$K \rightarrow \pi$ transition generalized form factors and transverse quark spin density from the instanton vacuum

Hyeon-Dong Son

Inha University, Korea

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in collaboration with
Seung-il Nam
Hyun-Chul Kim

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Motivation

- Kaon semileptonic decay (K_{l3} decay)

  - Provides tests of various features of the Standard Model(SM)

    e.g.) CKM matrix element $|V_{us}| = 0.2253 \pm 0.0014$ (PDG)

  - W-boson exchange: vector transition is well studied

  - Experimental results show consistency with the SM prediction

    scalar or tensor: 10^{-2} - 10^{-3} level relative to the SM

- Tensor operator

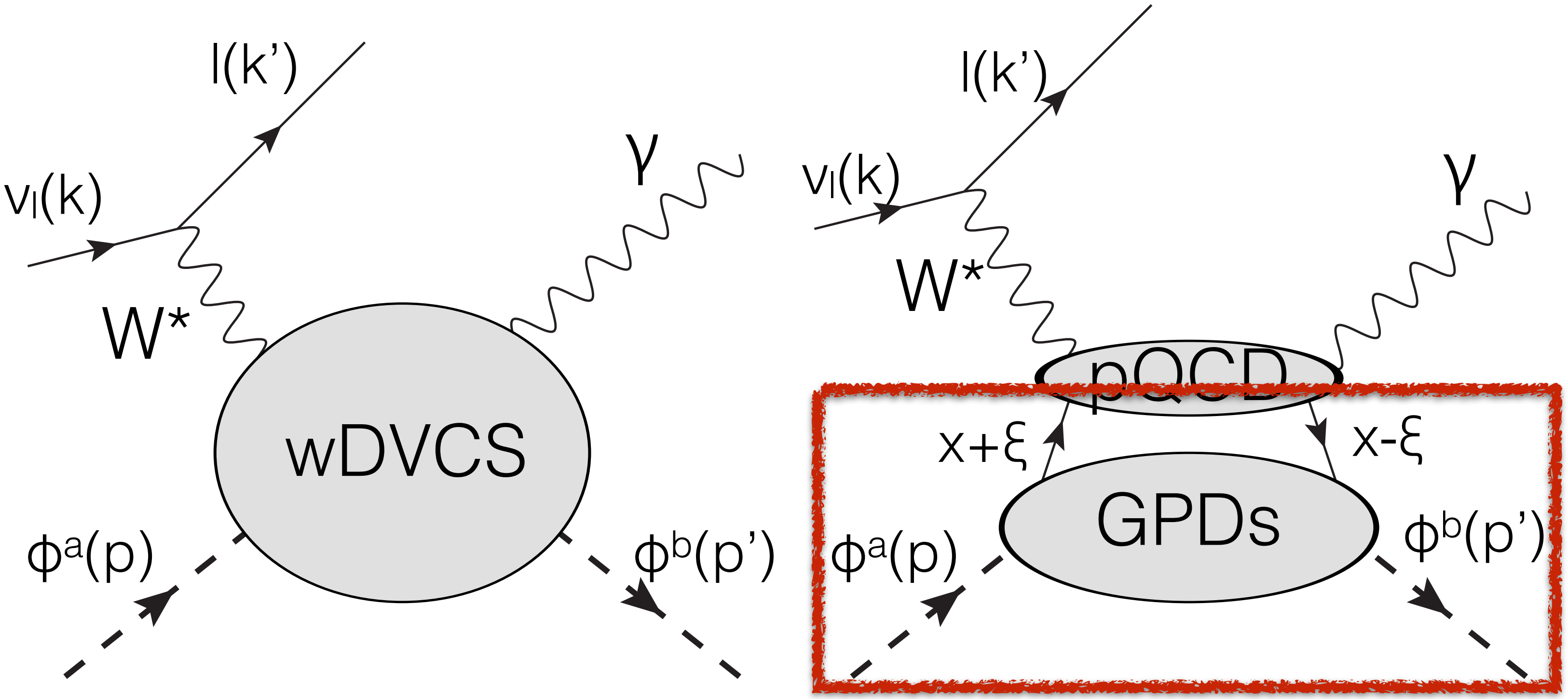
  - Test of nonstandard type interaction

  - Reveals the quark spin structure of hadrons

Motivation

- Weak generalized parton distributions
  - (semi-)Exclusive reactions for transition processes
  - Provide rich information
  - Polynomiality → GFFs
  - Transverse densities
- We study the vector and tensor $K^0 \to \pi^-$ transitions
  - Polynomiality for the wGPDs
  - Vector and tensor GFFs
  - Transverse charge and quark spin densities

Weak DVCS & GPDs



- Light cone frame

$$P^\mu = \frac{1}{2}\left(p^\mu + p'^\mu\right), \quad \Delta^\mu = p'^\mu - p^\mu, \quad t = \Delta^2 \qquad n_\pm = \frac{1}{\sqrt{2}}(1, 0, 0, \pm 1)$$

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3), \quad v_\perp = (v^1, v^2) \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

GPDs: Matrix elements

- Kaon Transition GPDs for $K^0 \to \pi^-$

$$2P^+ H^{K\pi}(x,\xi,t)$$

$$= \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^-(p')|\bar{s}(-\lambda n/2)\gamma^+[-\lambda n/2, \lambda n/2]u(\lambda n/2)|K^0(p)\rangle$$

$$\frac{P^+\Delta^j - \Delta^j P^+}{m_K} E_T^{K\pi}(x,\xi,t)$$

$$= \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^-(p')|\bar{s}(-\lambda n/2)i\sigma^{+j}[-\lambda n/2, \lambda n/2]u(\lambda n/2)|K^0(p)\rangle$$

- Polynomiality

Integrating over x with $x^n \to$
Finite polynomial of ξ with nth order Generalized Form Factors

