Stability of the pion beyond the chiral limit

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In collaboration with Hyeon-Dong Son
Traditional form factors
Traditional form factors

Probes examine the structure of a hadron
Traditional form factors

Probes examine the structure of a hadron

- Electromagnetic form factors (Vector form factors)
- Axial-vector form factors
- Scalar form factors
- Pseudoscalar form factors
Energy–Momentum Tensor Form Factors

- Given an action \( S = \int d^4 \sqrt{-g} \mathcal{L} \)

\[
T_{\mu\nu} = 2 \frac{\delta S}{\delta g_{\mu\nu}} \quad \text{or} \quad T_{\mu\nu} = -\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_a(x))} \partial_\nu \phi_a(x) + g_{\mu\nu} \mathcal{L}
\]

- EMT is a conserved quantity: \( \partial^\mu T_{\mu\nu} = 0 \)

  (EMTFFs are scale-independent quantities)

- Energy-momentum tensor form factors of the pion

\[
\langle \pi^a(p') | T_{\mu\nu}(0) | \pi^b(p) \rangle = \frac{\delta^{ab}}{2} \left[ (tg_{\mu\nu} - q_\mu q_\nu) \Theta_1(t) + 2P_\mu P_\nu \Theta_2(t) \right]
\]

(also known as the gravitational form factors)

Energy–Momentum Tensor Form Factors

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\[ \langle \pi^a(p')|T_{\mu \nu}(0)|\pi^b(p)\rangle = \delta^{ab} \frac{\delta}{2} \left[ (tg_{\mu \nu} - q_\mu q_\nu) \Theta_1(t) \right] + 2P_\mu P_\nu \Theta_2(t) \]

(also known as the gravitational form factors)

Energy-Momentum Tensor Form Factors

Energy-Momentum Tensor form factors & Tensor Form factors

These form factors are as equally important as vector and axial-vector form factors (Energy & Momentum distributions & transversity, resp.)!
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- A probe for the EMTFF: Graviton too weakly coupled to hadrons
- A probe for the tensor FF: Unknown so far
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• A probe for the tensor FF: Unknown so far

Then, how can we measure them?
Novel view on form factors

Deeply Virtual Compton Scattering

\[ 2\delta^{ab}H^q_\pi(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p')|\bar{\psi}^q(-\lambda n/2)\gamma[\lambda n/2, \lambda n/2]\psi^q(\lambda n/2)|\pi^b(p)\rangle \]
Deeply Virtual Compton Scattering

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Novel view on form factors

- Form factor as a Mellin moment of the GPD

\[ \int dxx^{n-1} H^q(x, \xi, t) = A_{n,0}(t) + \sum_{i=1, \text{odd}}^{n} (-2\xi)^{i+1} A_{n,i+1}(t) \]

- Generalized form factors of the pion
  - pion EM form factor as the first Mellin moment:
    \[ F_\pi(t) = A_{1,0}(t) \]
  - EMTFFs as the second Mellin moments, which are the subjects of the present talk.
Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors** but
Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors** but Form factors as Mellin moments of the GPDs and they can be measured!
Chiral quark model

Effective Chiral Action

\[ SU(2)_L \times SU(2)_R \rightarrow SU(2)_V \] by the quark condensate

\[ SU(2)_L \times SU(2)_R / SU(2)_V : \text{Goldstone bosons} \quad \Sigma \rightarrow L\Sigma R^+ \]
Chiral quark model

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\[ \text{SU}(2)_L \times \text{SU}(2)_R / \text{SU}(2)_V : \text{Goldstone bosons} \quad \Sigma \rightarrow L \Sigma R^\dagger \]

\[ S_{\text{eff}} = -N_c \text{Tr} \log \left[ i\phi + iM\Sigma P_L + iM\Sigma^\dagger P_R + im1 \right] \]

- \( N_c \): The number of colors
- \( M \): Dynamical quark mass
- \( \Sigma = \exp(i\pi \cdot \tau / f_\pi) \): Pion field as a pseudo-Goldstone boson
- \( P_L, P_R \): Chiral projection operators
- \( m = (m_u + m_d)/2 \): Current quark mass

