# Baryon resonances in a combined analysis of pion- and photon-induced reactions

## Recent results from the Juelich PWA

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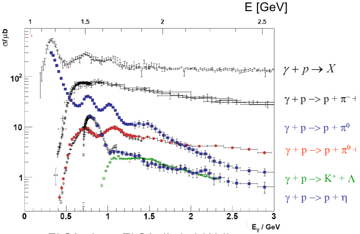
# The excited hadron spectrum:
## Connection between experiment and QCD in the non-perturbative regime

Experimental study of hadronic reactions




source: ELSA; data: ELSA, JLab, MAMI

Major progress in recent years:

- enlarged data base with high quality for different final states

  $\to$ alternative source of information besides $\pi N \to X$

- measurement of several (double) polarization observables in $\gamma N \to X$

  $\to$ towards a *complete experiment*: unambiguous determination of the amplitude (up to an overall phase)

Extract information from experimental data:

e.g. unitarized ChPT, "classical" PWA , K-Matrix, unitary isobar models ...

**Dynamical coupled channel (DCC) models:**

- combined analysis of different reactions
- wide energy range
- theoretical constraints of the $S$-matrix are met (or approximated)

This talk: $N$ and $\Delta$ resonances

# The excited hadron spectrum:
## Connection between experiment and QCD in the non-perturbative regime

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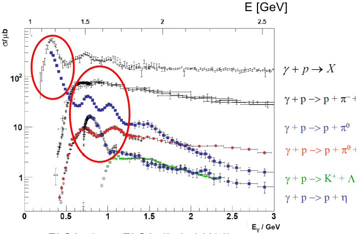


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This talk: $N$ and $\Delta$ resonances

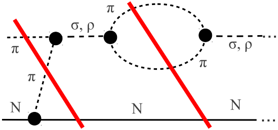
# Theoretical constraints of the $S$-matrix

**Unitarity:** probability conservation

- 2-body unitarity
- 3-body unitarity:

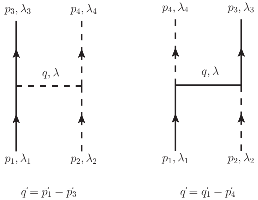
  discontinuities from $t$-channel exchanges

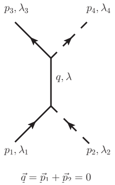
  → Meson exchange from requirements of the $S$-matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]




**Analyticity:** from unitarity and causality

- correct structure of branch point, right-hand cut (real, dispersive parts)
- to approximate left-hand cut → Baryon $u$-channel exchange



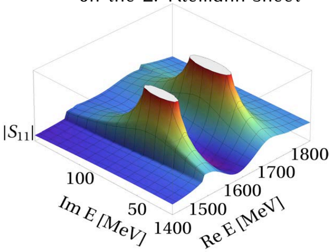




→ Resonances

# Analytic structure of the amplitude

important information for a reliable determination of the resonance spectrum

**Resonances:** poles in the $T$-matrix on the 2. Riemann sheet

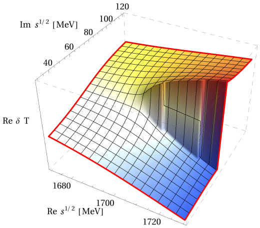



Re($E_0$) = "mass", -2Im($E_0$) = "width"

- pole position $E_0$ is the same in all channels
- residues→ branching ratios

Opening of **inelastic channels**:

⇒ **branch point** and new **Riemann sheet**




Example: $\rho N$ branch point at

$$M_N + m_{rho} = 1700 \pm i75 \text{ MeV}$$

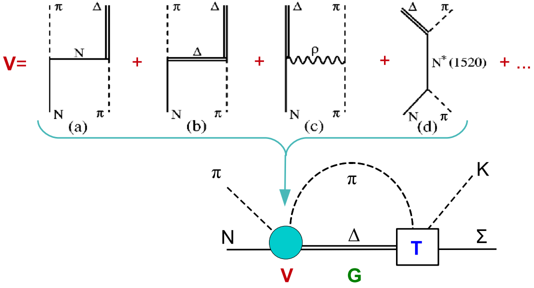
Inclusion of branch points important to avoid false resonance signal!

# A dynamical coupled-channel approach: the Jülich model

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

## The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$




- potentials $V$ constructed from effective $\mathcal{L}$
- $s$-channel diagrams: $T^P$
  genuine resonance states
- $t$- and $u$-channel: $T^{NP}$
  dynamical generation of poles
  partial waves strongly correlated

# A dynamical coupled-channel approach: the Jülich model

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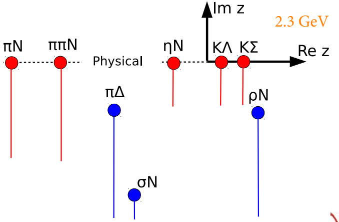
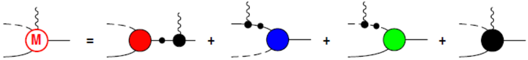
- (2-body) unitarity and analyticity respected
- 3-body $\pi\pi N$ channel:
  - parameterized effectively as $\pi\Delta$, $\sigma N$, $\rho N$
  - $\pi N/\pi\pi$ subsystems fit the respective phase shifts
  - ↳ branch points move into complex plane



# Photoproduction
## Different approaches

- **Field theoretical** approaches : DMT, ANL-Osaka, **Jülich-Athens-Washington**, ...

*Example*: Gauge invariant formulation by Haberzettl, Huang and Nakayama
Phys. Rev. C56 (1997), Phys. Rrev. C74 (2006), Phys. Rev. C85 (2012)




↓
complicated, involved construction/calculation

**Focus of the present analysis:**

extraction of resonance parameters

⇒ flexible, **phenomenological** parameterization of photo excitation

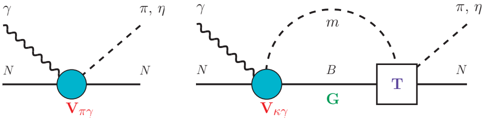
- Advantage: easy to implement, analyze large amounts of data
- Disadvantage: no information on microscopic reaction dynamics

# Photoproduction in a semi-phenomenological approach

## Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_{\kappa} V^{IJ}_{\kappa\gamma}$$

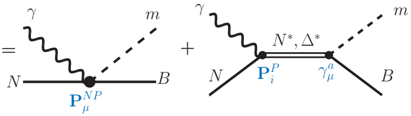
(partial wave basis)




$m = \pi, \eta$ , $B = N, \Delta$

$T_{\mu\kappa}$: Jülich hadronic $T$-matrix → Watson's theorem fulfilled by construction
→ **analyticity of T**: extraction of resonance parameters