GFFs for the Kaon Transition

- Generalized form factors for the kaon transition $K^0 \rightarrow \pi^-$

$$\langle \pi^-(p')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_n}|K^0(p)\rangle = \mathcal{S}\left[2P^\mu P^{\mu_1}\cdots P^{\mu_n}A_{n+1,0}^{K\pi}(t)\right.$$

$$+2\sum_{i=1,\mathrm{odd}}^{n} \underbrace{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_i}}_{\text{Even \# of }\Delta} P^{\mu_{i+1}}\cdots P^n A_{n+1,i+1}^{K\pi}(t)$$

$$\left.+2\sum_{i=0,\mathrm{even}}^{n} \underbrace{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_i}}_{\text{Odd \# of }\Delta} P^{\mu_{i+1}}\cdots P^n C_{n+1,i+1}^{K\pi}(t)\right]$$

$$\mathcal{O}_V^{\mu\mu_1\cdots\mu_n} = \mathcal{S}\left[\bar{s}(\gamma^\mu i\overleftrightarrow{D}^{\mu_1})\cdots(i\overleftrightarrow{D}^{\mu_n})u\right]$$

GFFs for the Kaon Transition

- Generalized form factors for the kaon transition $K^0 \to \pi^-$

$$\langle \pi^-(p') | \mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} | K^0(p) \rangle =$$

$$\mathcal{AS}\left[\frac{(P^\mu \Delta^\nu - \Delta^\mu P^\nu)}{m_K} \sum_{i=0}^{n-1} \Delta^{\mu_1} \ldots \Delta^{\mu_i} P^{\mu_{i+1}} \ldots P^{\mu_{n-1}} B_{T\,n,i}^{K\pi}(t) \right]$$

$$\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} = \mathcal{AS}\left[\bar{s}\sigma^{\mu\nu}(i\overset{\leftrightarrow}{D}^{\mu_1}) \cdots (i\overset{\leftrightarrow}{D}^{\mu_{n-1}}) u \right]$$

Polynomiality

- Mellin moments of the $K^0 \to \pi^-$ transition GPDs

$$\int dx\, x^n H^{K\pi}(x,\xi,t)$$

$$= A^{K\pi}_{n+1,0}(t) + \sum_{i=1,\mathrm{odd}}^{n} (-2\xi)^{i+1} A^{K\pi}_{n+1,i+1}(t) + \sum_{i=1,\mathrm{odd}}^{n+1} (-2\xi)^{i} C^{K\pi}_{n+1,i}(t)$$

$$\int dx\, x^n E^{K\pi}_T(x,\xi,t) = \sum_{i=0}^{n} (-2\xi)^i B^{K\pi}_{T\,n+1,i}(t)$$

- n=0

$$\boxed{\int dx\, H^{K\pi}(x,\xi,t) = A^{K\pi}_{1,0} - 2\xi C^{K\pi}_{1,1}, \qquad \int dx\, E^{K\pi}_T(x,\xi,t) = B^{K\pi}_{T\,1,0}(t)}$$

Nonlocal Chiral Quark Model

$$S_{\text{eff}} = -N_c \text{Tr} \log \left[i\not{\partial} + i\hat{m} + i\sqrt{M(i\partial)} U^{\gamma_5} \sqrt{M(i\partial)} \right]$$

- ✤ The chiral effective action derived from the instanton vacuum

$$U^{\gamma_5} = \exp \left[\frac{i\gamma_5}{f_\phi} (\lambda \cdot \phi) \right]$$

- ✤ No free parameter
  - – Average Instanton size & separation

$$\bar{\rho} \approx \frac{1}{3}\ \text{fm} \quad \bar{\text{R}} \approx 1\ \text{fm}$$

- ✤ Nonlocality
  - – Momentum-dependent dynamical quark mass
- ✤ Nicely reproduces pion properties: Fpi, EMFF
- ✤ Explicit SU(3) symmetry breaking

$$\hat{m} = \text{diag}(m_u, m_d, m_s),\ m_u = m_d = 5\ \text{MeV},\ m_s = 150\ \text{MeV}$$

[D. Diakonov, Instantons at work, arXiv:hep-ph/0212026v4]

Nonlocal Chiral Quark Model

- Momentum-dependent dynamical quark mass

$$\sqrt{M(i\partial)} = \sqrt{M_0 f(m) F^2(i\partial)}$$

$$F(k) = \frac{k}{\Lambda}\left[I_0(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda}) - I_1(\frac{k}{2\Lambda})K_0(\frac{k}{2\Lambda}) - \frac{2\Lambda}{k}I_1(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda})\right]$$

$$F_N(k) = \left(\frac{2N\Lambda^2}{2N\Lambda^2 + k^2}\right)^N \qquad \Lambda = 1/\bar{\rho} = 600 \text{ MeV}$$

- Current quark mass correction f(m)

$$f(m) = \sqrt{1 + \frac{m^2}{d^2}} - \frac{m}{d}, \quad d \approx 198 \text{ MeV}$$

