OSAKA UNIVERSITY

Stability of the pion beyond the chiral limit

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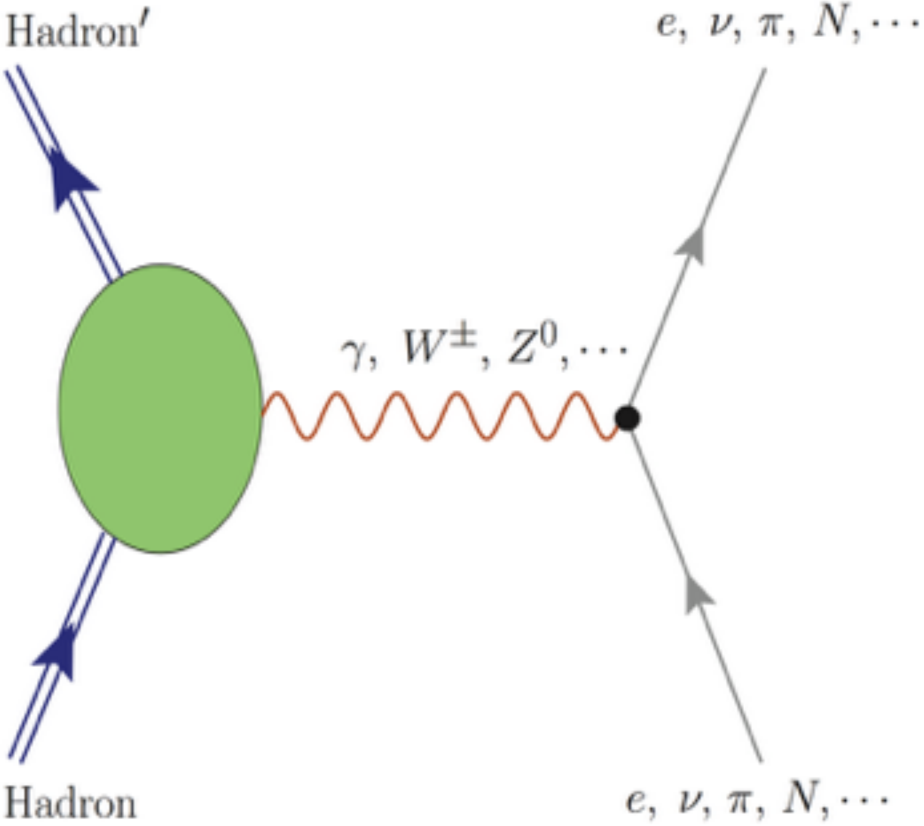
In colloboration with Hyeon-Dong Son

Traditional form factors



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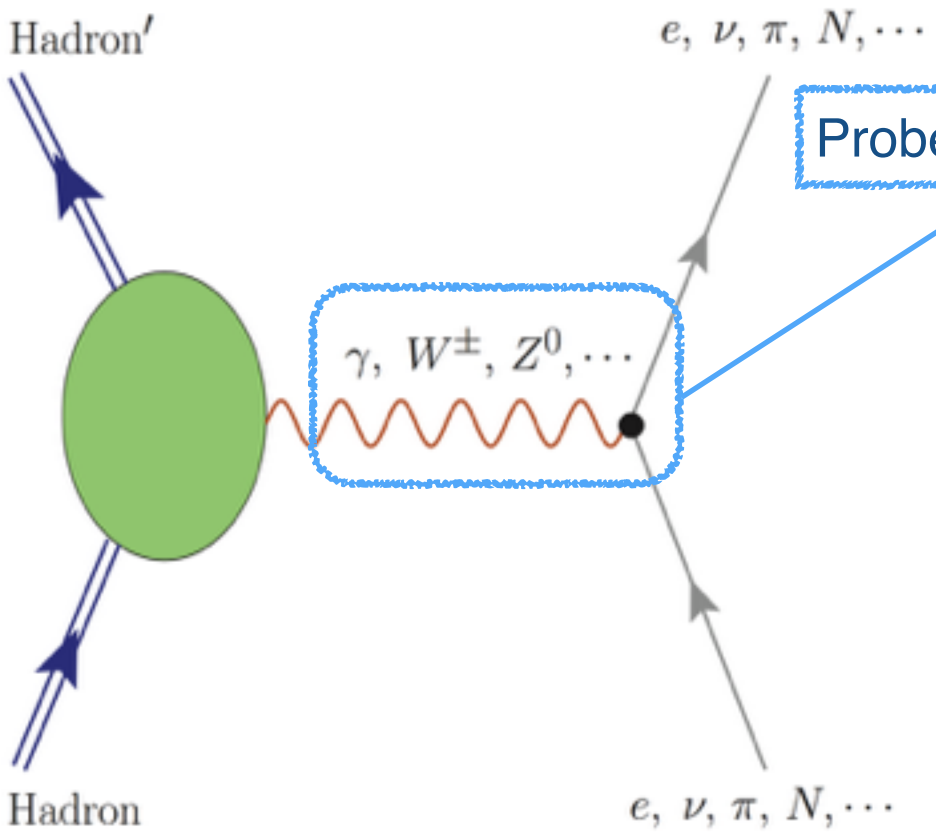

Probes examine the structure of a hadron

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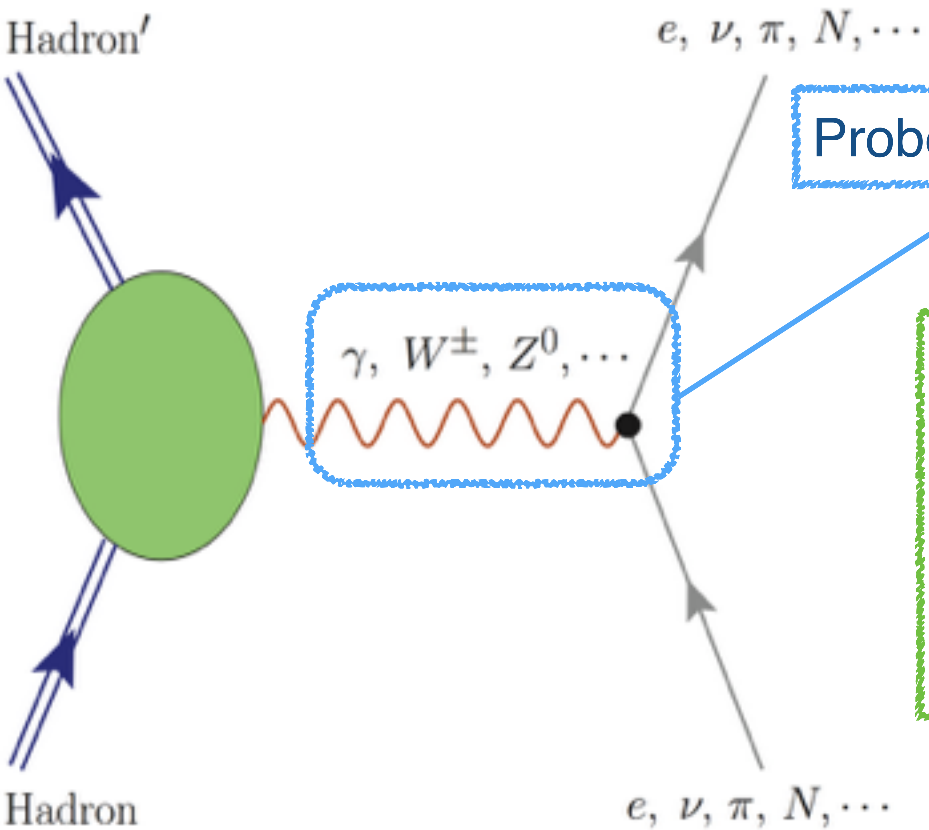

Probes examine the structure of a hadron

- Electromagnetic form factors (Vector form factors)
- Axial-vector form factors
- Scalar form factors
- Pseudoscalar form factors

Energy–Momentum Tensor Form Factors

- Given an action $S = \int d^4\sqrt{-g}\mathcal{L}$

$$T_{\mu\nu} = 2\frac{\delta S}{\delta g^{\mu\nu}} \quad \text{or} \quad T_{\mu\nu} = -\frac{\partial \mathcal{L}}{\partial(\partial^\mu \varphi_a(x))}\partial_\nu \phi_a(x) + g_{\mu\nu}\mathcal{L}$$

- EMT is a conserved quantity: $\partial^\mu T_{\mu\nu} = 0$

  (EMTFFs are scale-independent quantities)

- Energy-momentum tensor form factors of the pion

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 2P_\mu P_\nu \Theta_2(t)\right]$$

  (also known as the gravitational form factors)

H. Pagels, Phys. Rev. **144**, 1250 (1966).

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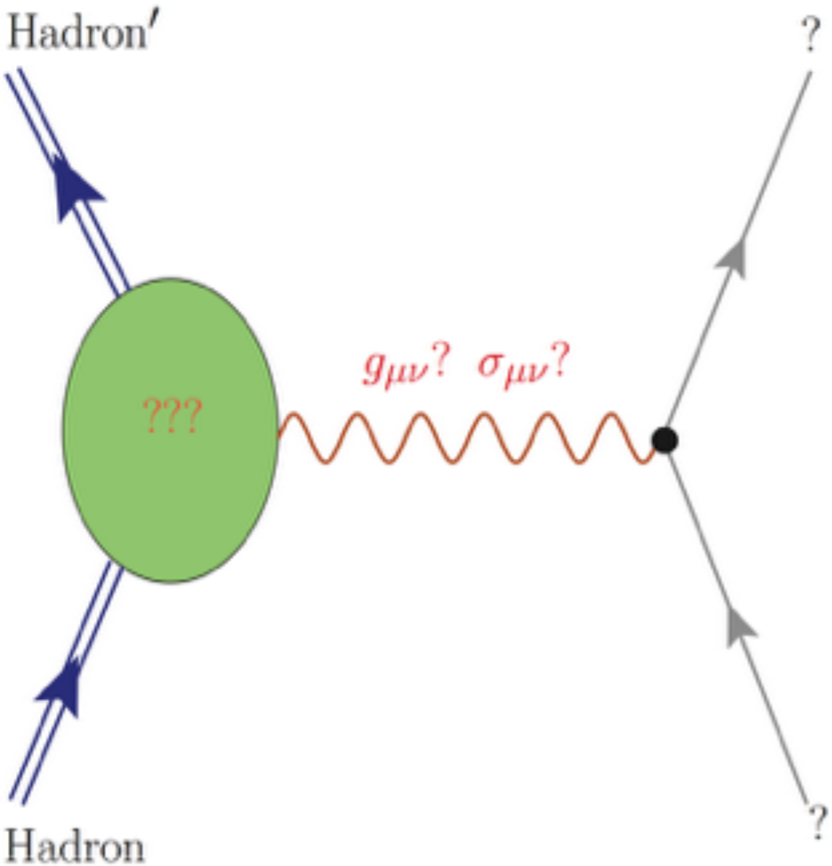
Energy-Momentum Tensor form factors & Tensor Form factors

These form factors are as equally important as vector and axial-vector form factors (Energy & Momentum distributions & transversity, resp.)!



