Nucleon structure functions at small x via holographic Pomeron exchange

Akira Watanabe (Academia Sinica)
Katsuhiko Suzuki (Tokyo University of Science)

Based on:
AW, K. Suzuki, Phys. Rev. D86, 035011 (2012)
AW, K. Suzuki, Phys. Rev. D89, 115015 (2014)

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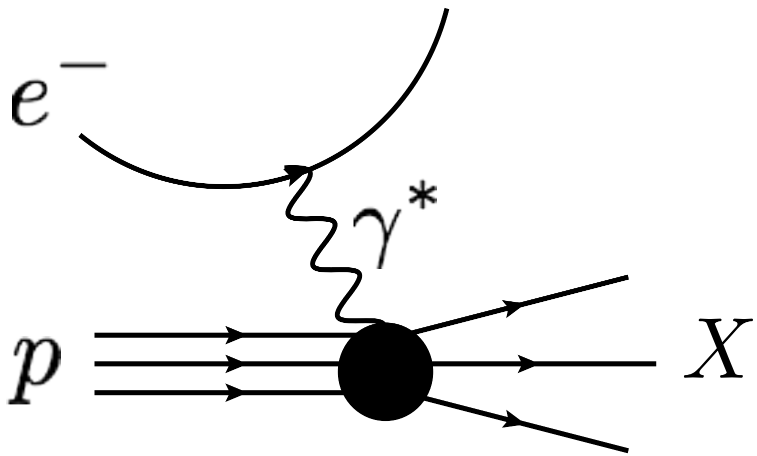
DIS structure functions

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[\left\{1+(1-y)^2\right\}F_2\left(x,Q^2\right)-y^2F_L\left(x,Q^2\right)\right]$$



$$F_2\left(x,Q^2\right)=\frac{Q^2}{4\pi^2\alpha}\sigma_{tot}\left(x,Q^2\right)$$

$$F_L\left(x,Q^2\right)=\frac{Q^2}{4\pi^2\alpha}\sigma_L\left(x,Q^2\right)$$

- Structure functions are physical quantities which have information on the internal structure of hadrons.
- They depend on two kinematic variables, Bjorken-*x* and photon 4-momentum squared Q^2.

Longitudinal structure function F_L

- In the quark-parton model, F_2 can be written as:

$$F_2 = x\sum_q e_q^2 q_i(x) \qquad F_L = 0$$

- F_L is expressed, for example by Altarelli-Martinelli equation, as:

Altarelli-Martinelli (1978)

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3}\left[\frac{8}{3}F_2(y,Q^2) + 4\sum_q e_q^2\left(1-\frac{x}{y}\right) yg(y,Q^2)\right]$$

At small x, the second term becomes dominant and F_L is approximately expressed by

Cooper-Sarkar et al. (1988)

$$F_L \approx 0.3\frac{4\alpha_s}{3\pi} xg(2.5x,Q^2)$$

Hence, studying F_L is related to the investigations on the gluon distribution and the higher order contribution in QCD.

Holographic QCD

- A generic name of QCD-like theories constructed based on the AdS/CFT correspondence.
- Holographic QCD has a potential to be a powerful tool for analysis on hadron physics.

type IIB supergravity theory on $S^5 \times AdS_5$	⟺	strong coupling 4D N=4 supersymmetric Yang-Mills (SYM) theory
supergravity theory (classical theory) on AdS_5	⟺	usual 4D QCD at strong coupling

Motivation

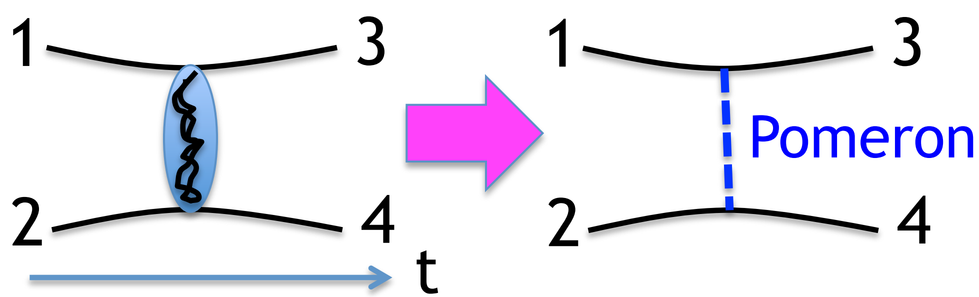
- Our understandings of hadron structures at small x are poor. Further studies are needed.
- So far, holographic technique has been considered to be suitable only for qualitative studies, because the original gauge theory is the large-N theory. There is a fundamental uncertainty of 30%.
- However, if it works well in nonperturbative regions as an effective model and predicts some unknown quantities, it will be a powerful theoretical tool.
- As our first trial, we try to study DIS at small x in the framework of holographic QCD, considering a more realistic model.

Pomeron exchange picture

- A description of high energy scattering before QCD
- Pomeron: a color singlet gluonic object
- Total cross sections for the high energy two-to-two elastic scattering can be well reproduced by this picture

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$



can be expressed by a single parameter (Pomeron intercept)

Therefore, F_2 structure function can be written as

$$F_2(x, Q^2) \sim x^{1-\alpha_0}$$

← depends on the scale in fact

$\alpha_0 \rightarrow \alpha_0(Q^2)$

(this is effective only in the small *x* region)

Holographic description of structure functions

- **Polchinski-Strassler (2003)**
- **Brower-Polchinski-Strassler-Tan (2007)**
- **Brower-Djuric-Sarcevic-Tan (2010)**

→ derived Pomeron exchange kernel

→ studied nucleon structure functions

$$\mathcal{A}(s,t) = 2is\int d^2b e^{iq\cdot b}\int dz dz' P_{13}(z) P_{24}(z')\left\{1 - e^{i\chi(s,b,z,z')}\right\}$$

$$F_2\left(x,Q^2\right) = \frac{Q^2}{2\pi^2}\int dz dz' P_{13}\left(z,Q^2\right) P_{24}\left(z'\right) \mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$

