Properties of baryons from the Bonn-Gatchina partial wave analysis

A. Sarantsev

Petersburg Nuclear Physics Institute

HISKP, Bonn (Bonn University)

PNPI, Gatchina (Russia)

25-28 May 2015, Osaka

Bonn-Gatchina partial wave analysis group:

A. Anisovich, E. Klempt, V. Nikonov, A. Sarantsev, U. Thoma.

http://pwa.hiskp.uni-bonn.de/

Petersburg Nuclear Physics Institute

Bonn-Gatchina Partial Wave Analysis

Address: Nussallee 14-16, D-53115 Bonn   Fax: (+49) 228 / 73-2505

Data Base | Meson Spectroscopy | Baryon Spectroscopy | NN-interaction | Formalism

Analysis of Other Groups

- SAID
- MAID
- Giessen Uni

BG PWA

- Publications
- Talks
- Contacts

Useful Links

- SPIRES
- PDG Homepage
- Durham Data Base
- Bonn Homepage

CB-ELSA Homepage

Responsible: Dr. V. Nikonov, E-mail: *nikonov@hiskp.uni-bonn.de*
Last changes: January 26th, 2010.

The new solutions BG2014-01 and BG2014-02 are released

Energy dependent approach

In many cases an unambiguous partial wave decomposition at fixed energies is impossible. Then the energy and angular parts should be analyzed together:

$$A(s,t) \;=\; \sum_{\beta\beta' n} A_n^{\beta\beta'}(s) Q^{(\beta)+}_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} Q^{(\beta')}_{\nu_1\ldots\nu_n}$$

1. **Correlations between angular part and energy part are under control.**
2. **Unitarity and analyticity can be introduced from the beginning.**
3. **Parameters can be fixed from a combined fit of many reactions.**

1 **C. Zemach, Phys. Rev. 140, B97 (1965); 140, B109 (1965)**

2 **S.U.Chung, Phys. Rev. D 57, 431 (1998)**

3 **A. V. Anisovich, V. V. Anisovich, V. N. Markov, M. A. Matveev and A. V. Sarantsev, J. Phys. G G 28, 15 (2002)**

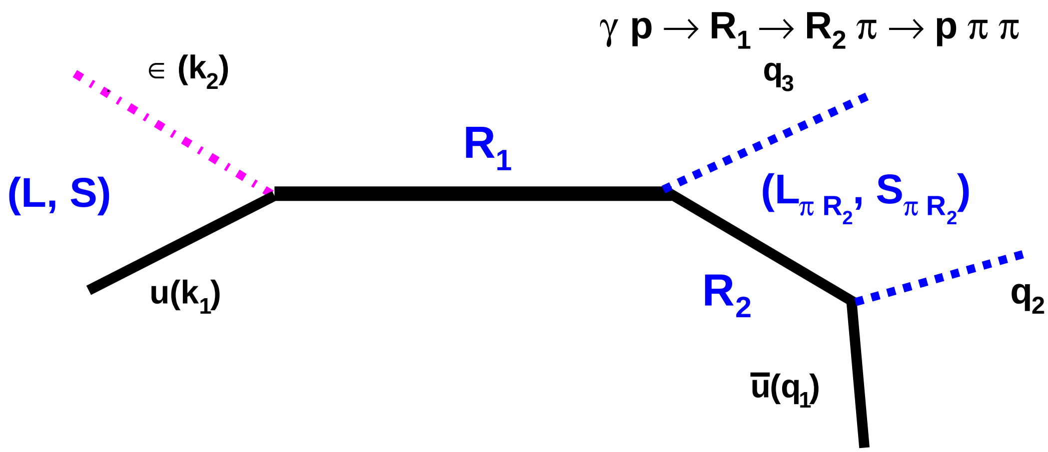
4 **B. S. Zou and D. V. Bugg, Eur. Phys. J. A 16, 537 (2003)**

5 **A. Anisovich, E. Klempt, A. Sarantsev and U. Thoma, Eur. Phys. J. A 24, 111 (2005)**

6 **A. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A 30, 427 (2006)**

7 **A. V. Anisovich, V. V. Anisovich, E. Klempt, V. A. Nikonov and A. V. Sarantsev, Eur. Phys. J. A 34, 129 (2007).**

Resonance amplitudes for meson photoproduction



General form of the angular dependent part of the amplitude:

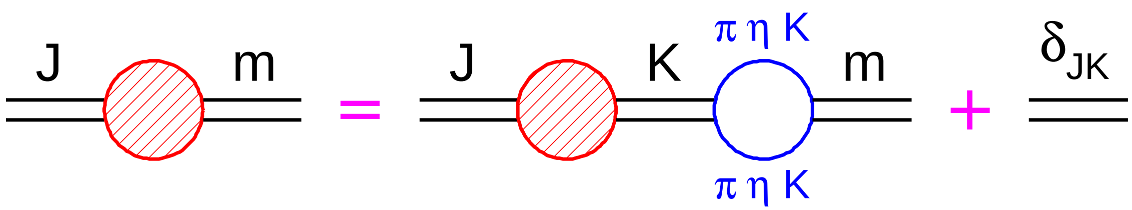
$$\bar{u}(q_1)\tilde{N}_{\alpha_1\ldots\alpha_n}(R_2\to\mu N)F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}(q_1+q_2)\tilde{N}^{(j)\beta_1\ldots\beta_n}_{\gamma_1\ldots\gamma_m}(R_1\to\mu R_2)$$

$$F^{\gamma_1\ldots\gamma_m}_{\xi_1\ldots\xi_m}(P)V^{(i)\mu}_{\xi_1\ldots\xi_m}(R_1\to\gamma N)u(k_1)\varepsilon_\mu$$

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat{p})O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\left(g^{\perp}_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

$$\sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$

N/D based (D-matrix) analysis of the data



$$D_{jm} = D_{jk}\sum_\alpha B^{km}_\alpha(s)\frac{1}{M_m - s} + \frac{\delta_{jm}}{M_j^2 - s} \qquad \hat{D} = \hat{\kappa}(I - \hat{B}\hat{\kappa})^{-1}$$

$$\hat{\kappa} = diag\left(\frac{1}{M_1^2 - s}, \frac{1}{M_2^2 - s}, \ldots, \frac{1}{M_N^2 - s}, R_1, R_2 \ldots\right)$$

$$\hat{B}_{ij} = \sum_\alpha B^{ij}_\alpha = \sum_\alpha \int \frac{ds'}{\pi} \frac{g^{(R)i}_\alpha \rho_\alpha(s', m_{1\alpha}, m_{2\alpha}) g^{(L)j}_\alpha}{s' - s - i0}$$

In the present fits we calculate the elements of the B^{ij}_α using one subtraction taken at the channel threshold $M_\alpha = (m_{1\alpha} + m_{2\alpha})$:

$$B^{ij}_\alpha(s) = B^{ij}_\alpha(M^2_\alpha) + (s - M^2_\alpha) \int\limits_{m_a^2}^{\infty} \frac{ds'}{\pi} \frac{g^{(R)i}_\alpha \rho_\alpha(s', m_{1\alpha}, m_{2\alpha}) g^{(L)j}_\alpha}{(s' - s - i0)(s' - M^2_\alpha)}.$$

In this case the expression for elements of the $\hat{B}$ matrix can be rewritten as:

$$B^{ij}_\alpha(s) = g^{(R)i}_a \left(b^\alpha + (s - M^2_\alpha) \int\limits_{m_a^2}^{\infty} \frac{ds'}{\pi} \frac{\rho_\alpha(s', m_{1\alpha}, m_{2\alpha})}{(s' - s - i0)(s' - M^2_\alpha)} \right) g^{(L)j}_\beta = g^{(R)i}_a B_\alpha g^{(L)j}_\beta$$

and D-matrix method equivalent to the K-matrix method with loop diagram with real part taken into account:

$$A = \hat{K}(I - \hat{B}\hat{K})^{-1} \qquad B_{\alpha\beta} = \delta_{\alpha\beta} B_\alpha$$

Minimization methods

1. **The two body final states $\pi N, \gamma N \to \pi N, \eta N, K\Lambda, K\Sigma, \omega N, K^*\Lambda$: χ^2 method.**

   For n measured bins we minimize

$$\chi^2 = \sum_j^n \frac{(\sigma_j(PWA) - \sigma_j(exp))^2}{(\Delta\sigma_j(exp))^2}$$

   Present solution $\chi^2 = 54634$ for 33988 points. $\chi^2/N_F = 1.6$

2. **Reactions with three or more final states are analyzed with logarithm likelihood method. $\pi N, \gamma N \to \pi\pi N, \pi\eta N, \omega p, K^*\Lambda$. The minimization function:**

$$f = -\sum_j^{N(data)} ln \frac{\sigma_j(PWA)}{\sum\limits_m^{N(rec\,MC)} \sigma_m(PWA)}$$

   This method allows us to take into account all correlations in many dimensional phase space. Above 1 000 000 data events are taken in the fit.