**Phenomenological potential:**

$V_{\mu\gamma}(E,q)=$ 


$$= \frac{\tilde{\gamma}^a_\mu(q)}{m_N} P^{NP}_\mu(E) + \sum_i \frac{\gamma^a_{\mu;i}(q) P^P_i(E)}{E - m^b_i}$$

$\tilde{\gamma}^a_\mu$, $\gamma^a_{\mu;i}$: hadronic vertices → correct threshold behaviour, cancellation of singularity at $E = m^b_i$
→ $\gamma^a_{\mu;i}$ affects pion- and photon-induced production of final state $mB$

$i$: resonance number per multipole; $\mu$: channels $\pi N$, $\eta N$, $\pi\Delta$

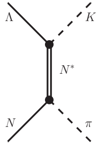
Polynomials

# Data analysis and fit results

# Combined analysis of pion- and photon-induced reactions
## Simultaneous fit

**Fit parameter:**

s-channel: resonances ($T^P$)




$m_{bare}$ + $f_{\pi NN*}$

- $\pi N \to \ \pi N$

  $\pi^- p \to \ \eta n, \ K^0\Lambda, \ K^0\Sigma^0, \ K^+\Sigma^-$

  $\pi^+ p \to \ K^+\Sigma^+$

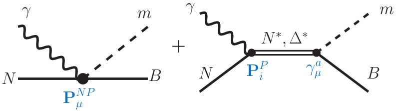
$\Rightarrow$ 128 free parameters

11 $N^*$ resonances $\times$ (1 $m_{bare}$+ couplings to $\pi N$, $\rho N$, $\eta N$, $\pi\Delta$, $K\Lambda$, $K\Sigma$))
+ 10 $\Delta$ resonances $\times$ (1 $m_{bare}$+ couplings to $\pi N$, $\rho N$, $\pi\Delta$, $K\Sigma$)

- $\gamma p \to \pi^0 p$, $\pi^+ n$, $\eta p$

  $\Rightarrow$ up to 456 free parameters

  couplings of the polynomials




↳ calculations on the JUROPA supercomputer: parallelization in energy (∼ 300 - 400 processes)

# Data base

simultaneous fit to $\pi$- and $\gamma$-induced reactions

<table>
<tr><th></th><th>Fit A</th><th>Fit B</th></tr>
<tr><td></td><td>(dashed blue line)</td><td>(solid red line)</td></tr>
<tr><td>$\pi N \to \pi N$</td><td colspan="2">PWA GW-SAID WI08 [Arndt et al., PRC 86 (2012)]</td></tr>
<tr><td>$\pi^- p \to \eta n$</td><td colspan="2">$d\sigma/d\Omega$, $P$</td></tr>
<tr><td>$\pi^- p \to K^0\Lambda$</td><td colspan="2">$d\sigma/d\Omega$, $P$, $\beta$</td></tr>
<tr><td>$\pi^- p \to K^0\Sigma^0$</td><td colspan="2">$d\sigma/d\Omega$, $P$</td></tr>
<tr><td>$\pi^- p \to K^+\Sigma^-$</td><td colspan="2">$d\sigma/d\Omega$</td></tr>
<tr><td>$\pi^+ p \to K^+\Sigma^+$</td><td colspan="2">$d\sigma/d\Omega$, $P$ , $\beta$</td></tr>
<tr><td></td><td colspan="2">$\sim$ 6000 data points</td></tr>
<tr><td>$\gamma p \to \pi^0 p$</td><td colspan="2">$d\sigma/d\Omega$, $\Sigma$, $P$, $T$, $\Delta\sigma_{31}$, $G$, $H$</td></tr>
<tr><td>$\gamma p \to \pi^+ n$</td><td colspan="2">$d\sigma/d\Omega$, $\Sigma$, $P$, $T$, $\Delta\sigma_{31}$, $G$, $H$</td></tr>
<tr><td>$\gamma p \to \eta p$</td><td>$d\sigma/d\Omega$, $P$, $\Sigma$</td><td>$d\sigma/d\Omega$, $P$, $\Sigma$, $T$, $F$</td></tr>
<tr><td></td><td>29,392 data points</td><td>29,680 data points</td></tr>
</table>

- More single/double polarization:
  $E$, $C_{x'L}$, $C_{z'L}$,
  $T$, $P$, $H$ (ELSA 2014)
  $(\gamma p \to \pi^0 p)$
  $\Rightarrow$ predictions

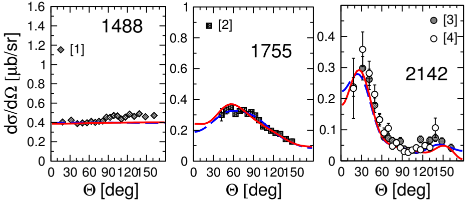
$\to$ $T$, $F$: Akondi *et al.* (A2 at MAMI)
PRL 113 10, 102001 (2014)

# Fit results $\gamma p \to \eta p$

selected results, arXiv:1504.01643 [nucl-th]

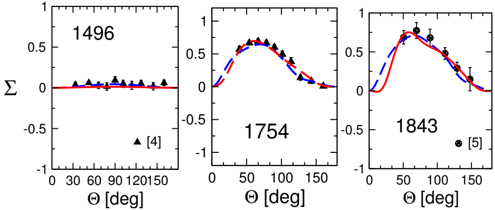
$T$, $F$ not included — $T$, $F$ included

- Differential cross section



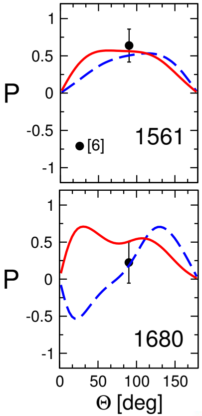

[1] McNicoll 2010 (MAMI), [2] Williams 2009 (JLab), [3] Credé 2009 (ELSA), [4] Credé 2005 (ELSA)

- Beam asymmetry




[4] Bartalini 2007 (GRAAL), [5] Elsner 2007 (ELSA)

- Recoil polarization
  - only 7 data points in total -

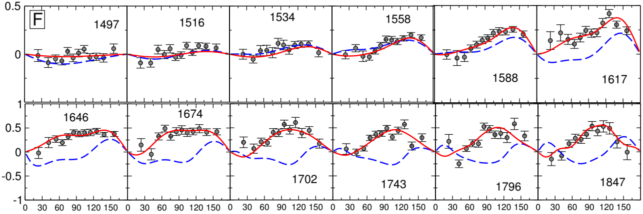



[6] Heusch (Caltech), PRL25, 1381 (1970)