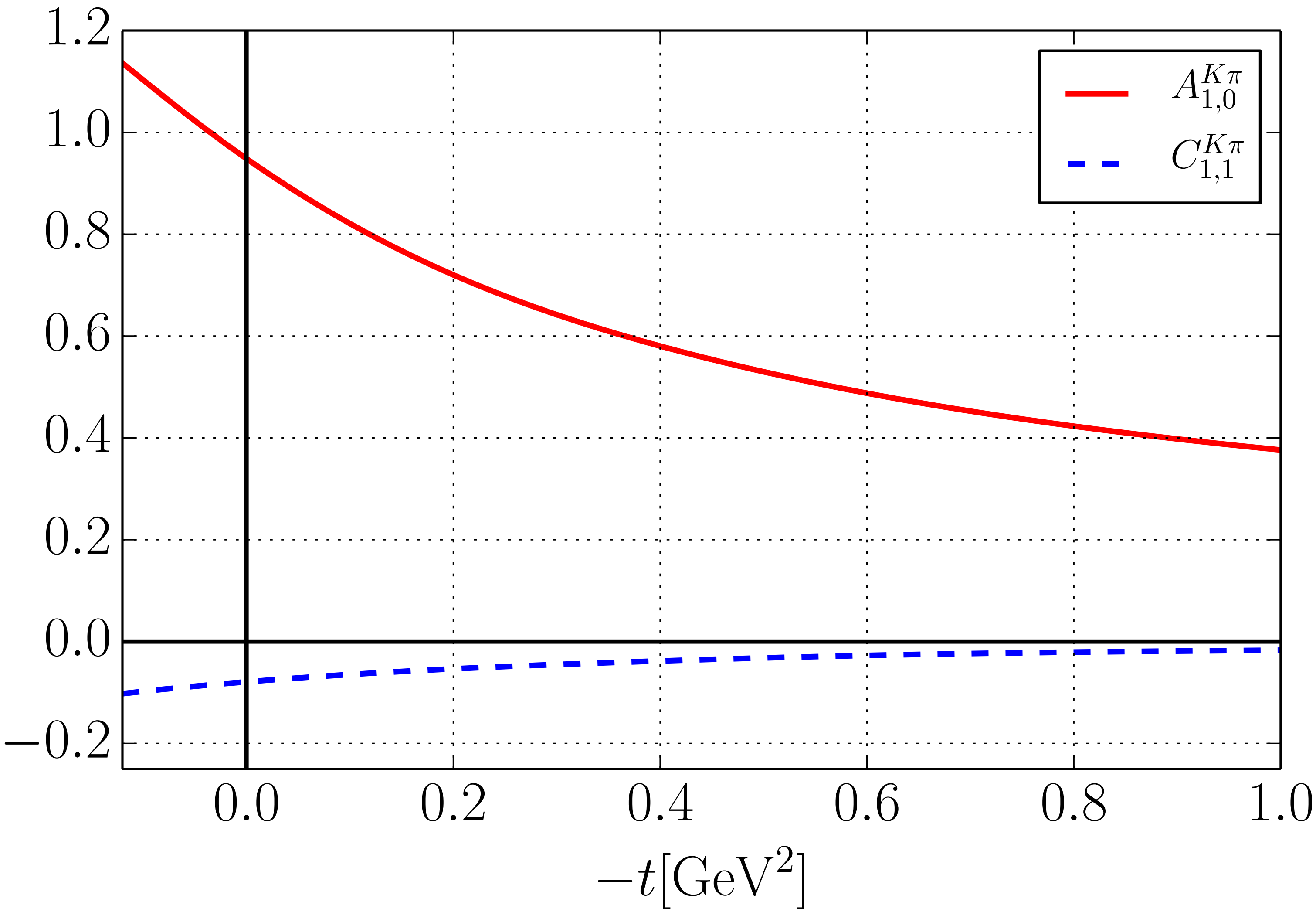
- Dynamical quark mass at k=0

$$M_0 \approx 350 \text{ MeV}$$

[M. Musakhanov Eur.Phys.J.C9,235(1999)]

Calculation of the Vector Form Factors

$$\langle \pi^-(p')|\bar{s}\gamma_\mu u|K^0(p)\rangle = 2P_\mu A_{1,0}^{K\pi}(t) + 2\Delta_\mu C_{1,1}^{K\pi}(t)$$

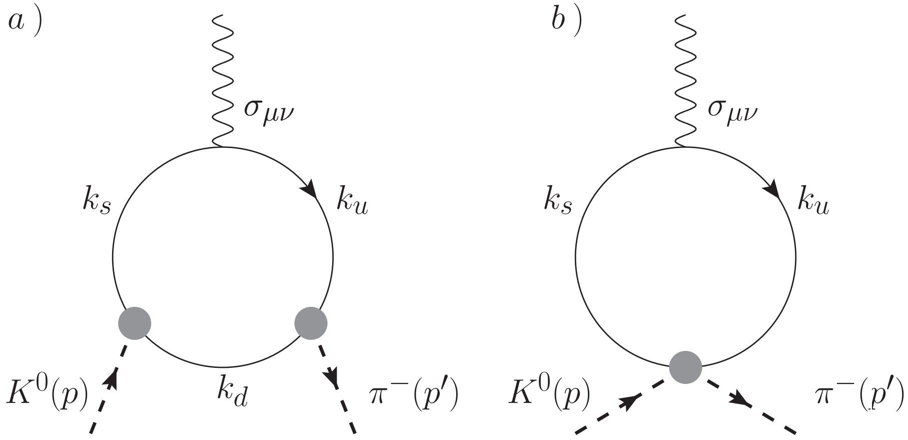


[Nam, S.-I., & Kim, H.-Ch. (2007) Phys. Rev. D, 75(9), 094011.]

Calculation of the Tensor Form Factor

$$\langle \pi^-(p')|\bar{s}\sigma_{\mu\nu}u|K^0(p)\rangle =$$

$$-\frac{8N_c}{F_\pi F_K}\int\frac{d^4l}{(2\pi)^4}\left[\frac{\sqrt{M_d^2 M_u M_s}}{G_u G_d G_s}\epsilon^{ijk}k_{i\mu}k_{j\nu}\bar{M}_{kf_k} - \frac{\sqrt{M_u M_s}}{2G_u G_s}\left(k_{s\mu}k_{u\nu} - k_{s\nu}k_{u\mu}\right)\right]$$



$$G_f = k_f^2 + \bar{M}_f^2$$

$$\bar{M}_f = m_f + M(k_f^2, m_f)$$

$$k_u = l + \frac{p}{2} + \frac{\Delta}{2} \qquad k_d = l - \frac{p}{2} - \frac{\Delta}{2}$$

$$k_s = l + \frac{p}{2} - \frac{\Delta}{2}$$

Pseudoscalar meson decay constants and masses

$$f_\pi = 93 \text{ MeV}, \quad f_K = 113 \text{ MeV}$$
$$m_\pi = 140 \text{ MeV}, \quad m_K = 495 \text{ MeV}$$