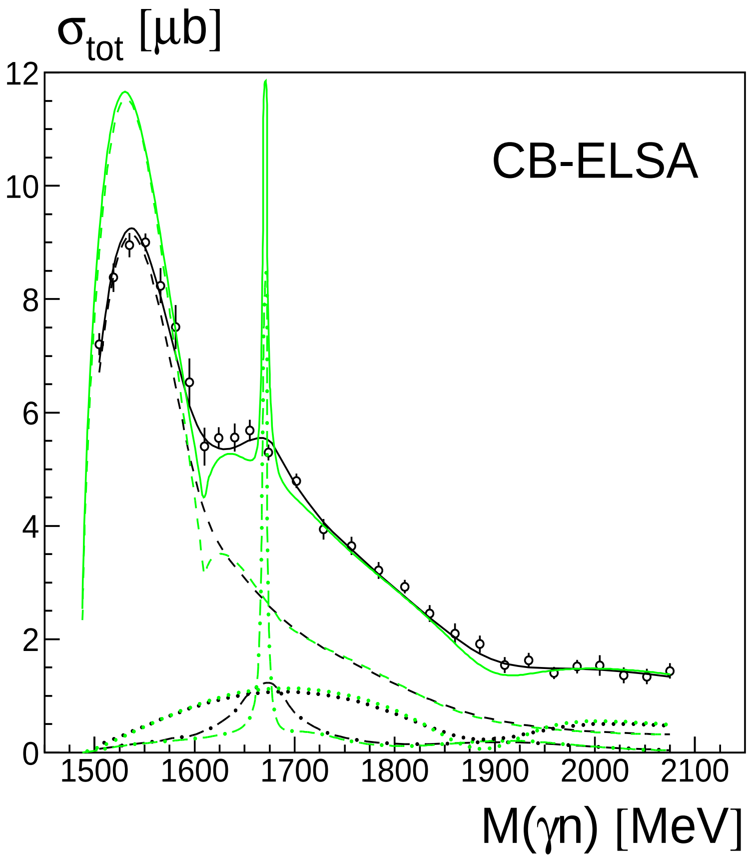
Baryon data base

DATA	BG2013-2014	added in BG2014-2015
$\pi N \to \pi N$ **ampl.**	**SAID or Hoehler energy fixed**	
$\gamma p \to \pi N$	$\frac{d\sigma}{d\Omega}, \Sigma, T, P, E, G, H$	E **(CB-ELSA, CLAS)**
$\gamma n \to \pi N$	$\frac{d\sigma}{d\Omega}, \Sigma, T, P$	$\frac{d\sigma}{d\Omega} (MAMI)$
$\gamma n \to \eta n$	$\frac{d\sigma}{d\Omega}, \Sigma$	$\frac{d\sigma}{d\Omega}$ **(MAMI)**
$\gamma p \to \eta p$	$\frac{d\sigma}{d\Omega}, \Sigma$	T, P, H, E **(CB-ELSA)**
$\gamma p \to \eta' p$		$\frac{d\sigma}{d\Omega}, \Sigma$
$\gamma p \to K^+ \Lambda$	$\frac{d\sigma}{d\Omega}, \Sigma, P, T, C_x, C_z, O_{x'}, O_{z'}$	Σ, P, T, O_x, O_z **(CLAS)**
$\gamma p \to K^+ \Sigma^0$	$\frac{d\sigma}{d\Omega}, \Sigma, P, C_x, C_z$	Σ, P, T, O_x, O_z **(CLAS)**
$\gamma p \to K^0 \Sigma^+$	$\frac{d\sigma}{d\Omega}, \Sigma, P$	
$\pi^- p \to \eta n$	$\frac{d\sigma}{d\Omega}$	
$\pi^- p \to K^0 \Lambda$	$\frac{d\sigma}{d\Omega}, P, \beta$	
$\pi^- p \to K^0 \Sigma^0$	$\frac{d\sigma}{d\Omega}, P\ (K^0\Sigma^0)\ \frac{d\sigma}{d\Omega}\ (K^+\Sigma^-)$	
$\pi^+ p \to K^+ \Sigma^+$	$\frac{d\sigma}{d\Omega}, P, \beta$	
$\pi^- p \to \pi^0 \pi^0 n$	$\frac{d\sigma}{d\Omega}$ **(Crystal Ball)**	
$\pi^- p \to \pi^+ \pi^- n$		$\frac{d\sigma}{d\Omega}$ **(HADES)**
$\gamma p \to \pi^0 \pi^0 p$	$\frac{d\sigma}{d\Omega}, \Sigma, E, I_c, I_s$	
$\gamma p \to \pi^0 \eta p$	$\frac{d\sigma}{d\Omega}, \Sigma, I_c, I_s$	
$\gamma p \to \pi^+ \pi^- p$		$\frac{d\sigma}{d\Omega}, I_c, I_s$ **(CLAS)**
$\gamma p \to \omega p$		$\frac{d\sigma}{d\Omega}, \Sigma, \rho^0_{ij}, \rho^1_{ij}, \rho^2_{ij}, E, G$ **(CB-ELSA)**
$\gamma p \to K^*(890)\Lambda$		$\frac{d\sigma}{d\Omega}, \Sigma, \rho^0_{ij}$ **(CLAS)**

New MAMI Data on $\gamma n \to \eta n$ reaction

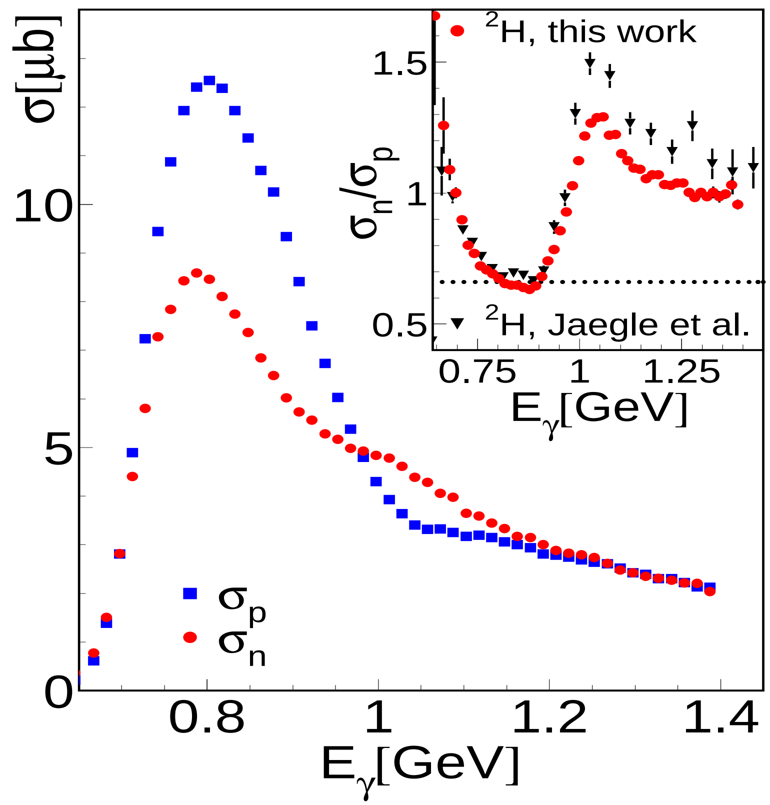
Fermi motion smearing

CB-ELSA



B. Krusche group

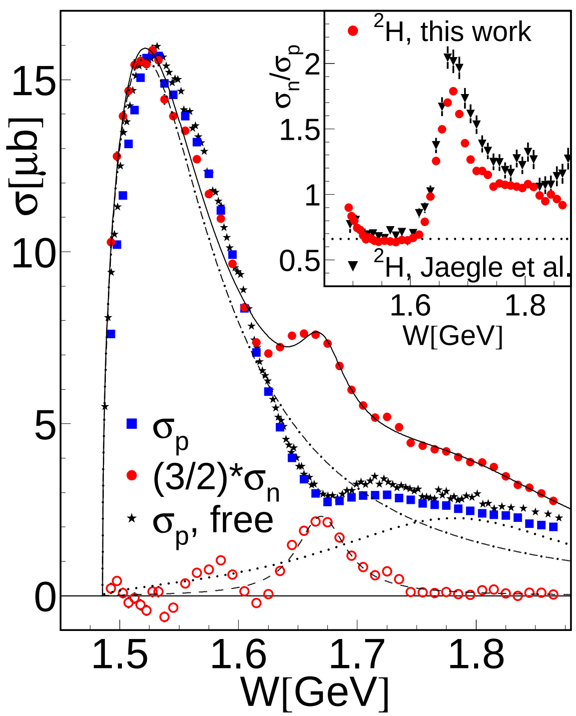
MAMI

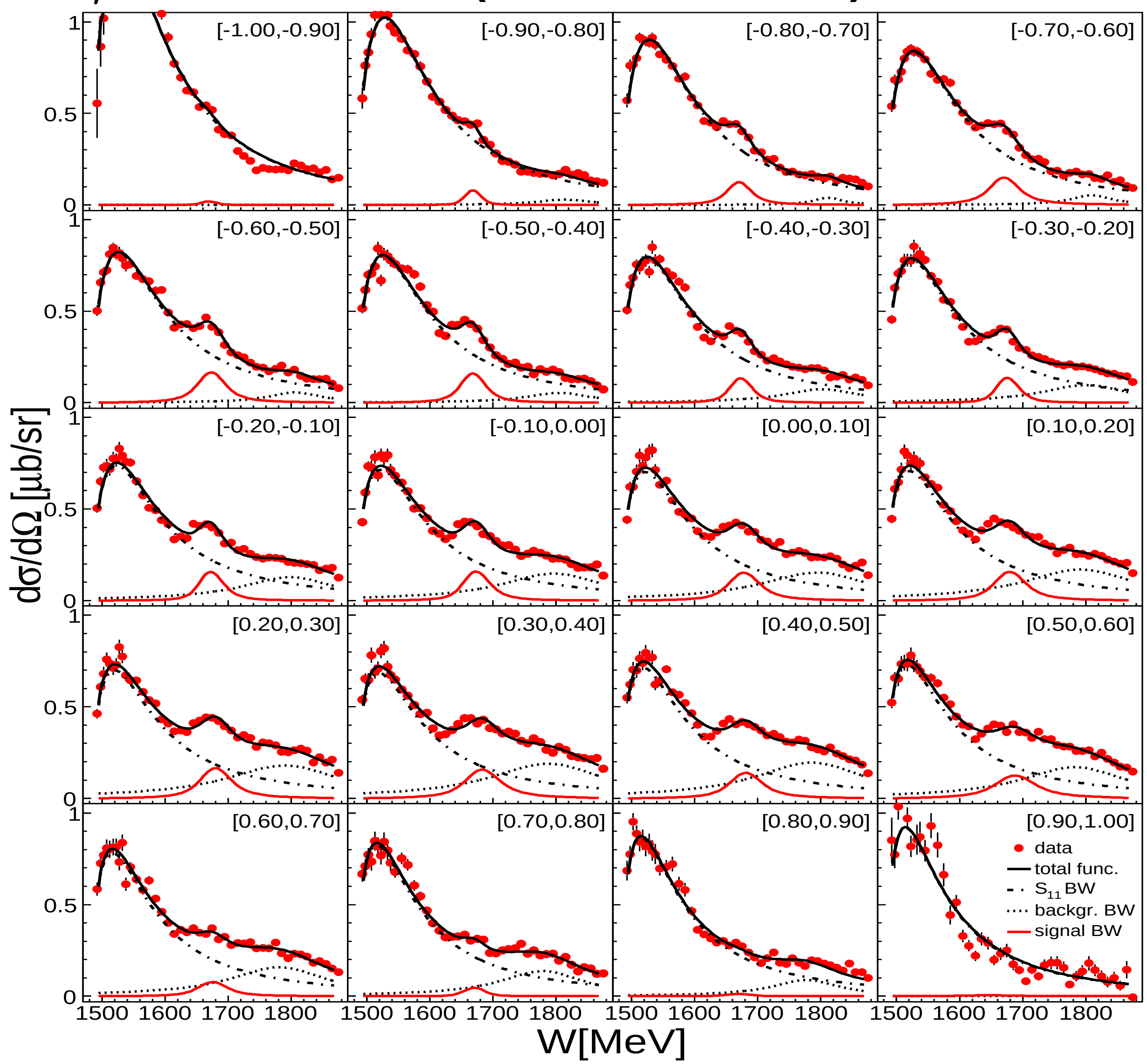


B. Krusche group

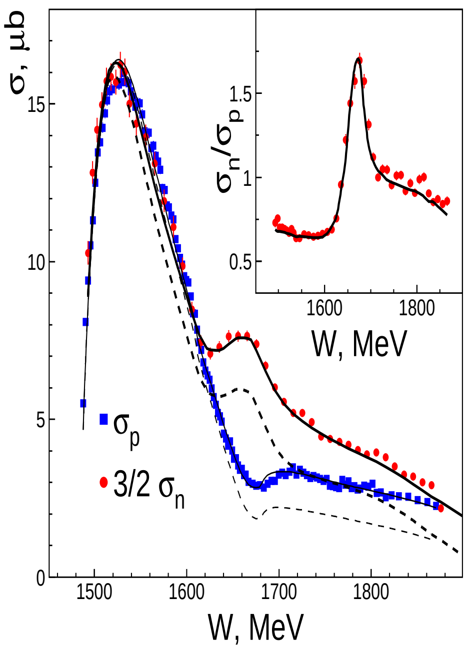
Full event reconstruction

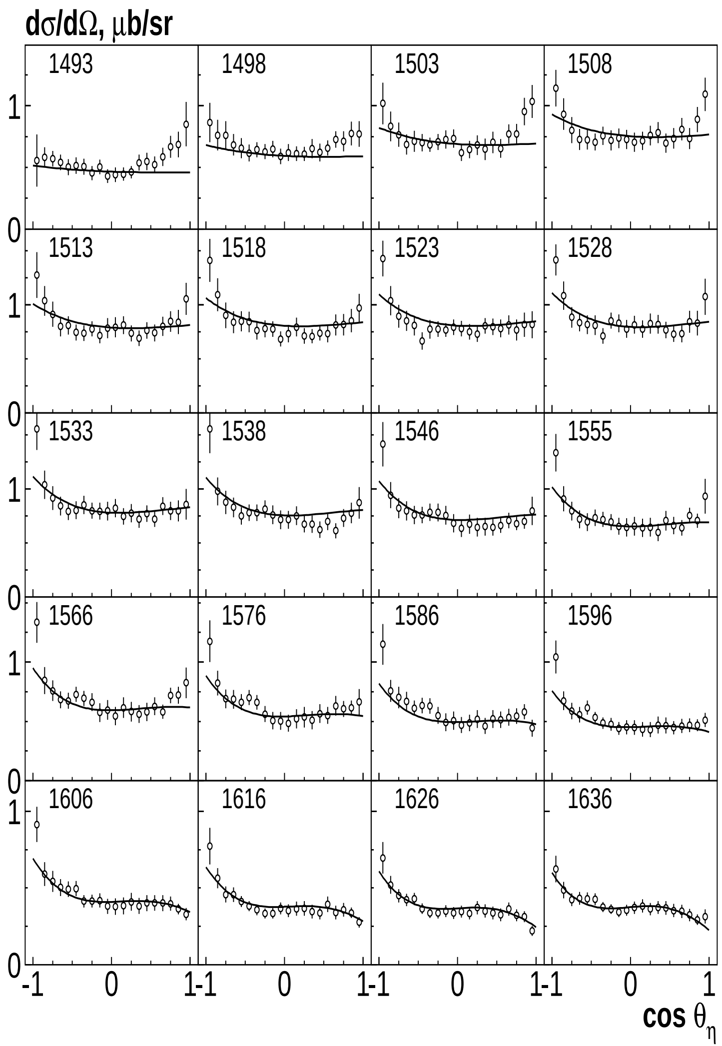
Energy resolution of η: ΔW=10-42 MeV (W=1500-1850 MeV)