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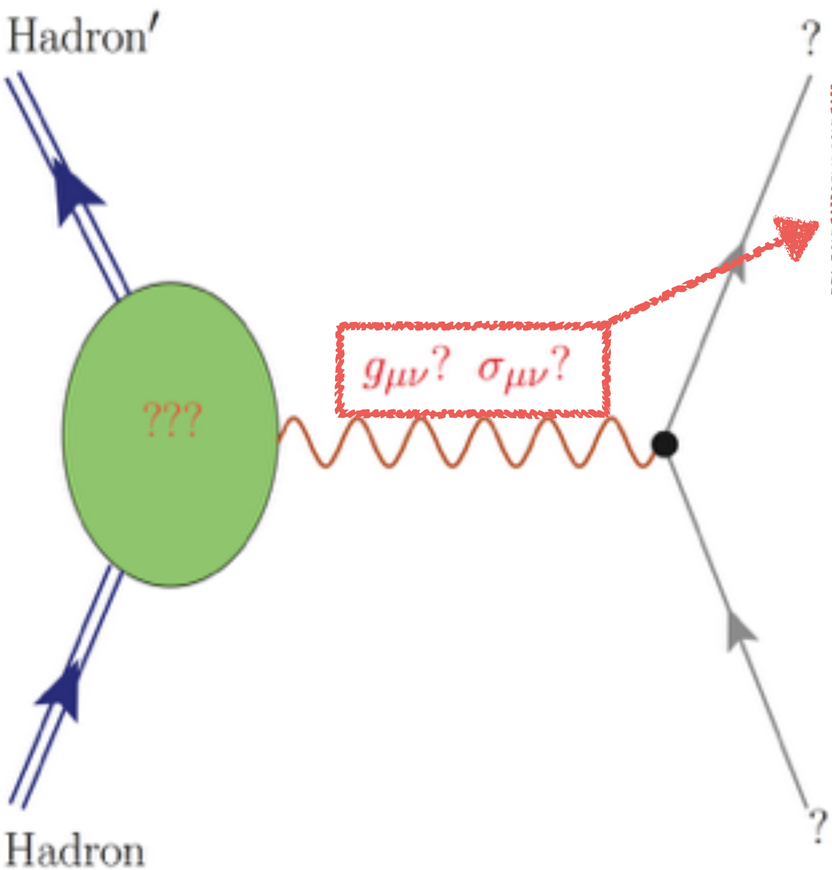

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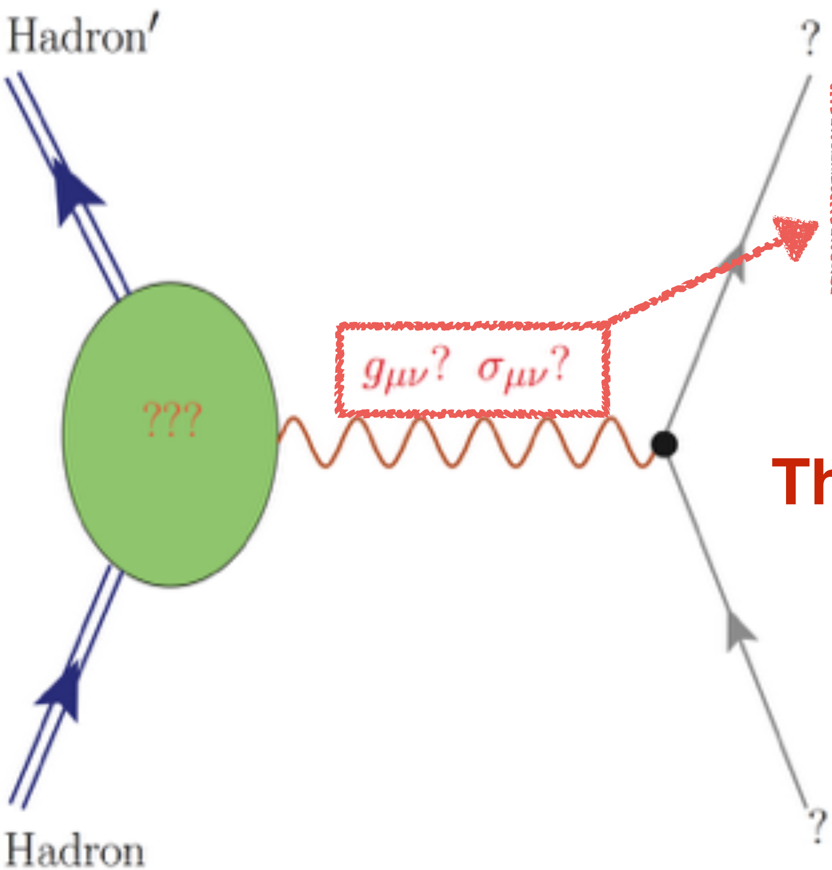

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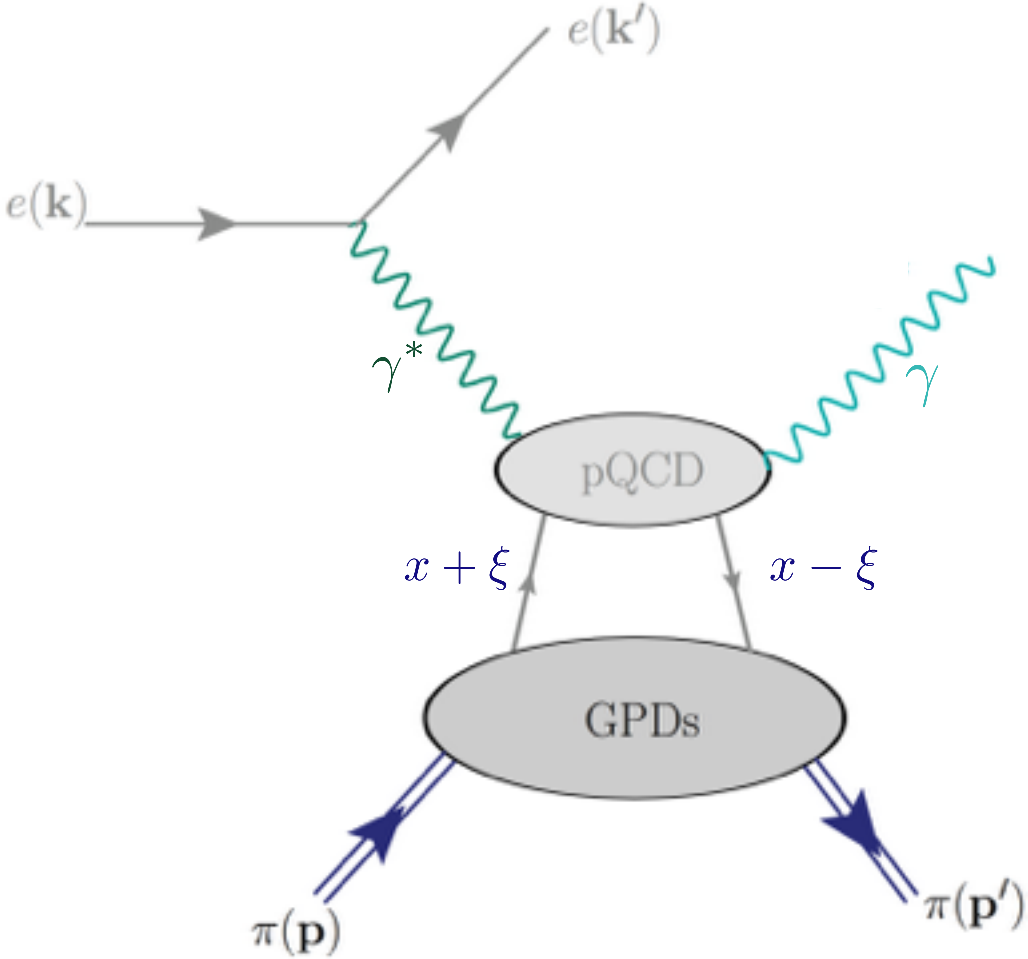
Then, how can we measure them?

Novel view on form factors

Deeply Virtual Compton Scattering

$$2\delta^{ab} H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p')|\bar{\psi}^q(-\lambda n/2)\not{n}[-\lambda n/2, \lambda n/2]\psi^q(\lambda n/2)|\pi^b(p)\rangle$$



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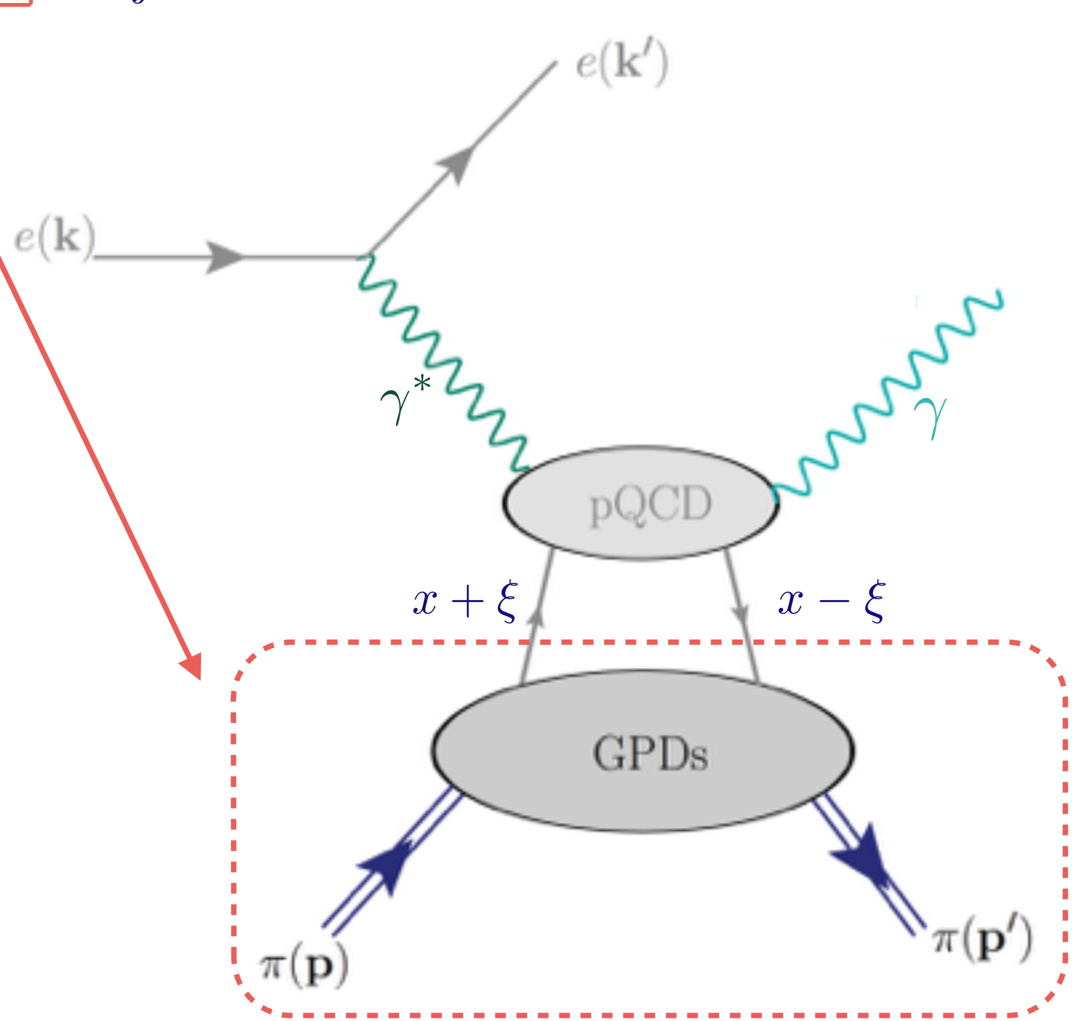

