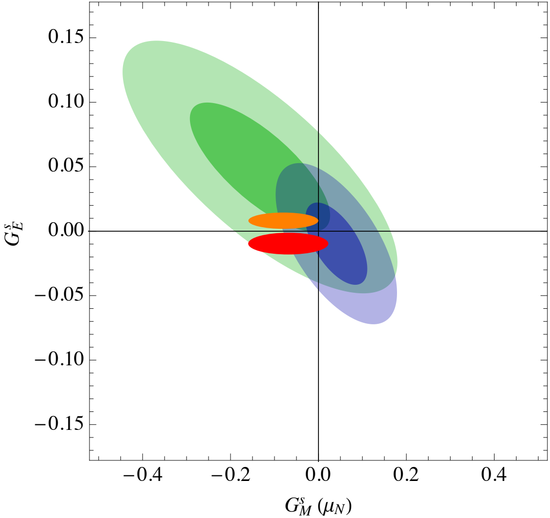
This calculation (and experiment) assumes **charge symmetry**.

Good approximation
arXiv:1503.01142

At $Q^2 \approx 0.26$ GeV^2



This work: 0.26 GeV^2
A4: 0.23 GeV^2
G0: 0.23 GeV^2

Dark: 1σ
Light: 2σ

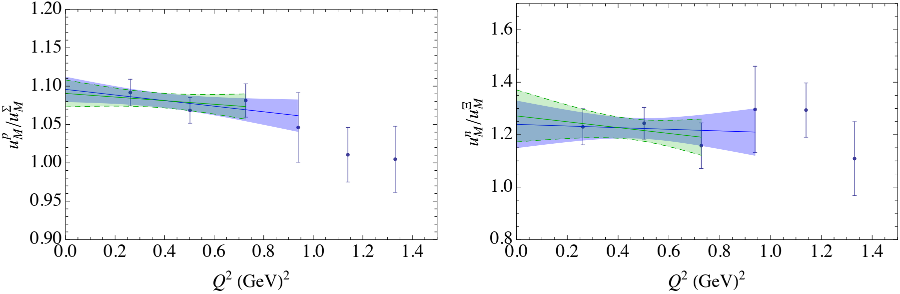
Strange magnetic moment $G_M(Q^2 = 0)$

Additional information: hyperon magnetic moments have been measured.
Use the assumption of charge symmetry:

$$^{\ell}G^s = \left(\frac{^{\ell}R_d^s}{1 - {}^{\ell}R_d^s} \right) \left[2p + n - \frac{u^p}{u^\Sigma} \left(\Sigma^+ - \Sigma^- \right) \right],$$

$$^{\ell}G^s = \left(\frac{^{\ell}R_d^s}{1 - {}^{\ell}R_d^s} \right) \left[p + 2n - \frac{u^n}{u^\Xi} \left(\Xi^0 - \Xi^- \right) \right].$$

Take ratios of form factors u^p/u^Σ, u^n/u^Ξ from lattice QCD.



Experimental determinations of $G^s_{E/M}$

EM and **weak** vector currents give access to different combinations of $G^{p,(u/d/s)}$:

$$G^{p,\gamma} = \frac{2}{3}G^{p,u} - \frac{1}{3}\left(G^{p,d} + G^{p,s}\right)$$

$$G^{p,Z} = \left(1 - \frac{8}{3}\sin^2\theta_W\right)G^{p,u} - \left(1 - \frac{4}{3}\sin^2\theta_W\right)\left(G^{p,d} + G^{p,s}\right)$$

Assume **charge symmetry** ($G^{p,u} = G^{n,d}$, $G^{p,d} = G^{n,u}$, $G^{p,s} = G^{n,s}$)

$$G^{p,s}_{E/M} = \left(1 - 4\,\underline{\sin^2\theta_W}\right)\underline{G^{p,\gamma}_{E/M}} - \underline{G^{n,\gamma}_{E/M}} - \underline{G^{p,Z}_{E/M}}$$

well determined

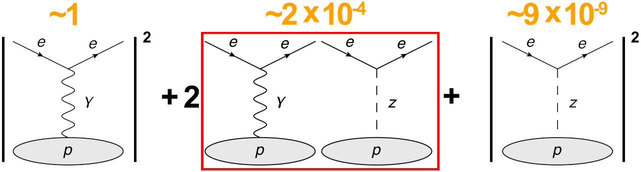
PVES

Parity-violating electron scattering

JLab (*G0, HAPPEX*), MIT-Bates (*SAMPLE*), Mainz (*A4*)