z and z' : 5th coordinate

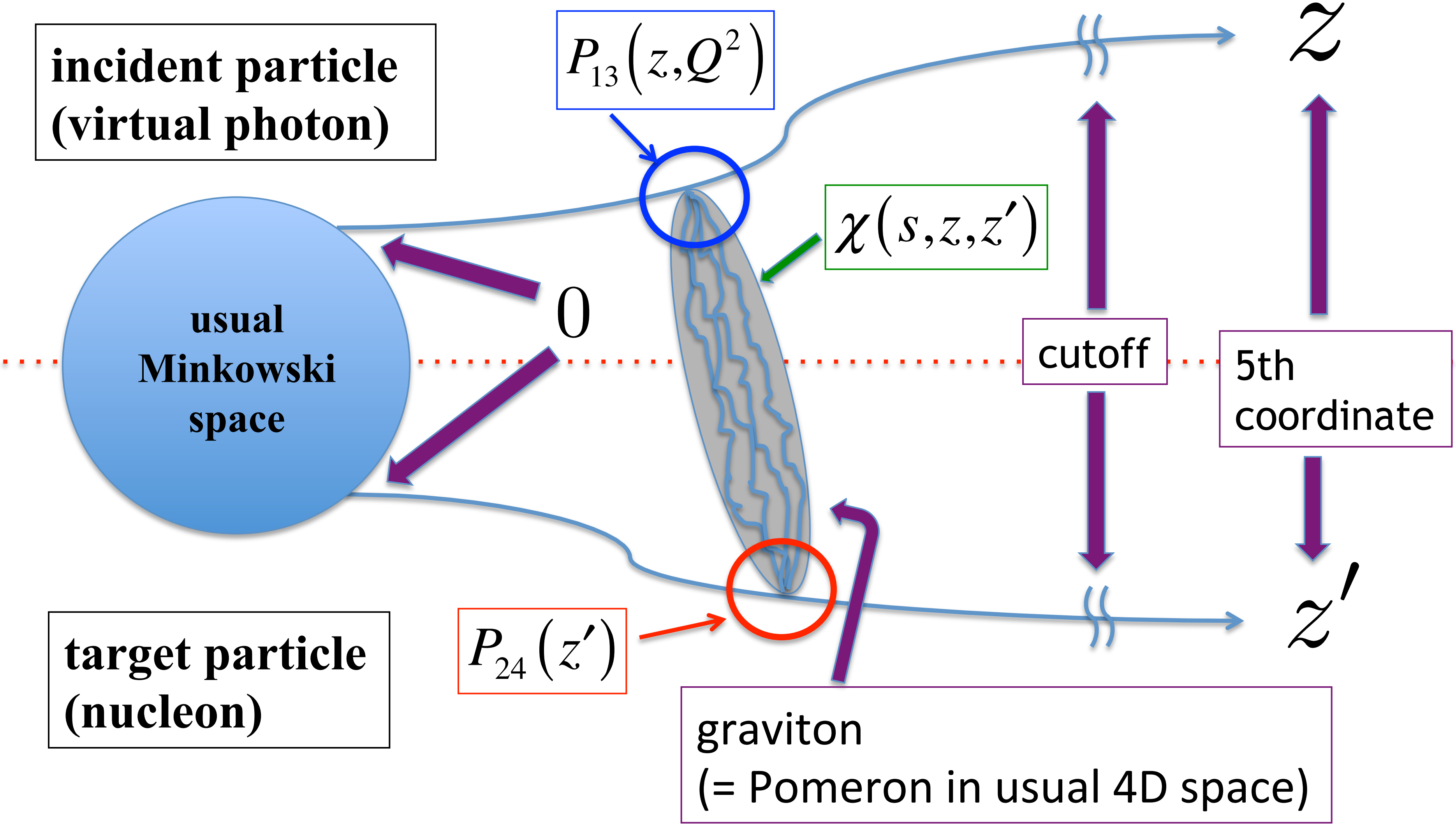
χ : Pomeron exchange kernel in the AdS space

$P_{13}(z,Q^2)$: incident particle (virtual photon, 4-momentum Q)

$P_{24}(z')$: target particle

} overlap functions (density distributions in the AdS space)

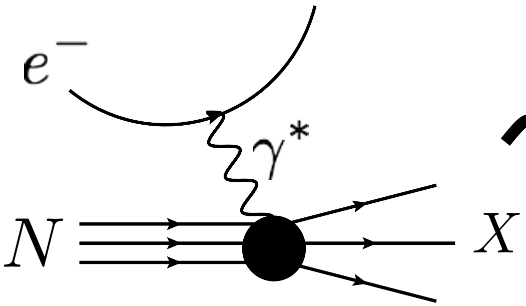
5D background spacetime (AdS_5)



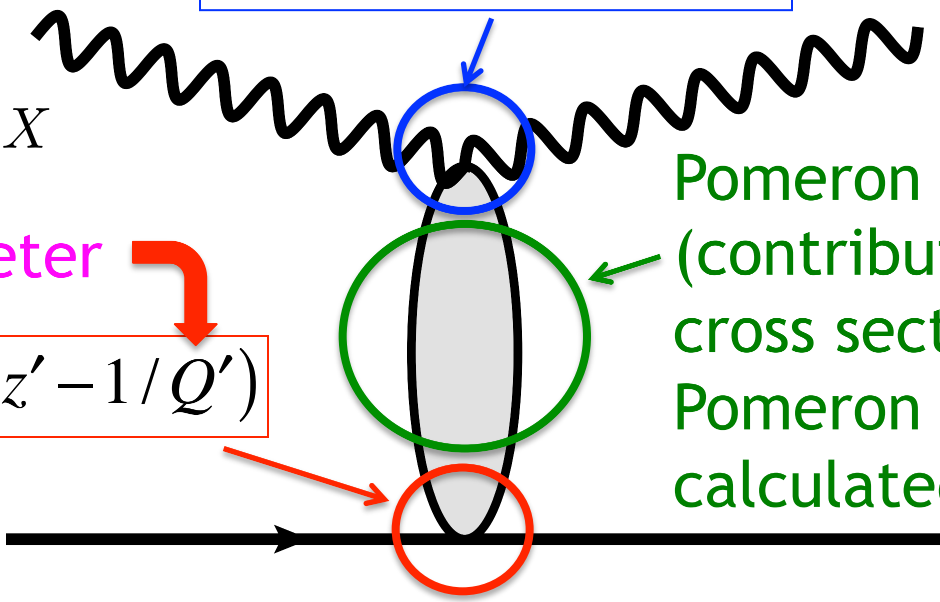
A preceding study

Brower-Djuric-Sarcevic-Tan (2010)

$$F_2\left(x,Q^2\right)=\frac{Q^2}{2\pi^2}\int dz dz' P_{13}\left(z,Q^2\right)P_{24}\left(z'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$