QCD RG Evolution for the tensor form factor

✤ Next-to-leading order

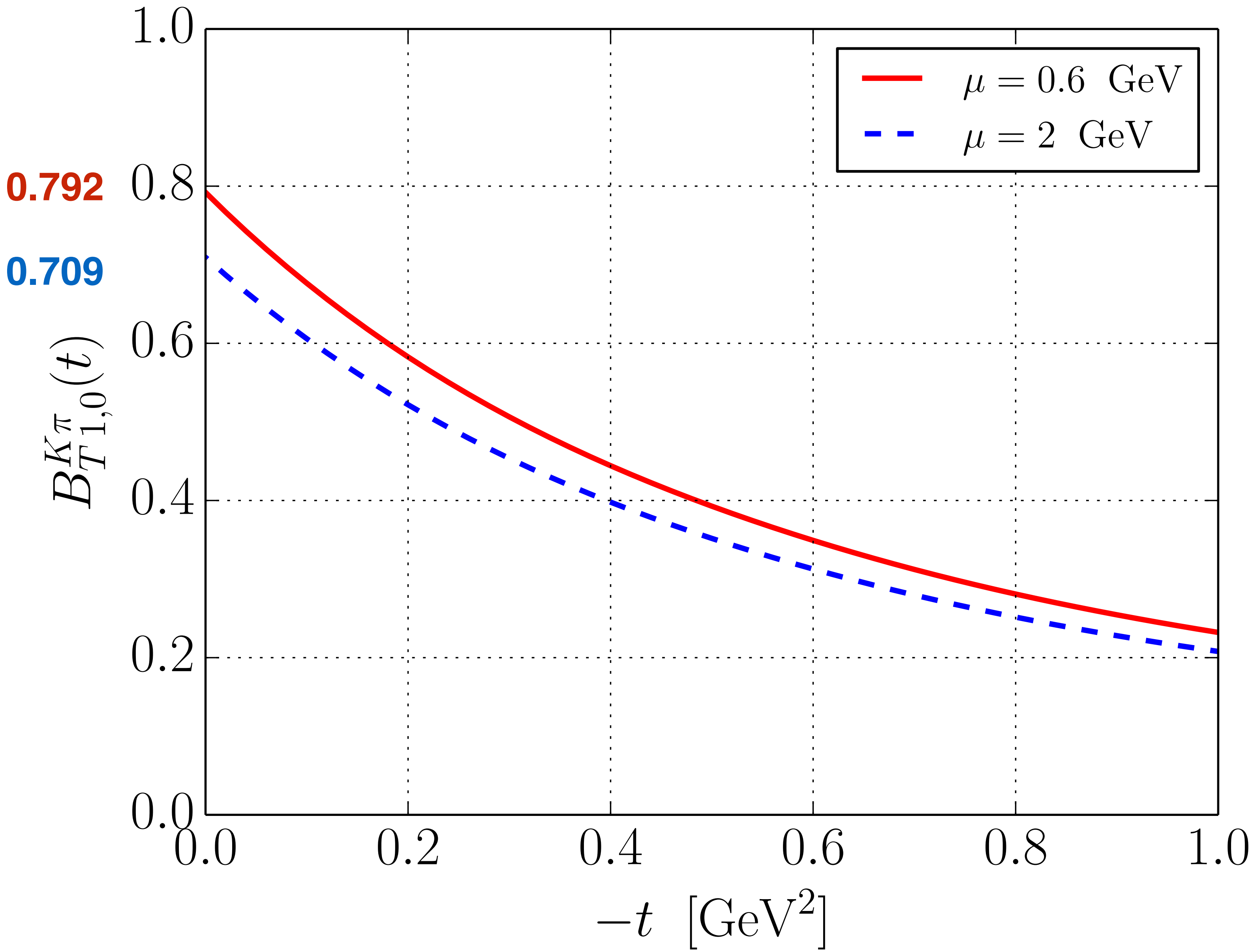
$$B^{K\pi}_{T\,1,0}(\mu^2) = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_i^2)}\right)^{4/27} \left[1 - \frac{337}{486\pi}\left(\alpha_s(\mu_i^2) - \alpha_s(\mu^2)\right)\right] B^{K\pi}_{T\,1,0}(\mu_1)$$

$$\alpha_s^{\mathrm{NLO}}(\mu^2) = \frac{4\pi}{9\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}\left[1 - \frac{64}{81}\frac{\ln\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}{\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}\right]$$

$\Lambda_{\mathrm{QCD}} = 0.25\ \mathrm{GeV},\ N_f = 3$

[Glück et al. Zeits.Für.Phys. C.67, 433
Barone et al. Phys.Repts., 359, 1.]