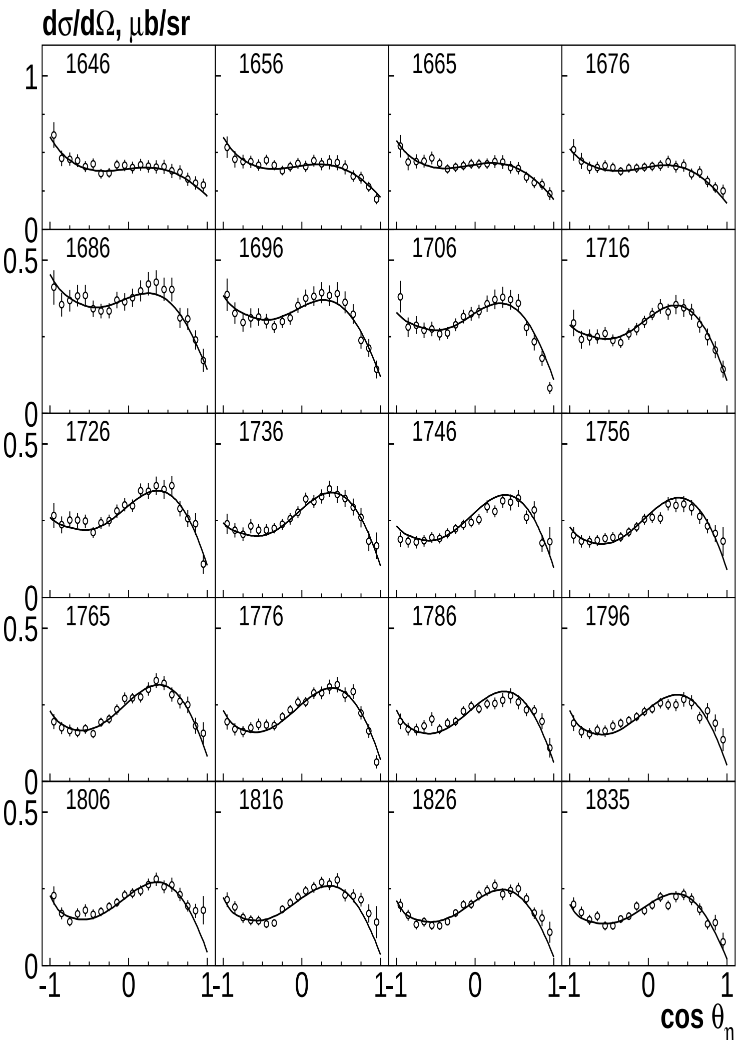




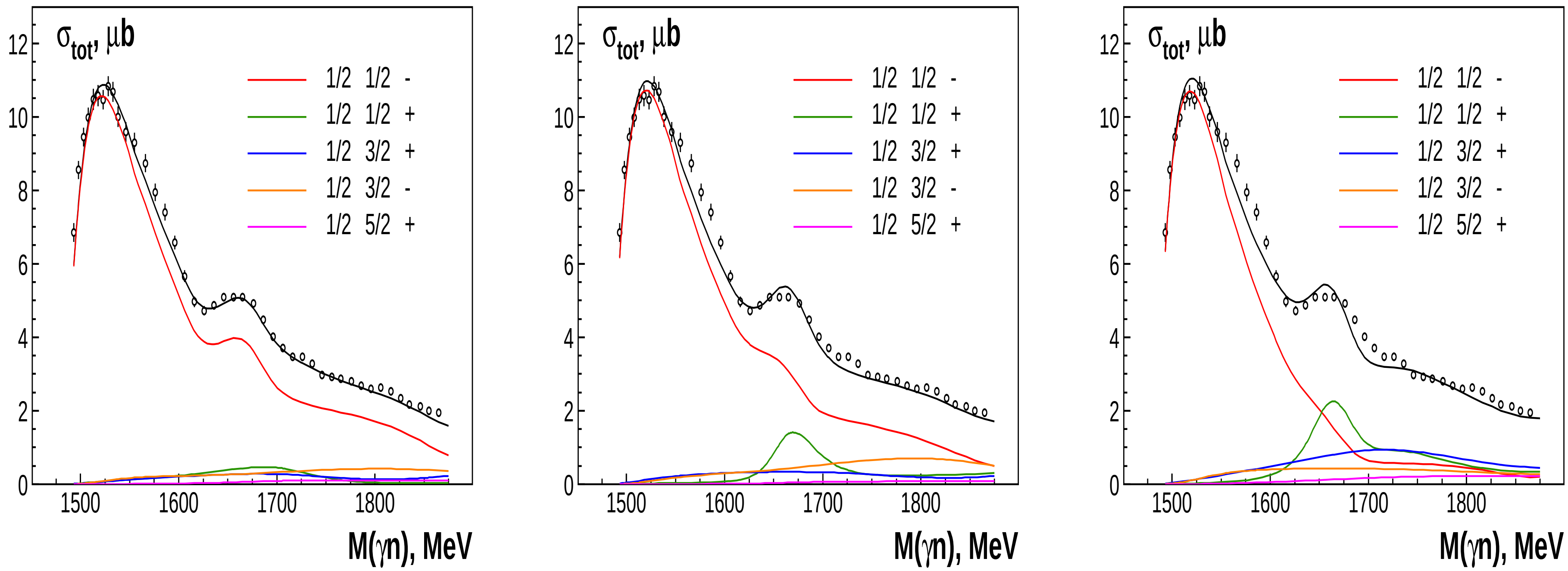
Solution with interference between S_{11} states



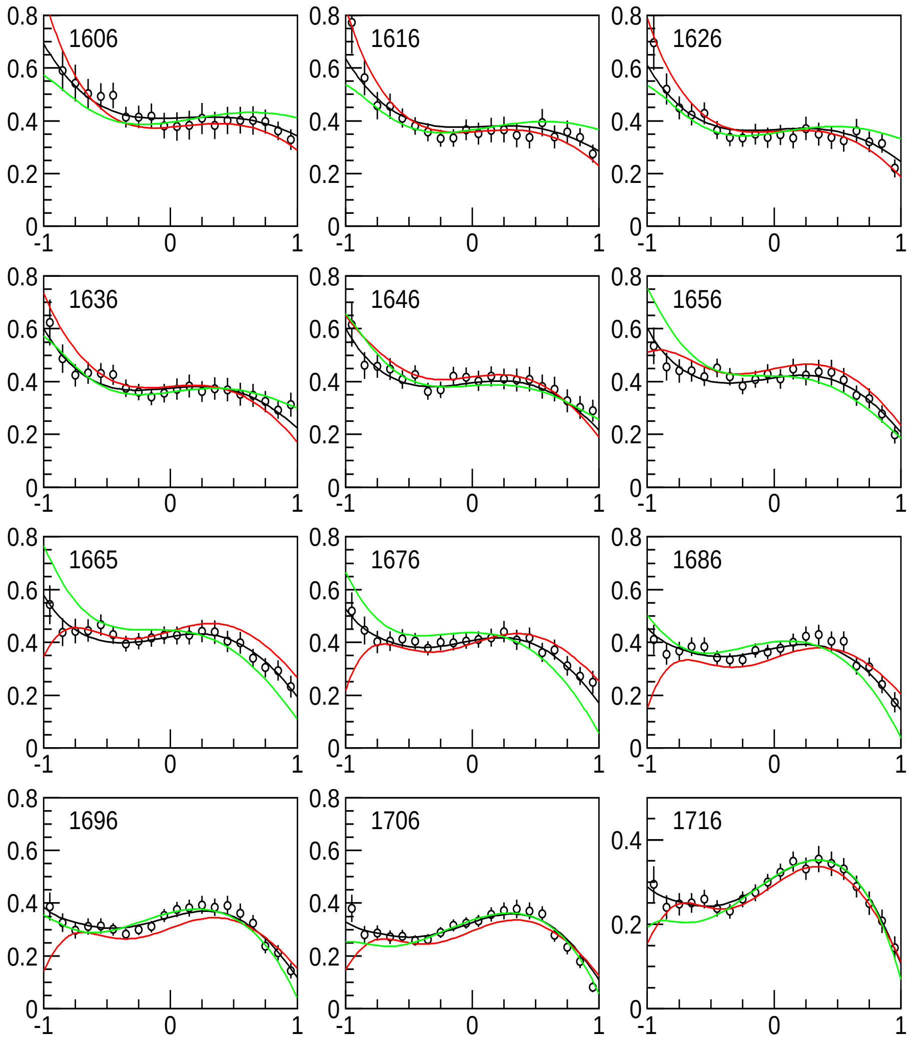


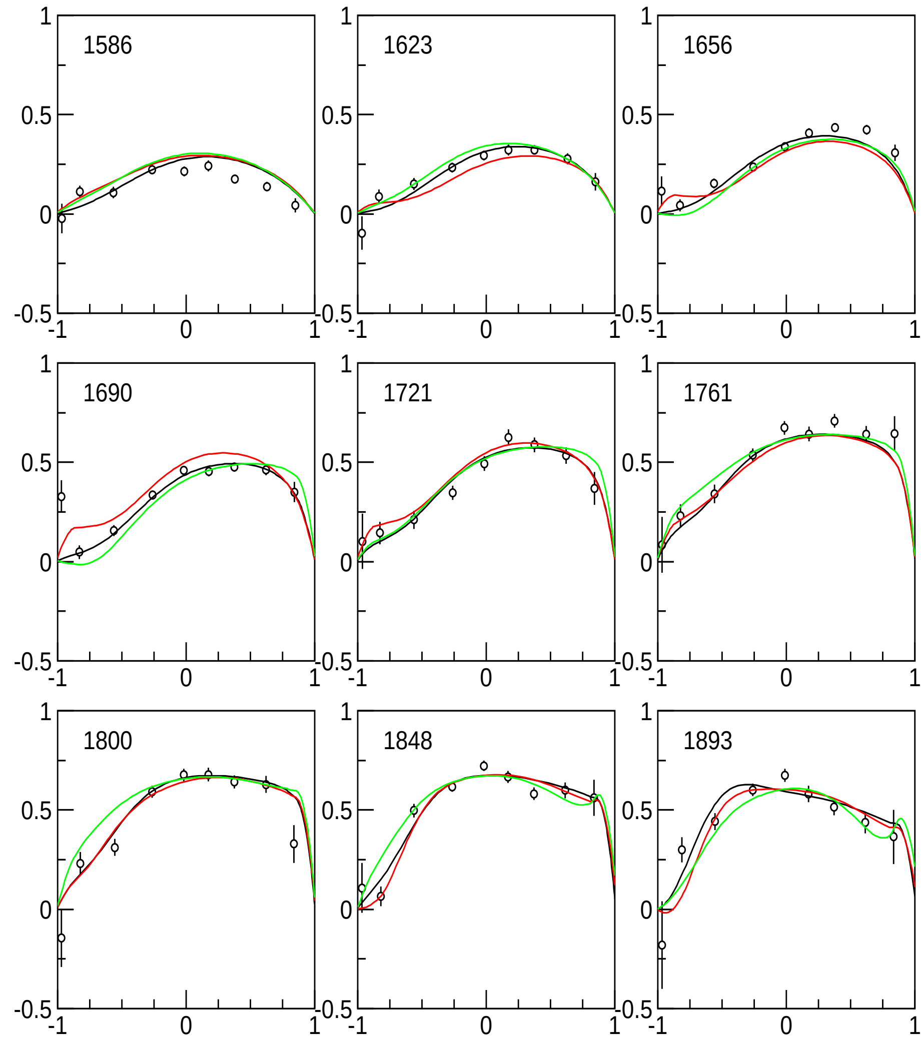


Solutions with the $P_{11}(1680)$ states



The description of the new data as well as GRAAL data is notably worse





Limit for the production of $P_{11}(1680)$**:** $|A^{\frac{1}{2}}|Br(\eta n) <$ **5 Gev**$^{-\frac{1}{2}}10^{-3}$

Helicity amplitudes for S_{11} states. The results is compared with Single-Quark-Transition model calculations V. D. Burkert, R. De Vita, M. Battaglieri, M. Ripani and V. Mokeev, Phys. Rev. C 67, 035204 (2003)

	$A^{\frac{1}{2}}(\gamma p)$ GeV$^{-\frac{1}{2}}$		$A^{\frac{1}{2}}(\gamma n)$ GeV$^{-\frac{1}{2}}$	
	$N(1535)1/2^-$	$N(1650)1/2^-$	$N(1535)1/2^-$	$N(1650)1/2^-$
T-matrix	0.114 ± 0.008	0.032 ± 0.007	-0.095 ± 0.006	0.019 ± 0.006
Bare states	0.096 ± 0.007	0.075 ± 0.007	-0.120 ± 0.006	-0.052 ± 0.006
SQT	0.097 ± 0.007	0.053 ± 0.004	-0.090 ± 0.006	-0.031 ± 0.003

1) The coupling of bare N/D states can be fixed at SQT values.