# $F$ and $T$ in $\gamma p \to \eta p$

arXiv:1504.01643 [nucl-th]

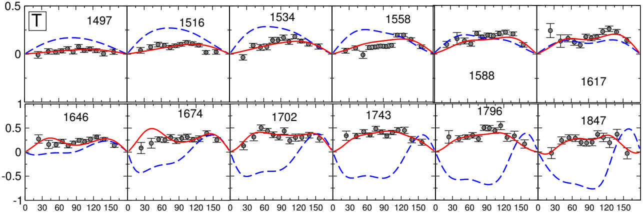
Data: Akondi *et al.* (A2 at MAMI) PRL 113 10, 102001 (2014)




prediction

fit

| Polarization: | | |
|---|---|---|
| Beam | Target | Recoil |
| $+1$ | $+x$ | 0 |
| $-1$ | $+x$ | 0 |




| Polarization: | | |
|---|---|---|
| Beam | Target | Recoil |
| 0 | $+y$ | 0 |
| 0 | $-y$ | 0 |

# Resonance content: I=1/2

arXiv:1504.01643 [nucl-th]

Pole search on the $2^{nd}$ sheet of the scattering matrix $T_{\mu\nu}$

Resonance parameter:

- "mass" $= Re(E_0)$
- "width"$= -2Im(E_0)$
- Residues $\to$ branching ratios

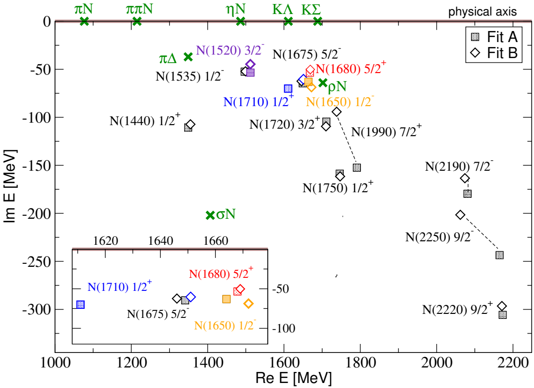
$E_0$: pole position

$\to$ no new states compared to Jülich2012

(Jülich2012: only pion-induced data)

$\to$ no narrow structure at 1.68 GeV

(seen in eta photoproduction on the neutron)




x: branch points

Notation: $N(\text{"name"})\, J^{parity}$

# Resonance parameters

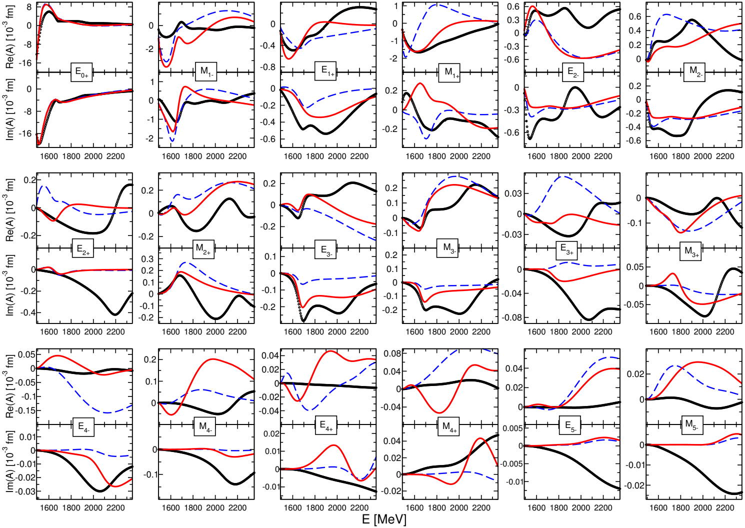
selected results, arXiv:1504.01643 [nucl-th]

| | fit | Re $E_0$ [MeV] | -2Im $E_0$ [MeV] | $\lvert r_{\pi N}\rvert$ [MeV] | $\theta_{\pi N\to\pi N}$ [deg] | $\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{tot}}$ [%] | $\theta_{\pi N\to\eta N}$ [deg] |
|---|---|---|---|---|---|---|---|
| $N(1535)$ $1/2^-$ | A | 1497 | 105 | 23 | $-48$ | 51 | 110 |
| | B | 1499 | 104 | 22 | $-46$ | 51 | 112 |
| | $A_{had}$ | 1498 | 74 | 17 | $-37$ | 51 | 120 |
| $N(1710)$ $1/2^+$ | A | 1611 | 140 | 2.7 | $-40$ | 6.1 | 175 |
| | B | 1651 | 121 | 3.2 | 55 | 16 | $-180$ |
| | $A_{had}$ | 1637 | 97 | 4 | $-30$ | 24 | 130 |

fit A: $T$, $F$ not included   fit B: $T$, $F$ included   fit $A_{had}$: Jülich2012, only pion-induced data

# Multipoles for $\gamma p \to \eta p$

## Comparison with the Bonn-Gatchina 2014-02 solution



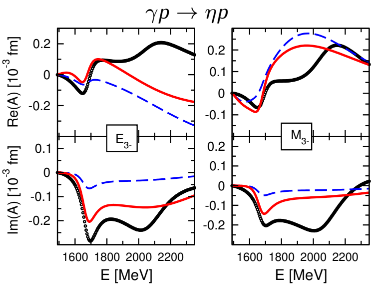

(Black) solid line: BG 2014-02. Dashed (blue) line: fit A; solid (red) line: fit B.

$\gamma p \to \pi^0 p$

# Multipoles: $\gamma p \to \eta p$ vs $\gamma p \to \pi^0 p$

## Comparison with the Bonn-Gatchina 2014-02 solution

- Example: $E_{3-}$ and $M_{3-}$ multipoles ($F_{15}$, $F_{35}$)

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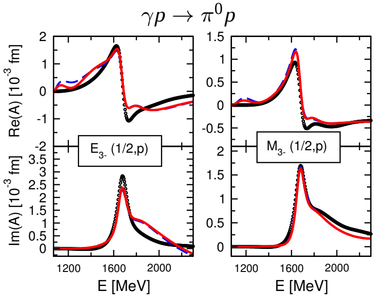



(Black) solid line: BG 2014-02. Dashed (blue) line: fit A; solid (red) line: fit B.

$\Rightarrow$ Multipole content of $\gamma p \to \eta p$ seems less well established than for $\gamma p \to \pi N$

$\Rightarrow$ Convergence with larger data base of $\gamma p \to \eta p$?