Numerical Results



Comparison to Lattice Result

- Present work @ μ = 2 GeV

$$f_T^{K\pi}(t) = \frac{m_K + m_\pi}{2m_K} B_{T1,0}^{K\pi}(t)$$

$$B_{T1,0}^{K\pi}(0) = 0.71 \rightarrow \boxed{f_T^{K\pi}(0) = 0.45}$$

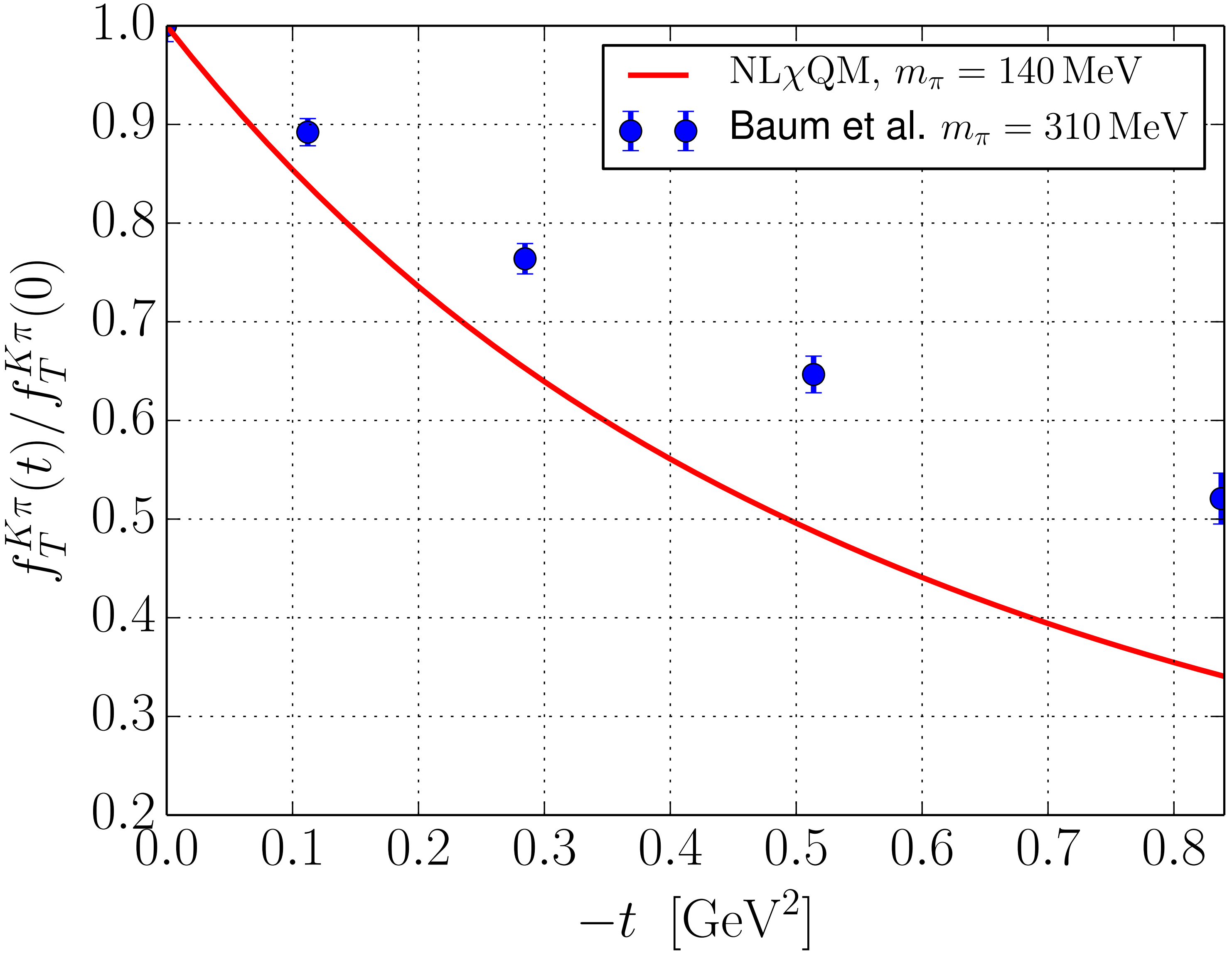
- I. Baum et al, μ = 2 GeV

$$\langle \pi^0 | \bar{s}\sigma^{\mu\nu} d | K^0 \rangle = (p_\pi^\mu p_K^\nu - p_\pi^\nu p_K^\mu) \frac{\sqrt{2} f_T^{K\pi}(q^2)}{M_K + M_\pi}$$

$$\boxed{f_T^{K\pi}(0) = 0.417\ (14_{\text{stat}})\ (5_{\text{syst}}) = 0.417\ (15)}$$

(Extrapolated to Physical meson masses)

Comparison to Lattice Result



Transverse Density

is this slide necessary?

- P-pole parametrization

$F(t) = \frac{F(0)}{\left(1 - \frac{t}{pM^2}\right)^p}$	$A_{1,0}^{K\pi}(t)$	$B_{T1,0}^{K\pi}(t)$
p	1.31	2.2
M [GeV]	0.85	0.78
t=0	0.95	0.71

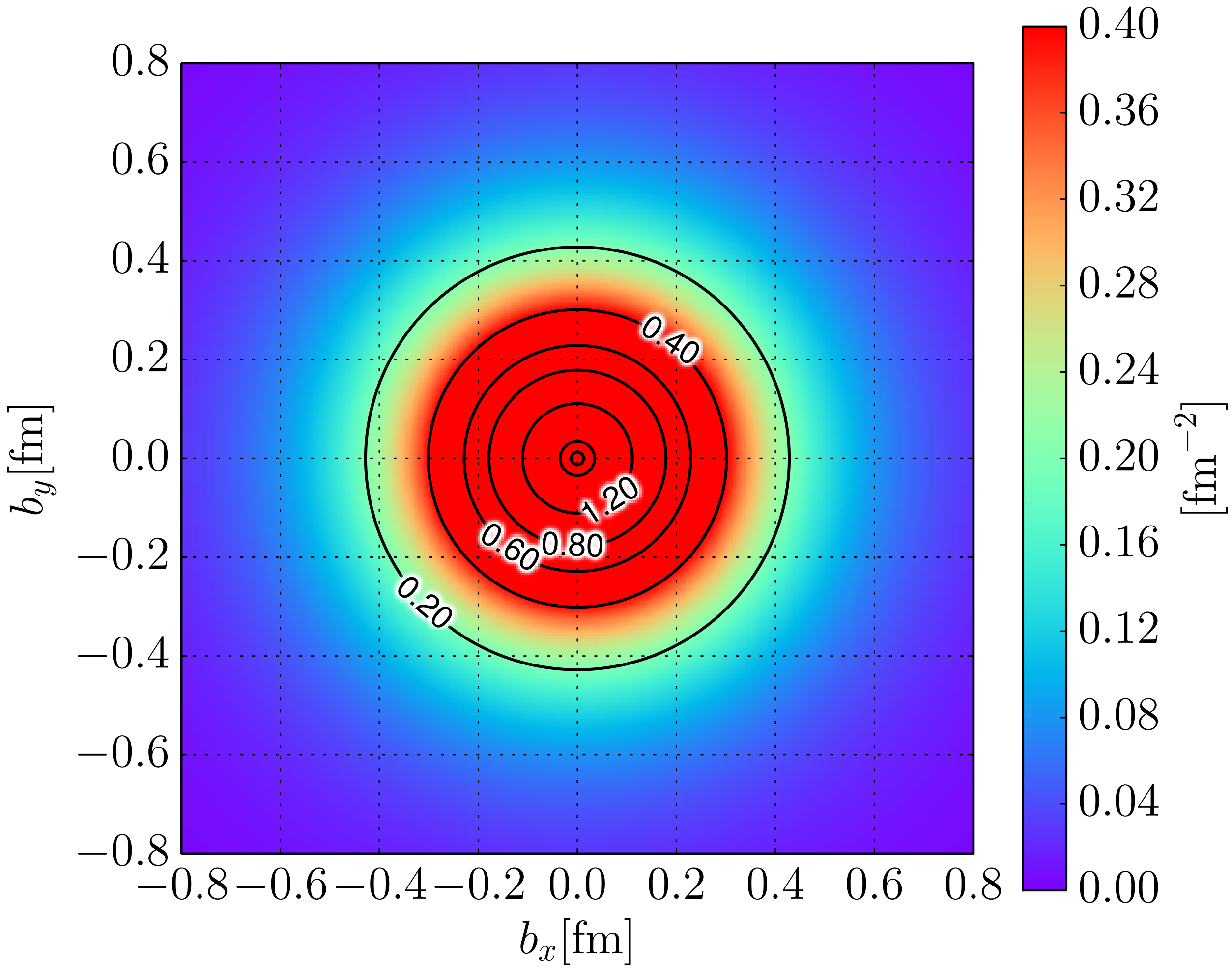
- Fourier transform into 2D transverse plane