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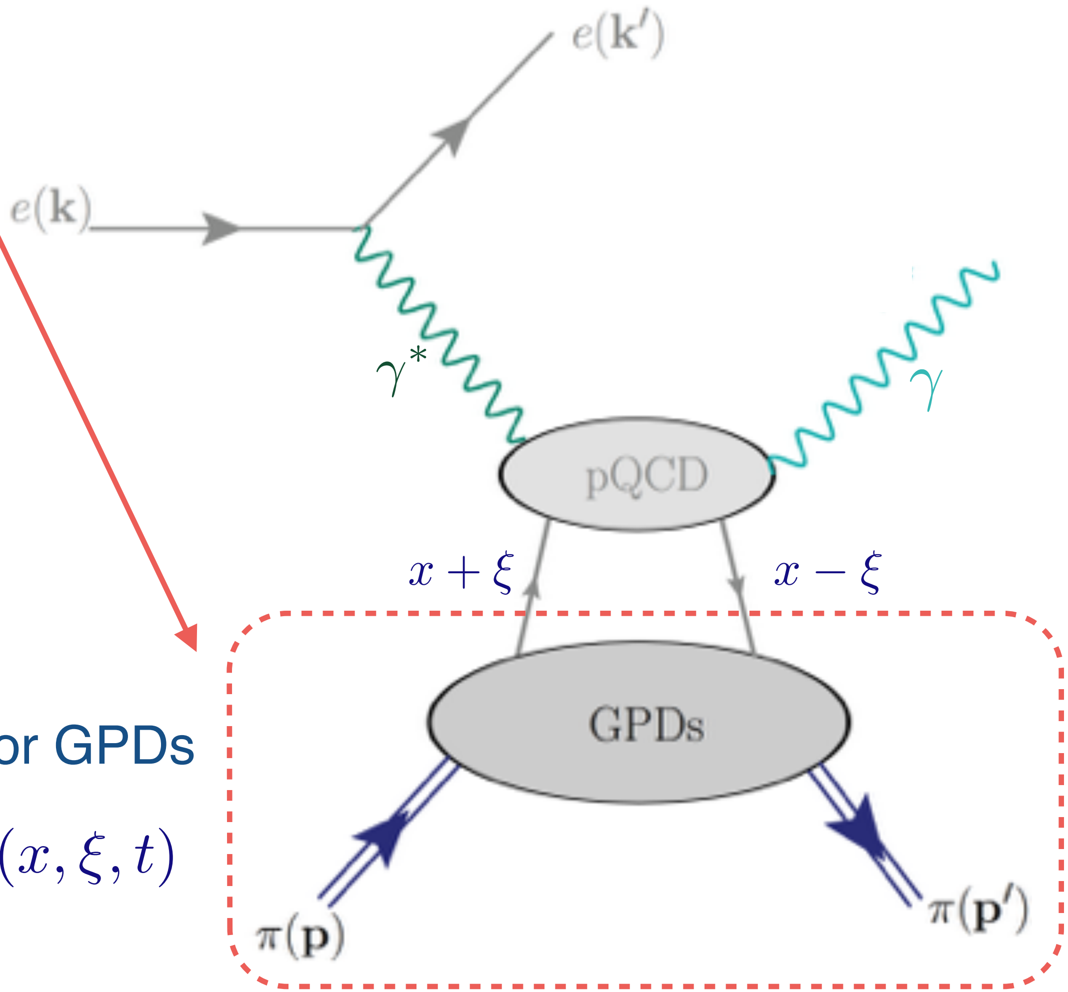


Vector GPDs

$H_\pi^q(x,\xi,t)$

Novel view on form factors

- Form factor as a Mellin moment of the GPD

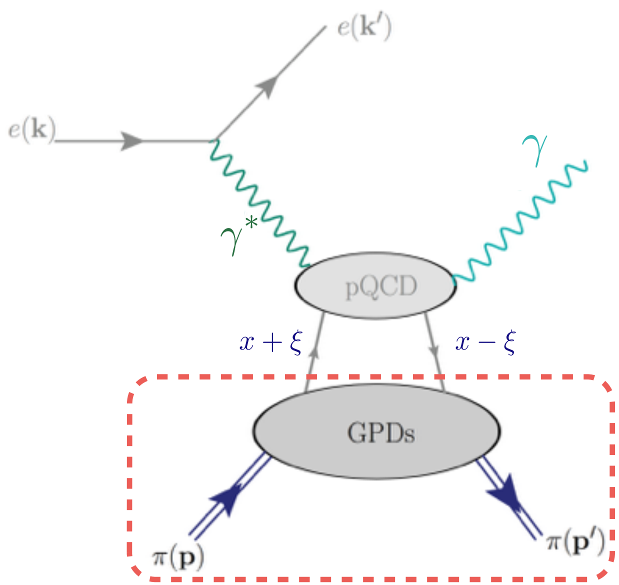
$$\int dx x^{n-1} H_\pi^q(x,\xi,t) = A_{n,0}(t) + \sum_{i=1,\,\mathrm{odd}}^{n} (-2\xi)^{i+1} A_{n,i+1}(t)$$

- Generalized form factors of the pion

  - pion EM form factor as the first Mellin moment:

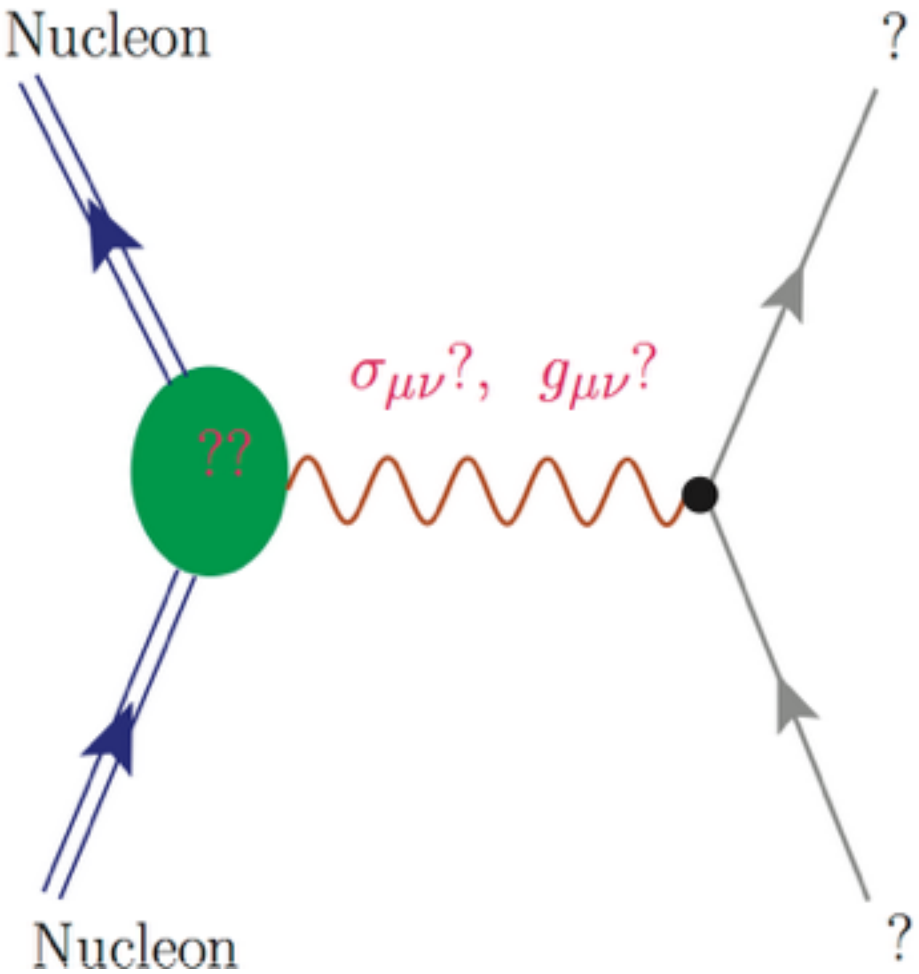
$$F_\pi(t) = A_{1,0}(t)$$

- **EMTFFs as the second Mellin moments, which are the subjects of the present talk.**



Generalised Parton Distributions

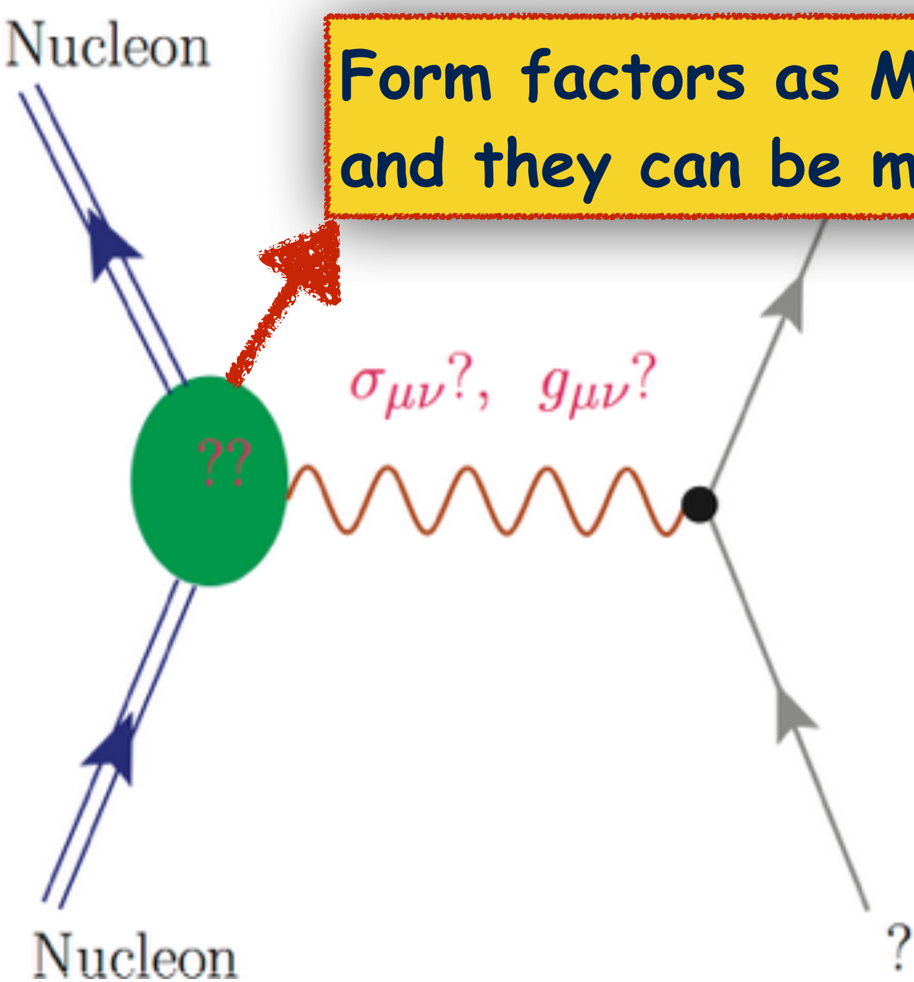
Probes are unknown for **Tensor form factors**
and the **Energy-Momentum Tensor form factors but**



Generalised Parton Distributions

Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors** but

Form factors as Mellin moments of the GPDs and they can be measured!