2) The mass of second bare state $S_{11}(1650)_{bare} = 1400 - 1480$ MeV while the mass of Roper bare state $P_{11}(1440)_{bare} = 1550 - 1590$ MeV.

3) The ηN coupling of $S_{11}(1650)_{bare}$ is almost 0 (and can be fixed at 0) as expected from SU(3) calculations. While ηN branching ratio calculated at pole is 30 ± 5%.

Parity doublets of N and Δ resonances at high mass region

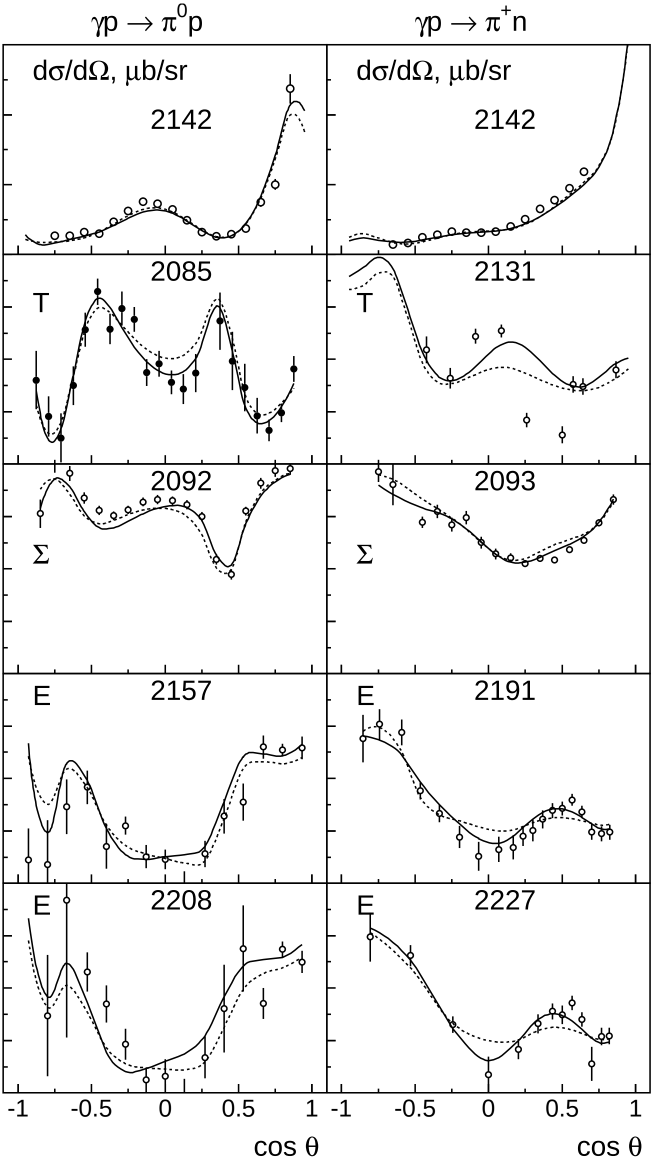
Parity doublets must not interact by pion emission

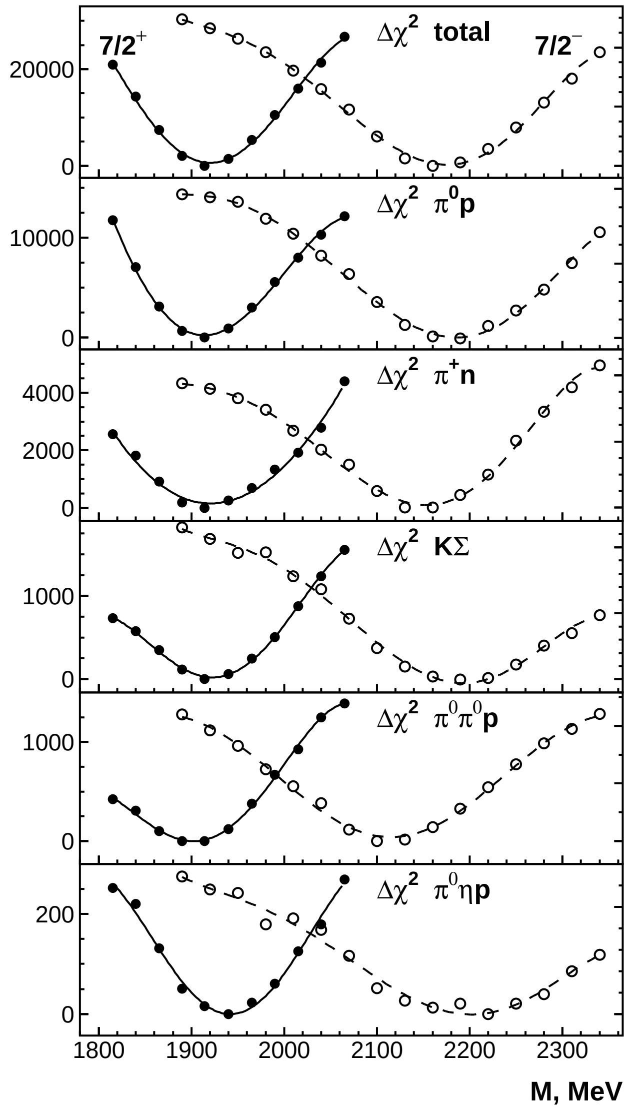
and could have a small coupling to πN.

$J=\frac{1}{2}$	$N_{1/2+}(1880)$	**	$N_{1/2-}(1890)$	**	$\Delta_{1/2+}(1910)$	****	$\Delta_{1/2-}(1900)$	**
$J=\frac{3}{2}$	$N_{3/2+}(1900)$	***	$N_{3/2-}(1875)$	**	$\Delta_{3/2+}(1940)$	***	$\Delta_{3/2-}(1990)$	**
$J=\frac{5}{2}$	$N_{5/2+}(1880)$	**	$N_{5/2-}(2060)$	**	$\Delta_{5/2+}(1940)$	****	$\Delta_{5/2-}(1930)$	***
$J=\frac{7}{2}$	$N_{7/2+}(1980)$	**	$N_{7/2-}(2170)$	****	$\Delta_{7/2+}(1920)$	****	$\Delta_{7/2-}(2200)$	*
$J=\frac{9}{2}$	$N_{9/2+}(2220)$	****	$N_{9/2-}(2250)$	****	$\Delta_{9/2+}(2300)$	**	$\Delta_{9/2-}(2400)$	**

$J=\frac{5}{2}$	$N_{5/2+}(2090)$	**	$N_{5/2-}(2060)$	**	$\Delta_{5/2+}(1940)$	****	$\Delta_{5/2-}(1930)$	***
$J=\frac{7}{2}$	$N_{7/2+}(2100)$	**	$N_{7/2-}(2150)$	****	$\Delta_{7/2+}(1950)$	****	$\Delta_{7/2-}(2200)$	*
$J=\frac{9}{2}$	$N_{9/2+}(2220)$	****	$N_{9/2-}(2250)$	****	$\Delta_{9/2+}(2300)$	**	$\Delta_{9/2-}(2400)^{a}$	**

Data from CLAS and CBELSA/TAPS (E-preliminary) reveal $\Delta(2200)7/2^-$





Data on $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ reveal the existence of the one-star $\Delta_{7/2^-}(2200)$. Its mass and width are determined to

$$M = 2176 \pm 40\,\mathrm{MeV}$$

$$\Gamma = 210 \pm 70\,\mathrm{MeV}$$

This value is compatible with $a \cdot (L + N)$ predicting 2195 MeV and not with parity doubling predicting 1950 MeV.

Both data (CLAS and CBELSA/TAP) are required to achieve this result!

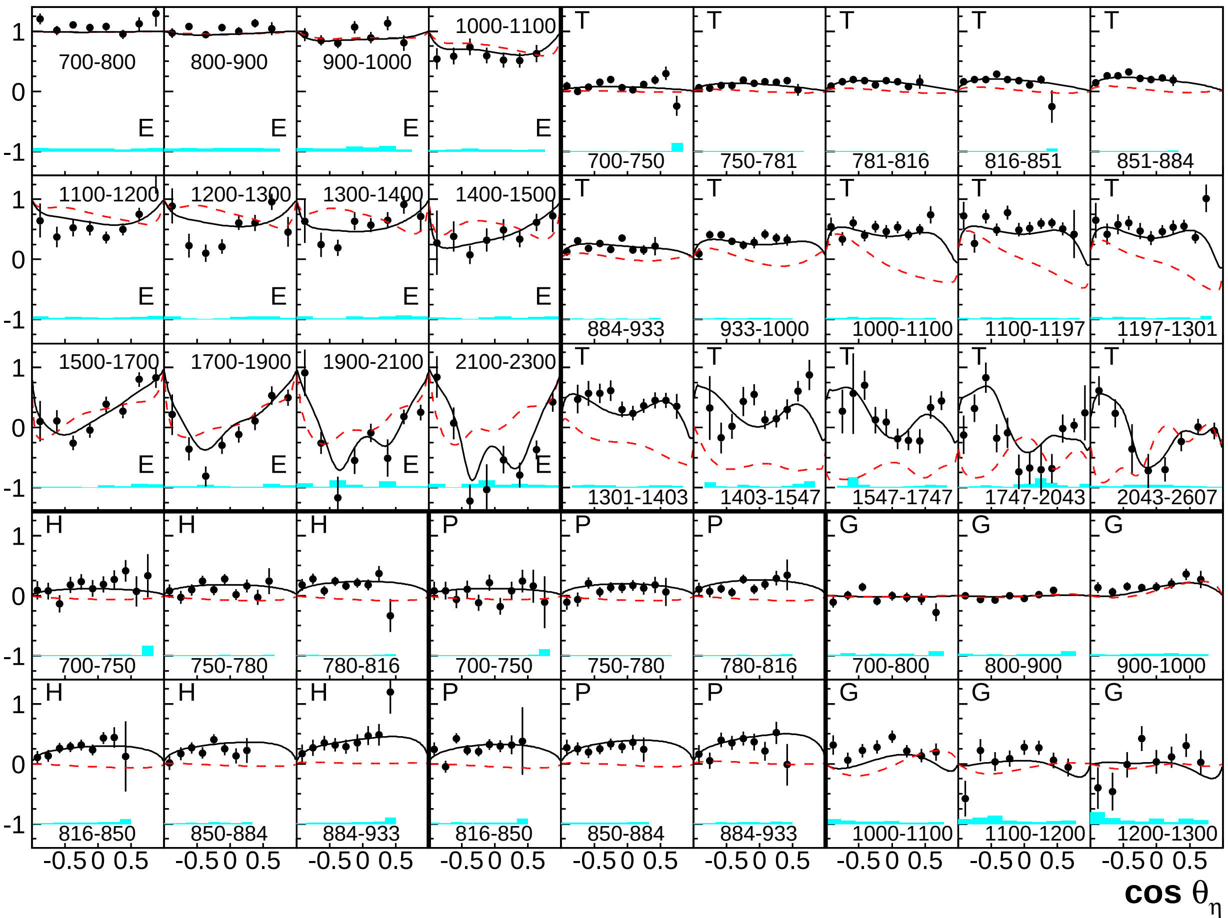
Chiral symmetry is not restored in high-mass hadrons.