$$\mathcal{F}_{1,0}^{K\pi}(b_\perp) = \frac{1}{(2\pi)^2} \int d^2\Delta^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}} \mathcal{F}_{1,0}^{K\pi}(t) = \frac{1}{2\pi} \int_0^\infty Q dQ J_0(bQ) \mathcal{F}_{1,0}^{K\pi}(Q^2)$$

Transverse Charge Density

- Transverse charge density for the kaon transition $K^0 \to \pi^-$

$$\rho_1^{K\pi} = \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta} \int dx\, H^{K\pi}(x, \xi = 0, t) \;\; = \frac{1}{(2\pi)^2} \int d^2\Delta e^{-i\mathbf{b}_\perp \cdot \Delta} A_{1,0}^{K\pi}(t)$$



Transverse Quark Spin Density

- Quarks with definite transverse polarization **s**

$$\frac{1}{2}\bar{\psi}\left[\gamma^{+} - s^{j} i\sigma^{+j}\gamma_5\right]\psi \qquad\qquad \sigma^{\mu\nu}\gamma_5 = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} i\sigma_{\alpha\beta}$$

- Transverse quark spin density, ξ=0

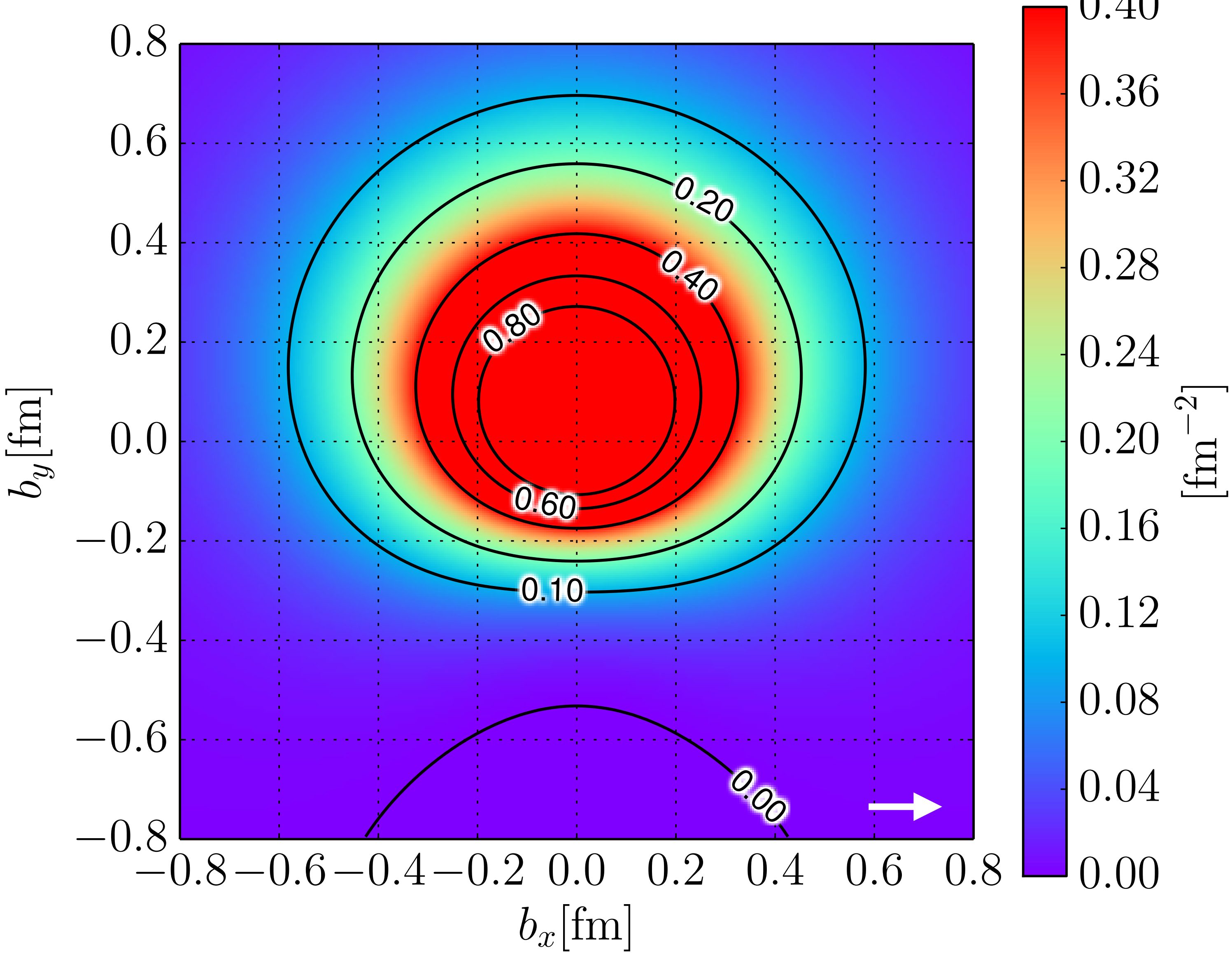
$$\rho_1^{K\pi}(b, \mathbf{s}_\perp) = \frac{1}{2}\left[A_{1,0}^{K\pi}(b^2) - \frac{s_\perp^i \epsilon^{ij} b^j}{m_K}\frac{\partial B_{T1,0}^{K\pi}(b^2)}{\partial b^2}\right]$$

