Baryon resonances in a combined analysis of pion- and photon-induced reactions

Recent results from the Juelich PWA

Deborah Rönchen

HISKP, Bonn University

In collaboration with:

NSTAR2015
May 25, 2015, Osaka, Japan
The excited hadron spectrum: Connection between experiment and QCD in the non-perturbative regime

**Experimental study of hadronic reactions**

- $\gamma + p \rightarrow X$
- $\gamma + p \rightarrow p + \pi^+$
- $\gamma + p \rightarrow p + \pi^0$
- $\gamma + p \rightarrow p + \pi^-$
- $\gamma + p \rightarrow K^+ + \Lambda$
- $\gamma + p \rightarrow p + \eta$

source: ELSA; data: ELSA, JLab, MAMI

**Major progress in recent years:**

- enlarged data base with high quality for different final states
- alternative source of information besides $\pi N \rightarrow X$
- measurement of several (double) polarization observables in $\gamma N \rightarrow X$
- towards a complete experiment: unambiguous determination of the amplitude (up to an overall phase)

**Extract information from experimental data:**

e.g. unitarized ChPT, “classical” PWA, K-Matrix, unitary isobar models ...

**Dynamical coupled channel (DCC) models:**

- combined analysis of different reactions
- wide energy range
- theoretical constraints of the $S$-matrix are met (or approximated)
The excited hadron spectrum: Connection between experiment and QCD in the non-perturbative regime

Experimental study of hadronic reactions

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Dynamical coupled channel (DCC) models:

- combined analysis of different reactions
- wide energy range
- theoretical constraints of the \( S \)-matrix are met (or approximated)
Theoretical constraints of the $S$-matrix

**Unitarity:** probability conservation
- 2-body unitarity
- 3-body unitarity: discontinuities from $t$-channel exchanges
  $\rightarrow$ Meson exchange from requirements of the $S$-matrix  
  [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]

**Analyticity:** from unitarity and causality
- correct structure of branch point, right-hand cut (real, dispersive parts)
- to approximate left-hand cut $\rightarrow$ Baryon $u$-channel exchange

$$\vec{q} = \vec{p}_1 - \vec{p}_3$$

$\vec{q} = \vec{q}_1 - \vec{q}_3$

$\vec{q} = \vec{p}_1 + \vec{p}_2 = 0$

$\rightarrow$ Resonances
Analytic structure of the amplitude
important information for a reliable determination of the resonance spectrum

**Resonances:** poles in the $T$-matrix on the 2. Riemann sheet

- Re($E_0$) = “mass”, -2Im($E_0$) = “width”
- Pole position $E_0$ is the same in all channels
- Residues $\rightarrow$ branching ratios

**Opening of inelastic channels:**
⇒ branch point and new Riemann sheet

Example: $\rho N$ branch point at $M_N + m_{\rho} = 1700 \pm 75$ MeV

Inclusion of branch points important to avoid false resonance signal!
Dynamical coupled-channels (DCC): \textit{simultaneous} analysis of different reactions

The scattering equation \textit{in} partial-wave basis

$$
\langle L'S'p'|T_{\mu\nu}^{ll}|LSp \rangle = \langle L'S'p'|V_{\mu\nu}^{ll}|LSp \rangle + \sum_{\gamma,L''S''} \int_{0}^{\infty} dq \, q^2 \langle L'S'p'|V_{\mu\gamma}^{ll}|L''S''q \rangle \frac{1}{E - E_{\gamma}(q) + i\epsilon} \langle L''S''q|T_{\gamma\nu}^{ll}|LSp \rangle
$$

- potentials $V$ constructed from effective $\mathcal{L}$
- $s$-channel diagrams: $T^P$
  - genuine resonance states
- $t$- and $u$-channel: $T^{NP}$
  - dynamical generation of poles
  - partial waves strongly correlated

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Combined analysis of pion- and photon-induced reactions
Dynamical coupled-channels (DCC): simultaneous analysis of different reactions

The scattering equation in partial-wave basis

\[
\langle L' S' p' | T_{\mu \nu}^{IJ} | L S p \rangle = \langle L' S' p' | V_{\mu \nu}^{IJ} | L S p \rangle + \sum_{\gamma, L'' S''} \int_0^{\infty} dq \, q^2 \langle L' S' p' | V_{\mu \gamma}^{IJ} | L'' S'' q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L'' S'' q | T_{\gamma \nu}^{IJ} | L S p \rangle
\]

- (2-body) unitarity and analyticity respected
- 3-body ππN channel:
  - parameterized effectively as πΔ, σN, ρN
  - πN/ππ subsystems fit the respective phase shifts
  - branch points move into complex plane
**Motivation and Introduction**

Data analysis and fit results

Extracting information from experiment

The Jülich model

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**Photoproduction**

Different approaches

- **Field theoretical approaches**: DMT, ANL-Osaka, Jülich-Athens-Washington, ...