# Summary

Extraction of the $N^*$ and $\Delta$ resonance spectrum

from a simultaneous analysis of pion- and photon-induced reactions

- DCC analysis of $\pi N \to \pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$
- $\pi$ and $\eta$ photoproduction in a semi-phenomenological approach

Comparison of 3 different fits:

- simultaneous fit of $\pi N$, $\gamma N \to$ X **without** recent MAMI $T$ and $F$ data
- simultaneous fit of $\pi N$, $\gamma N \to$ X **with** recent MAMI $T$ and $F$ data
- earlier fit (Jülich2012), only $\pi N \to X$

$\Rightarrow$ noticeable influence of photoproduction data in general / new polarization observables

- on pole positions and photocouplings
- on hadronic couplings

Thank you for your attention!

# Photocouplings at the pole

selected results, arXiv:1504.01643 [nucl-th]

$$\tilde{A}^h_{pole} = A^h_{pole} e^{i\vartheta^h}$$
$$h = 1/2,\ 3/2$$

$$\tilde{A}^h_{pole} = I_F \sqrt{\frac{q_p}{k_p} \frac{2\pi\,(2J+1)\mathrm{E_0}}{m_N\ \mathrm{r_{\pi N}}}}\ \mathrm{Res}\ A^h_{L\pm}$$

$I_F$: isospin factor
$q_p$ ($k_p$): meson (photon) momentum at the pole
$J = L \pm 1/2$ total angular momentum
$E_0$: pole position
$r_{\pi N}$: elastic $\pi N$ residue

| | fit | $\mathrm{A}^{1/2}_{\mathrm{pole}}$ $[10^{-3}\ \mathrm{GeV}^{-\frac{1}{2}}]$ | $\vartheta^{1/2}$ [deg] |
|---|---|---|---|
| $N(1535)\ 1/2^-$ | A | 107 | 4.6 |
| | B | 106 | 5.2 |
| | 2 | $50^{+4}_{-4}$ | $-14^{+12}_{-10}$ |
| $N(1710)\ 1/2^+$ | A | 7.1 | $-177$ |
| | B | 20 | $-83$ |
| | 2 | $28^{+9}_{-2}$ | $103^{+20}_{-6}$ |

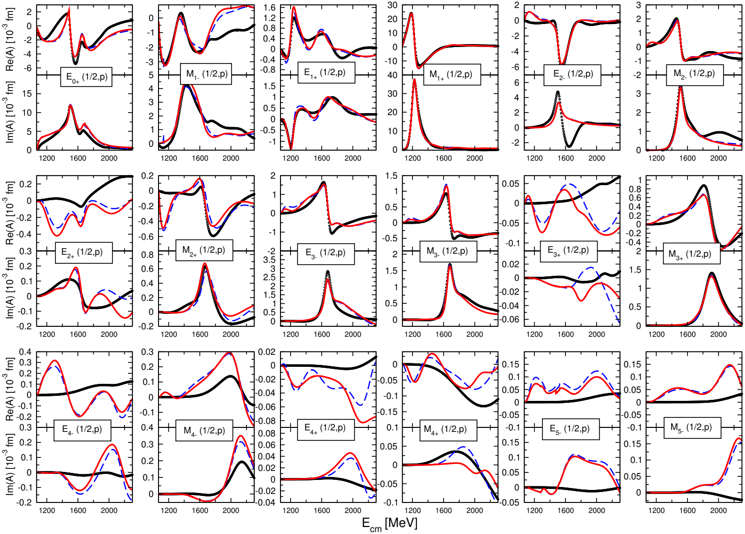
fit A: $T$, $F$ not included

it B: $T$, $F$ included

fit 2: Jülich2013, only pion photoproduction
(same pole positions as fit $\mathrm{A_{had}}$)

# Multipoles: $\gamma p \to \pi^0 p$

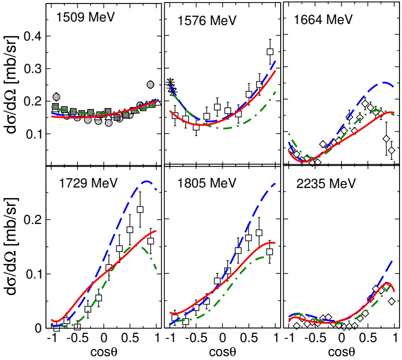
## Comparison with the Bonn-Gatchina 2014-02 solution

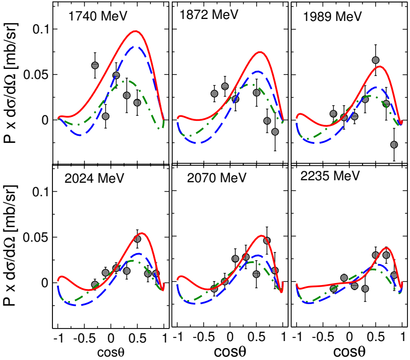



(Black) solid line: BG 2014-02. Dashed (blue) line: fit A; solid (red) line: fit B. back

# Pion-induced eta production: $\pi^- p \rightarrow \eta n$

arXiv:1504.01643 [nucl-th]




fit A: $T$, $F$ not included  fit B: $T$, $F$ included  fit $A_{had}$: Jülich2012, only pion-induced data

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Resonance parameter:

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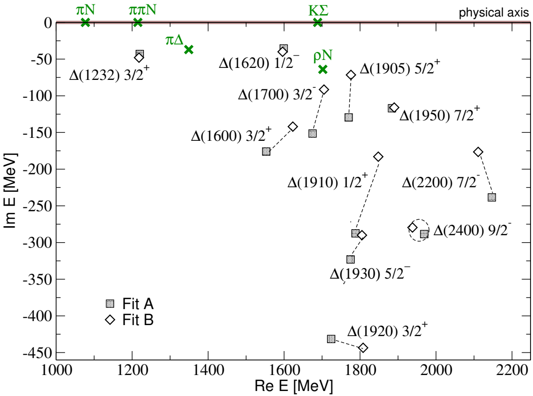
- Residues $\rightarrow$ branching ratios

$E_0$: pole position

$\rightarrow$ no new states compared to Jülich2012

(Jülich2012: only pion-induced data)