Accessing the neutral weak current G^Z

Elastic $e-p$ scattering cross sections $\propto |\mathcal{M}_\gamma + \mathcal{M}_Z|^2$, **BUT** γ dominates



Parity-violating cross-term → form observable sensitive to G^Z:

$$A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \sim \frac{2M_\gamma^* M_Z^{PV}}{|M_\gamma|^2} \sim 10^{-5}$$

spin-dependent DIS

$$= -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1-4\mathrm{Sin}^2\theta_W)\epsilon' G_M^\gamma G_A^e}{\epsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

Different targets (proton, deuteron, helium-4), different kinematic configurations → different ϵ, ϵ', i.e., different linear combinations of G_E^s and G_M^s

Finite-range regularisation (FRR)

Physically motivated:

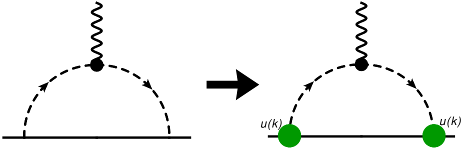
Mesons emitted/absorbed by composite objects made from quarks, gluons
$\Rightarrow$ Form factors suppress these processes for momenta $k > R^{-1}$

DR

- Large contributions from $k \to \infty$ portion of integral
- Short distance physics highly overestimated
- Baryons are hard point particles

FRR

- Remove the incorrect short distance contribution:
  Introduce finite UV cutoff $u(k)$ into loop integrals
- Baryons are soft particles with structure



$$u(k) = \left(\frac{\Lambda^2}{\Lambda^2 + k^2} \right)^2$$

Finite-range regularization (FRR)

e.g., Young, Leinweber, Thomas, Prog.Part.Nucl.Phys. 50, 399 (2003), PR D66, 094507 (2002)

Within the 'Power Counting Regime'

- FRR is NOT a model
- Equivalent to any other regularization scheme

Outside the 'Power Counting Regime'

- FRR IS a model
    - Physically motivated way of re-summing higher terms of chiral expansion
    - Better than systematically setting higher terms to zero (wrong)
    - One model parameter, constrained by lattice data
      Different regulators: check model-dependent uncertainty (small)

Lattice simulation details

- Non-perturbatively $\mathcal{O}(\alpha)$-improved Wilson fermions
- Clover action: tree-level Symanzik improved gluon action together with a mild stout smeared fermion action
- $\beta = 5.50 \Leftrightarrow a = 0.074(2)$ fm. The scale is set using various singlet quantities.
- $L^3 \times T = 32^3 \times 64$

	κ_0	κ_l	κ_s	m_π (MeV)	m_K (MeV)	$m_\pi L$
1	0.120900	0.120900	0.120900	465	465	5.6
2		0.121040	0.120620	360	505	4.3
3		0.121095	0.120512	310	520	3.7
4	0.120920	0.120920	0.120920	440	440	5.3
5	0.120950	0.120950	0.120950	400	400	4.8
6		0.121040	0.120770	330	435	4.0

κ_0 denotes the value of $\kappa_l = \kappa_s$ at the SU(3) symmetric point.

The lattice simulations: momentum transfer Q^2

- Zero sink momentum
- Several values of the three momentum transfer $\vec{q} = \vec{p}' - \vec{p}$

Boundary conditions $\psi(x+L) = \psi(x)$
$\Rightarrow$ momentum is quantised on the lattice: $k = \frac{2\pi n}{L}$

$$\vec{q}^{\,2} = \{1,2,3,4,5,6\ldots\} \times \left(\frac{2\pi}{32a}\right)^2$$

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Relate to 4-momentum transfer q^2 using the dispersion relation:

$$\vec{q}^2 = \left(\frac{q^2}{2M_B}\right)^2 - q^2$$

Physical values of $Q^2 = -q^2$ vary with different baryon masses M_B

The lattice simulations: momentum transfer Q^2

- Zero sink momentum
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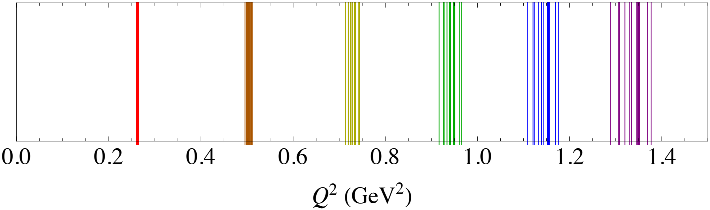
Each colour denotes a single value of the momentum transfer in lattice units