  *Example*: Gauge invariant formulation by Haberzettl, Huang and Nakayama
  

  - complicated, involved construction/calculation

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**Focus of the present analysis:**

- **Extraction of resonance parameters**
  
  ⇒ flexible, **phenomenological** parameterization of photo excitation

  - Advantage: easy to implement, analyze large amounts of data
  - Disadvantage: no information on microscopic reaction dynamics
Motivation and Introduction
Data analysis and fit results

Photoproduction in a semi-phenomenological approach

**Multipole amplitude**

\[
M_{\mu\gamma}^{ij} = V_{\mu\gamma}^{ij} + \sum_{\kappa} T_{\mu\kappa}^{ij} G_{\kappa} V_{\kappa\gamma}^{ij}
\]

(partial wave basis)

\[
T_{\mu\kappa}: \text{Jülich hadronic } T\text{-matrix} \quad \rightarrow \text{Watson's theorem fulfilled by construction}
\]

**phenomenological potential:**

\[
V_{\mu\gamma}(E, q) = \begin{cases} 
\gamma \quad m \\
N \quad B
\end{cases} + \begin{cases} 
\gamma \quad N^*, \Delta^* \\
N \quad P_i^P \quad \gamma_{\mu} \\
N \quad B
\end{cases} = \frac{\tilde{\gamma}_{\mu}^a(q)}{m_N} P^{NP}_{\mu} (E) + \sum_i \frac{\gamma_{\mu;i}^a(q) P_i^P (E)}{E - m_i^b}
\]

\[
\tilde{\gamma}_{\mu}, \gamma_{\mu;i}^a: \text{hadronic vertices} \quad \rightarrow \text{correct threshold behaviour, cancellation of singularity at } E = m_i^b
\]

\[
\rightarrow \gamma_{\mu;i}^a \text{ affects pion- and photon-induced production of final state } mB
\]

\[
i: \text{resonance number per multipole; } \quad \mu: \text{channels } \pi N, \eta N, \pi \Delta
\]

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Data analysis and fit results
Combined analysis of pion- and photon-induced reactions

Simultaneous fit

Fit parameter:

- $\pi N \rightarrow \pi N$
  $\pi^- p \rightarrow \eta n, \ K^0 \Lambda, \ K^0 \Sigma^0, \ K^+ \Sigma^-$
  $\pi^+ p \rightarrow K^+ \Sigma^+$

$\Rightarrow$ 128 free parameters

- 11 $N^*$ resonances $\times$ (1 $m_{bare} +$ couplings to $\pi N, \rho N, \eta N, \pi \Delta, K \Lambda, K \Sigma$))
- 10 $\Delta$ resonances $\times$ (1 $m_{bare} +$ couplings to $\pi N, \rho N, \pi \Delta, K \Sigma$)

- $\gamma p \rightarrow \pi^0 p, \pi^+ n, \eta p$
  $\Rightarrow$ up to 456 free parameters
  couplings of the polynomials

$\downarrow$ calculations on the JUROPA supercomputer: parallelization in energy ($\sim 300 - 400$ processes)
## Data base

Data base

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Fit A</th>
<th>Fit B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi N \to \pi N$</td>
<td>PWA GW-SAID WI08 [Arndt et al., PRC 86 (2012)]</td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to \eta n$</td>
<td>$d\sigma/d\Omega$, $P$</td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to K^0\Lambda$</td>
<td>$d\sigma/d\Omega$, $P$, $\beta$</td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to K^0\Sigma^0$</td>
<td>$d\sigma/d\Omega$, $P$</td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to K^+\Sigma^-$</td>
<td>$d\sigma/d\Omega$</td>
<td></td>
</tr>
<tr>
<td>$\pi^+ p \to K^+\Sigma^+$</td>
<td>$d\sigma/d\Omega$, $P$, $\beta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sim 6000$ data points</td>
<td></td>
</tr>
<tr>
<td>$\gamma p \to \pi^0 p$</td>
<td>$d\sigma/d\Omega$, $\Sigma$, $P$, $T$, $\Delta\sigma_{31}$, $G$, $H$</td>
<td></td>
</tr>
<tr>
<td>$\gamma p \to \pi^+ n$</td>
<td>$d\sigma/d\Omega$, $\Sigma$, $P$, $T$, $\Delta\sigma_{31}$, $G$, $H$</td>
<td></td>
</tr>
<tr>
<td>$\gamma p \to \eta p$</td>
<td>$d\sigma/d\Omega$, $P$, $\Sigma$</td>
<td>$d\sigma/d\Omega$, $P$, $\Sigma$, $T$, $F$</td>
</tr>
<tr>
<td></td>
<td>29,392 data points</td>
<td>29,680 data points</td>
</tr>
</tbody>
</table>