Chiral quark model

Effective Chiral Action

$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$ by the quark condensate

↓

$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R/\mathrm{SU}(2)_V$: Goldstone bosons $\Sigma \to L\Sigma R^\dagger$

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$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R / \mathrm{SU}(2)_V$: Goldstone bosons $\qquad \Sigma \to L \Sigma R^\dagger$

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr} \log \left[i\partial\!\!\!/ + iM\Sigma P_L + iM\Sigma^\dagger P_R + im\mathbf{1} \right]$$

N_c : The number of colors

M : Dynamical quark mass

$\Sigma = \exp(i\pi \cdot \tau / f_\pi)$: Pion field as a pseudo-Goldstone boson

$P_L,\ P_R$: Chiral projection operators

$m = (m_u + m_d)/2$: Current quark mass

A. Manohar, H. Georgi, NPB**234**, 189 (1984) D. Diakonov and V.Y. Petrov, NPB**272**, 457 (1986)

EMT form factors

Energy-momentum Tensor Form factors (Pagels, 1966)

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 2P_\mu P_\nu \Theta_2(t)\right]$$

$$T_{\mu\nu}(x) = \frac{1}{2}\bar{\psi}(x)\gamma_{\{}i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$: EMT operator

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Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi}e^{ix\lambda(P\cdot n)}\langle \pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2, \lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

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The second moment of the GPD

$$\int dx\, xH_\pi^{I=0}(x,\xi,t) = A_{20}(t) + 4\xi^2 A_{22}(t)$$

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The second moment of the GPD

$$\int dx\, xH_\pi^{I=0}(x,\xi,t) = \boxed{A_{20}(t)} + 4\xi^2\boxed{A_{22}(t)}$$: Generalized form factors of the pion

$$\boxed{\Theta_1 = -4A_{22}^{I=0},\ \Theta_2 = A_{20}^{I=0}}$$

$$\Theta_1(0) - \Theta_2(0) = \mathcal{O}(m_\pi^2)$$

EMT form factors

Time component of the EMT matrix element gives the pion mass.

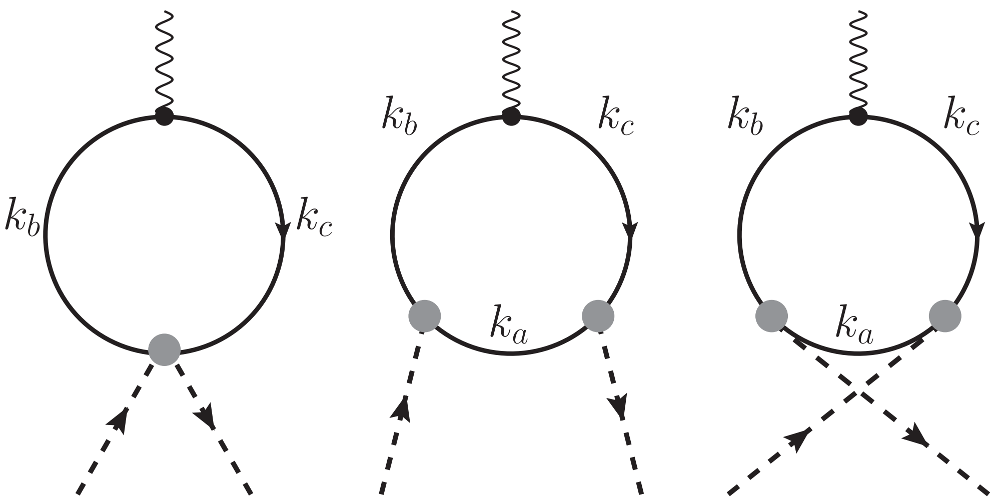
$$\langle \pi^a(p)|T_{44}(0)|\pi^b(p)\rangle\Big|_{t=0} = -2m_\pi^2\Theta_2(0)\delta^{ab}$$

The sum of the spatial component of the EMT matrix element gives the pressure of the pion, which should vanish!

$$\langle \pi^a(p)|T_{ii}(0)|\pi^b(p)\rangle\Big|_{t=0} = \frac{3}{2}\delta^{ab} t\,\Theta_1(t)\Big|_{t=0}$$ Zero in the chiral limit

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

EMT form factors



$$k_{a\mu} = k_\mu - p_\mu/2 - q_\mu/2$$
$$k_{b\mu} = k_\mu + p_\mu/2 - q_\mu/2$$
$$k_{c\mu} = k_\mu + p_\mu/2 + q_\mu/2$$
$$k_d = k_b + k_c$$
$$k_{ij} = k_i \cdot k_j$$

$$\left\langle \pi^a(p') | \Theta_{\mu\nu}(0) | \pi^b(p) \right\rangle = \delta^{ab} \frac{2N_c}{f_\pi^2} \int d\tilde{k} \sum_i \mathcal{F}_i(k,p,q)_{\mu\nu} + (\mu \leftrightarrow \nu)$$

$$\mathcal{F}_{a\mu\nu} = -\frac{M\overline{M} k_{d\mu} k_{d\nu}}{D_b D_c} \qquad (\overline{M} = m + M)$$

$$\mathcal{F}_{b\mu\nu} = \frac{2M^2 k_{d\nu}}{D_a D_b D_c} \left[-k_{a\mu}\left(k_{bc} + \overline{M}^2\right) + k_{b\mu}\left(k_{ac} + \overline{M}^2\right) + k_{c\mu}\left(k_{ab} + \overline{M}^2\right) \right]$$

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

Pressure of the pion beyond the chiral limit

$$\mathcal{P} = \langle \pi^a(p)|T_{ii}(0)|\pi^a(p)\rangle$$

$$= \frac{12N_c m M}{f_\pi^2}\int d\tilde{l}\,\frac{-l^2}{[l^2+\overline{M}^2]^2} + \frac{12N_c M^2}{f_\pi^2}\int d\tilde{l}\int_0^1 dx\,\frac{-p^2 l^2}{[l^2+x(1-x)p^2+\overline{M}^2]^3}$$

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$$i\langle\psi^\dagger\psi\rangle = 8N_c\int d\tilde{l}\frac{\overline{M}}{[l^2+\overline{M}^2]}$$

Quark condensate

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

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$$i\langle\psi^\dagger\psi\rangle = 8N_c\int d\tilde{l}\frac{\overline{M}}{[l^2+\overline{M}^2]}$$

$$f_\pi^2 = 4N_c\int_0^1 dx\int d\tilde{l}\,\frac{M\overline{M}}{[l^2+\overline{M}^2+x(1-x)p^2]^2}$$

Quark condensate

Pion decay constant

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

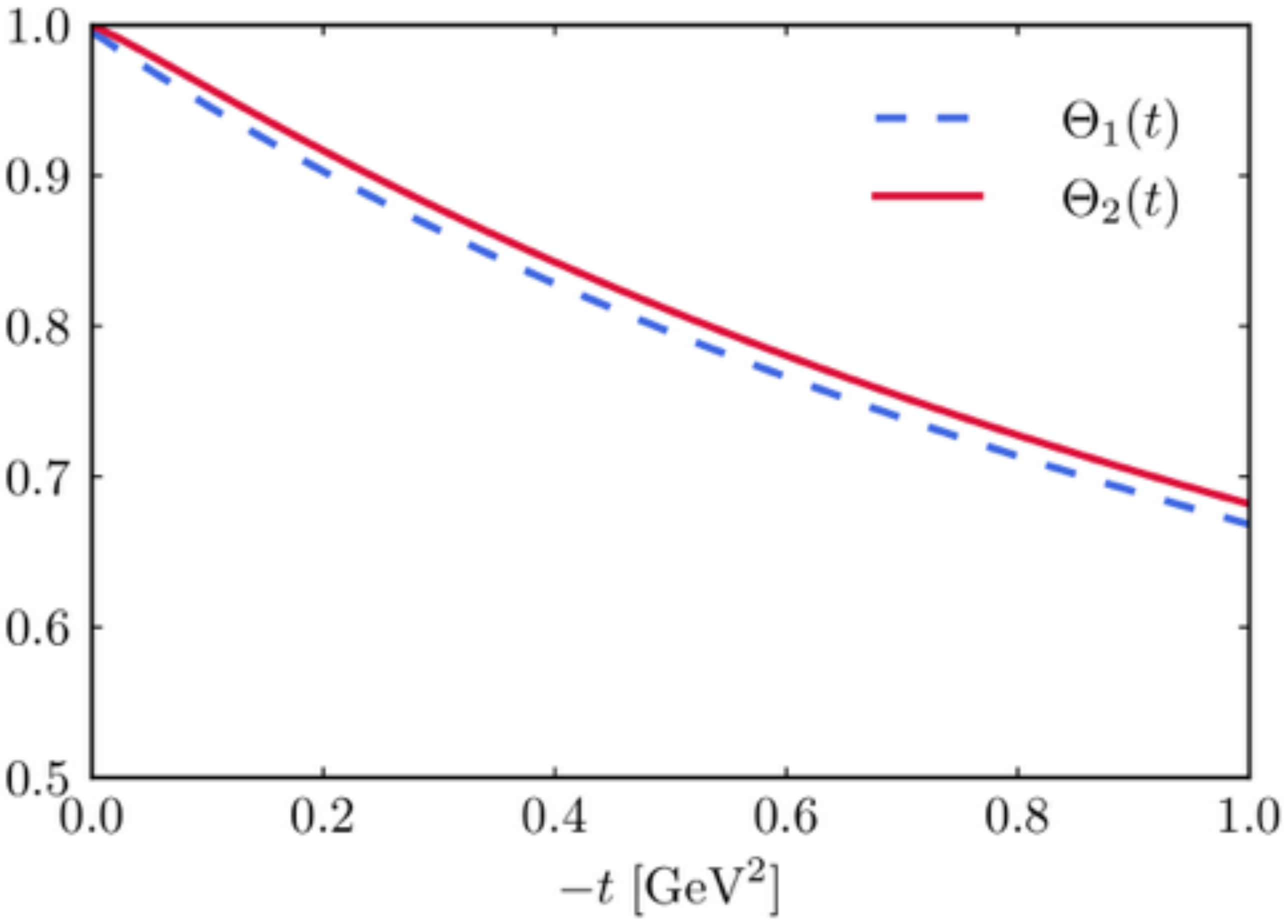
$$\mathcal{P} = \frac{3M}{f_\pi^2 \overline{M}} \left(m \left\langle \bar{\psi}\psi \right\rangle + m_\pi^2 f_\pi^2 \right) = 0 \; !$$

by the Gell-Mann-Oakes-Renner relation to linear m order

Physical implications: The stability of the pion should be deeply rooted spontaneous breakdown of chiral symmetry and the pattern of explicit chiral symmetry breaking.