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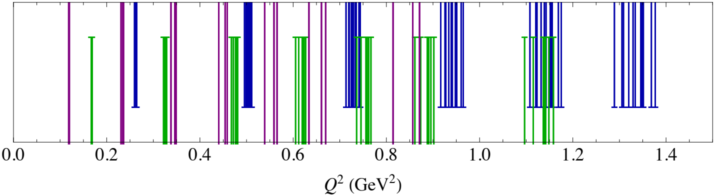
L	32	48	48
a(fm)	0.074	0.074	0.062



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Chiral Lagrangian - Magnetic, leading order

‘Magnetic Lagrangian density’:

$$\mathcal{L} = \frac{e}{4m_N} F_{\mu\nu}\sigma^{\mu\nu}\left[\mu_\alpha\left(\overline{B}BQ\right) + \mu_\beta\left(\overline{B}QB\right) + \mu_\gamma\left(\overline{B}B\right)\mathrm{Str}(Q)\right]$$

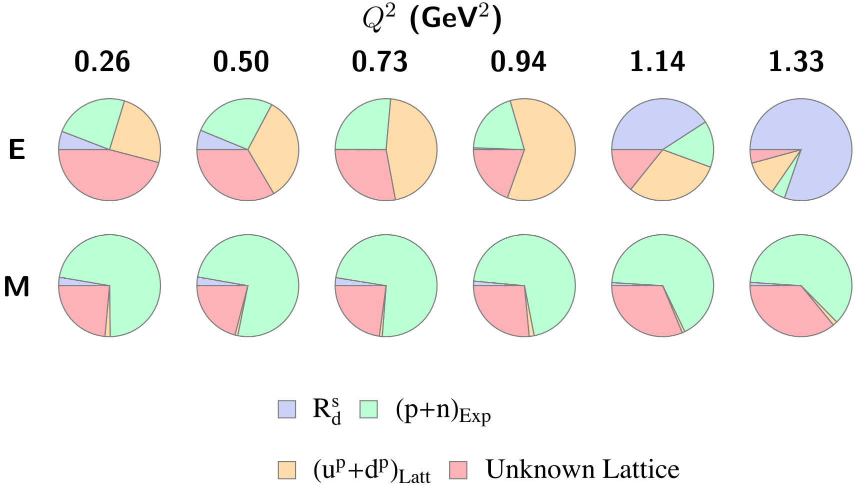
Interpret $\mu_{\alpha/\beta/\gamma}$ as chiral-limit form factors at some *fixed* Q^2.

Explicit symmetry breaking at leading order in the quark masses:

$$\begin{aligned}\mathcal{L}_{\mathrm{lin}} = \mathcal{B}\frac{e}{2m_N}\Big[& c_1\left(\overline{B}m_\psi B\right)\mathrm{Str}(Q) + c_2\left(\overline{B}Bm_\psi\right)\mathrm{Str}(Q) + c_3\left(\overline{B}QB\right)\mathrm{Str}(m_\psi) \\ & + c_4\left(\overline{B}BQ\right)\mathrm{Str}(m_\psi) + c_5\left(\overline{B}Qm_\psi B\right) + c_6\left(\overline{B}BQm_\psi\right) + c_7\left(\overline{B}B\right)\mathrm{Str}(Qm_\psi) \\ & + c_8\left(\overline{B}B\right)\mathrm{Str}(Q)\mathrm{Str}(m_\psi) + c_9(-1)^{\eta_l(\eta_j+\eta_m)}\left(\overline{B}^{kji}(m_\psi)^l_i Q^m_j B_{lmk}\right) \\ & + c_{10}(-1)^{\eta_j\eta_m+1}\left(\overline{B}^{kji}(m_\psi)^m_i Q^l_j B_{lmk}\right) + c_{11}(-1)^{\eta_l(\eta_j+\eta_m)}\left(\overline{B}^{kji}Q^l_i(m_\psi)^m_j B_{lmk}\right) \\ & + c_{12}(-1)^{\eta_j\eta_m+1}\left(\overline{B}^{kji}Q^m_i(m_\psi)^l_j B_{lmk}\right)\Big] F_{\mu\nu}\sigma^{\mu\nu}\end{aligned}$$