$$P_{13}(z,Q^2)\approx\delta\left(z-1/Q\right)$$



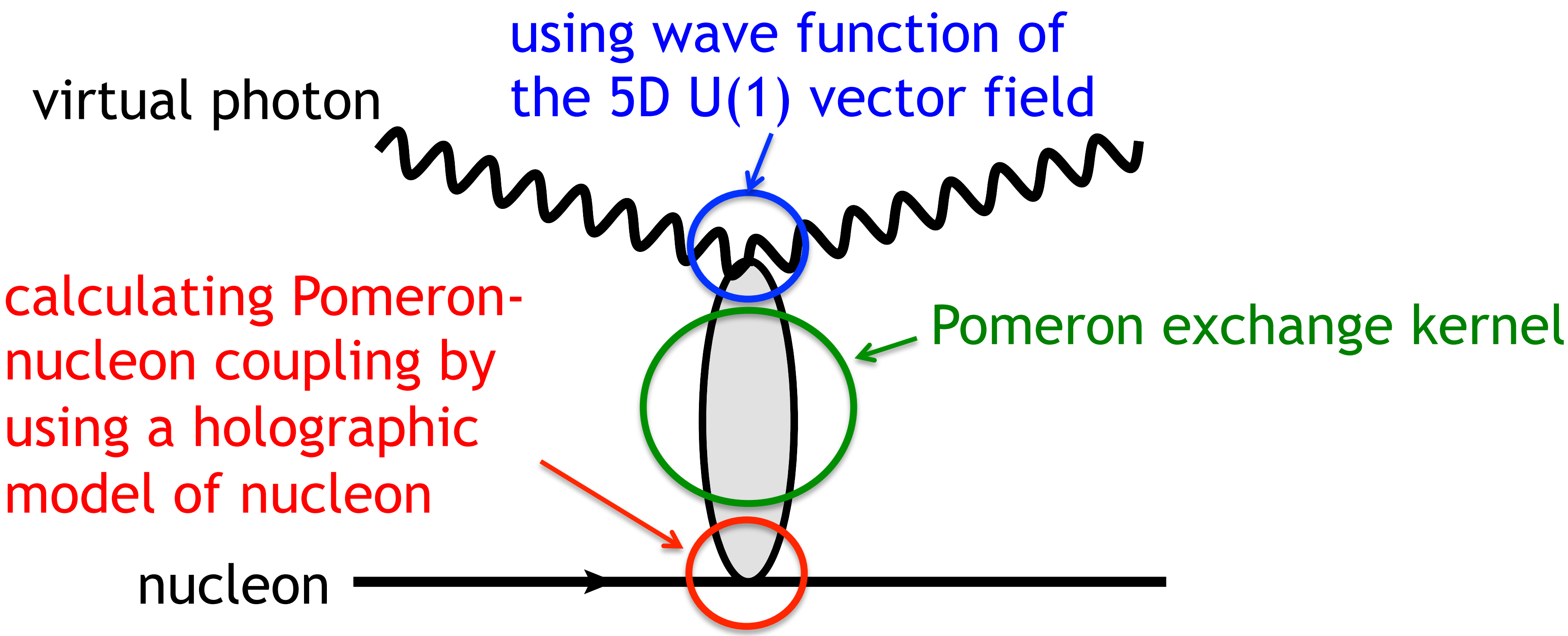
Pomeron exchange kernel (contribution to the total cross section for the Pomeron exchange, calculated by AdS/CFT)

parameter

$$P_{24}(z')\approx\delta\left(z'-1/Q'\right)$$

- Incident and target particles are simply replaced with delta functions.

Our model setup



- This is a more consistent description of structure functions.
- In this model, we can consider structure functions of various hadrons and longitudinal structure functions, which can not be considered in the preceding model setup.

Pomeron exchange kernel

Brower-Polchinski-Strassler-Tan (2007)
Brower-Strassler-Tan (2009)

$$F_i\left(x,Q^2\right)=\frac{g_0^2\rho^{3/2}Q^2}{32\pi^{5/2}}\int dz\,dz' P_{13}^{(i)}\left(z,Q^2\right)P_{24}\left(z'\right)\left(zz'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$

$i = 2$ or L

$$\mathrm{Im}\left[\chi_c\left(s,z,z'\right)\right]\equiv e^{(1-\rho)\tau}e^{-\frac{\log^2 z/z'}{\rho\tau}}/\tau^{1/2}$$

$$\tau=\log\left(\rho zz's/2\right)$$

$$\mathrm{Im}\left[\chi_{\mathrm{mod}}\left(s,z,z'\right)\right]\equiv\mathrm{Im}\left[\chi_c\left(s,z,z'\right)\right]+\mathcal{F}\left(z,z',\tau\right)\mathrm{Im}\left[\chi_c\left(s,z,z_0^2/z'\right)\right]$$

$$\mathcal{F}\left(z,z',\tau\right)=1-2\sqrt{\rho\pi\tau}\,e^{\eta^2}\mathrm{erfc}\left(\eta\right)$$

$$\eta=\left(-\log\frac{zz'}{z_0^2}+\rho\tau\right)/\sqrt{\rho\tau}$$

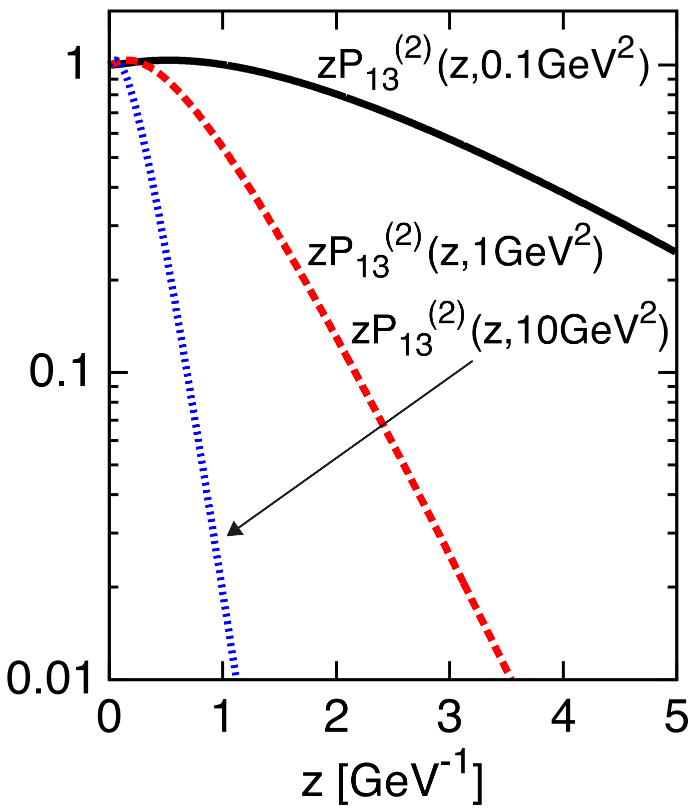
confinement effect

3 adjustable parameters : $\rho,\ g_0^2,\ z_0$

Density distribution of the probe photon

Polchinski-Strassler (2003)

- As a density distribution of the virtual photon, we use wave function of the 5D U(1) vector field



$$P_{13}^{(2)}(z,Q^2) = Q^2 z\left(K_0^2(Qz) + K_1^2(Qz)\right)$$

(to calculate F_2)

$$P_{13}^{(L)}(z,Q^2) = Q^2 z K_0^2(Qz)$$

(to calculate F_L)