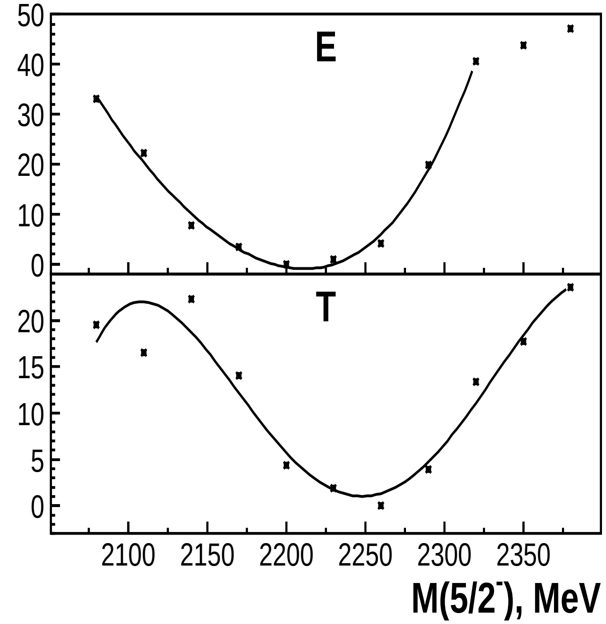
A. V. Anisovich, V. Burkert, E. Klempt, V. A. Nikonov, E. Pasyuk, A. V. Sarantsev, S. Strauch, and U. Thoma, arXiv:1503.05774 [nucl-ex].

Fit of the new polarization data on $\gamma p \to \eta p$. (CB ELSA, Preliminary)

J. Müller, J. Hartmann, M. Grüner

The fit is improved if a new D_{15} state is introduced to the fit



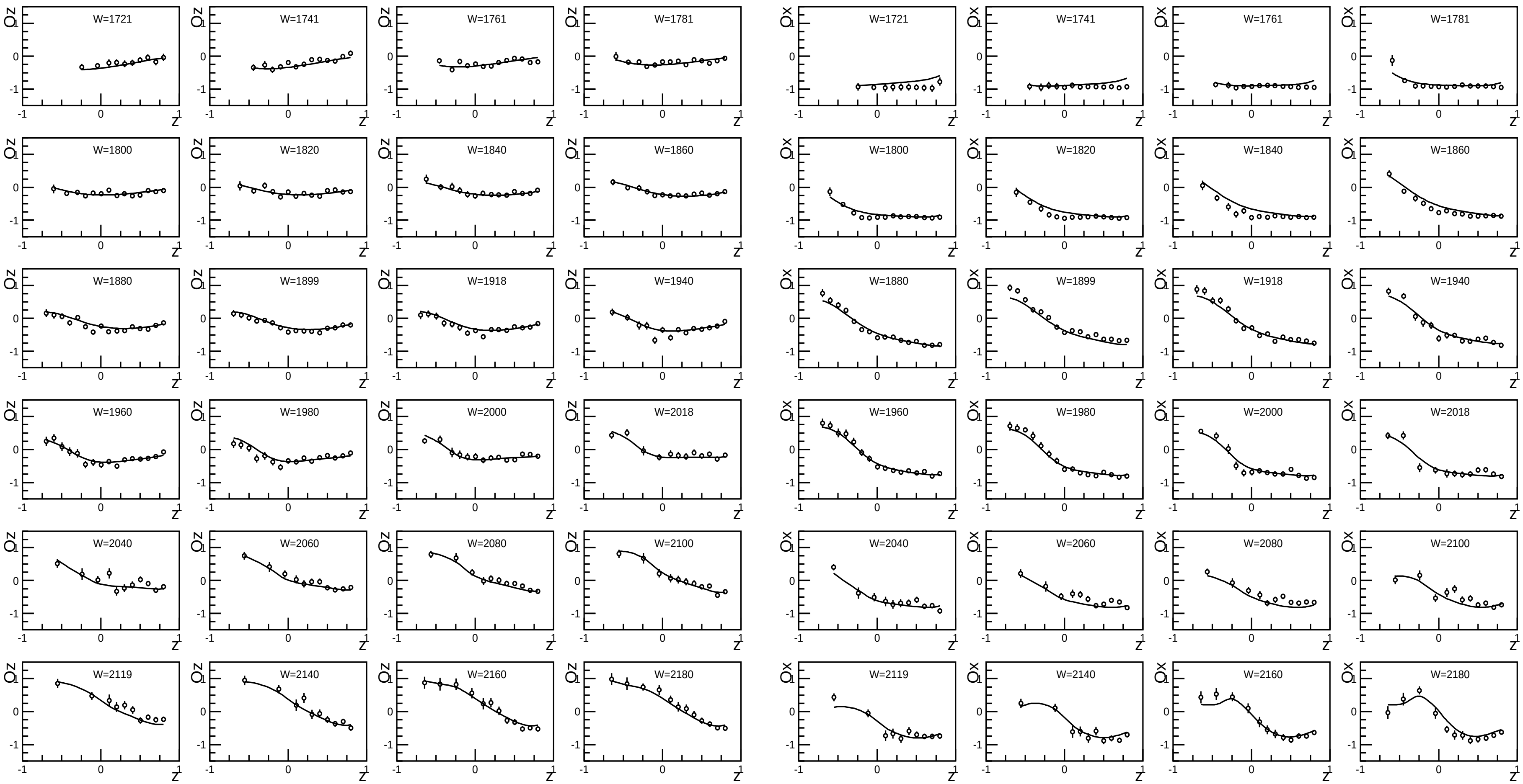


D_{15}: M=2200 ± 25 MeV, $\Gamma = 260 \pm 50$ MeV, $A^{\frac{1}{2}}/A^{\frac{3}{2}}$ $\sim$-0.5

Fit of the new polarization data on $\gamma p \to K\Lambda$ (CLAS Preliminary, courtesy of D. Ireland)

The best improvement is also from D_{15} state

O_Z O_x



D_{15}: M∼2260 MeV, $\Gamma \sim 300$ MeV, $A^{\frac{1}{2}}/A^{\frac{3}{2}}$ ∼-1.0

Photoproduction of vector mesons. $\gamma p \to K^*\Lambda$

Density matrices

$$\frac{d\sigma}{d\Omega_{K^*}\, d\Omega_{dec}} = \frac{d\sigma}{d\Omega_{K^*}} W(\cos\Theta_{dec}, \Phi_{dec})$$

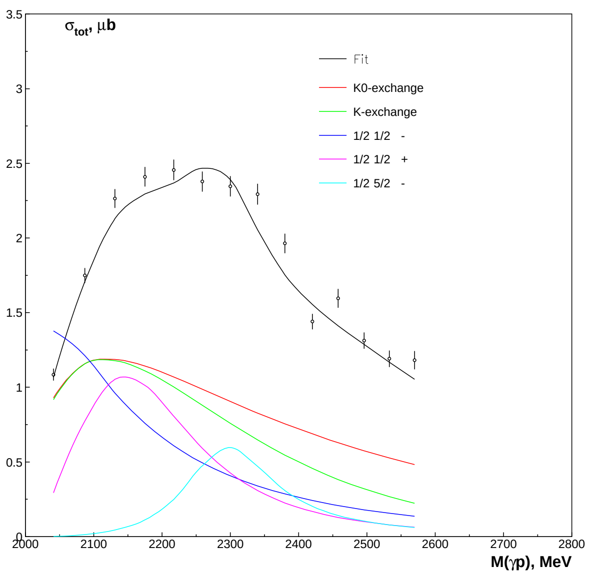
$$\gamma p \to \Lambda K^*(\pi K)$$

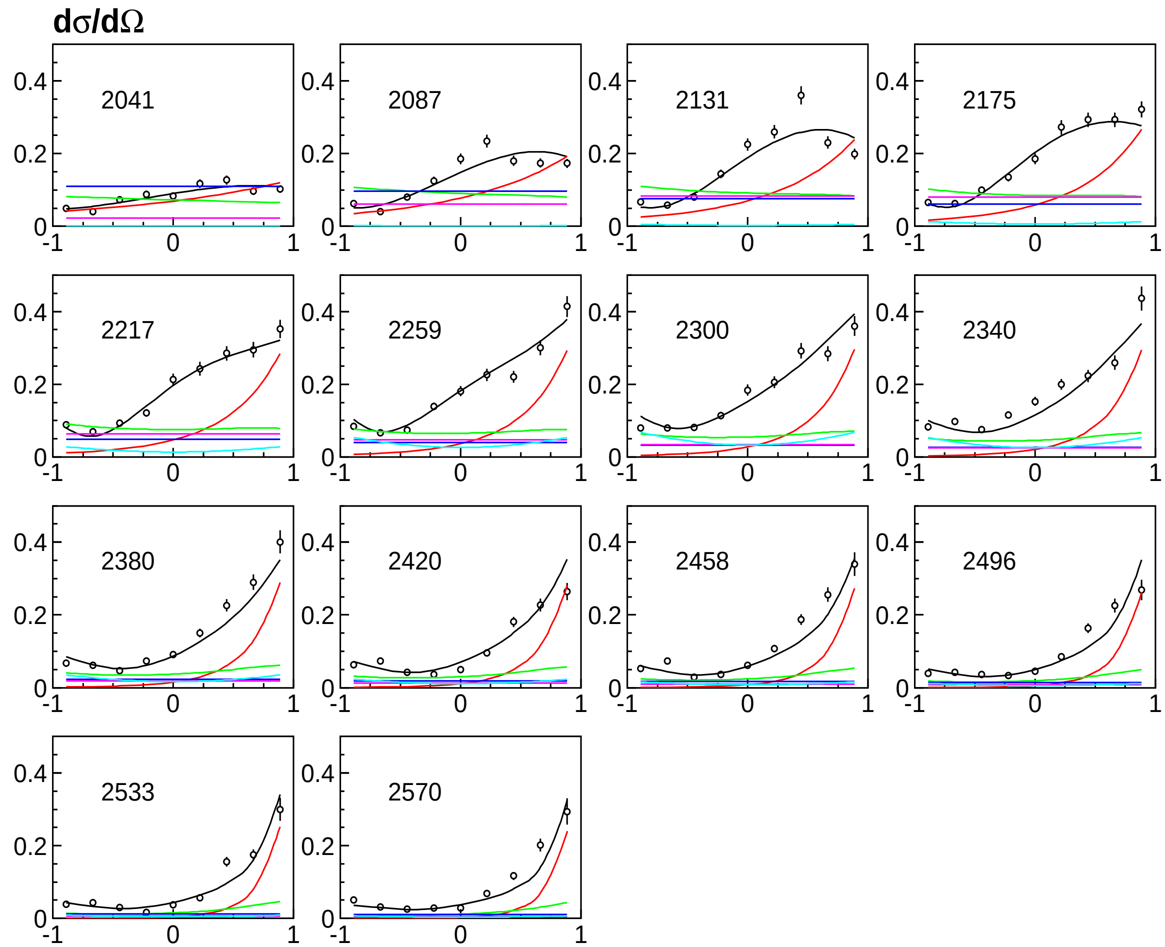
$$W(\cos\Theta, \Phi) = \frac{3}{4\pi}\left(\frac{1}{2}(1-\rho_{00}) + \frac{1}{2}(3\rho_{00}-1)\cos^2\Theta - \sqrt{2}Re\rho_{10}\sin 2\Theta\cos\Phi - \rho_{1-1}\sin^2\Theta\cos 2\Phi\right).$$