Q=quark charge matrix (diagonal), m_ψ= quark mass matrix (diagonal)

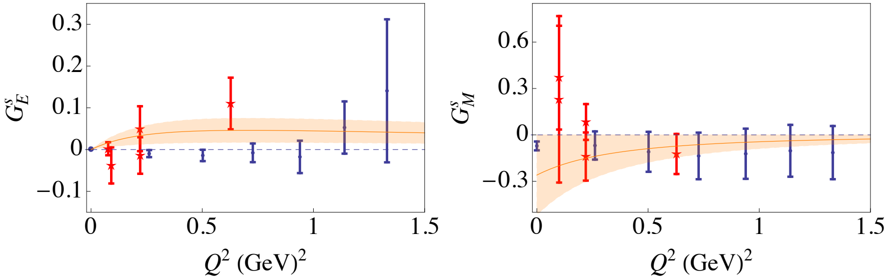
Q^2-dependence of uncertainties



Unknown lattice: finite-a, excited state contamination. . .

Comparison with global fit - strange FFs

Global analysis of parity-violating asymmetry data for elastic electron scattering
R. González-Jiménez, J.A. Caballero, T.W. Donnelly arXiv:1403.5119



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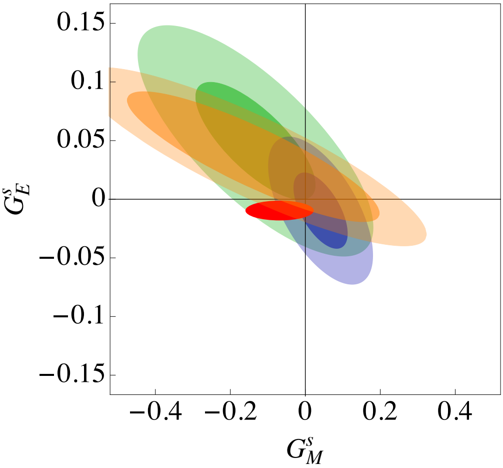
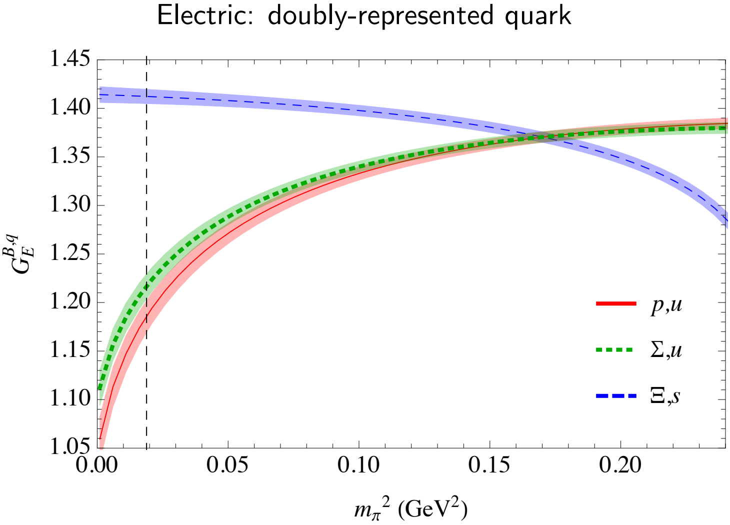
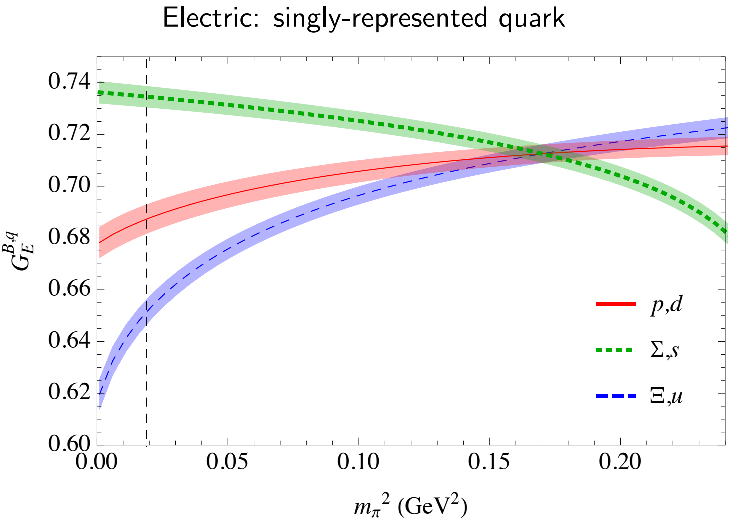