- More single/double polarization:
  - $E$, $C_{x'L}$, $C_{z'L}$,
  - $T$, $P$, $H$ (ELSA 2014)
  - ($\gamma p \to \pi^0 p$)
  - $\Rightarrow$ predictions

$\Rightarrow$ predictions
Motivation and Introduction
Data analysis and fit results

Fit results \( \gamma p \rightarrow \eta p \)

selected results, arXiv:1504.01643 [nucl-th]

- T, F not included
- T, F included

**Differential cross section**

![Graphs of differential cross sections](image)

1488 \( [1] \)
1755 \( [2] \)
2142 \( [3] \)


**Recoil polarization**

- only 7 data points in total -

![Graphs of recoil polarization](image)

1496 \( [4] \)
1754 \( [4] \)
1843 \( [5] \)
1843 \( [5] \)


**Beam asymmetry**

![Graphs of beam asymmetry](image)

1496 \( [4] \)
1754 \( [4] \)
1843 \( [5] \)
1843 \( [5] \)


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Combined analysis of pion- and photon-induced reactions
Motivation and Introduction
Data analysis and fit results

Fit results
Resonance parameters

$F$ and $T$ in $\gamma p \rightarrow \eta p$

Data: Akondi et al. (A2 at MAMI) PRL 113 10, 102001 (2014)

Prediction
Fit

Polarization:

<table>
<thead>
<tr>
<th>Beam</th>
<th>Target</th>
<th>Recoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+1$</td>
<td>$+x$</td>
<td>0</td>
</tr>
<tr>
<td>$-1$</td>
<td>$+x$</td>
<td>0</td>
</tr>
</tbody>
</table>

Polarization:

<table>
<thead>
<tr>
<th>Beam</th>
<th>Target</th>
<th>Recoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$+y$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$-y$</td>
<td>0</td>
</tr>
</tbody>
</table>
Resonance content: I=1/2

Pole search on the 2nd sheet of the scattering matrix $T_{\mu\nu}$

Resonance parameter:
- "mass" = $\text{Re}(E_0)$
- "width" = $-2\text{Im}(E_0)$
- Residues $\rightarrow$ branching ratios

$E_0$: pole position

→ no new states compared to Jülich2012
(Jülich2012: only pion-induced data)

→ no narrow structure at 1.68 GeV
(seen in eta photoproduction on the neutron)
## Resonance parameters

selected results, arXiv:1504.01643 [nucl-th]

|            | $\text{Re } E_0$ | $-2\text{Im } E_0$ | $|r_{\pi N}|$ | $\theta_{\pi N \rightarrow \pi N}$ | $\Gamma_{\pi N}^{1/2} \Gamma_{\eta N}^{1/2}$ | $\Gamma_{\text{tot}}$ | $\theta_{\pi N \rightarrow \eta N}$ |
|------------|------------------|--------------------|--------------|----------------|-----------------------------|----------------|----------------|
|            | [MeV]            | [MeV]              | [MeV]        | [deg]          | [%]                        |               | [deg]          |
| fit        |                  |                    |             |                |                            |               |                |
| $N(1535) \, 1/2^-$ |                 |                    |             |                |                            |               |                |
| A          | 1497             | 105                | 23           | $-48$          | 51                         |                 | 110            |
| B          | 1499             | 104                | 22           | $-46$          | 51                         |                 | 112            |
| $A_{\text{had}}$ | 1498             | 74                 | 17           | $-37$          | 51                         |                 | 120            |
| $N(1710) \, 1/2^+$ |                 |                    |             |                |                            |               |                |
| A          | 1611             | 140                | 2.7          | $-40$          | 6.1                        |                 | 175            |
| B          | 1651             | 121                | 3.2          | 55             | 16                         |                 | $-180$         |
| $A_{\text{had}}$ | 1637             | 97                 | 4            | $-30$          | 24                         |                 | 130            |

fit A: $T, F$ not included  
fit B: $T, F$ included  
fit $A_{\text{had}}$: Jülich2012, only pion-induced data
Motivation and Introduction
Data analysis and fit results
Fit results
Resonance parameters

Multipoles for $\gamma p \rightarrow \eta p$

Comparison with the Bonn-Gatchina 2014-02 solution

(Black) solid line: BG 2014-02. Dashed (blue) line: fit A; solid (red) line: fit B.