$P_{13}^{(2)}$ are localized at the origin with Q^2 increasing (the behavior of $P_{13}^{(L)}$ is similar)

Density distribution for the target particle (1)

Abidin-Carlson (2009)

The matrix element of the energy momentum tensor in respect to spin 1/2 particle:

$$\left\langle p_2, s_2 \middle| T^{\mu\nu}(0) \middle| p_1, s_1 \right\rangle = u(p_2, s_2)\left(A(t)\gamma^{(\mu}p^{\nu)} + B(t)\frac{ip^{(\mu}\sigma^{\nu)\alpha}q_\alpha}{2m} + C(t)\frac{q^\mu q^\nu - q^2\eta^{\mu\nu}}{m} \right)u(p_1, s_1)$$

To calculate A(t) etc. with the holographic model of nucleons,

$$S_F = \int d^5x\sqrt{g}e^{-\kappa^2 z^2}\left(\frac{i}{2}\bar{\Psi}e_A^N\Gamma^A D_N\Psi - \frac{i}{2}(D_N\Psi)^\dagger\Gamma^0 e_A^N\Gamma^A\Psi - (M + \kappa^2 z^2)\bar{\Psi}\Psi \right)$$

we introduce the metric perturbation, $\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$, in the 5D classical action, and pick up the hψψ terms.
By comparing the Lorentz structure of them, we can obtain the form factors. (in this case, only A(t) remains)

Density distribution for the target particle (2)

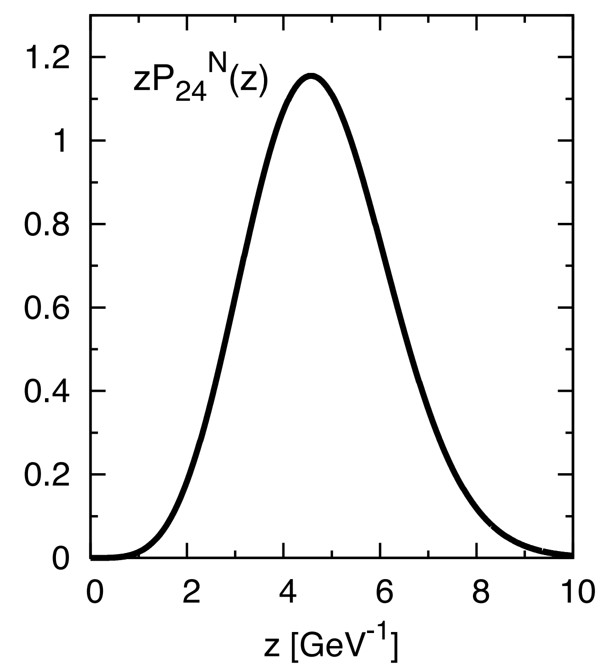
Finally, one can obtain the density distribution for the target nucleon:

$$P_{24}(z') = \frac{e^{-\kappa^2 z^2}}{2z'^3}\left(\psi_L^2(z') + \psi_R^2(z')\right)$$

$$\int dz' P_{24}(z') = 1$$

where $\psi_{L,R}$ are 5D wave functions describing a nucleon as a 5D Dirac fermion with chiral symmetry breaking.

- The peak position of P_{24} is in the large z region, which is obviously different from P_{13}.

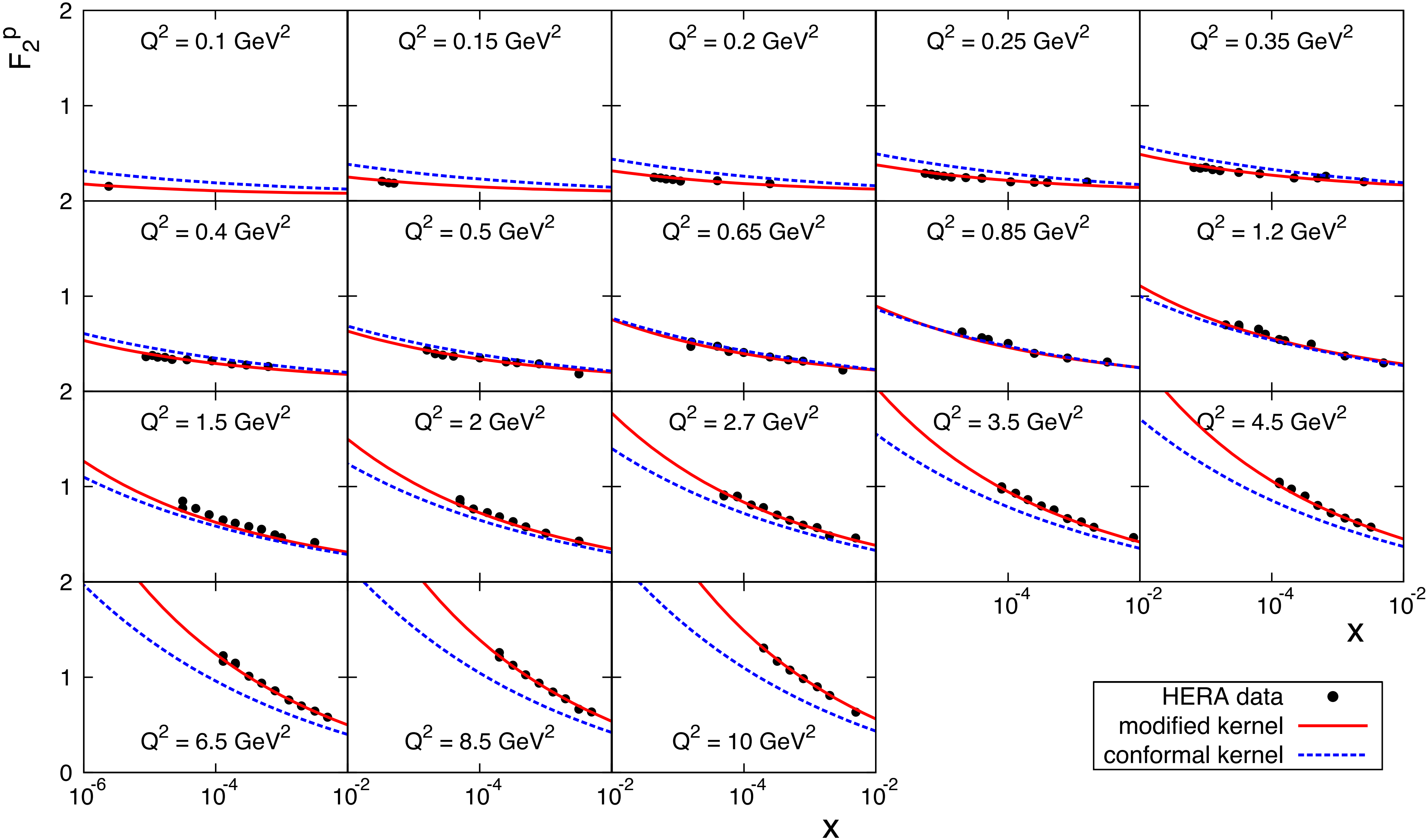


Fixing parameters

- There are 3 adjustable parameters (ρ, g_0, and z_0)
- They are fixed with the experimental data for F_2^p measured at HERA