Environment sensitivity



Trajectory: fixed (physical) singlet pseudoscalar mass $(m_K^2 + m_\pi^2/2)$

Environment sensitivity



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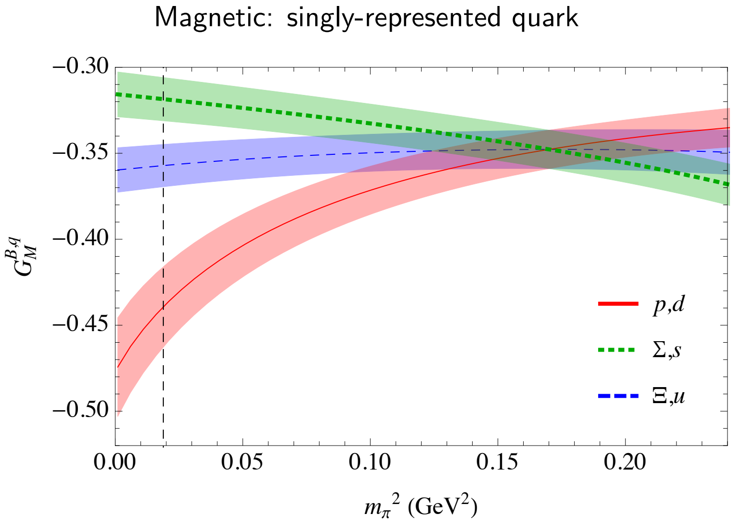
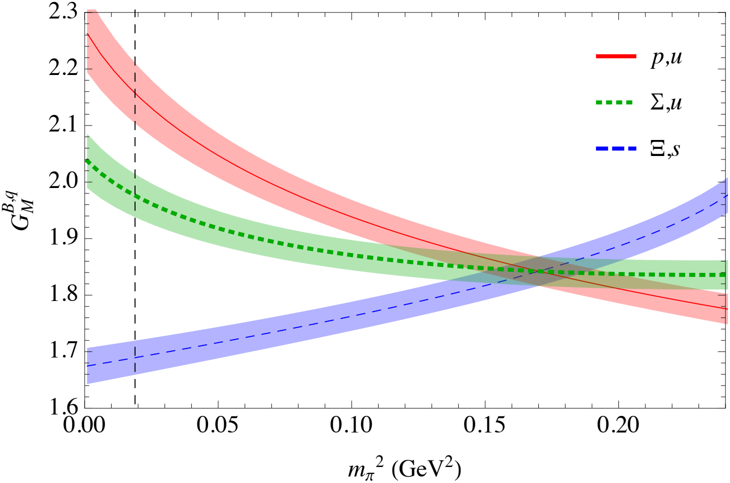