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Motivation and Introduction

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Resonance parameters

**Multipoles:** $\gamma p \rightarrow \eta p$ vs $\gamma p \rightarrow \pi^0 p$

Comparison with the Bonn-Gatchina 2014-02 solution

- Example: $E_{3-}$ and $M_{3-}$ multipoles ($F_{15}$, $F_{35}$)

$\Rightarrow$ Multipole content of $\gamma p \rightarrow \eta p$ seems less well established than for $\gamma p \rightarrow \pi^0 p$

$\Rightarrow$ Convergence with larger data base of $\gamma p \rightarrow \eta p$?
Summary

Extraction of the $N^*$ and $\Delta$ resonance spectrum
from a simultaneous analysis of pion- and photon-induced reactions

- DCC analysis of $\pi N \rightarrow \pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$
- $\pi$ and $\eta$ photoproduction in a semi-phenomenological approach

Comparison of 3 different fits:

- simultaneous fit of $\pi N$, $\gamma N \rightarrow X$ without recent MAMI $T$ and $F$ data
- simultaneous fit of $\pi N$, $\gamma N \rightarrow X$ with recent MAMI $T$ and $F$ data
- earlier fit (Jülich2012), only $\pi N \rightarrow X$

$\Rightarrow$ noticeable influence of photoproduction data in general / new polarization observables
- on pole positions and photocouplings
- on hadronic couplings
Thank you for your attention!
Photocouplings at the pole

\[ \tilde{A}_{\text{pole}}^h = A_{\text{pole}}^h e^{i\vartheta^h} \]

\[ h = 1/2, 3/2 \]

\[ \tilde{A}_{\text{pole}}^h = I_F \sqrt{\frac{q_p}{k_p} \frac{2\pi (2J+1)E_0}{m_N r_{\pi N}}} \text{Res} A_{L\pm}^h \]

\[ I_F: \text{isospin factor} \]
\[ q_p (k_p): \text{meson (photon) momentum at the pole} \]
\[ J = L \pm 1/2 \text{ total angular momentum} \]
\[ E_0: \text{pole position} \]
\[ r_{\pi N}: \text{elastic } \pi N \text{ residue} \]

<table>
<thead>
<tr>
<th></th>
<th>( A_{\text{pole}}^{1/2} )</th>
<th>( \vartheta^{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([10^{-3} \text{ GeV}^{-1/2}])</td>
<td>[deg]</td>
</tr>
<tr>
<td>fit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N(1535) ) 1/2(^-)</td>
<td>A</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(50^{+4}_{-4})</td>
</tr>
<tr>
<td>( N(1710) ) 1/2(^+)</td>
<td>A</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(28^{+9}_{-2})</td>
</tr>
</tbody>
</table>

Fit A: \( T, F \) not included

Fit B: \( T, F \) included

Fit 2: Jülich2013, only pion photoproduction
(same pole positions as fit \( A_{\text{had}} \))
Multipoles: $\gamma p \rightarrow \pi^0 p$

Comparison with the Bonn-Gatchina 2014-02 solution

(Black) solid line: BG 2014-02. Dashed (blue) line: fit A; solid (red) line: fit B.
Resonance parameters
Photon-induced reactions

Pion-induced eta production: $\pi^- p \rightarrow \eta n$

$\frac{d\sigma}{d\Omega}$ [mb/sr]

- 1509 MeV
- 1576 MeV
- 1664 MeV
- 1729 MeV
- 1805 MeV
- 2235 MeV
- 1740 MeV
- 1872 MeV
- 1989 MeV
- 2024 MeV
- 2070 MeV
- 2235 MeV

$P \times \frac{d\sigma}{d\Omega}$ [mb/sr]

- 1740 MeV
- 1872 MeV
- 1989 MeV
- 2024 MeV
- 2070 MeV
- 2235 MeV

fit A: $T, F$ not included

fit B: $T, F$ included

fit $A_{\text{had}}$: Jülich2012, only pion-induced data
Resonance parameters

Photon-induced reactions

Resonance content: \( I=3/2 \)