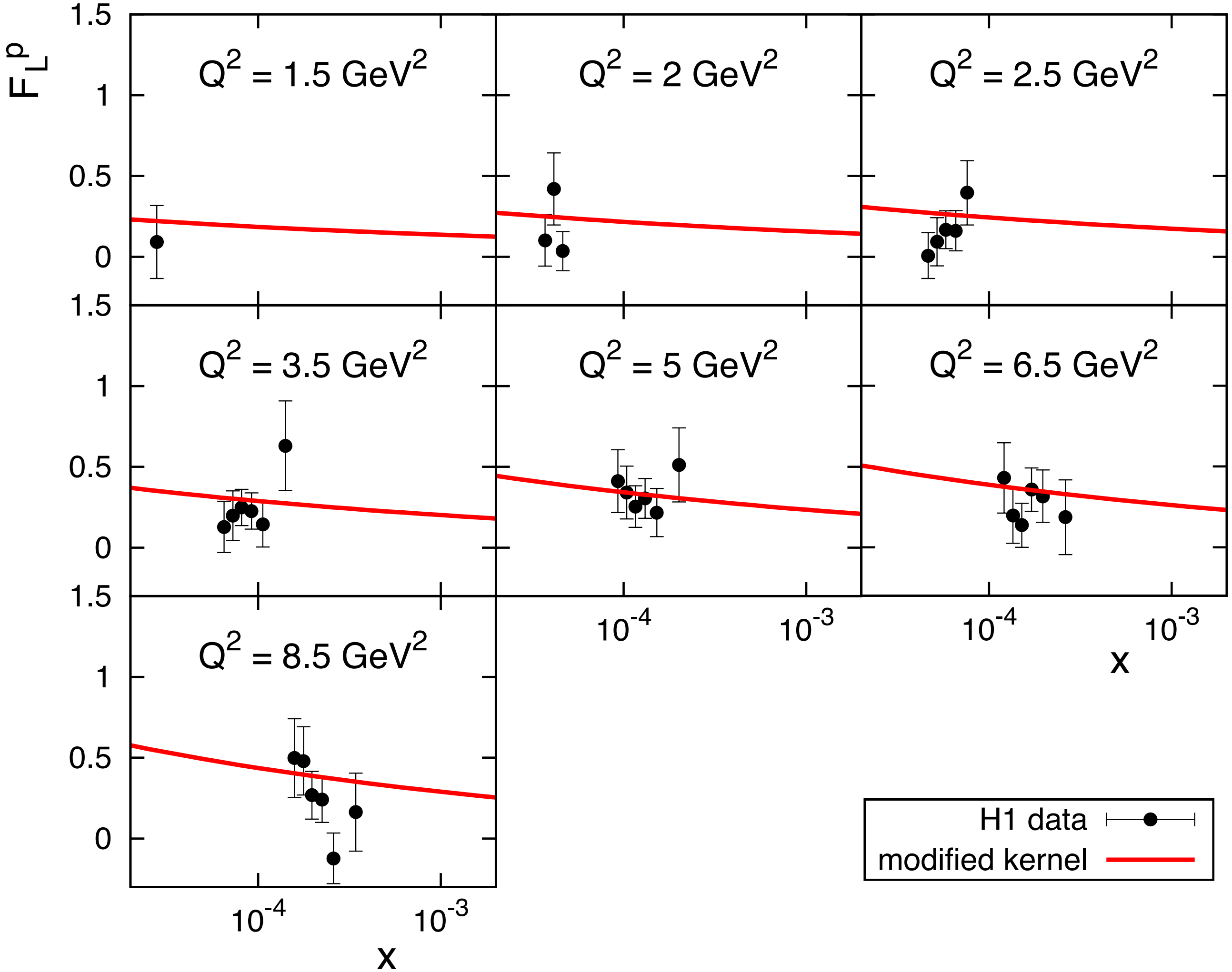
ZEUS collaboration (1999)
H1 and ZEUS collaborations (2010)