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Energy-momentum Tensor FFs



$$\Theta_1 = \Theta_2$$

in the chiral limit

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

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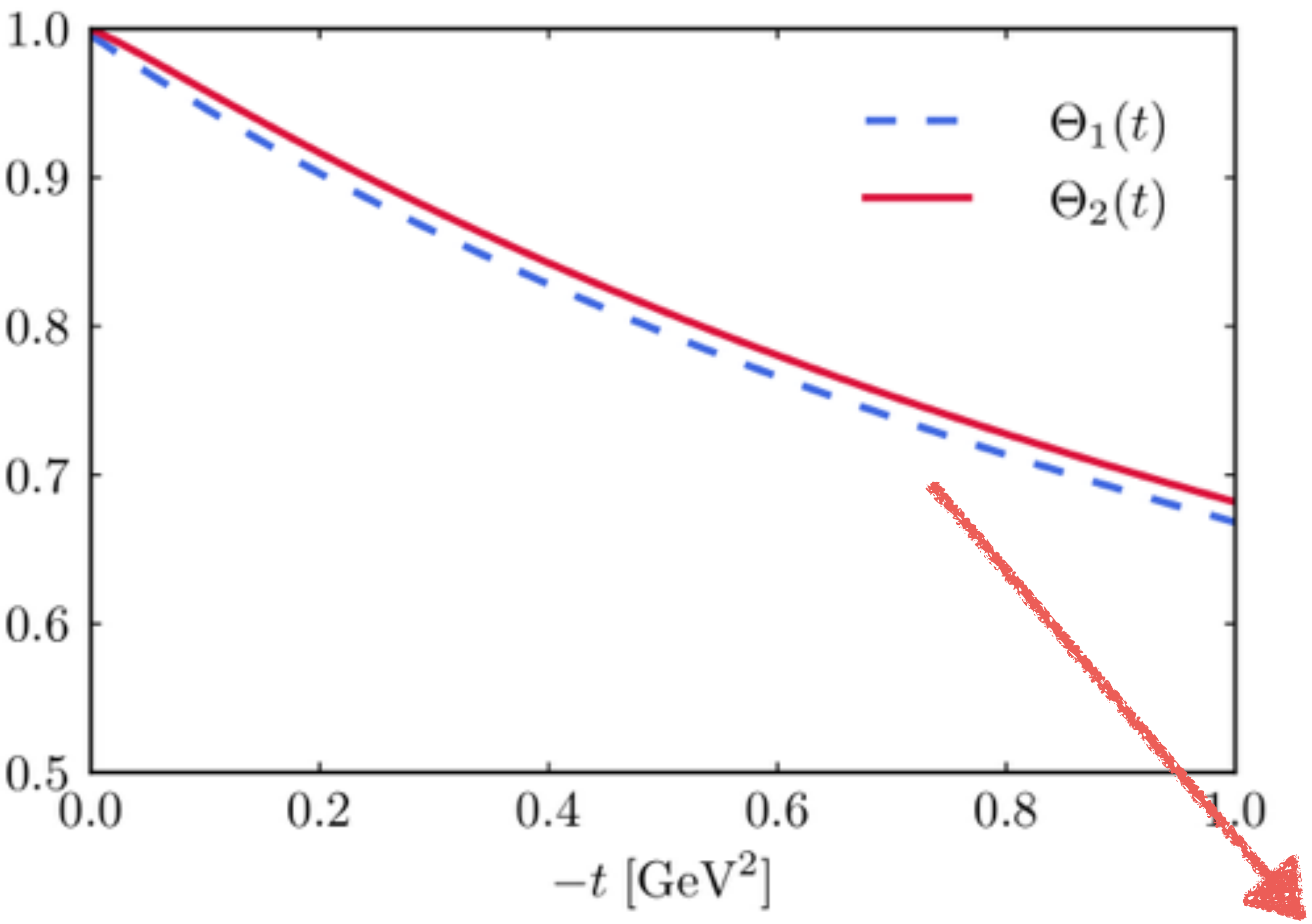

$$\Theta_1 = \Theta_2$$

in the chiral limit

The difference arises from the explicit chiral symmetry breaking.

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Low-Energy Constants in curved space

Chiral Lagrangian in flat space

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger)$$
$$+ L_1[\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger)]^2 + L_2 \mathrm{Tr}(D_\mu \Sigma D_\nu U^\dagger)\mathrm{Tr}(D^\mu \Sigma D^\nu \Sigma^\dagger)$$
$$+ L_3 \mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger D_\nu \Sigma D^\nu \Sigma^\dagger) + \cdots$$

Chiral Lagrangian in curved space

$$\mathcal{L} = L_{11} R \mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger) + L_{12} R^{\mu\nu} \mathrm{Tr}(D_\mu \Sigma D_\nu \Sigma^\dagger)$$
$$+ L_{13} R \mathrm{Tr}(\chi \Sigma^\dagger + \Sigma \chi^\dagger) + \cdots$$

The Low-Energy constants can be derived by the Derivative expansion.
(small pion momentum, small pion mass)

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

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H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Low-Energy Constants in curved space

$$\Theta_1(q^2) = 1 + \frac{2q^2}{f_\pi^2}(4L_{11} + L_{12}) - \frac{16m_\pi^2}{f_\pi^2}(L_{11} - L_{13}) + \ldots$$

$$\Theta_2(q^2) = 1 - \frac{2q^2}{f_\pi^2}L_{12} + \ldots$$

[J.F. Donoghue and H. Leutwyler, Z.Phys.C(1991) 52, 343]

- Derivative expansion in curved space

$$L_{11} = \frac{N_c}{192\pi^2} = 1.6 \times 10^{-3}$$

$$L_{12} = -2L_{11} = -3.2 \times 10^{-3}$$

$$L_{13} = -\frac{N_c}{96\pi^2}\frac{M}{B_0}\Gamma\left(0, \frac{M^2}{\Lambda^2}\right) = 0.84 \times 10^{-3}$$

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

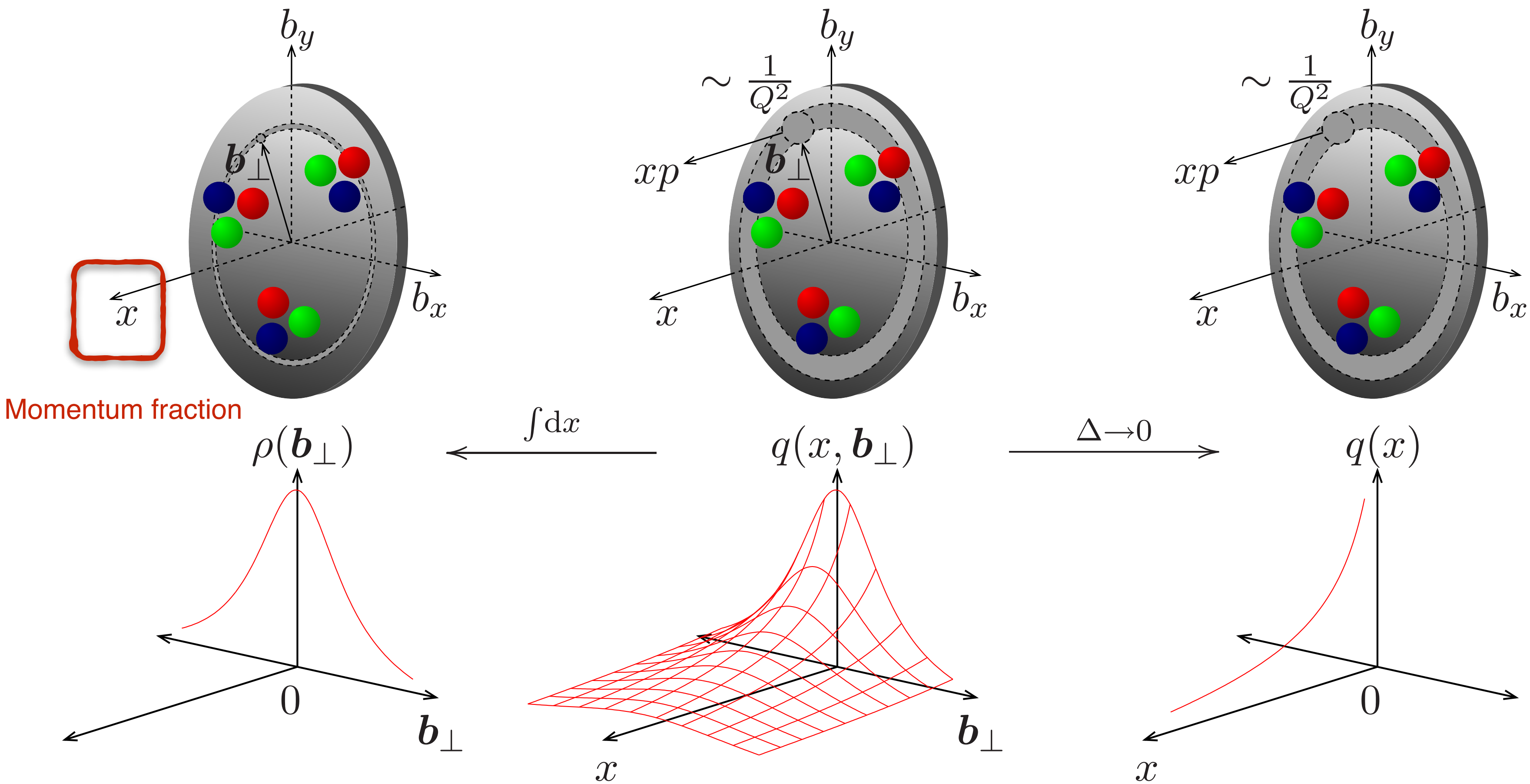
Low-Energy Constants in curved space

	L_{11}	L_{12}	L_{13}
Present Work	**1.6*10^{-3}**	**-3.2*10^{-3}**	**0.84*10^{-3}**
SQM*	1.6*10^{-3}	-3.2*10^{-3}	0.3*10^{-3}
XPT**	1.4*10^{-3}	-2.7*10^{-3}	0.9*10^{-3}

[*Megias *et al*. PRD **70**, 034031 (2004)]
[**J.F. Donoghue and H. Leutwyler, Zeit. PC **52**, 343 (1991)]

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Pion Tomography



Transverse densities
of Form factors

GPDs
Pion Tomography

Structure functions
Parton distributions

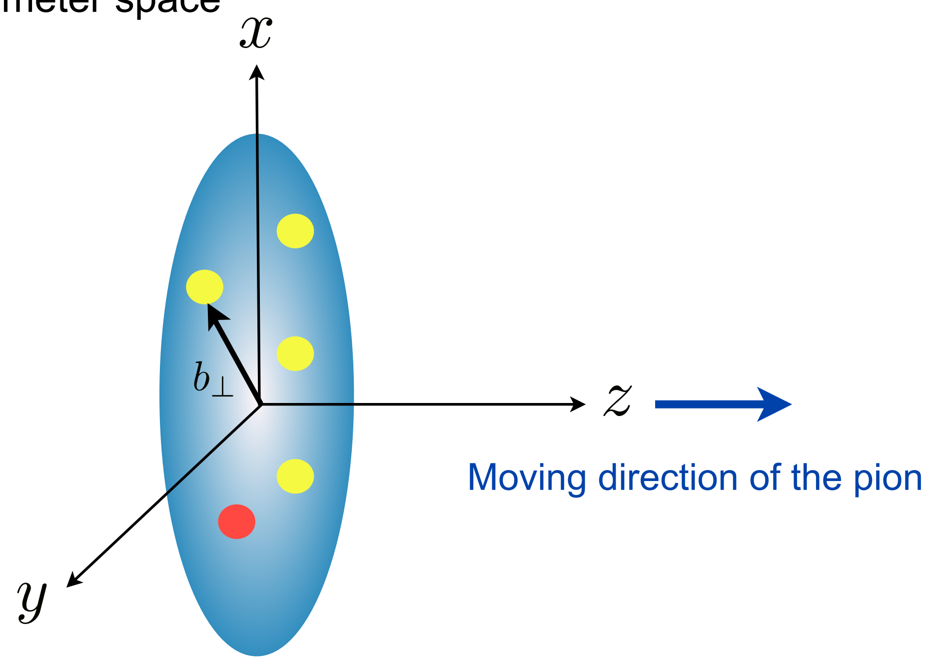
D. Brömmel, Dissertation (Regensburg U.)