Trajectory: fixed (physical) singlet pseudoscalar mass $(m_K^2 + m_\pi^2/2)$

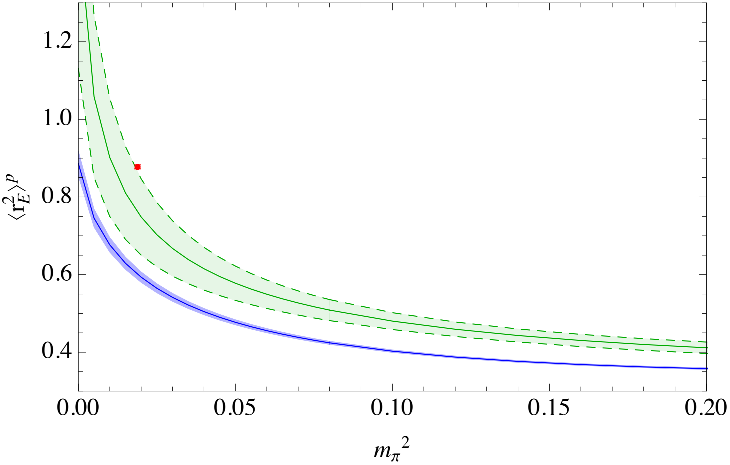
Environment sensitivity

Magnetic: doubly-represented quark



Trajectory: fixed (physical) singlet pseudoscalar mass $(m_K^2 + m_\pi^2/2)$

Electric radii - proton



Dipole fit in Q^2

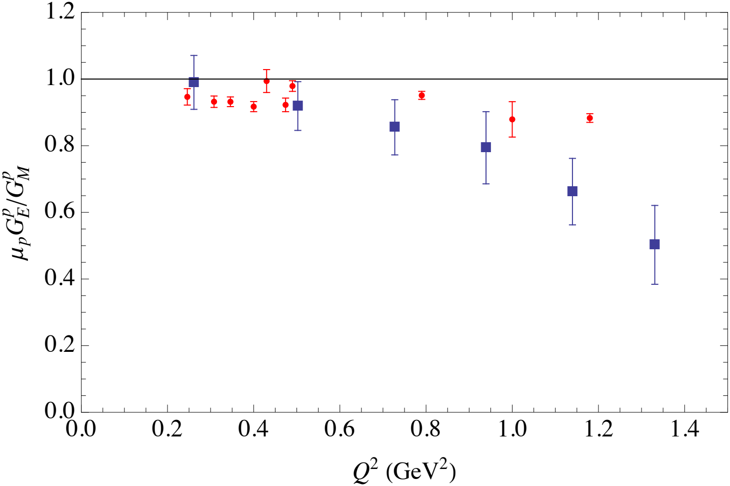
[DASHED] More general fit in Q^2: $G_E^{\rm fit}(Q^2) = \frac{G_E(Q^2{=}0)}{1{+}d_1Q^2{+}d_2Q^4{+}d_3Q^6}$

Electric radii - charged octet baryons

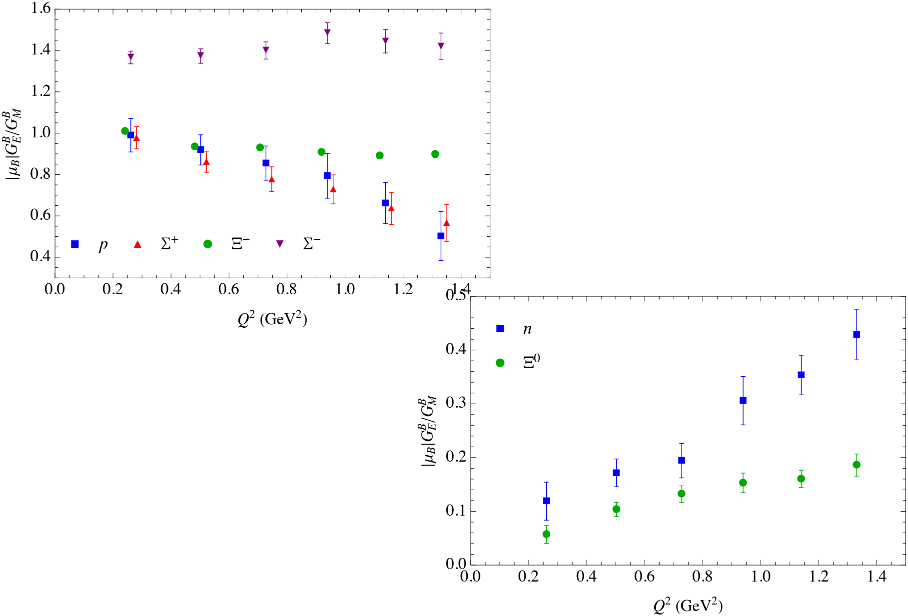
	$\langle r_E^2\rangle^B$ (fm^2)			
	p	Σ^+	Σ^-	Ξ^-
Dipole ansatz in Q^2	$0.601(14)$	$0.598(12)$	$0.414(5)$	$0.352(3)$
General ansatz in Q^2	$0.76(10)$	$0.61(8)$	$0.45(3)$	$0.37(2)$
Experimental	$0.878(5)$		$0.780(10)$	