D. Diakonov and V.Y. Petrov, *NPB* 272, 457 (1986)
EMT form factors

Energy-momentum Tensor Form factors (Pagels, 1966)

\[ \langle \pi^a(p') | T_{\mu\nu}(0) | \pi^b(p) \rangle = \frac{\delta^{ab}}{2} \left[ (tg_{\mu\nu} - q_{\mu}q_{\nu}) \Theta_1(t) + 2P_{\mu}P_{\nu} \Theta_2(t) \right] \]

\[ T_{\mu\nu}(x) = \frac{1}{2} \bar{\psi}(x) \gamma^\nu \{ i \leftrightarrow \partial_{\nu} \} \psi(x) : EMT \text{ operator} \]
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EMTFFs (Gravitational FFs)

\[ T_{\mu\nu}(x) = \frac{1}{2} \bar{\psi}(x)\gamma^i \overleftrightarrow{\partial^\nu} \psi(x) : \text{EMT operator} \]
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T_{\mu\nu}(x) = \frac{1}{2} \bar{\psi}(x)\{i\not{\partial}_\nu\} \psi(x) : \text{EMT operator}
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Isoscalar vector GPDs of the pion

\[
2\delta_{ab} H_{\pi}^{I=0}(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p')|\bar{\psi}(-\lambda n/2)\psi[-\lambda n/2, \lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle
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**EMT form factors**

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**EMTFFs (Gravitational FFs)**

\[ T_{\mu\nu}(x) = \frac{1}{2} \bar{\psi}(x) \gamma \{ i \partial_\nu \} \psi(x) \quad \text{: EMT operator} \]

**Isoscalar vector GPDs of the pion**

\[ 2\delta^{ab} H_\pi^{I=0}(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda (p \cdot n)} \langle \pi^a(p') | \bar{\psi}(-\lambda n/2) \phi[-\lambda n/2, \lambda n/2] \psi(\lambda n/2) | \pi^b(p) \rangle \]

**The second moment of the GPD**

\[ \int dx \, x H_\pi^{I=0}(x, \xi, t) = A_{20}(t) + 4\xi^2 A_{22}(t) \]
EMT form factors

Energy-momentum Tensor Form factors (Pagels, 1966)

\[
\langle \pi^a(p') | T_{\mu\nu}(0) | \pi^b(p) \rangle = \frac{\delta^{ab}}{2} \left[ (t g_{\mu\nu} - q_\mu q_\nu) \Theta_1(t) + 2 P_\mu P_\nu \Theta_2(t) \right]
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\[T_{\mu\nu}(x) = \frac{1}{2} \bar{\psi}(x) \gamma \{ i \vec{\partial}_\nu \} \psi(x) \] : EMT operator

Isoscalar vector GPDs of the pion

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\[\int dx \, x H_{\pi}^{I=0}(x, \xi, t) = A_{20}(t) + 4\xi^2 A_{22}(t) \] : Generalized form factors of the pion
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EMTFFs (Gravitational FFs)

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The second moment of the GPD

\[ \int dx x H_{\pi}^{I=0}(x, \xi, t) = [A_{20}(t)] + 4\xi^2 A_{22}(t) \] : Generalized form factors of the pion

\[ \Theta_1 = -4A_{22}^{I=0}, \quad \Theta_2 = A_{20}^{I=0} \]

\[ \Theta_1(0) - \Theta_2(0) = O(m_\pi^2) \]
EMT form factors

Time component of the EMT matrix element gives the pion mass.

$$\left. \langle \pi^a(p)|T_{44}(0)|\pi^b(p) \rangle \right|_{t=0} = -2m_\pi^2 \Theta_2(0) \delta^{ab}$$

The sum of the spatial component of the EMT matrix element gives the pressure of the pion, which should vanish!

$$\left. \langle \pi^a(p)|T_{\mathit{ii}}(0)|\pi^b(p) \rangle \right|_{t=0} = \frac{3}{2} \delta^{ab} t \Theta_1(t) \bigg|_{t=0} \text{ Zero in the chiral limit}$$
EMT form factors

\[ k_{a\mu} = k_{\mu} - p_{\mu}/2 - q_{\mu}/2 \]
\[ k_{b\mu} = k_{\mu} + p_{\mu}/2 - q_{\mu}/2 \]
\[ k_{c\mu} = k_{\mu} + p_{\mu}/2 + q_{\mu}/2 \]
\[ k_{d} = k_{b} + k_{c} \]
\[ k_{ij} = k_{i} \cdot k_{j} \]