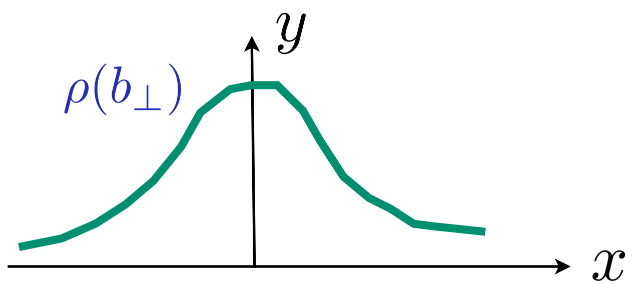
Transverse charge densities

Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

$$q(x,b) = \int \frac{d^2}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_\pi^q(x,0,t)$$





Transverse charge densities

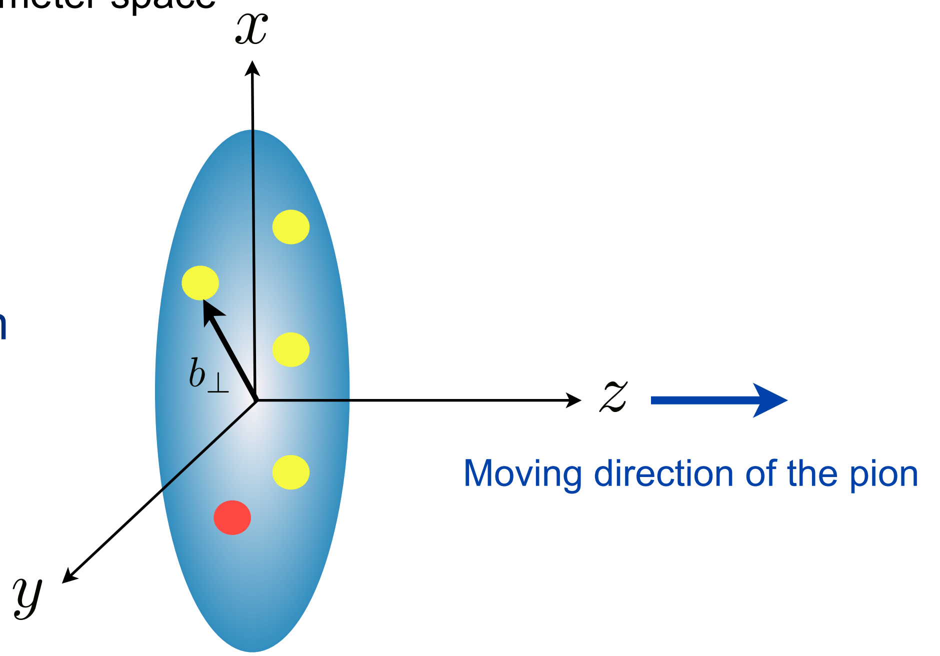
Why transverse charge densities?

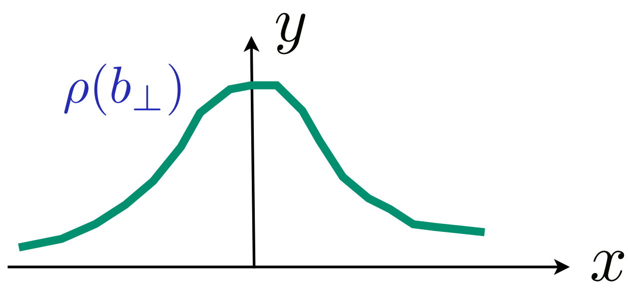
2-D Fourier transform of the GPDs in impact-parameter space

$$q(x,b) = \int \frac{d^2}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_\pi^q(x,0,t)$$

⇨ It can be interpreted as the probability distribution of a quark in the transverse plane.

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).





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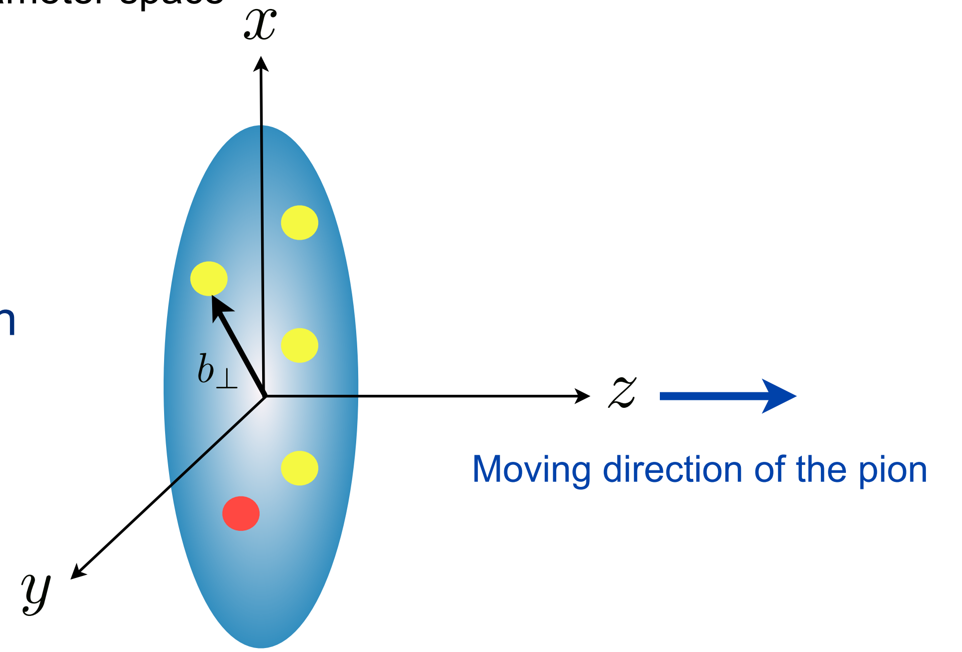
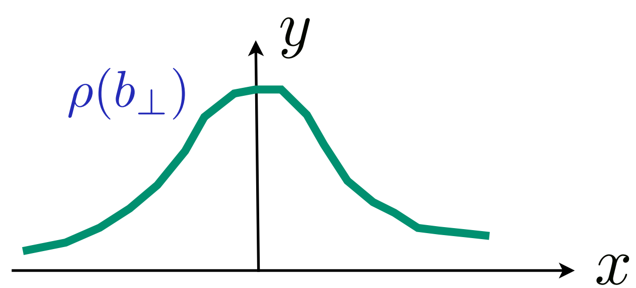


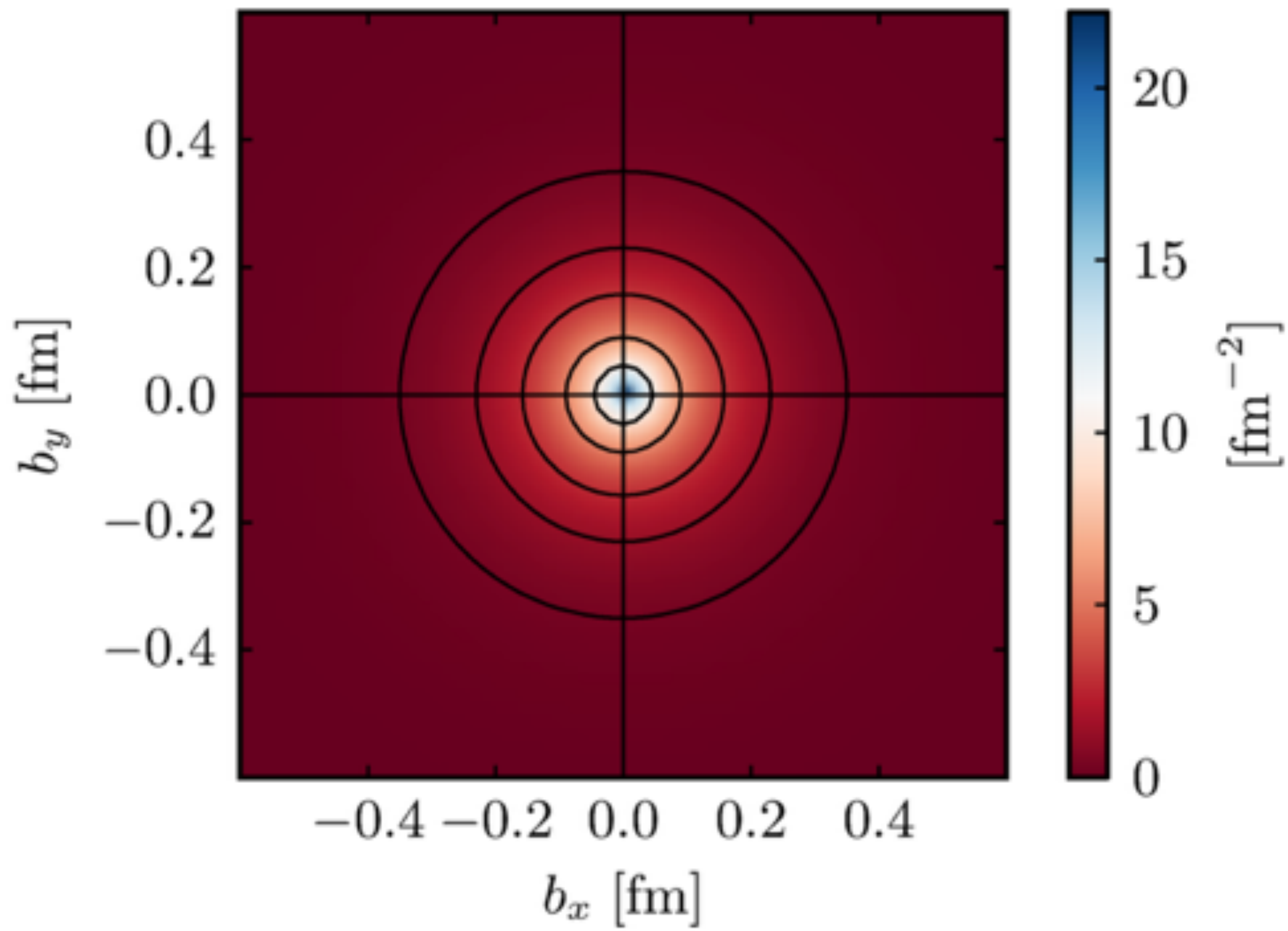
Pion transverse charge densities

$$\rho_{ni}(\mathbf{b}) := \int \frac{d^2q}{(2\pi)^2} A_{ni}(t) e^{i\mathbf{q}\cdot\mathbf{b}}$$