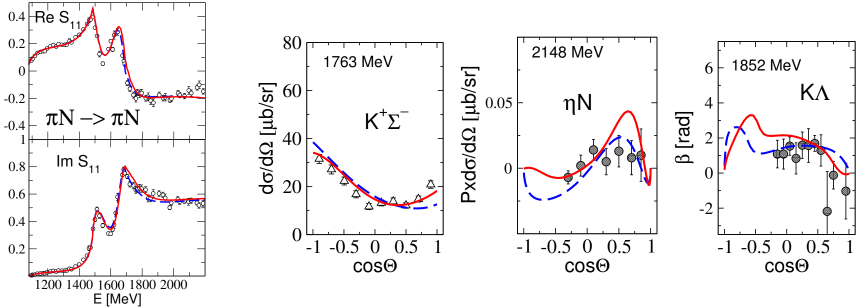



x: branch points  Notation: $N(\text{"name"})\, J^{parity}$

# Pion-induced reactions $\pi N \to \pi N$, $\eta N$, $K\Lambda$, $K\Sigma$

arXiv:1504.01643 [nucl-th]

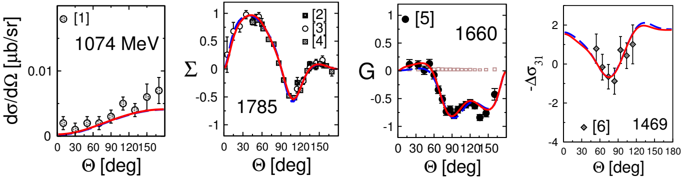
**Selected results:** Fit A and Fit B



# Pion photoproduction: selected fit results

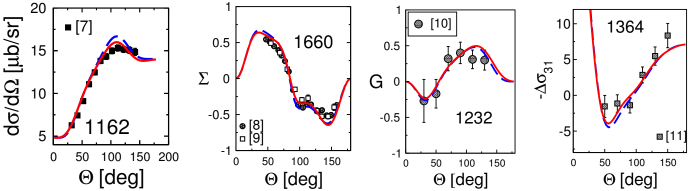
arXiv:1504.01643 [nucl-th]

- $\gamma p \to \pi^0 p$




[1] Schmidt 2001 (MAMI)
[2] Elsner 2009 (ELSA)
[3] Sparks 2010 (ELSA)
[4] Bartalini 2005 (GRAAL)
[5] Thiel 2012 (ELSA)
[6] Ahrens 2002 (MAMI)

- $\gamma p \to \pi^+ n$




[7] Ahrens 2004 (MAMI)
[8] Bartalini 2002 (GRAAL)
[9] Ajaka 2000 (GRAAL)
[10] Ahrens 2005 (MAMI)
[11] Ahrens 2006 (MAMI)

# Double polarization in $\gamma p \to \pi^0 p$

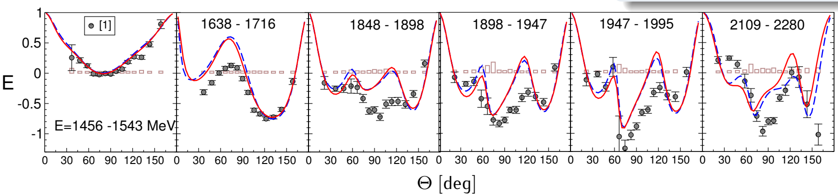
Data NOT included in fit

selected results, arXiv:1504.01643 [nucl-th]

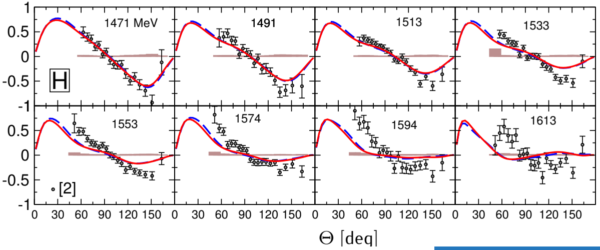
**Polarization**

| Beam | Target | Recoil |
|---|---|---|
| $+1$ | $-z$ | 0 |
| $-1$ | $-z$ | 0 |

[1] Gottschall *et al.* 2013 (ELSA) PRL. 112 1, 012003




[2] Hartmann *et al.* 2014 (ELSA) PRL 113, 062001




**Polarization**

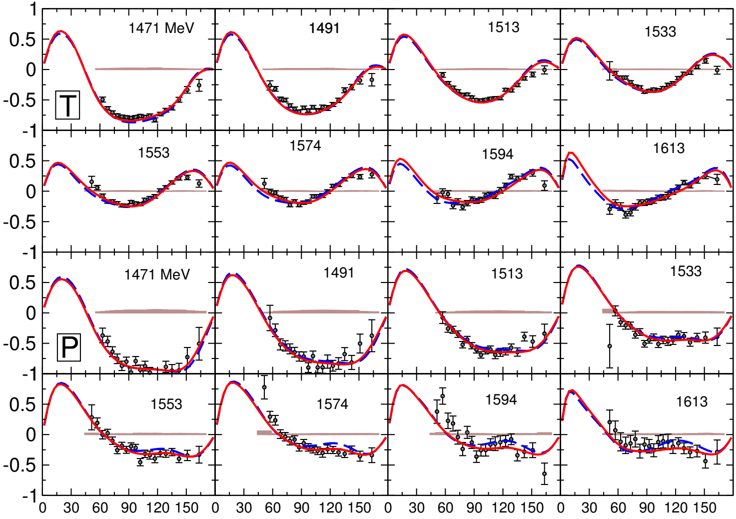
| Beam | Target | Recoil |
|---|---|---|
| $\perp'$ | $x$ | 0 |
| $\parallel'$ | $x$ | 0 |

new ELSA T,P data

# $T$, $P$ in $\gamma p \to \pi^0 p$

## Data NOT included in the Fit

Data: J. Hartmann *et al.* 2014 (ELSA)

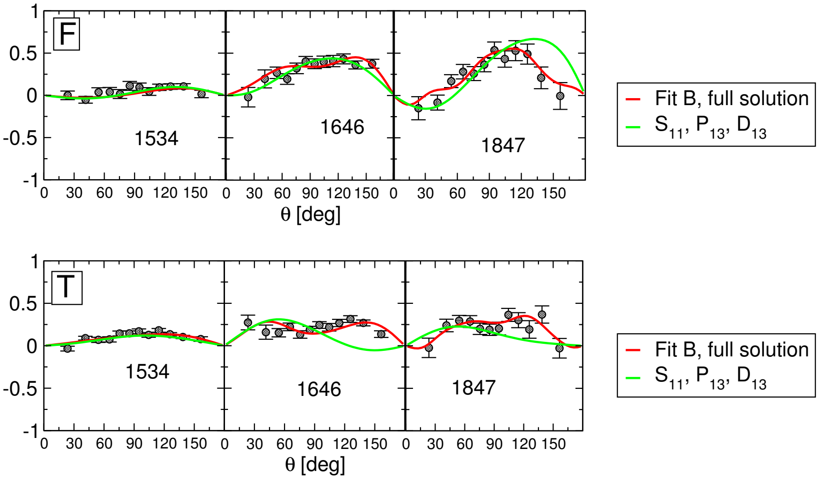



arXiv:1504.0164
[nucl-th]