Pole search on the 2\(^{nd}\) sheet of the scattering matrix \( T_{\mu\nu} \)

Resonance parameter:
- "mass" = \( \text{Re}(E_0) \)
- "width" = \(-2\text{Im}(E_0)\)
- Residues \( \rightarrow \) branching ratios

\( E_0 \): pole position

\[ \rightarrow \text{no new states compared to J"ulich2012} \]

(J"ulich2012: only pion-induced data)

\[ \times : \text{branch points} \quad \text{Notation: } N(\text{"name"}) J^{\text{parity}} \]
Selected results: Fit A and Fit B
**Pion photoproduction: selected fit results**

\[ \gamma p \rightarrow \pi^0 p \]

- \( 0.01 \text{ d}\sigma/\text{d}\Omega \) [\( \mu b/\text{sr} \)]
- 1074 MeV
- \( \Theta [\text{deg}] \)
- [1] Schmidt 2001 (MAMI)
- [3] Sparks 2010 (ELSA)
- [5] Thiel 2012 (ELSA)

\[ \gamma p \rightarrow \pi^+ n \]

- \( 1162 \) [\( \mu b/\text{sr} \)]
- \( \Theta [\text{deg}] \)
- [8] Bartalini 2002 (GRAAL)
- [10] Ahrens 2005 (MAMI)
Double polarization in $\gamma p \to \pi^0 p$

Data NOT included in fit

selected results, arXiv:1504.01643 [nucl-th]

[1] Gottschall et al. 2013 (ELSA) PRL. 112 1, 012003


Polarization

<table>
<thead>
<tr>
<th>Beam</th>
<th>Target</th>
<th>Recoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+1$</td>
<td>$-z$</td>
<td>$0$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-z$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Polarization

<table>
<thead>
<tr>
<th>Beam</th>
<th>Target</th>
<th>Recoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\perp'$</td>
<td>$x$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\parallel'$</td>
<td>$x$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

new ELSA T,P data

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Resonance parameters
Photon-induced reactions

$T, P$ in $\gamma p \rightarrow \pi^0 p$
Data NOT included in the Fit

Data: J. Hartmann et al. 2014 (ELSA)
Partial wave contribution in $F$ and $T$ in $\gamma p \rightarrow \eta p$

Fit B, full solution
$S_{11}$, $P_{13}$, $D_{13}$
Partial wave contribution in $F$ and $T$ in $\gamma p \rightarrow \eta p$

Fit B, full solution
$S_{11}, P_{13}, D_{13}$
all s-, p-, d-, f-waves
Details of the formalism

Polynomials:

\[ P^P_i(E) = \sum_{j=1}^{n} g^P_{i,j} \left( \frac{E - E_0}{m_N} \right)^j e^{-g^P_{i,n+1}(E-E_0)} \]

\[ P^{NP}_\mu(E) = \sum_{j=0}^{n} g^{NP}_{\mu,j} \left( \frac{E - E_0}{m_N} \right)^j e^{-g^{NP}_{\mu,n+1}(E-E_0)} \]

- \( E_0 = 1077 \text{ MeV} \)
- \( g^P_{i,j}, g^{NP}_{\mu,j} \): fit parameter
- \( e^{-g(E-E_0)} \): appropriate high energy behavior
- \( n = 3 \)
## World data base on $\eta N$, $K\Lambda$, $K\Sigma$

<table>
<thead>
<tr>
<th>Reaction</th>
<th>PWA</th>
<th>$\sigma_{tot}$</th>
<th>$d\sigma/d\Omega$</th>
<th>$P$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi N \to \pi N$</td>
<td>GWU/SAID 2006 up to $J=9/2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to \eta n$</td>
<td>62 data points</td>
<td>38 energy points</td>
<td>12 energy points</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z=1489$ to 2235 MeV</td>
<td>1740 to 2235 MeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to K^0\Lambda$</td>
<td>66 data points</td>
<td>46 energy points</td>
<td>27 energy points</td>
<td>7 energy points</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1626 to 1405 MeV</td>
<td>1633 to 2208 MeV</td>
<td>1852 to 2262 MeV</td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to K^0\Sigma^0$</td>
<td>16 data points</td>
<td>29 energy points</td>
<td>19 energy points</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1694 to 2405 MeV</td>
<td>1694 to 2316 MeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^- p \to K^+\Sigma^-$</td>
<td>14 data points</td>
<td>15 energy points</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1739 to 2405 MeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^+ p \to K^+\Sigma^+$</td>
<td>18 data points</td>
<td>32 energy points</td>
<td>32 energy points</td>
<td>2 energy points</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1729 to 2318 MeV</td>
<td>1729 to 2318 MeV</td>
<td>2021 and 2107 MeV</td>
<td></td>
</tr>
</tbody>
</table>