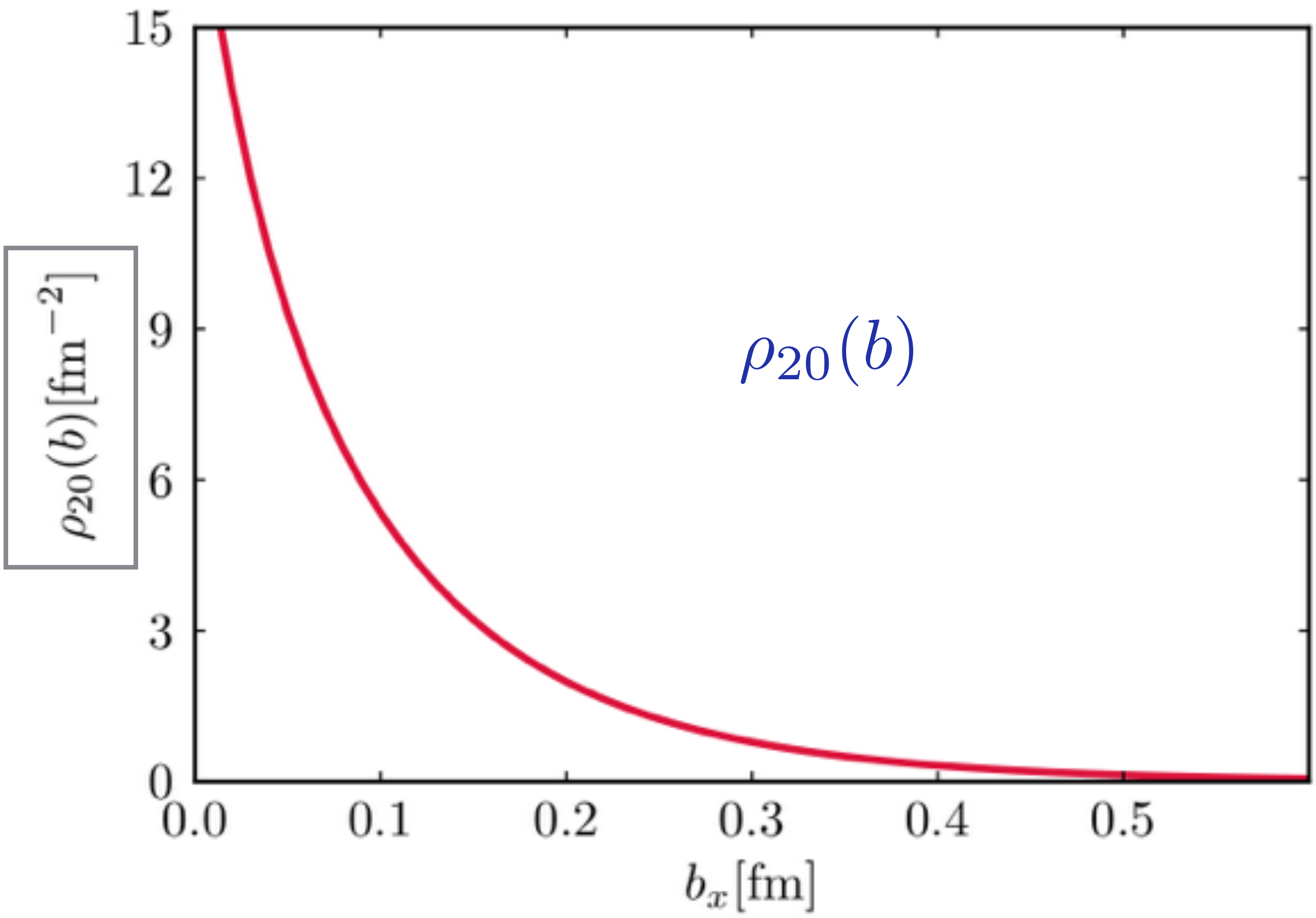
Transverse charge density of the pion

$$\rho_{20}(b) = \int_0^\infty \frac{QdQ}{2\pi} J_0(bQ)\Theta_2(t)$$



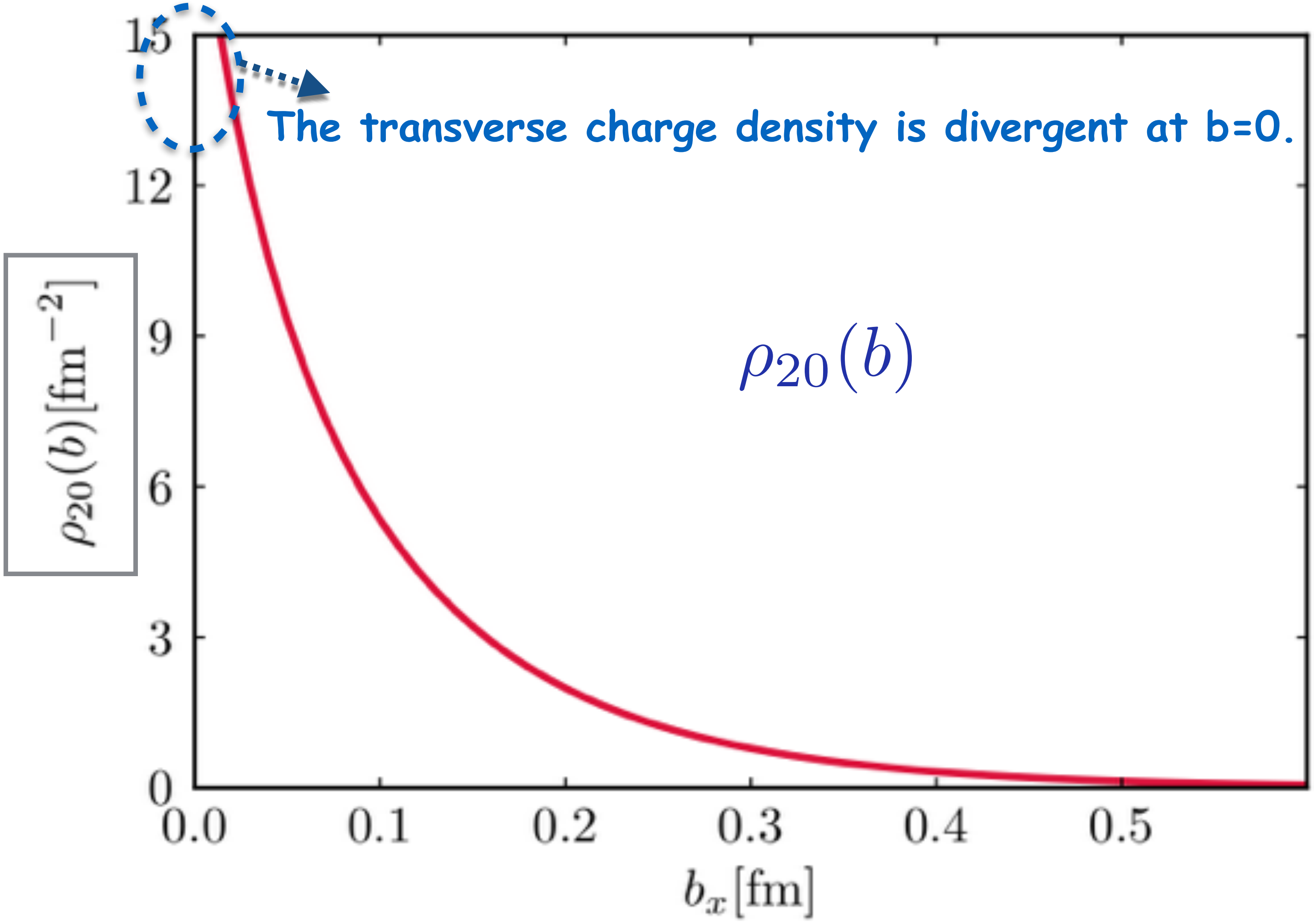
H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion



H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion



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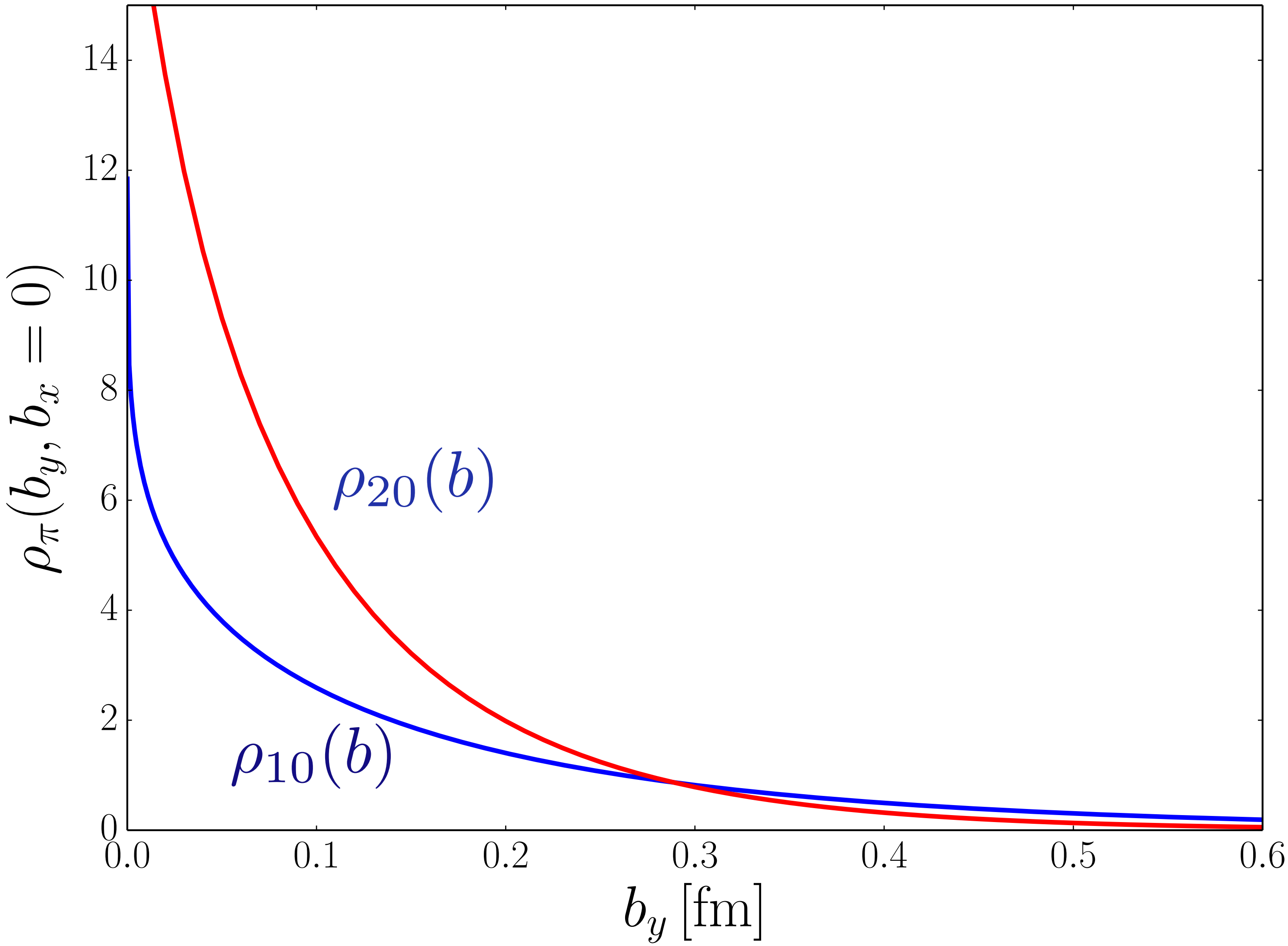
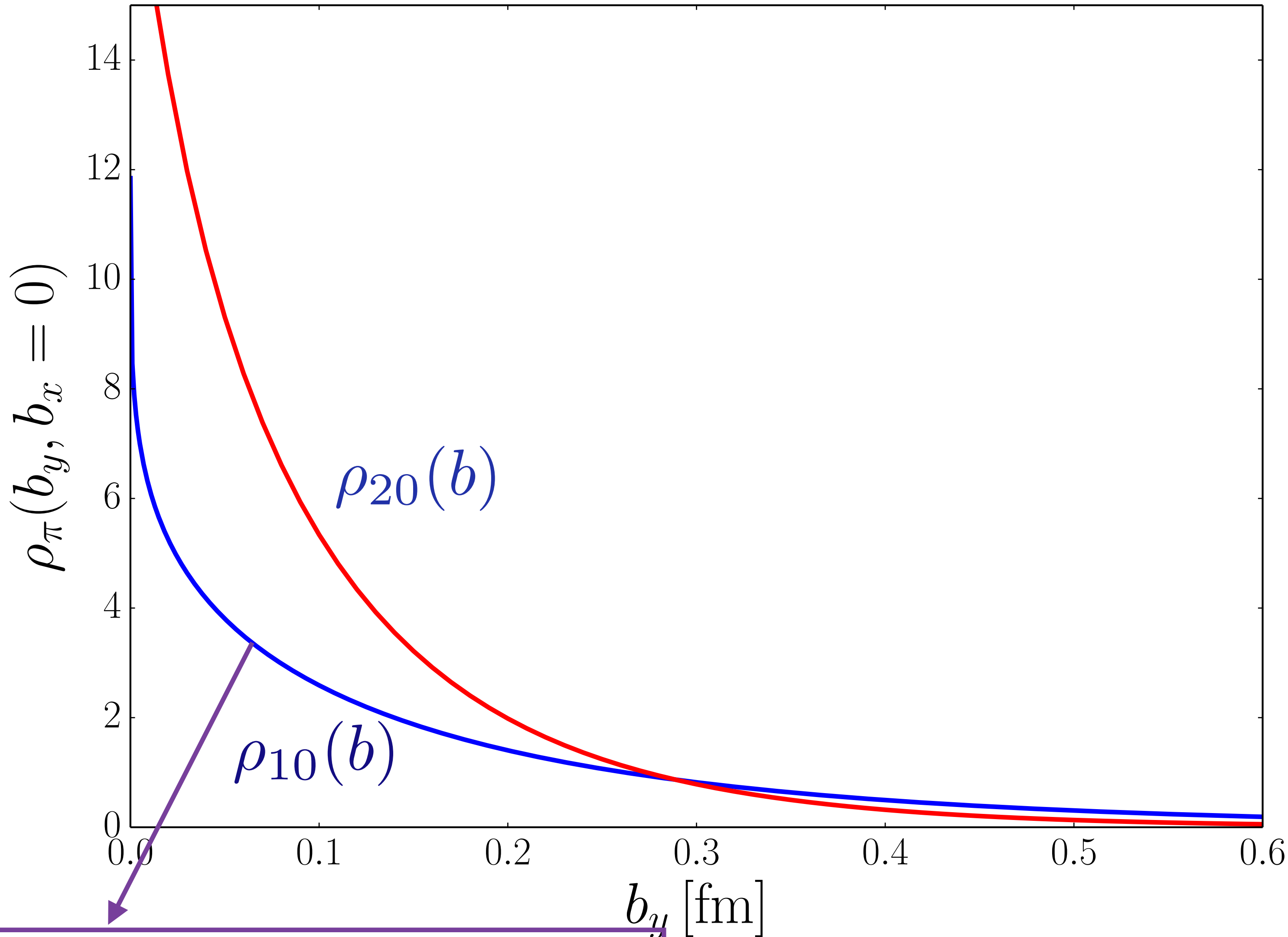
Transverse charge density of the pion

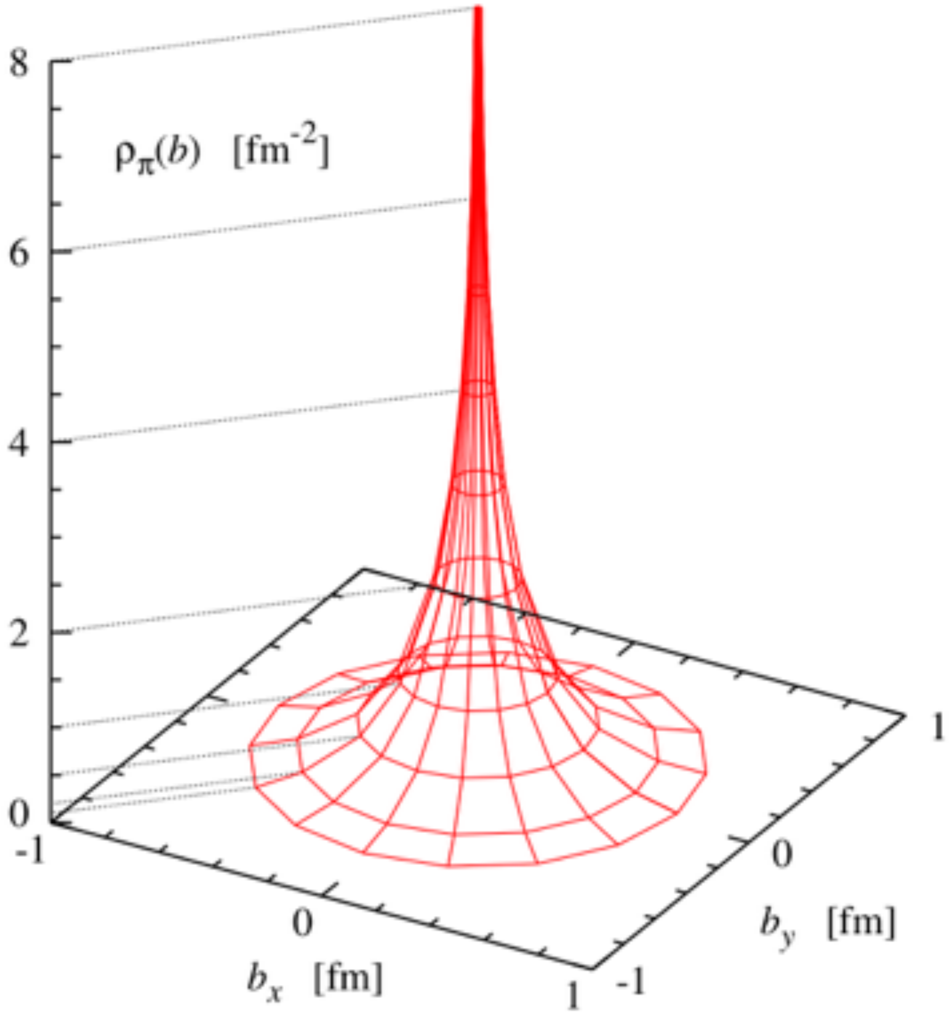


Transverse charge density of the pion



Transverse charge density from the EMFF

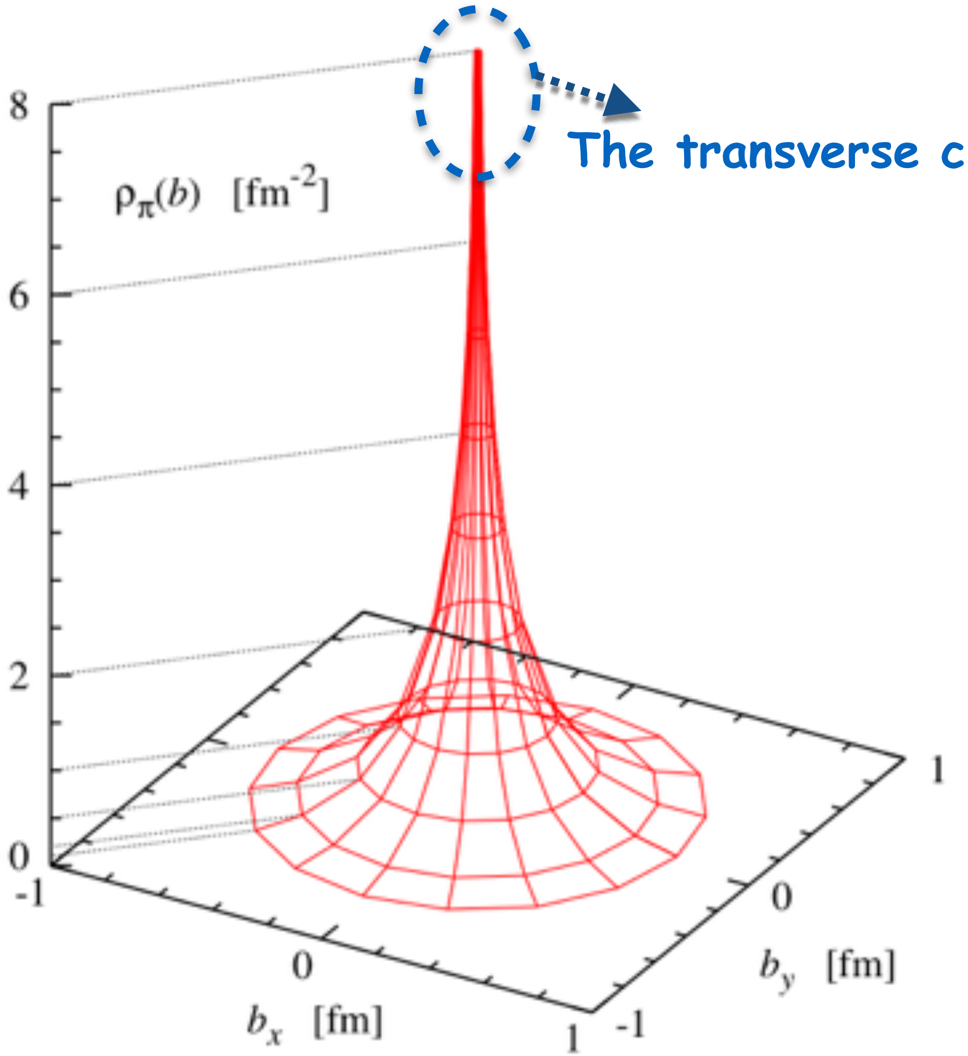
Transverse charge density of the pion



$\rho_{10}(b)$

G. Miller, A. Strikman, C. Weiss, PRD83, 013001 (2011)

Transverse charge density of the pion



The transverse charge density is divergent at b=0.

$$\rho_{10}(b)$$

G. Miller, A. Strikman, C. Weiss, PRD83, 013001 (2011)

Summary

- We also showed the energy-momentum tensor form factors of the pion. The stability of the pion beyond the chiral limit was shown to be secured by the Gell-Mann-Oakes-Renner relation, which implies that the stability of the pion is deeply related to the spontaneous breakdown of chiral symmetry and the pattern of chiral symmetry breaking.

- We also discussed the low-energy constants for the effective chiral Lagrangian in curved space, and the transverse charge densities of the pion in the transverse plane.

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!