\[ \langle \pi^{a}(p')|\Theta_{\mu\nu}(0)|\pi^{b}(p)\rangle = \delta^{ab} \frac{2N_{c}}{f_{\pi}^{2}} \int d\tilde{k} \sum_{i} F_{i}(k, p, q)_{\mu\nu} + (\mu \leftrightarrow \nu) \]

\[ F_{a\mu\nu} = -\frac{M\overline{M}k_{d\mu}k_{d\nu}}{D_{b}D_{c}} \]

\[ (\overline{M} = m + M) \]

\[ F_{b\mu\nu} = \frac{2M^{2}k_{d\nu}}{D_{a}D_{b}D_{c}} \left[ -k_{a\mu} \left( k_{bc} + \overline{M}^{2} \right) + k_{b\mu} \left( k_{ac} + \overline{M}^{2} \right) + k_{c\mu} \left( k_{ab} + \overline{M}^{2} \right) \right] \]

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Stability of the pion

Pressure of the pion beyond the chiral limit

\[
\mathcal{P} = \langle \pi^a(p) | T_{ii}(0) | \pi^a(p) \rangle \\
= \frac{12N_c m M}{f^2 \pi} \int d\vec{l} \frac{-l^2}{[l^2 + \overline{M}^2]^2} + \frac{12N_c M^2}{f^2 \pi} \int d\vec{l} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + \overline{M}^2]^3}
\]
Stability of the pion

Pressure of the pion beyond the chiral limit

\[ P = \langle \pi^a(p)|T_{ii}(0)|\pi^a(p)\rangle \]

\[ = \frac{12N_c m_M}{f^2} \int d\tilde{l} \frac{-l^2}{[l^2 + M^2]^2} + \frac{12N_c M^2}{f^2} \int d\tilde{l} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + M^2]^3} \]

\[ i\langle \psi^\dagger \psi \rangle = 8N_c \int d\tilde{l} \frac{M}{[l^2 + M^2]} \]

Quark condensate

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Stability of the pion

Pressure of the pion beyond the chiral limit

\[ \mathcal{P} = \langle \pi^a(p) | T_{ii}(0) | \pi^a(p) \rangle \]
\[ = \frac{12N_c M}{f_\pi^2} \int d\tilde{l} \frac{-l^2}{[l^2 + M^2]^2} \]
\[ + \frac{12N_c M^2}{f_\pi^2} \int d\tilde{l} \int_0^1 dx \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + M^2]^3} \]

\[ i \langle \psi^\dagger \psi \rangle = 8N_c \int d\tilde{l} \frac{M}{[l^2 + M^2]} \]

Quark condensate

Pion decay constant

\[ f_\pi^2 = 4N_c \int_0^1 dx \int d\tilde{l} \frac{M M}{[l^2 + M^2 + x(1-x)p^2]^2} \]

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Stability of the pion

\[ \mathcal{P} = \frac{3M}{f_\pi^2 M} \left( m \langle \bar{\psi} \psi \rangle + m^2 \frac{f_\pi^2}{f_\pi} \right) = 0! \]

by the Gell-Mann-Oakes-Renner relation to linear m order

Physical implications: The stability of the pion should be deeply rooted spontaneous breakdown of chiral symmetry and the pattern of explicit chiral symmetry breaking.
\[ \Theta_1 = \Theta_2 \]
in the chiral limit
Energy–momentum Tensor FFs

The difference arises from the explicit chiral symmetry breaking.

\[ \Theta_1 = \Theta_2 \]

in the chiral limit

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Chiral Lagrangian in flat space

\[ \mathcal{L} = \frac{f^2}{4} \text{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger) + L_1 [\text{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger)]^2 + L_2 \text{Tr}(D_\mu \Sigma D_\nu U^\dagger) \text{Tr}(D^{\mu} \Sigma D^{\nu} \Sigma^\dagger) + L_3 \text{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger D_\nu \Sigma D^{\nu} \Sigma^\dagger) + \cdots \]

Chiral Lagrangian in curved space

\[ \mathcal{L} = L_{11} R \text{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger) + L_{12} R^{\mu \nu} \text{Tr}(D_\mu \Sigma D_\nu \Sigma^\dagger) + L_{13} R \text{Tr}(\chi \Sigma^\dagger + \Sigma \chi^\dagger) + \cdots \]