General form: $G_E^{\mathrm{fit}}(Q^2) = \frac{G_E(Q^2=0)}{1+d_1Q^2+d_2Q^4+d_3Q^6}$

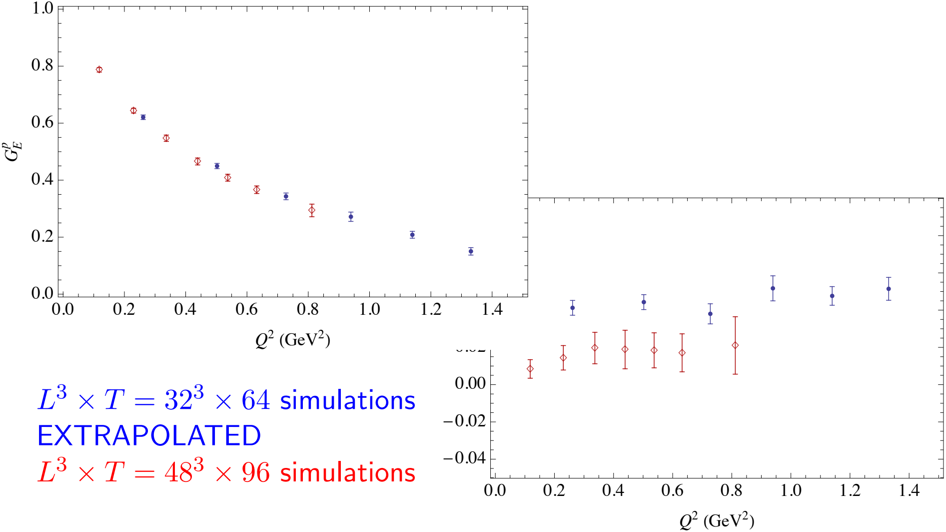
Ratio of electric and magnetic form factors - proton



Ratio of electric and magnetic form factors - hyperons



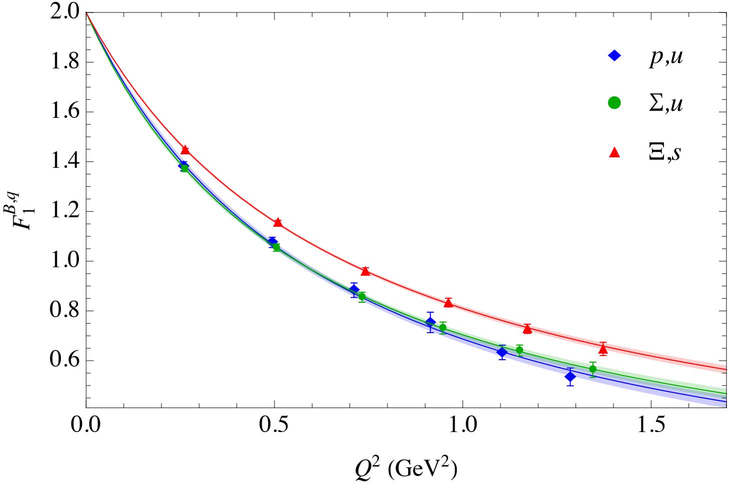
Comparison with larger volume $L^3 \times T = 48^3 \times 96$ simulations



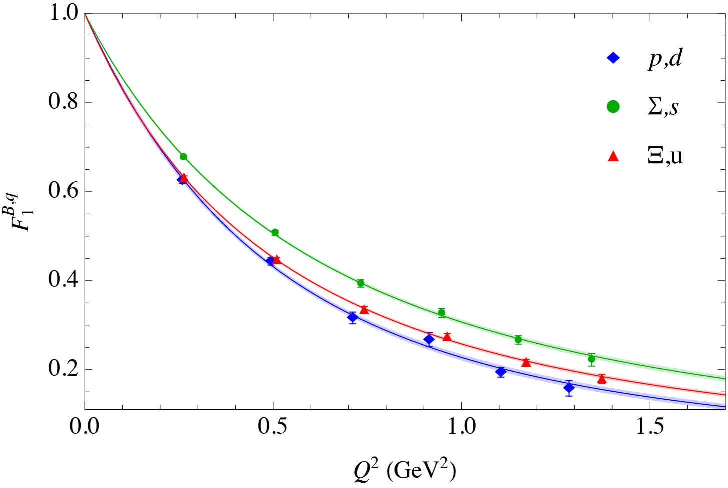
$L^3 \times T = 32^3 \times 64$ simulations EXTRAPOLATED