How the quark with polarized spin is distributed
in the transverse plane during the K-π transition process

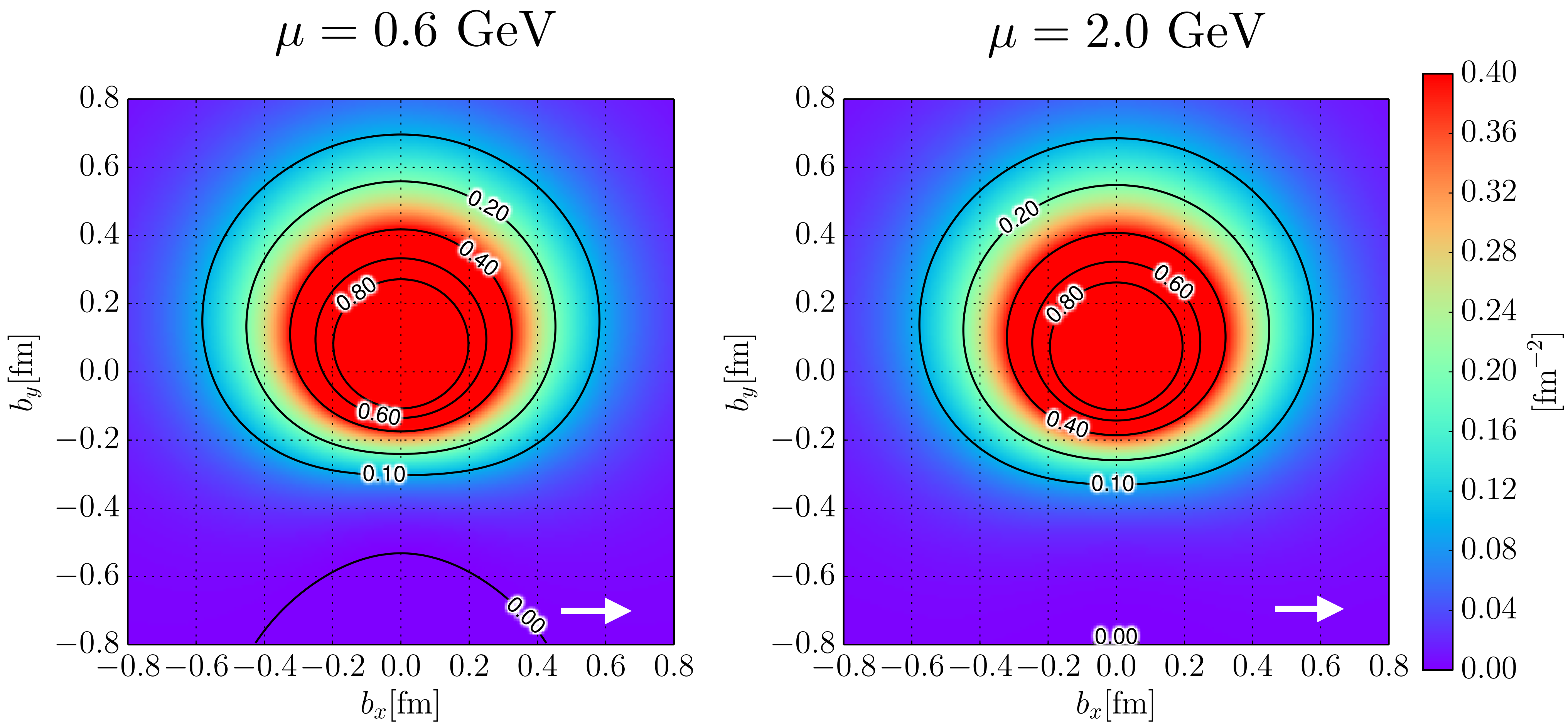
[M. Diehl & Ph. Hägler, Eur. Phys. J. C 44, 87–101 (2005)]