Proton structure function



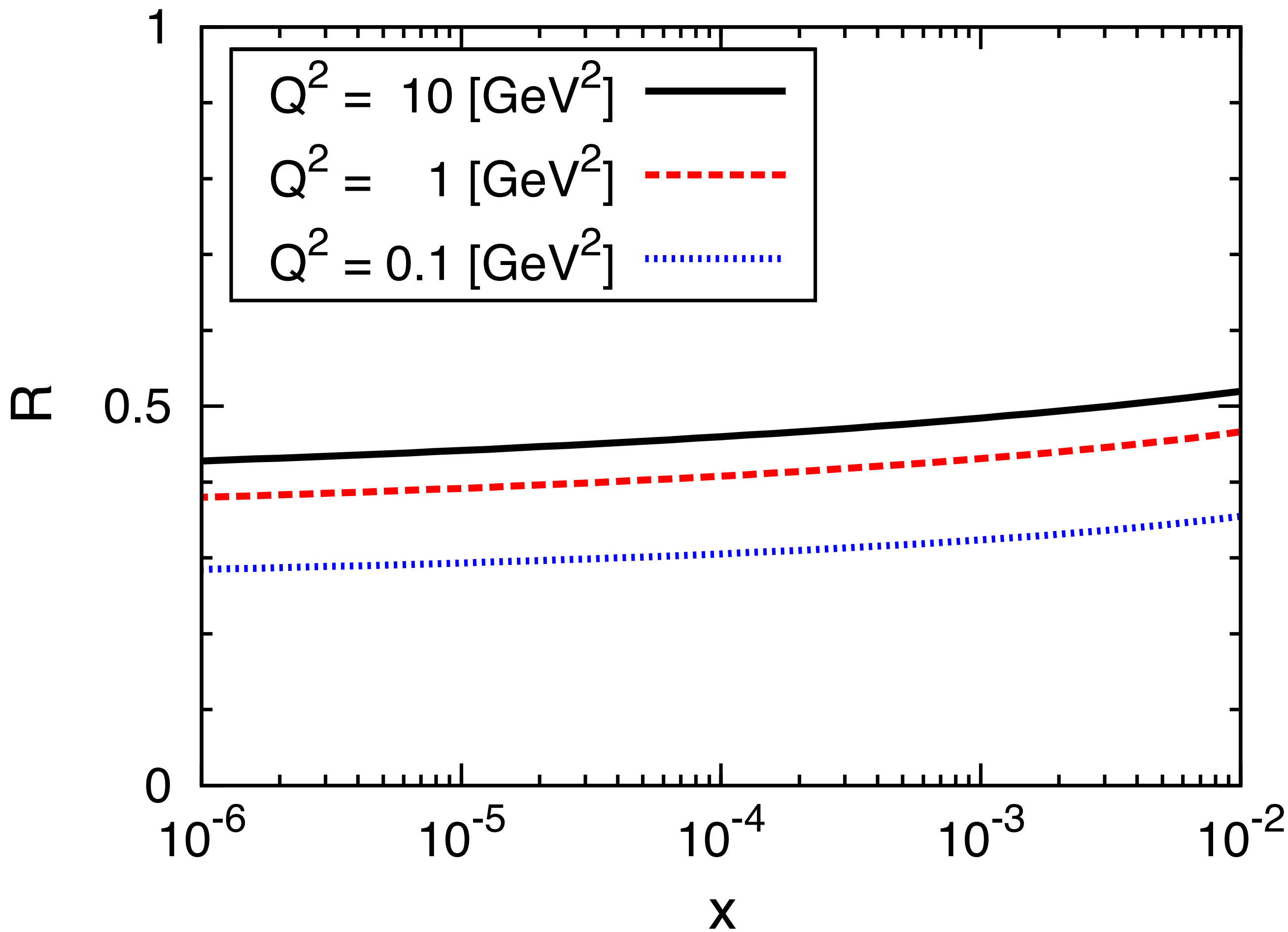
Proton longitudinal structure function

- Replacing the density distribution of the probe photon with the longitudinal component



Scale dependence of R

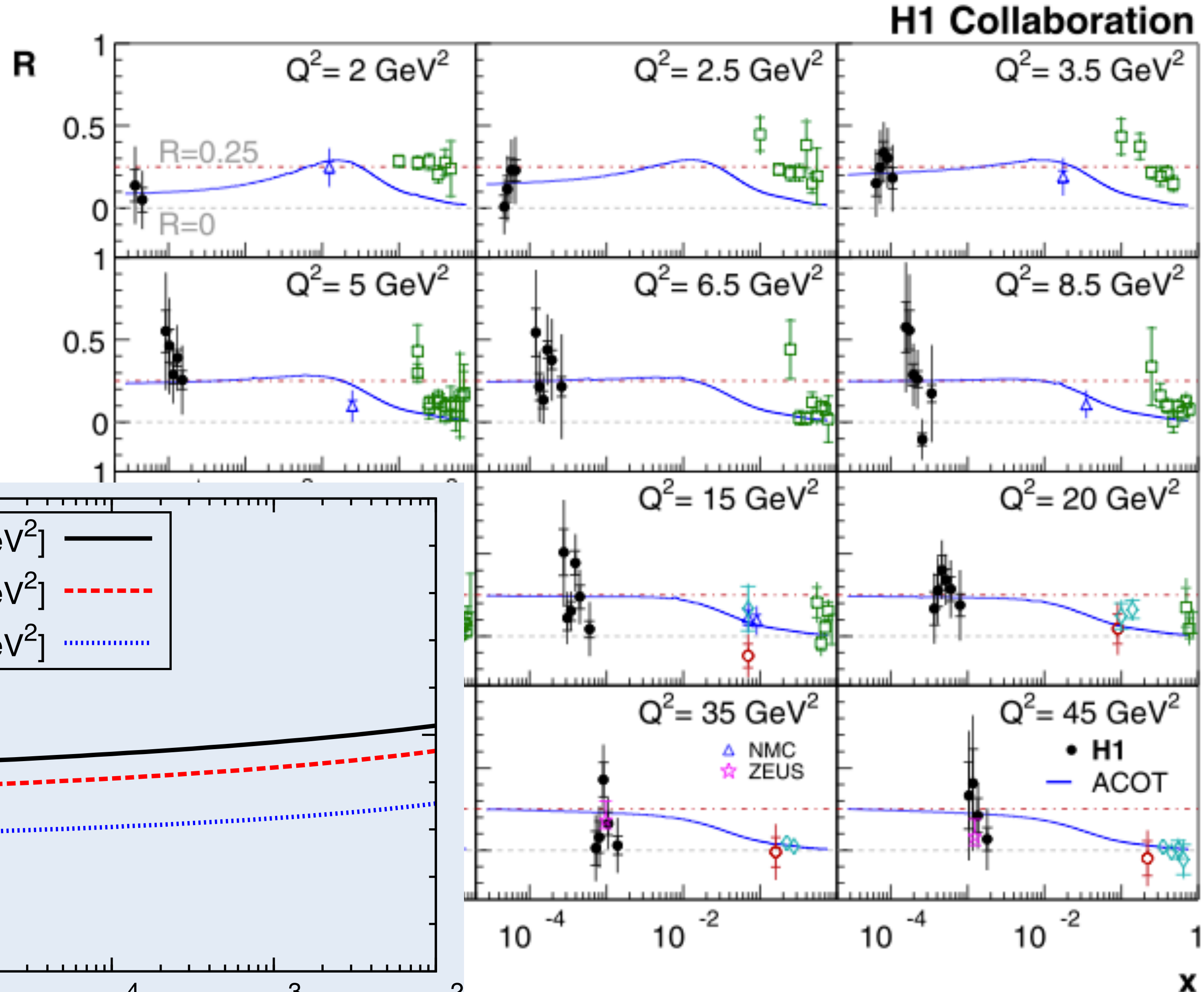
longitudinal-to-transverse ratio : $R \equiv F_L^p / F_T^p \left(= F_2^p - F_L^p\right)$

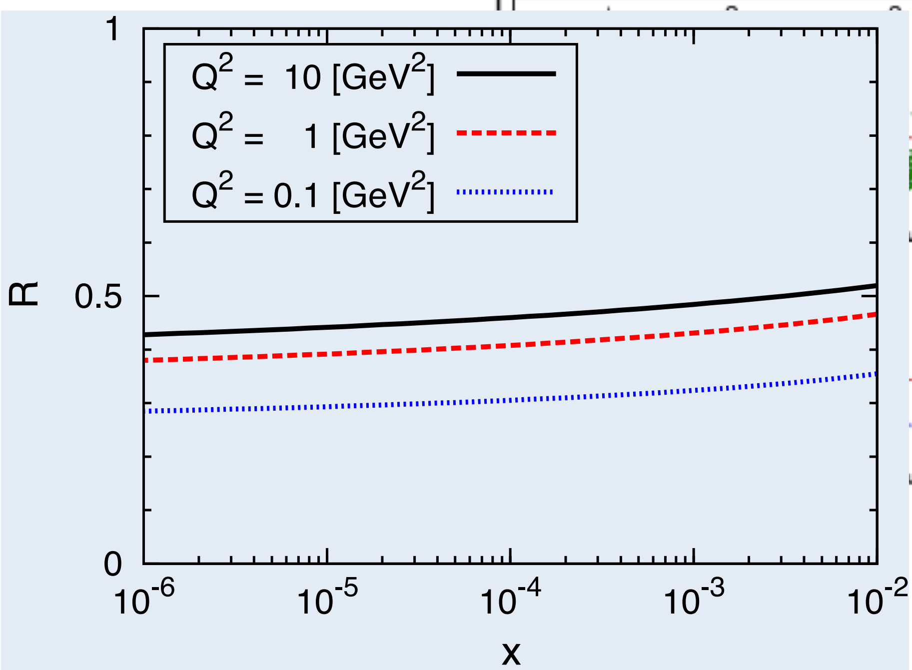


x dependence of R

H1 Collaboration (2011)