$L^3 \times T = 48^3 \times 96$ simulations

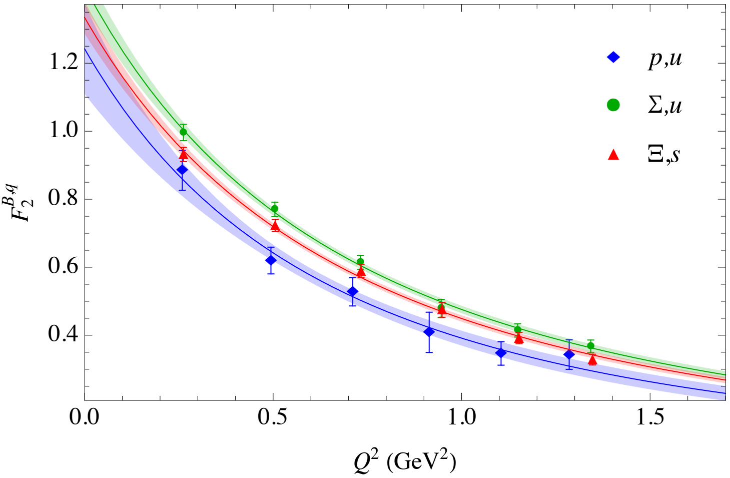
Dirac and Pauli form factors - no chiral extrapolation or FV corrections



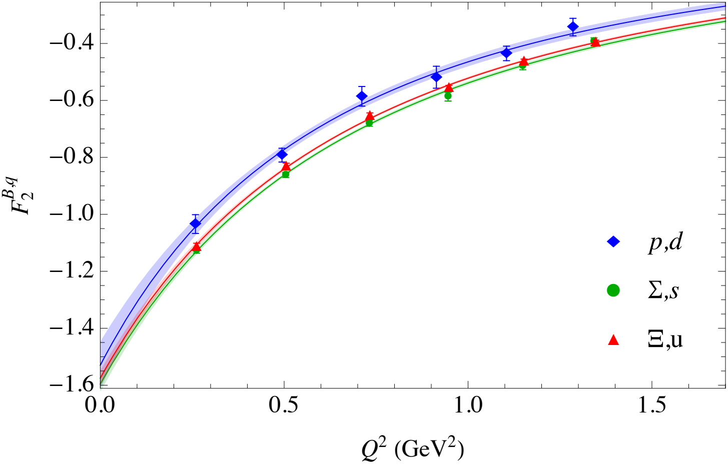
Dirac and Pauli form factors - no chiral extrapolation or FV corrections



Dirac and Pauli form factors - no chiral extrapolation or FV corrections



Dirac and Pauli form factors - no chiral extrapolation or FV corrections



Dispersion relation

$$\begin{aligned} p'^2 = M_B^2 &= (M_B + q_0)^2 - \vec{q}^2 \\ &= M_B^2 + 2M_B q_0 + q_0^2 - \vec{q}^2 \\ &= M_B^2 + 2M_B q_0 + q^2 \\ \Rightarrow q_0 = -\frac{q^2}{2M_B} \end{aligned}$$

$$\begin{aligned} \vec{q}^2 &= q_0^2 - q^2 \\ &= \left(\frac{q^2}{2M_B}\right)^2 - q^2 \end{aligned}$$

Strangeness

- Mass of the H dibaryon
  P. Shanahan *et al.* PRL **107** 092004 (2011)
- Sigma terms of octet baryons - particularly σ_s
  P. Shanahan *et al.* PR **D86** 074503 (2013)
- Strange EM form factors
  P. Shanahan *et al.* arXiv:1403.6537

Charge symmetry violation

- Strong baryon mass splittings
  P. Shanahan *et al.* PLB **718** 1148 (2013)
- CSV in the octet baryon PDF moments
  P. Shanahan *et al.* PR **D87** 094515 (2013), PR **D87** 114515 (2013)

EM form factors

- Hyperon EM form factors
  P. Shanahan *et al.* PR **D89** 074511 (2014), arXiv:1401.5862

Octet spin fractions and the proton spin problem
P. Shanahan *et al.* PRL **110** 202001 (2013)