back

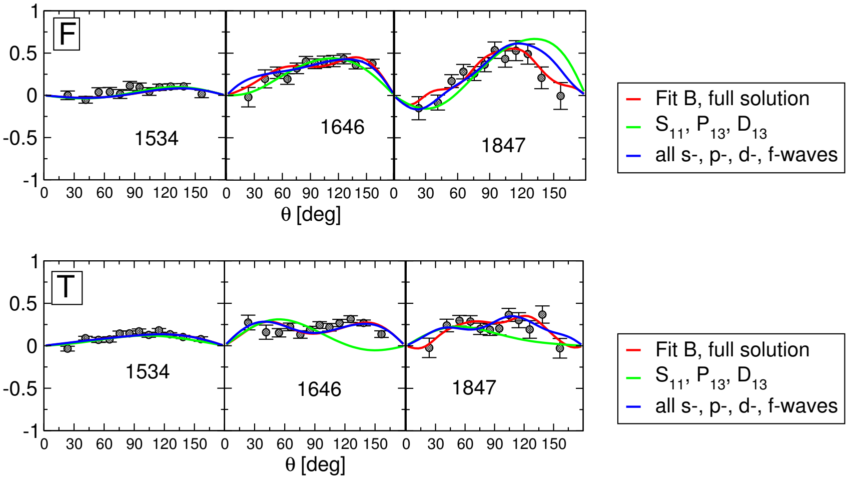
# Partial wave contribution in $F$ and $T$ in $\gamma p \to \eta p$

arXiv:1504.01643 [nucl-th]



# Partial wave contribution in $F$ and $T$ in $\gamma p \to \eta p$

arXiv:1504.01643 [nucl-th]



# Details of the formalism

**Polynomials:**

$$P_i^{\mathrm{P}}(E) = \sum_{j=1}^{n} g_{i,j}^{\mathrm{P}} \left(\frac{E-E_0}{m_N}\right)^j e^{-g_{i,n+1}^{P}(E-E_0)}$$

$$P_\mu^{\mathrm{NP}}(E) = \sum_{j=0}^{n} g_{\mu,j}^{\mathrm{NP}} \left(\frac{E-E_0}{m_N}\right)^j e^{-g_{\mu,n+1}^{\mathrm{NP}}(E-E_0)}$$

- $E_0 = 1077$ MeV
- $g_{i,j}^{\mathrm{P}}$, $g_{\mu,j}^{\mathrm{NP}}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

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# Data base

simultaneous fit to $\pi N \to \pi N$, $\eta N$, $K\Lambda$, $K\Sigma$

World data base on $\eta N$, $K\Lambda$, $K\Sigma$

| | PWA | $\sigma_{tot}$ | $\frac{d\sigma}{d\Omega}$ | P | $\beta$ |
|---|---|---|---|---|---|
| $\pi N \to \pi N$ | GWU/SAID 2006<br>up to J=9/2 | | | | |
| $\pi^- p \to \eta n$ | | 62 data points | 38 energy points<br>z=1489 to 2235 MeV | 12 energy points<br>1740 to 2235 MeV | |
| $\pi^- p \to K^0 \Lambda$ | | 66 data points | 46 energy points<br>1626 to 1405 MeV | 27 energy points<br>1633 to 2208 MeV | 7 energy points<br>1852 to 2262 MeV |
| $\pi^- p \to K^0 \Sigma^0$ | | 16 data points | 29 energy points<br>1694 to 2405 MeV | 19 energy points<br>1694 to 2316 MeV | |
| $\pi^- p \to K^+ \Sigma^-$ | | 14 data points | 15 energy points<br>1739 to 2405 MeV | | |
| $\pi^+ p \to K^+ \Sigma^+$ | | 18 data points | 32 energy points<br>1729 to 2318 MeV | 32 energy points<br>1729 to 2318 MeV | 2 energy pionts<br>2021 and 2107 MeV |