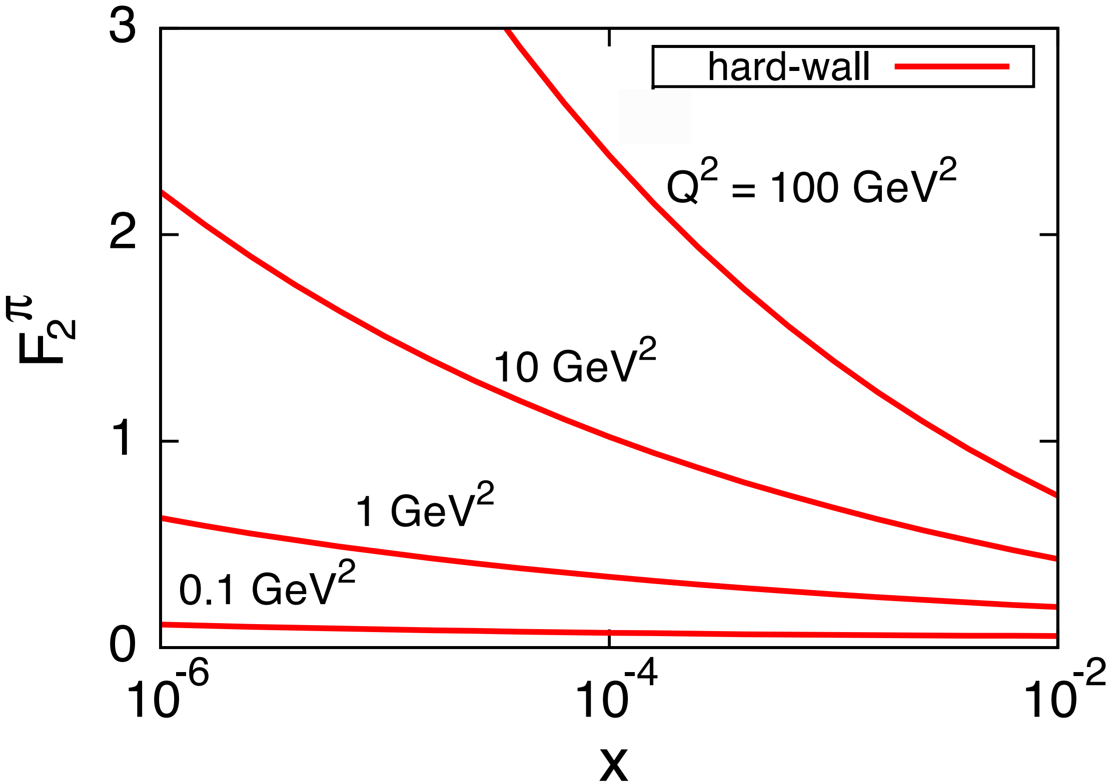
Fig. 14 The ratio R as a function of x in bins of Q^2. The inner error bars represent statistical error, the full error bars include the statistical and systematic uncertainties added in quadrature. The *solid curves* represent predictions of the DGLAP fit in the ACOT scheme. Results from other experiments are shown by the *open symbols*





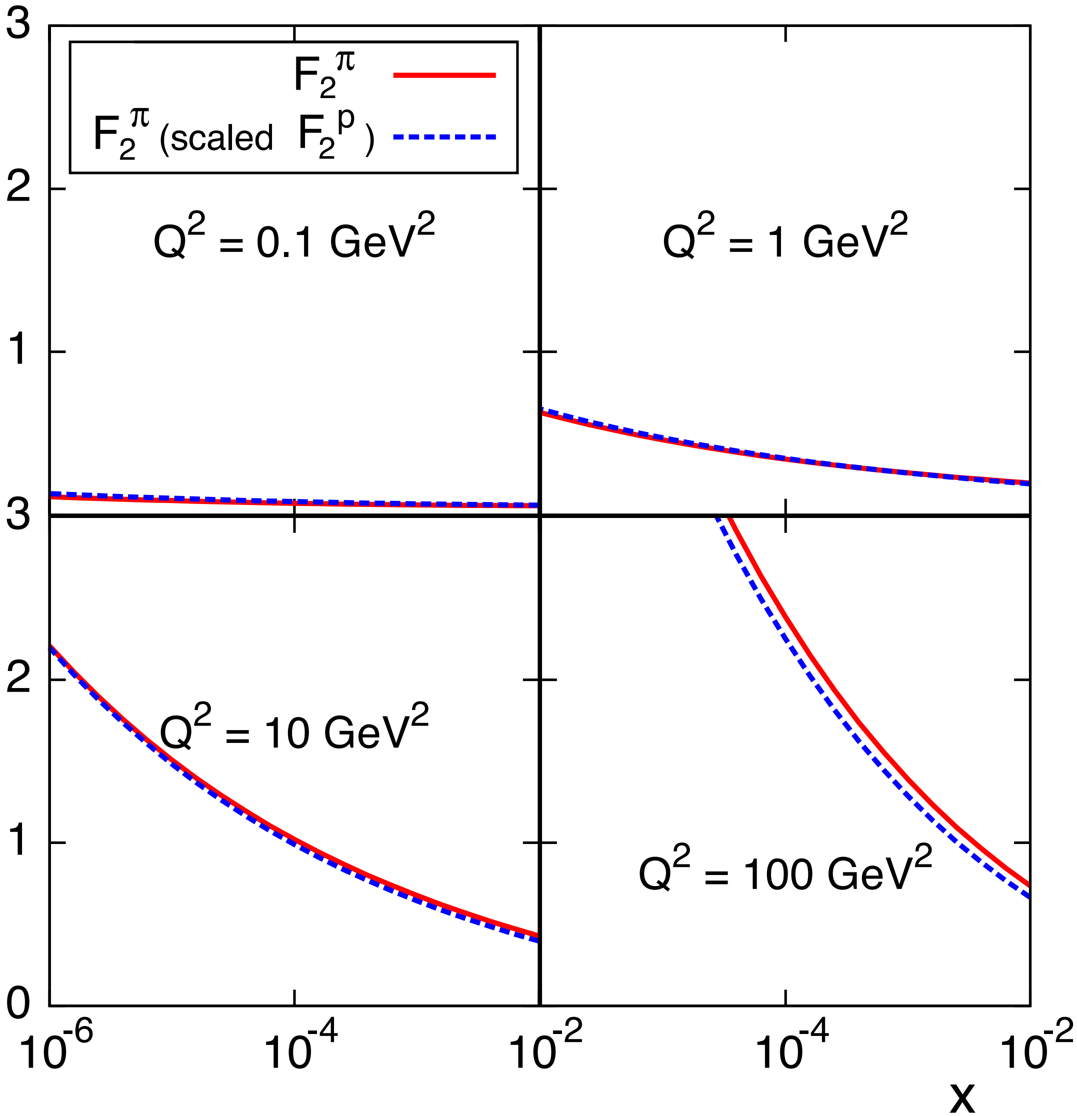
Pion structure function (from hard-wall model)