The Low-Energy constants can be derived by the Derivative expansion.
(small pion momentum, small pion mass)
Low-Energy Constants in curved space

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The Low-Energy constants can be derived by the Derivative expansion.
(small pion momentum, small pion mass)
Low-Energy Constants in curved space

\[ \Theta_1(q^2) = 1 + \frac{2q^2}{f_\pi^2}(4L_{11} + L_{12}) - \frac{16m_\pi^2}{f_\pi^2}(L_{11} - L_{13}) + \ldots \]

\[ \Theta_2(q^2) = 1 - \frac{2q^2}{f_\pi^2}L_{12} + \ldots \]


Derivative expansion in curved space

\[ L_{11} = \frac{N_c}{192\pi^2} = 1.6 \times 10^{-3} \]

\[ L_{12} = -2L_{11} = -3.2 \times 10^{-3} \]

\[ L_{13} = -\frac{N_c}{96\pi^2} \frac{M}{B_0}\Gamma \left( 0, \frac{M^2}{\Lambda^2} \right) = 0.84 \times 10^{-3} \]
## Low-Energy Constants in curved space

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<th>$L_{11}$</th>
<th>$L_{12}$</th>
<th>$L_{13}$</th>
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<td>$1.6 \times 10^{-3}$</td>
<td>$-3.2 \times 10^{-3}$</td>
<td>$0.84 \times 10^{-3}$</td>
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<td>SQM*</td>
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<td>$1.4 \times 10^{-3}$</td>
<td>$-2.7 \times 10^{-3}$</td>
<td>$0.9 \times 10^{-3}$</td>
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</tbody>
</table>

[*Megias et al. PRD 70, 034031 (2004)]
[**J.F. Donoghue and H. Leutwyler, Zeit. PC 52, 343 (1991)]
Pion Tomography

1 Generalised Parton Distributions

\[ \rho(b_\perp) \quad \int dx \quad q(x, b_\perp) \quad \Delta \to 0 \quad q(x) \]

Transverse densities of Form factors

GPDs

Structure functions

Parton distributions

D. Brömmel, Dissertation (Regensburg U.)
Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

\[ q(x, b) = \int \frac{d^2}{(2\pi)^2} e^{i\mathbf{q} \cdot \mathbf{b}} H^q_{\pi}(x, 0, t) \]
Transverse charge densities

Why transverse charge densities?

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It can be interpreted as the probability distribution of a quark in the transverse plane.

Transverse charge densities

Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

\[ q(x, b) = \int \frac{d^2 q}{(2\pi)^2} e^{i \mathbf{q} \cdot \mathbf{b}} H^q_\pi(x, 0, t) \]

It can be interpreted as the probability distribution of a quark in the transverse plane.


Pion transverse charge densities

\[ \rho_{ni}(b) := \int \frac{d^2 q}{(2\pi)^2} A_{ni}(t) e^{i \mathbf{q} \cdot \mathbf{b}} \]
Transverse charge density of the pion

\[ \rho_{20}(b) = \int_{0}^{\infty} \frac{Q dQ}{2\pi} J_0(bQ) \Theta_2(t) \]
Transverse charge density of the pion

\[ \rho_{20}(b) \]

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Transverse charge density of the pion

The transverse charge density is divergent at $b=0$. 

$$\rho_{20}(b)$$

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)
Transverse charge density of the pion

\[ \rho_{\pi}(b_y, b_x = 0) \]

\[ \rho_{20}(b) \]

\[ \rho_{10}(b) \]
Transverse charge density of the pion

\[ \rho_{\pi}(b_y, b_x = 0) \]

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\[ \rho_{10}(b) \]

Transverse charge density from the EMFF
Transverse charge density of the pion

\[ \rho_{10}(b) \]
The transverse charge density is divergent at $b=0$. 

$$\rho_{10}(b)$$
We also showed the energy-momentum tensor form factors of the pion. The stability of the pion beyond the chiral limit was shown to be secured by the Gell-Mann-Oakes-Renner relation, which implies that the stability of the pion is deeply related to the spontaneous breakdown of chiral symmetry and the pattern of chiral symmetry breaking.

We also discussed the low-energy constants for the effective chiral Lagrangian in curved space, and the transverse charge densities of the pion in the transverse plane.
Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!