$\sim 6000$ data points
Resonance parameters
Photon-induced reactions
Resonance parameters
Photon-induced reactions

Combined analysis of pion- and photon-induced reactions

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Resonance parameters
Photon-induced reactions

Combined analysis of pion- and photon-induced reactions
**Error analysis**

- $\chi^2 + 1$ criterion: determination of the non-linear parameter error
  - error of parameter $p_i$ determined by range of $p_i$ such that $\chi^2_{\text{min}}$ rises by less than 1
  - $\Rightarrow$ error on pole positions and residues.

**NPBA 851, 58 (2011)**

**BUT:** numerically not possible with $\geq 500$ free parameters

**Work in progress:** Developing of techniques to apply Monte-Carlo error propagation using bootstrap method (M. Döring et al.)
Matching to lattice
Prediction & analysis of lattice data

[Resonance parameters]
[Photon-induced reactions]

Matching to lattice
Prediction & analysis of lattice data

Scattering equation:

\[ T(q'', q') = V(q'', q') + \int_0^\infty dq q^2 V(q'', q) \frac{1}{z - E_1(q) - E_2(q) + i\epsilon} T(q, q') \]

\[ T(q'', q') = V(q'', q') + \frac{2\pi^2}{L^3} \sum_{i=0}^\infty \vartheta(i) V(q'', q_i) \frac{1}{z - E_1(q_i) - E_2(q_i)} T(q_i, q') \]

\( \vartheta^{(P)}(i) \) series

- Study finite-volume effects
- Predict lattice spectra

Discretization of momenta in the scattering equation:

\[ \int \frac{d^3 q}{(2\pi)^3} f(|\vec{q}|^2) \rightarrow \frac{1}{L^3} \sum_{i} f(|\vec{q}_i|^2), \quad \vec{q}_i = \frac{2\pi}{L} \vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3 \]

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**Notation:** $L_{2I/2J}$

- **Input to fit:** energy-dependent partial wave analysis, GWU/SAID 2006 up to $J = 9/2$ ($\sim H_{39}$)
\[ \pi N \rightarrow \eta N, K \Lambda \] selected results, arXiv:1504.01643 [nucl-th]
\[ \pi N \rightarrow K \Sigma \]

**Photon-induced reactions**

\[ \pi^- p \rightarrow K^0 \Sigma^0 \]

\[ \pi^- p \rightarrow K^+ \Sigma^- \]

\[ \pi^+ p \rightarrow K^+ \Sigma^+ \]

*No polarization data!*
Partial wave contribution to $F$ in $\gamma p \rightarrow \eta p$

Switch off different PWs in Fit B

$E_{0+}$ switched off
Switch off different PWs in Fit B

-1 -0.5 0 0.5 1
0 30 60 90 120 150

F

1497 1516 1534 1558
1588 1617 1646 1674
1702 1743 1796
1847

M_{1-} switched off
Switch off different PWs in Fit B

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Combined analysis of pion- and photon-induced reactions
Photoproduction of pseudoscalar meson

- Photocouplings of resonances
- High precision data from ELSA, MAMI, JLab... → resolve questionable/find new states

Photoproduction amplitude of pseudoscalar mesons:

\[ \hat{M} = F_1 \sigma \cdot \epsilon + i F_2 \epsilon \cdot (\hat{k} \times \hat{q}) + F_3 \sigma \cdot \hat{k} \hat{q} \cdot \epsilon + F_4 \sigma \cdot \hat{q} \hat{q} \cdot \epsilon \]

- \( F_i \): complex functions of the scattering angle, constructed from multipole amplitudes \( M_{\mu \gamma}^{IJ} \)

⇒ 16 polarization observables:
  - asymmetries composed of beam, target and/or recoil polarization measurements

⇒ **Complete Experiment**: unambiguous determination of the amplitude

8 carefully selected observables, including

- **single** and **double** polarization observables
- Measurement of beam, target and recoil polarization

\[ \downarrow \] easier to realize in \( K \) than in \( \pi \) or \( \eta \) photoproduction

\[ \leftarrow \] Caveat: in reality more observables needed (data uncertainties)