- Replacing the density distribution for target hadron with pion's, which can be calculated by using the holographic model of mesons



- There is no experimental data

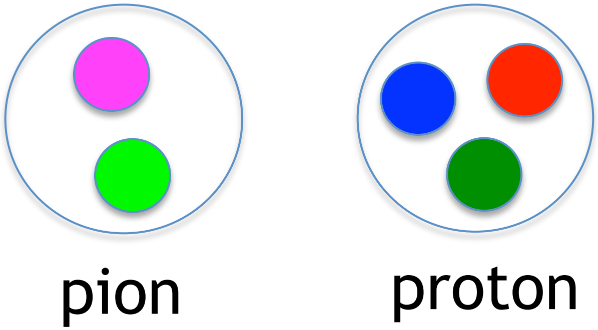
A relation between F_2^p and F_2^π (from hard-wall model)



Nikolaev-Speth-Zoller (2000)

$$F_2^\pi(x,Q^2) \simeq \frac{2}{3} F_2^p\left(\frac{2}{3}x,Q^2\right)$$

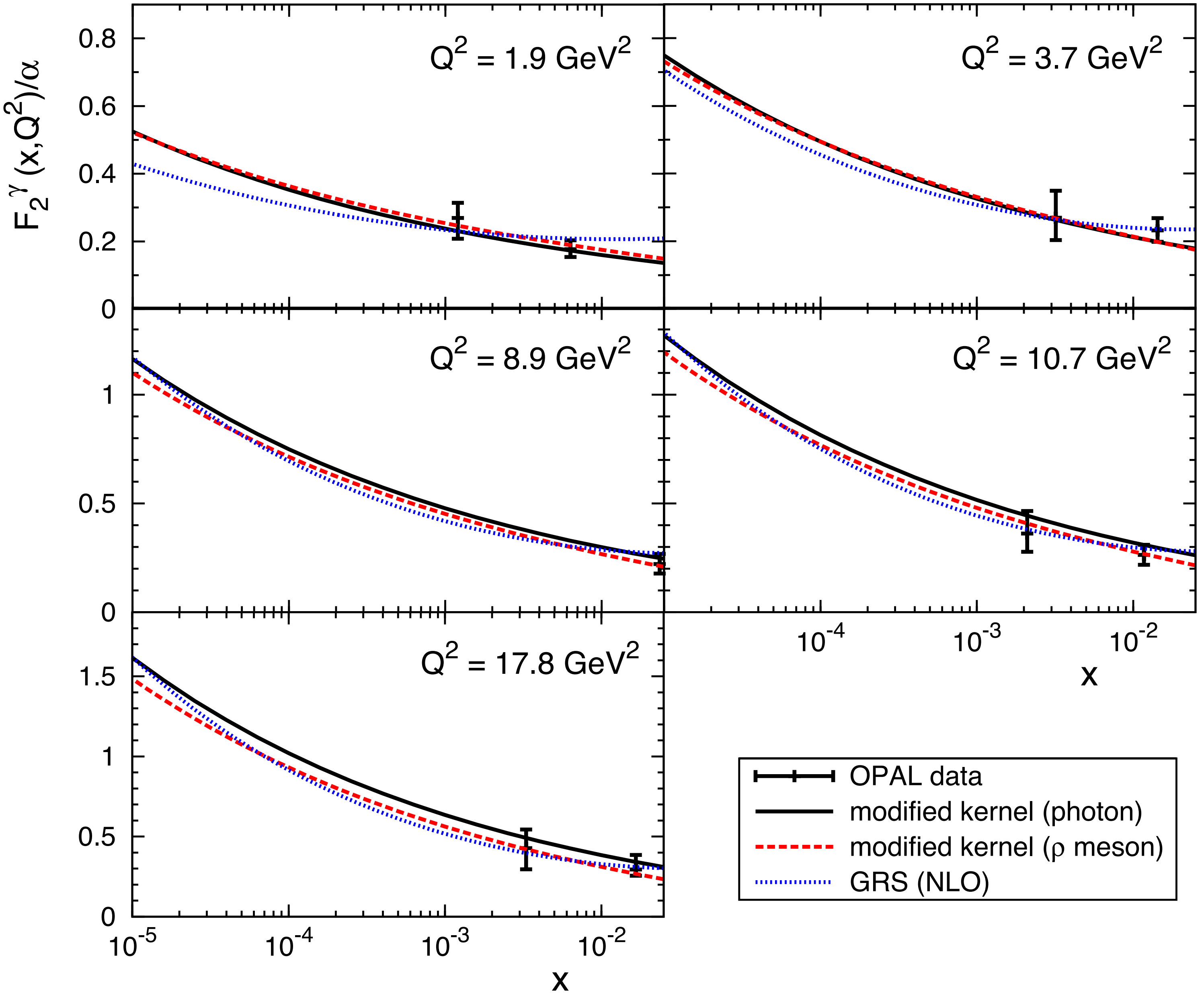
a relation based on the valence quark number



accidental coincidences?

Photon structure function at small x

AW-Li (2015; in preparation)



Summary

- We have studied nucleon structure functions at small *x* in the framework of holographic QCD.
- Results for F_2^p and F_L^p with the modified kernel are consistent with the experimental data, while those with the conformal kernel are not enough.
  -> QCD is significantly different from CFT at small *x*
  -> The holographic technique may be useful as a "building block" for building models to investigate the nonperturbative region in QCD.
- Various applications can be considered, which are not only for the improvement of the model itself, but also for other high energy scattering processes (DVCS, photoproduction of neutral vector mesons, and so on).