$\cos\Theta, \Phi$ **direction of the relative momentum of pion and kaon**

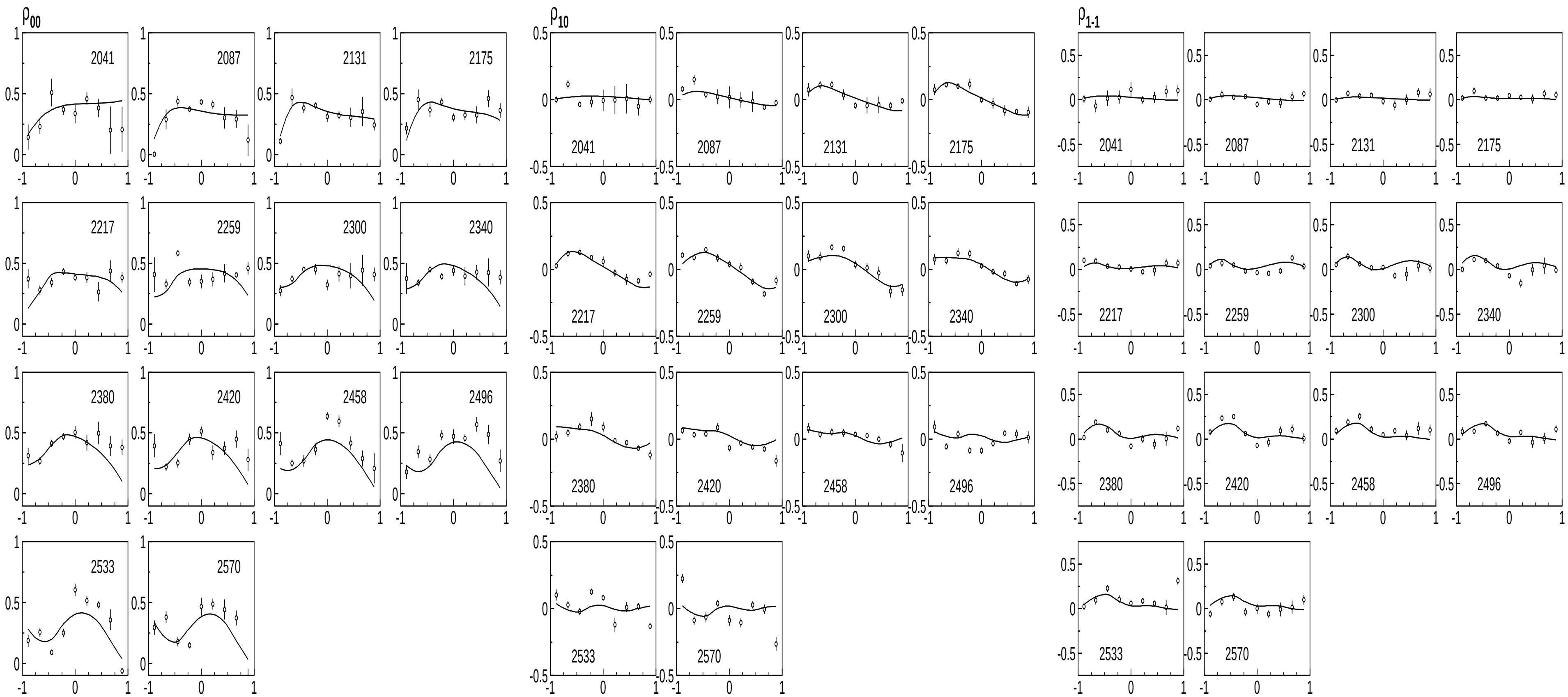
Capstick and Roberts: strong $K^*\Lambda$ decays expected for $N(1895)1/2^-$ and $N(1875)3/2^-$

$N5/2^-$, $N1/2^+$





Density matrix elements $\gamma p \to K^* \Lambda$ (CLAS, Preliminary)



D_{15}: M∼2280 MeV, $\Gamma \sim 170$ MeV, $A^{\frac{1}{2}}/A^{\frac{3}{2}}$ ∼-0.8

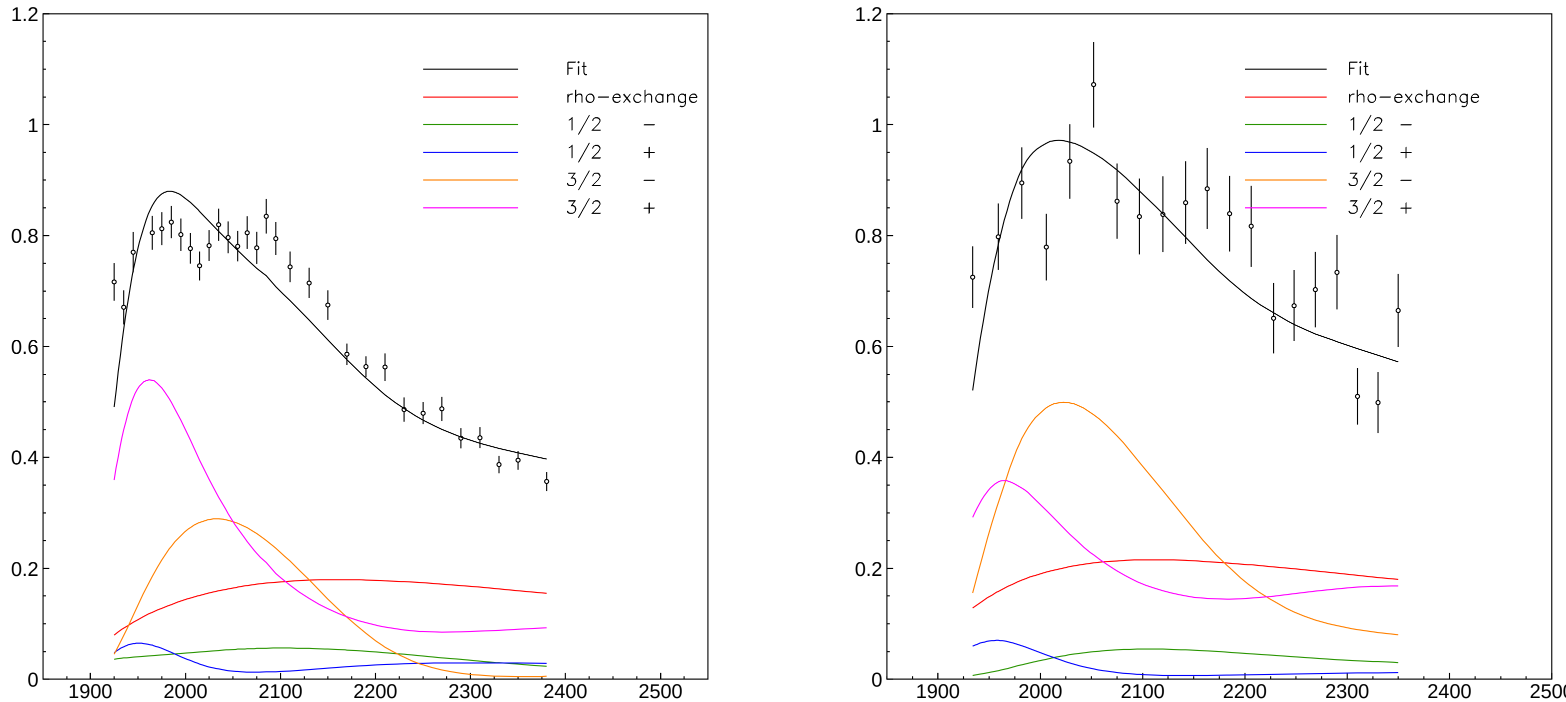
The third shell 30 N^*'s and 15 Δ^*'s expected in a large number of multiplets:

$(70,3^-)$; $(56,3^-)$; $(20,3^-)$; $(70,2^-)$; $(70,1^-)$; $(70,1^-)$; $(56,1^-)$; $(20,1^-)$

$(56,1^-)$:	$\Delta(1900)1/2^-$	$\Delta(1940)3/2^-$	$\Delta(1930)5/2^-$
	$N(1895)1/2^-$	$N(1875)3/2^-$	

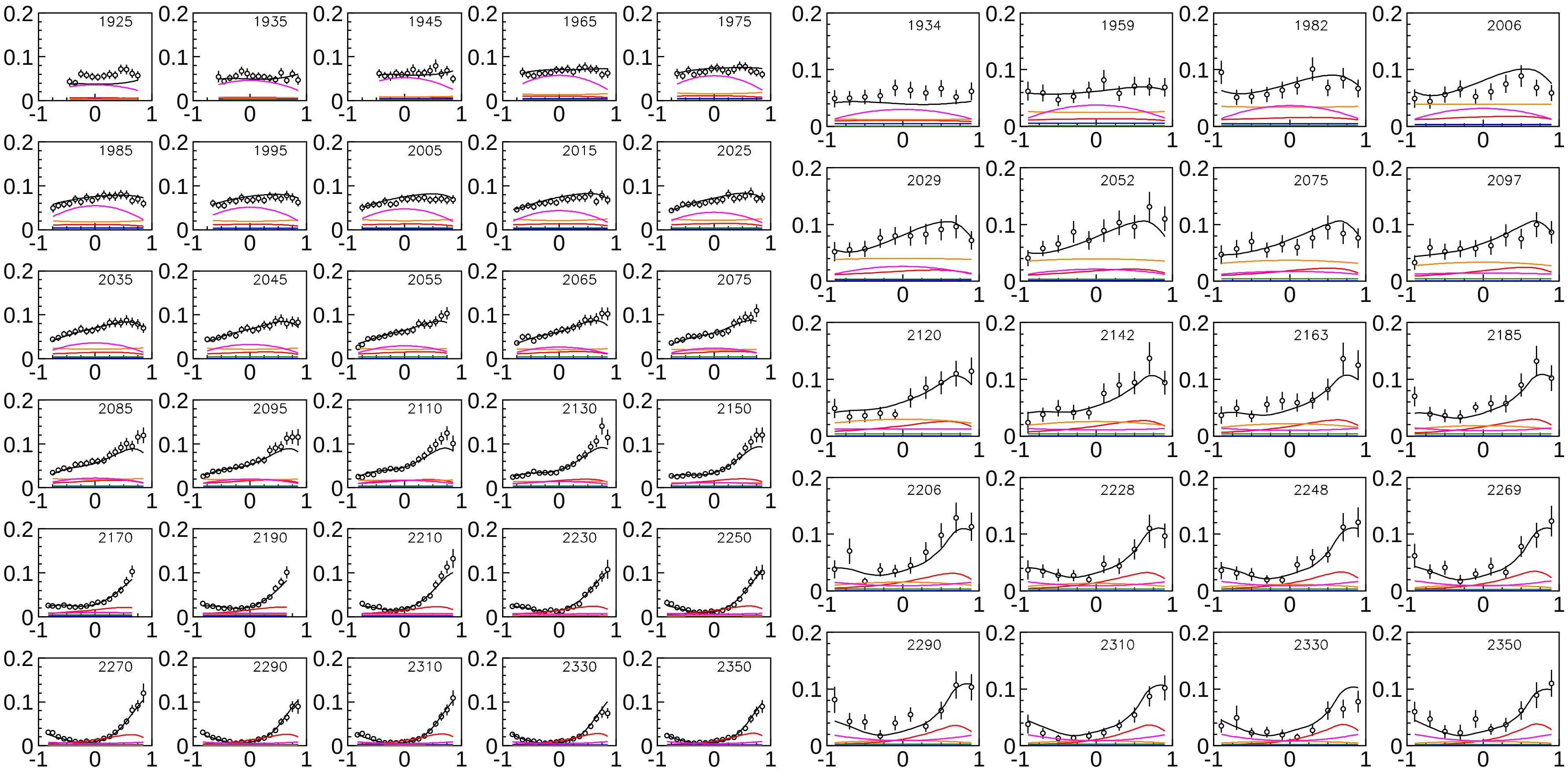
$(70,3^-)$:		$\Delta(2223)5/2^-$	$\Delta(2200)7/2^-$	
	$N(2150)3/2^-$	$N(2280)5/2^-$?	$N(2190)7/2^-$	$N(2250)9/2^-$
		$N(2060)5/2^-$	missing	

Do we have a proof for the resonances in the region 1.9 GeV from the $\gamma p \to \eta' p$ data?