Transverse Quark Spin Density

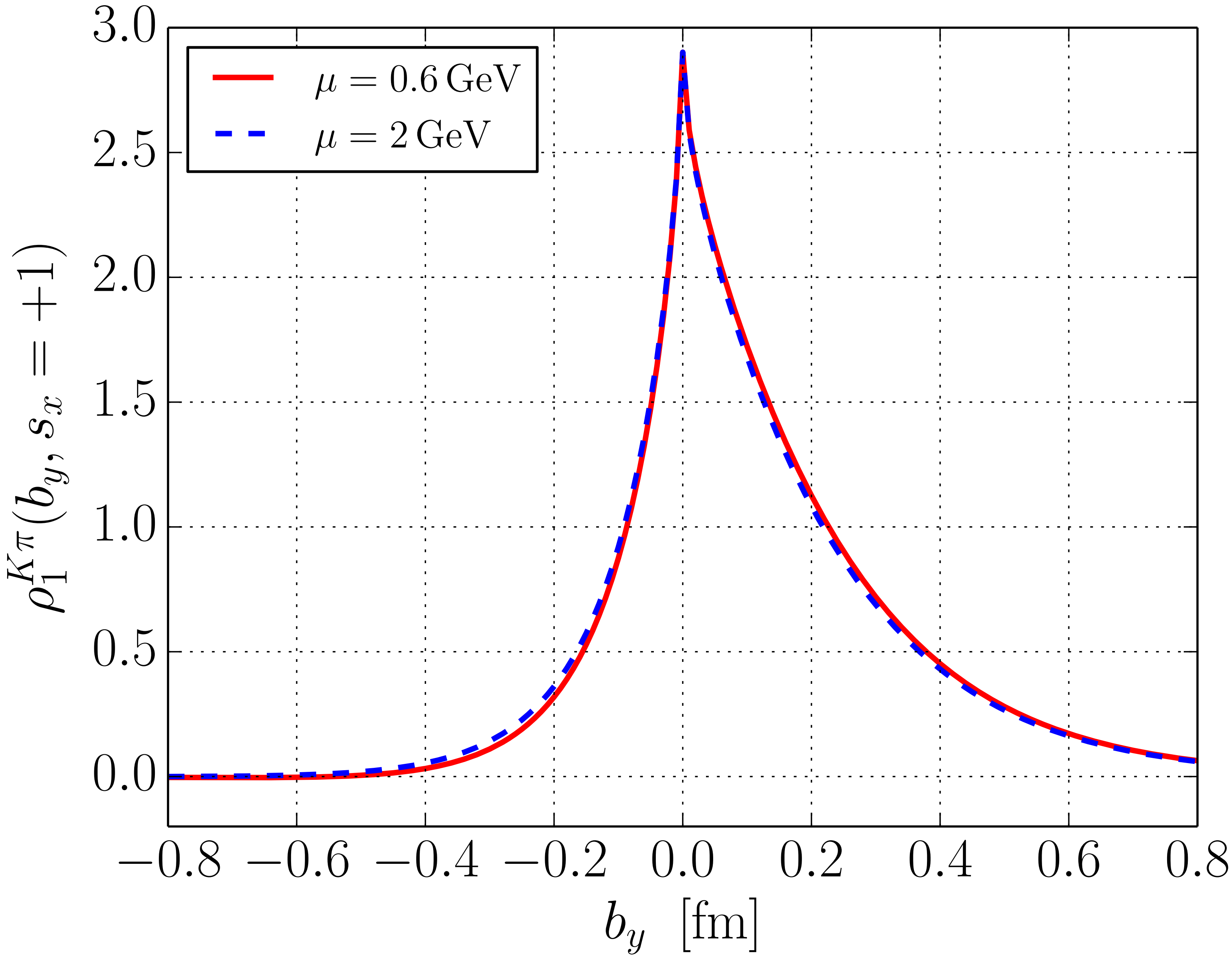
$$\rho_1^{K\pi}(b, \mathbf{s}_\perp) = \frac{1}{2}\left[A_{1,0}^{K\pi}(b^2) - \frac{s_\perp^i \epsilon^{ij} b^j}{m_K}\frac{\partial B_{T1,0}^{K\pi}(b^2)}{\partial b^2}\right]$$



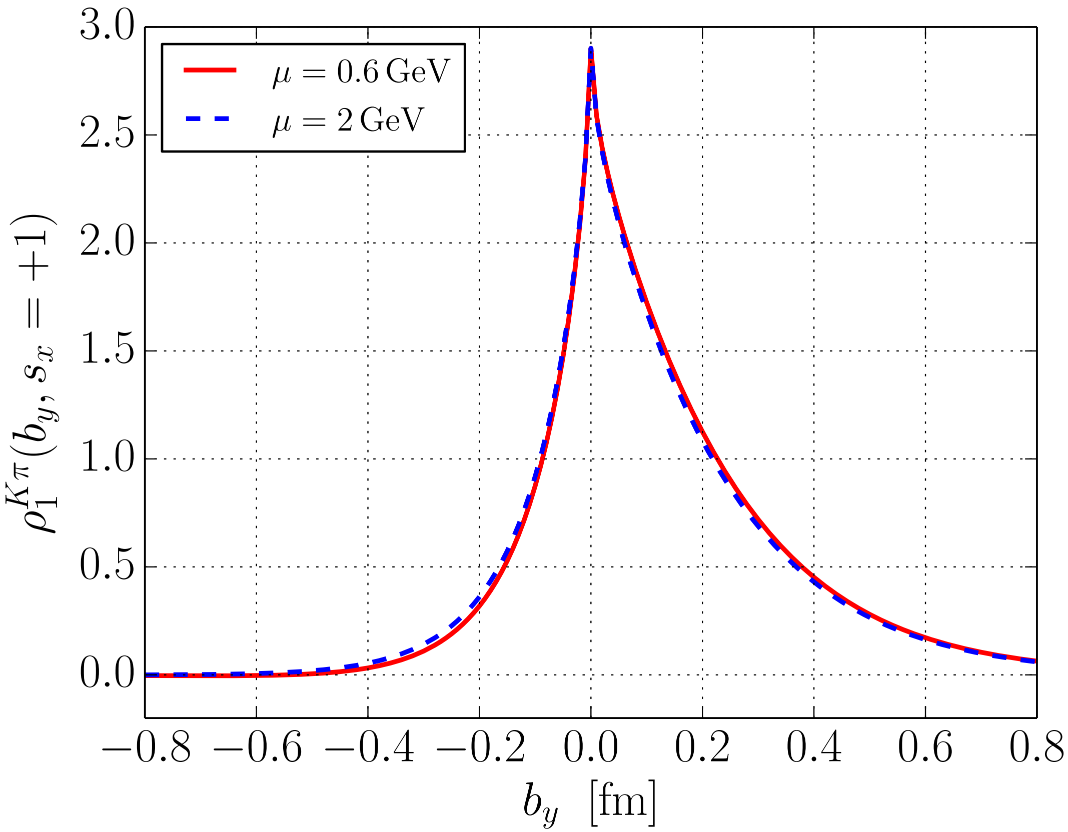
Transverse Quark Spin Density



Transverse Quark Spin Density



Transverse Quark Spin Density



- Average Shift

$$\langle b_y \rangle^{K\pi} = \frac{\int d^2b\, b_y\, \rho_1^{K\pi}(b, s_\perp)}{\int d^2b\, \rho_1^{K\pi}(b, s_\perp)} = \frac{1}{2m_K} \frac{B_{T\,1,0}^{K\pi}(0)}{A_{1,0}^{K\pi}(0)}$$

$$= (0.17, 0.15)\ \mathrm{fm}$$

Summary & Outlook

- Formulation of the wGPDs & GFFs for the K → π transition
- K → π generalized transition form factors (n=0)
- Good agreement with the lattice result
- Distorted quark spin structure when the quark spin is polarized
- Further studies on the wGPDs & GFFs

Thank you very much!