$\sim$ 6000 data points

back

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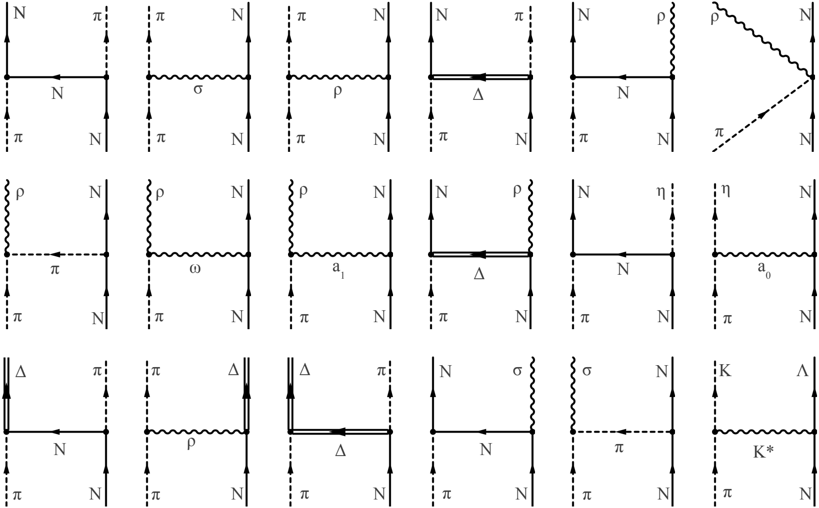
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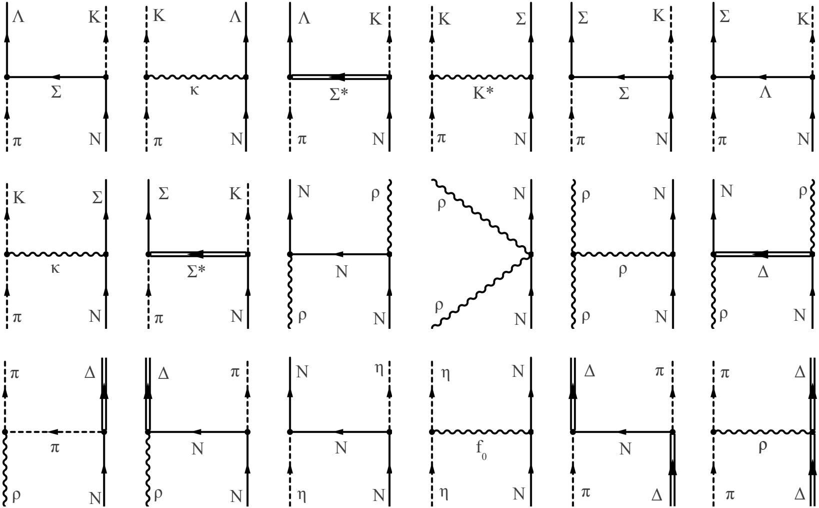
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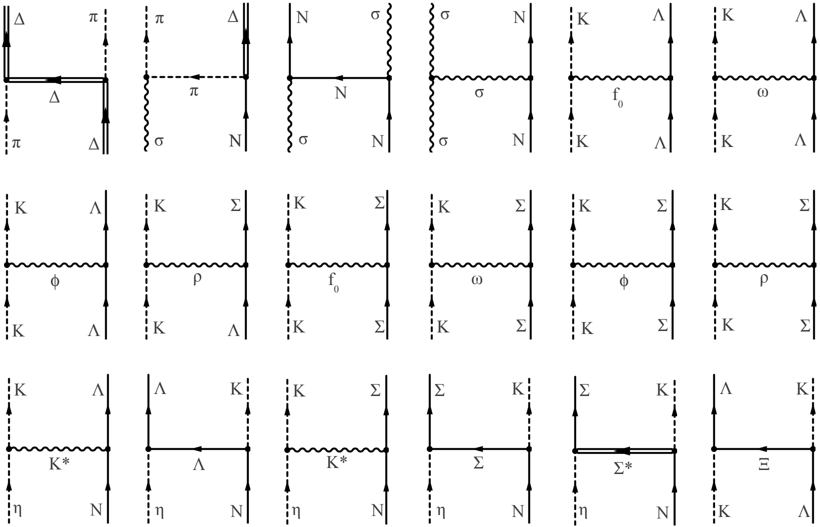
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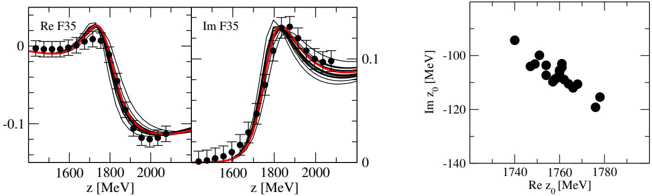
# Error analysis

- $\chi^2 + 1$ criterion: determination of the non-linear parameter error
  - error of parameter $p_i$ determined by range of $p_i$ such that $\chi^2_{min}$ rises by less than 1

$\Rightarrow$ error on pole positions and residues.




NPBA 851, 58 (2011)

BUT: numerically not possible with $\geq$ 500 free parameters

Work in progress: Developing of techniques to apply Monte-Carlo error propagation using bootstrap method (M. Döring et al.)

# Matching to lattice

## Prediction & analysis of lattice data

[M. Döring et al., EPJ A47, 163 (2011)]

Scattering equation:

$$T(q'', q') = V(q'', q') + \int_0^\infty dq\, q^2\, V(q'', q) \frac{1}{z - E_1(q) - E_2(q) + i\epsilon} T(q, q')$$

Discretization of momenta in the scattering equation:

$$\int \frac{\vec{d}\,^3 q}{(2\pi)^3} f(|\vec{q}\,|^2) \quad \rightarrow \quad \frac{1}{L^3} \sum_{\vec{n}_i} f(|\vec{q}_i|^2), \quad \vec{q}_i = \frac{2\pi}{L} \vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3$$

$$T(q'', q') = V(q'', q') + \frac{2\pi^2}{L^3} \sum_{i=0}^{\infty} \vartheta(i) V(q'', q_i) \frac{1}{z - E_1(q_i) - E_2(q_i)} T(q_i, q'),$$

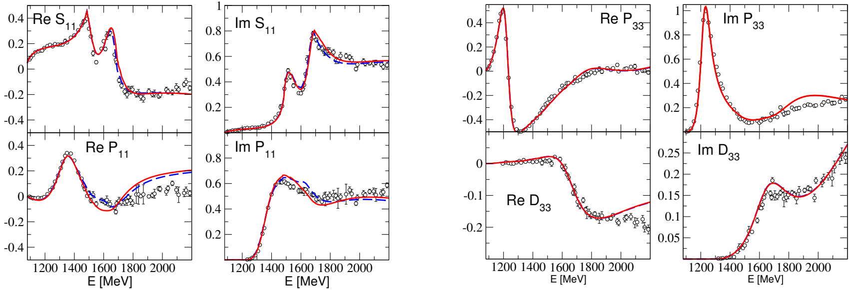
$\vartheta^{(P)}(i)$ series

- Study finite-volume effects
- Predict lattice spectra

# $\pi N \to \pi N$ partial wave amplitudes

selected results, arXiv:1504.01643 [nucl-th]

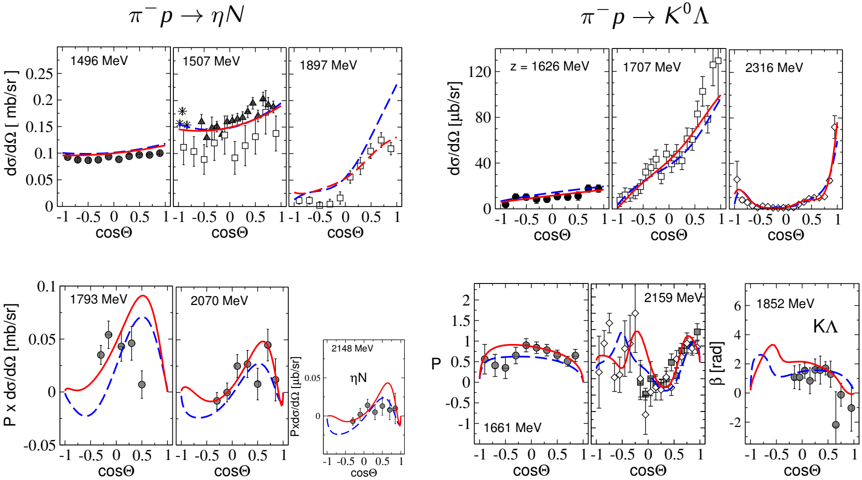
Fit A and Fit B




- Notation: $L_{2I\,2J}$
- Input to fit: energy-dependent partial wave analysis, GWU/SAID 2006 up to $J = 9/2$ ( $\sim H_{39}$)