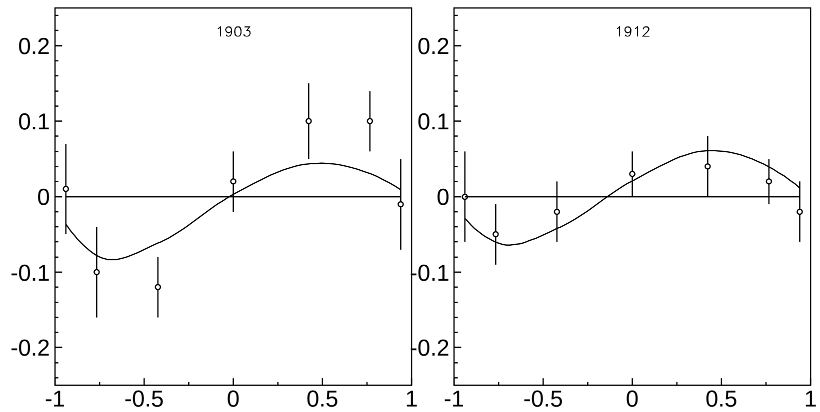
The contribution of the partial waves to the $\gamma p \to \eta' p$ total cross. Left panel shows CLAS and right-hand panel the CB-ELSA data.

The description of the CLAS and CB-ELSA differential cross section.

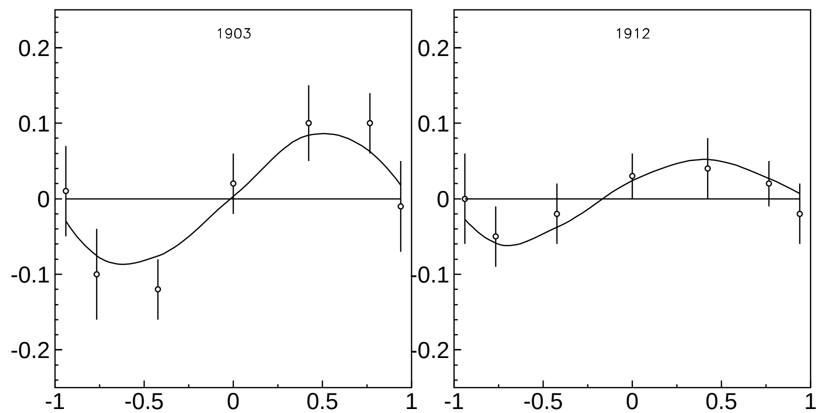


The description of the GRAAL beam asymmetry.

With CLASS differential cross setion

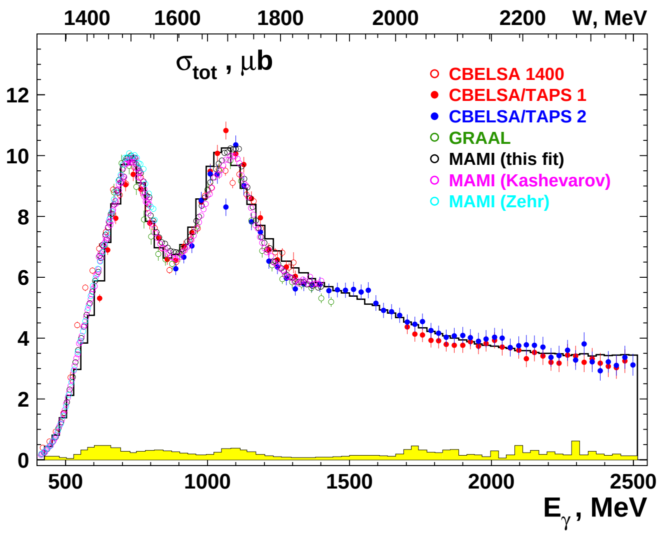


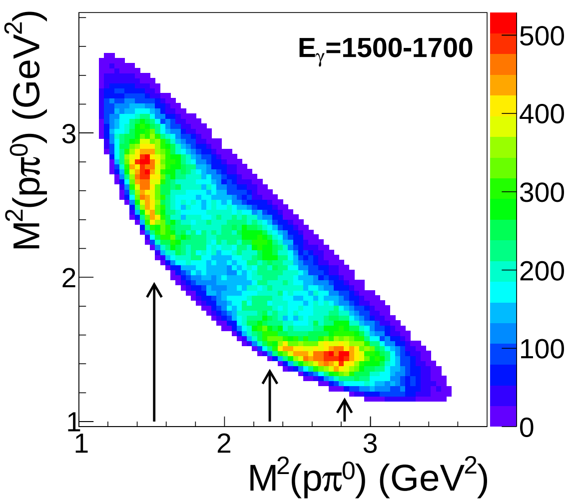
With CB-ELSA differential cross setion

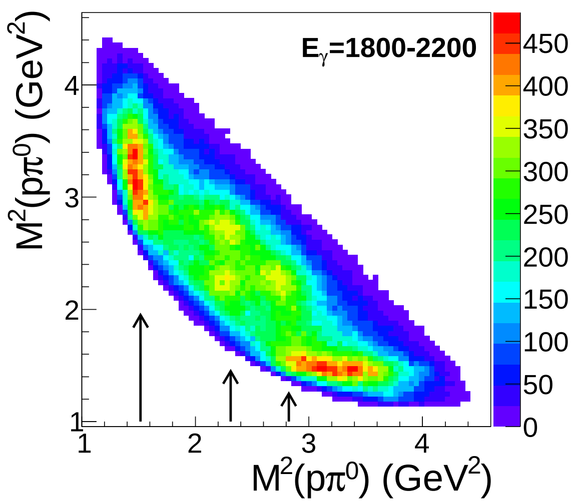


The data on $\gamma p \to \pi^0\pi^0 p$ and $\gamma p \to \pi^0\eta p$

(For details of the analysis see the talk presented by V.Nikonov, and new data were shown by A. Thiel and will be presented by Ph. Mahlberg)







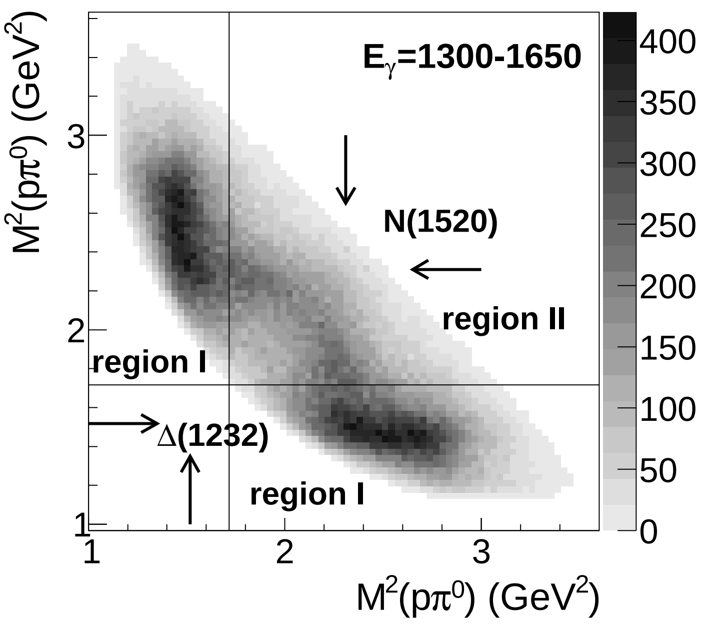
The $\gamma p \to \pi^+\pi^- p$ data should define the decay amplitudes of the resonances into $\rho(770) - N$ and practically saturate the unitarity condition in the region up to W=1.8 GeV. We include in our data base the data on:

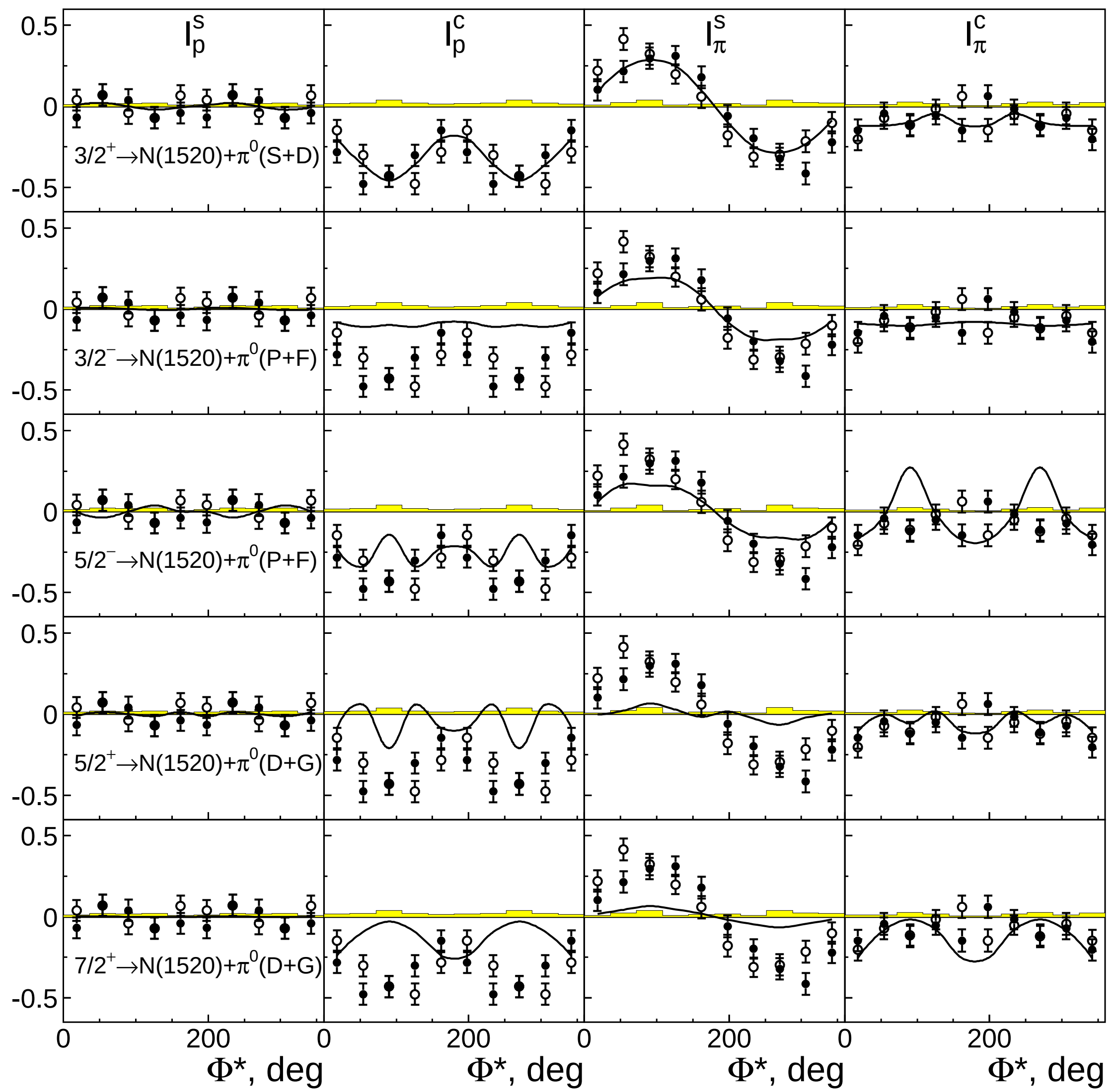
1) $\gamma p \to \pi^+\pi^- p$ differential cross section (SAPHIR, CLAS)

2) $\gamma p \to \pi^+\pi^- p$, I_c, I_s (CLAS)

3) New HADES data on $\gamma p \to \pi^+\pi^- n$ (See the talk presented by W. Przygoda).