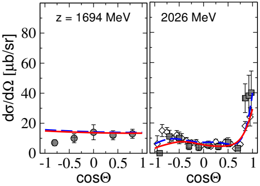
# $\pi N \to \eta N, K\Lambda$

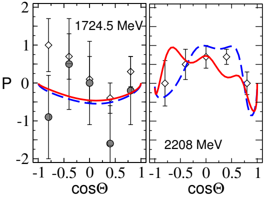
selected results, arXiv:1504.01643 [nucl-th]



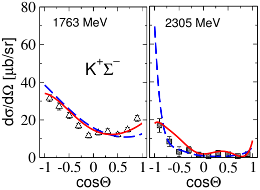
# $\pi N \to K\Sigma$ selected results, arXiv:1504.01643 [nucl-th]

$\pi^- p \to K^0\Sigma^0$



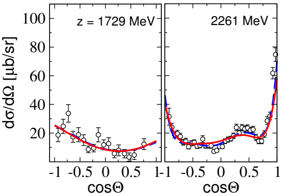



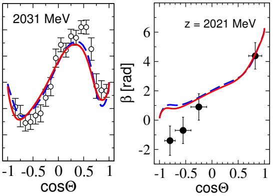

$\pi^- p \to K^+\Sigma^-$




No polarization data!

$\pi^+ p \to K^+\Sigma^+$

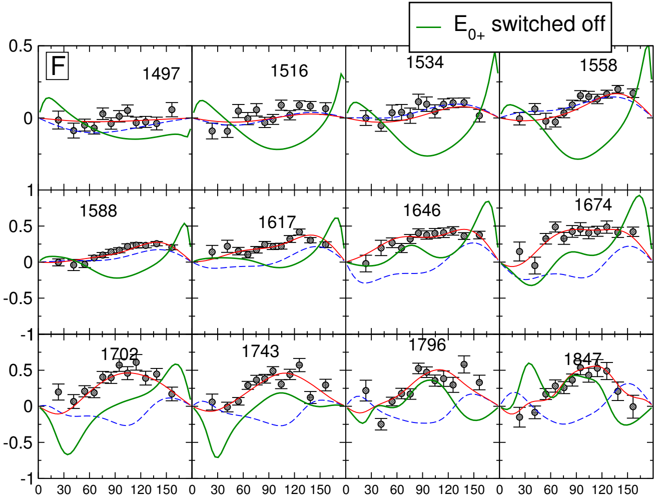


# Partial wave contribution to $F$ in $\gamma p \to \eta p$

arXiv:1504.01643 [nucl-th]

Switch off different PWs in **Fit B**



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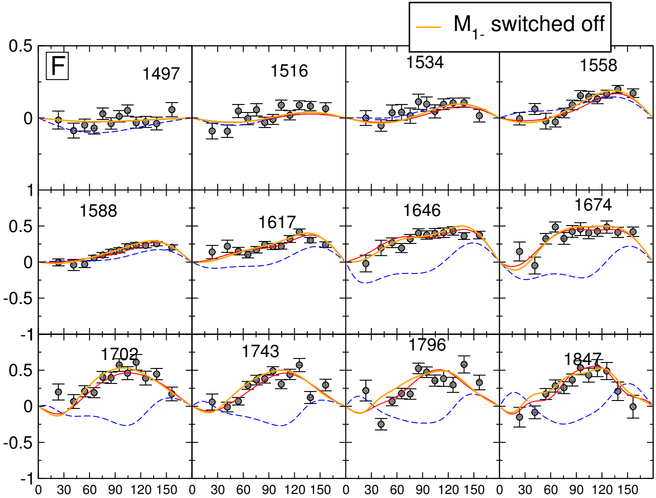
arXiv:1504.01643 [nucl-th]

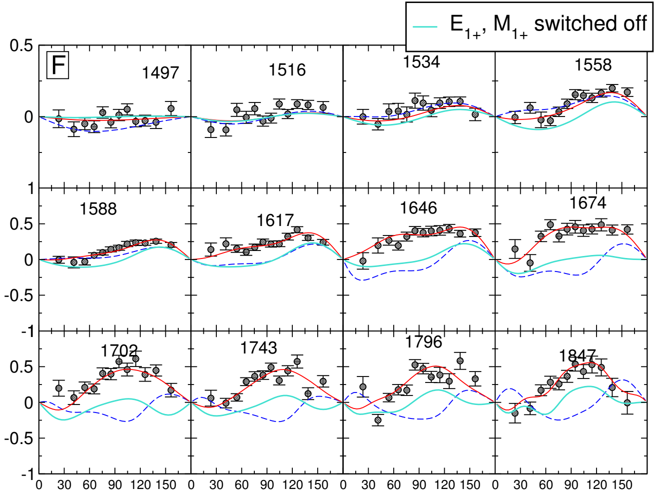
Switch off different PWs in **Fit B**



# Partial wave contribution to $F$ in $\gamma p \to \eta p$

arXiv:1504.01643 [nucl-th]

Switch off different PWs in **Fit B**



# Photoproduction of pseudoscalar meson

- Photocouplings of resonances
- high precision data from ELSA, MAMI, JLab...$\rightarrow$ resolve questionable/find new states

Photoproduction amplitude of pseudoscalar mesons:

Chew, Goldberger, Low, and Nambu, Phys. Rev. 106, 1345 (1957)

$$\hat{\mathcal{M}} = F_1 \vec{\sigma}\cdot\vec{\epsilon} + iF_2\vec{\epsilon}\cdot(\hat{k}\times\hat{q}) + F_3\vec{\sigma}\cdot\hat{k}\hat{q}\cdot\vec{\epsilon} + F_4\vec{\sigma}\cdot\hat{q}\hat{q}\cdot\vec{\epsilon}$$

$\vec{q}$: meson momentum
$\vec{k}$ ($\vec{\epsilon}$): photon momentum (polarization)

$F_i$: complex functions of the scattering angle, constructed from multipole amplitudes $M^{IJ}_{\mu\gamma}$

$\Rightarrow$ 16 polarization observables:
asymmetries composed of **beam**, **target** and/or **recoil** polarization measurements

$\Rightarrow$ **Complete Experiment**: unambiguous determination of the amplitude

8 carefully selected observables, including

Chiang and Tabakin, PRC 55, 2054 (1997)

- **single** and **double** polarization observbales
- measurement of **beam**, **target** and **recoil** polarization

↳ easier to realize in $K$ than in $\pi$ or $\eta$ photoproduction

$\hookrightarrow$ Caveat: in reality more observables needed (data uncertainties)