I_c and I_s polarization data are very important for the partial wave analysis



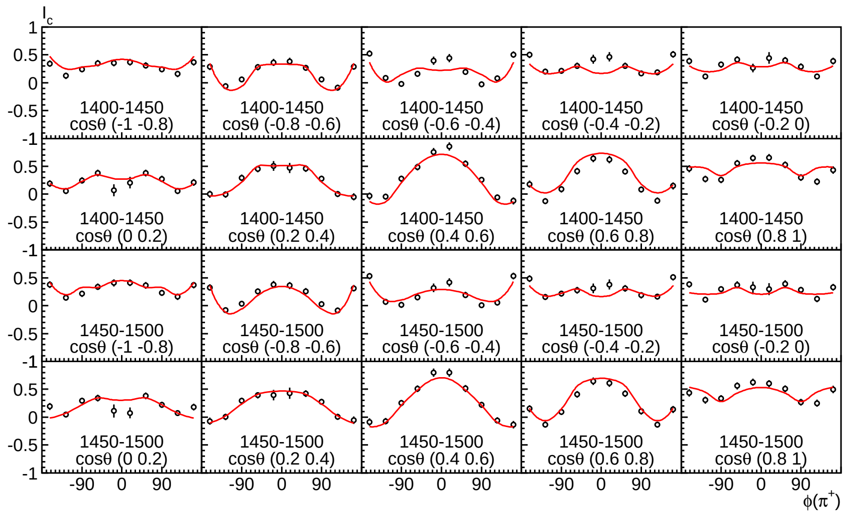


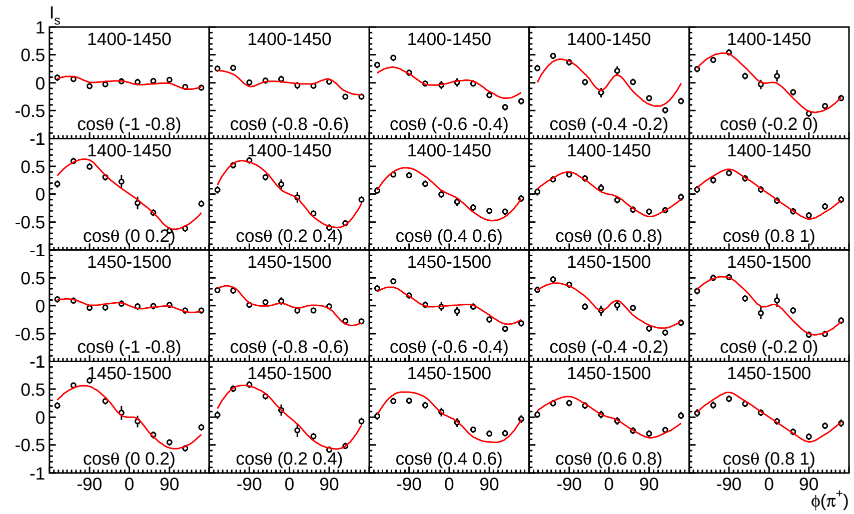
I_c and I_s for $\gamma p \to \pi^+\pi^-p$ from CLAS (Preliminary)

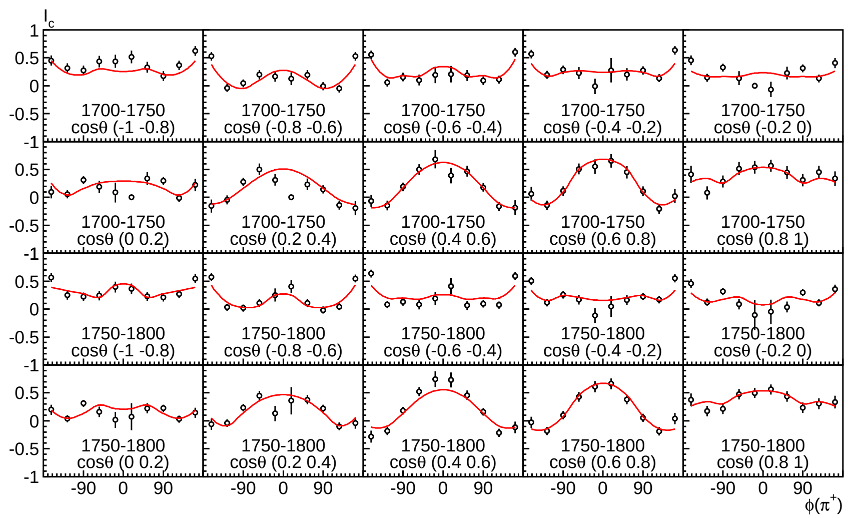
Courtesy of V. Crede, Florida State U

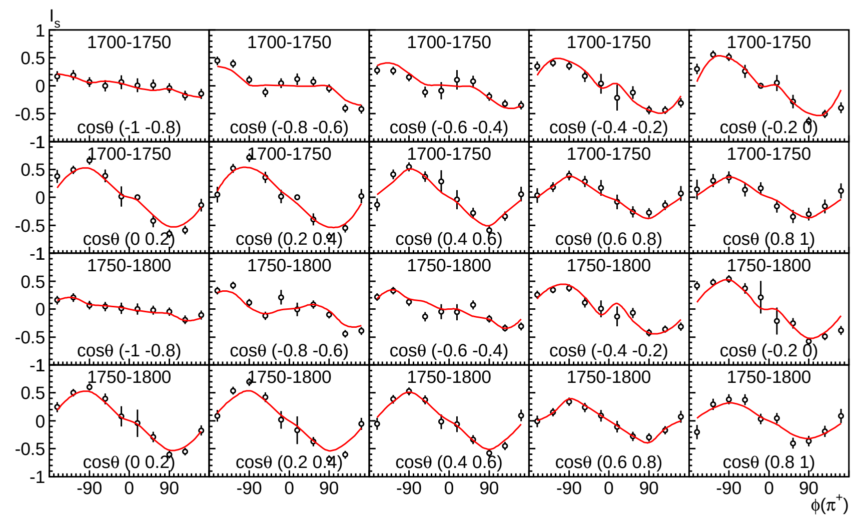
I_c

I_s









SUMMARY

- **The number of new data sets are included in the fit and are successfully described.**
- **The fit of the $\pi^0\pi^0$ and $\pi^+\pi^-$ final state should provide an important information about resonance properties and almost saturate the unitarity condition up to invariant masses 1.8 GeV**
- **The analysis of photoproduction of vector mesons like ωN and $K^*(890)\Lambda$ provides an important constraint on the branching ratios and reveals signals from resonances above 2 GeV.**
- **We have an indication for existence of the nucleon resonance with $J^P = 5/2^-$ in the mass region 2200-2300 MeV.**
- **There is a problem with description of the $\pi^+\pi^-$ data in the region above 2 GeV (and even below for some double polarization data). The new information about resonance properties will be obtained and hpefully new states will be discovered.**

1 Boson projection operators

In momentum representation:

$$P^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = (-1)^n O^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = \sum_{i=1}^{2n+1} u^{(i)}_{\mu_1\mu_2\ldots\mu_n} u^{(i)*}_{\nu_1\nu_2\ldots\nu_n}$$

The projection operator can depends only on the total momentum and the metric tensor.

For spin 0 it is a unit operator. For spin 1 the only possible combination is:

$$O^{\mu}_{\nu} = g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

The propagator for the particle with spin $S > 2$ must be constructed from the tensors $g^{\perp}_{\mu\nu}$: this is the only combination which satisfies:

$$p_\mu g^{\perp}_{\mu\nu} = 0.$$

Then for spin 2 state we obtain:

$$O^{\mu_1\mu_2}_{\nu_1\nu_2} = \frac{1}{2}(g^{\perp}_{\mu_1\nu_1} g^{\perp}_{\mu_2\nu_2} + g^{\perp}_{\mu_1\nu_2} g^{\perp}_{\mu_2\nu_1}) - \frac{1}{3} g^{\perp}_{\mu_1\mu_2} g^{\perp}_{\nu_1\nu_2}$$

Recurrent expression for the boson projector operator

$$O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} = \frac{1}{L^2}\left(\sum_{i,j=1}^{L} g^{\perp}_{\mu_i\nu_j} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_L}_{\nu_1\ldots\nu_{j-1}\nu j+1\ldots\nu_L} - \right.$$

$$\left. \frac{4}{(2L-1)(2L-3)} \sum_{i<j,k<m}^{L} g^{\perp}_{\mu_i\mu_j} g^{\perp}_{\nu_k\nu_m} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_L}_{\nu_1\ldots\nu_{k-1}\nu_{k+1}\ldots\nu_{m-1}\nu_{m+1}\ldots\nu_L}\right)$$

Normalization condition:

$$O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} O^{\nu_1\ldots\nu_L}_{\alpha_1\ldots\alpha_L} = O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}$$

Orbital momentum operator

The angular momentum operator is constructed from momenta of particles k_1, k_2 and metric tensor $g_{\mu\nu}$.

For $L=0$ this operator is a constant: $X^0=1$

The $L=1$ operator is a vector $X^{(1)}_\mu$, constructed from: $k_\mu=\frac{1}{2}(k_{1\mu}-k_{2\mu})$ and $P_\mu=(k_{1\mu}+k_{2\mu})$. Orthogonality:

$$\int \frac{d^4k}{4\pi} X^{(1)}_{\mu_1} X^{(0)} = \int \frac{d^4k}{4\pi} X^{(n)}_{\mu_1\ldots\mu_n} X^{(n-1)}_{\mu_2\ldots\mu_n} = \xi P_{\mu_1} = 0$$

Then:

$$X^{(1)}_\mu P_\mu = 0 \qquad\qquad X^{(n)}_{\mu_1\ldots\mu_n} P_{\mu_j} = 0$$

and:

$$X^{(1)}_\mu = k^\perp_\mu = k_\nu g^\perp_{\nu\mu}; \qquad\qquad g^\perp_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{p^2} \right) ;$$

in c.m.s $k^\perp = (0, \vec{k})$

Recurrent expression for the orbital momentum operators $X^{(n)}_{\mu_1\ldots\mu_n}$

$$X^{(n)}_{\mu_1\ldots\mu_n}=\frac{2n-1}{n^2}\sum_{i=1}^{n}k^\perp_{\mu_i}X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n}-\frac{2k^2_\perp}{n^2}\sum_{\substack{i,j=1\\i<j}}^{n}g_{\mu_i\mu_j}X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}$$

Taking into account the traceless property of $X^{(n)}$ we have:

$$X^{(n)}_{\mu_1\ldots\mu_n}X^{(n)}_{\mu_1\ldots\mu_n}=\alpha(n)(k^2_\perp)^n \qquad \alpha(n)=\prod_{i=1}^{n}\frac{2i-1}{i}=\frac{(2n-1)!!}{n!}.$$

From the recursive procedure one can get the following expression for the operator $X^{(n)}$:

$$X^{(n)}_{\mu_1\ldots\mu_n}=\alpha(n)\left[k^\perp_{\mu_1}k^\perp_{\mu_2}\ldots k^\perp_{\mu_n}-\frac{k^2_\perp}{2n-1}\left(g^\perp_{\mu_1\mu_2}k^\perp_{\mu_3}\ldots k^\perp_{\mu_n}+\cdots\right)+\frac{k^4_\perp}{(2n-1)(2n-3)}\left(g^\perp_{\mu_1\mu_2}g^\perp_{\mu_3\mu_4}k^\perp_{\mu_5}\cdots k_{\mu_4}+\cdots\right)+\cdots\right].$$

Scattering of two spinless particles

Denote relative momenta of particles before and after interaction as q and k, correspondingly. The structure of partial–wave amplitude with orbital momentum $L = J$ is determined by convolution of operators $X^{(L)}(k)$ and $X^{(L)}(q)$:

$$A_L = BW_L(s) X^{(L)}_{\mu_1\ldots\mu_L}(k) O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} X^{(L)}_{\nu_1\ldots\nu_L}(q) = BW_L(s) X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q)$$

$BW_L(s)$ **depends on the total energy squared only.**

The convolution $X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q)$ can be written in terms of Legendre polynomials $P_L(z)$:

$$X^{(L)}_{\mu_1\ldots\mu_L}(k) X^{(L)}_{\mu_1\ldots\mu_L}(q) = \alpha(L) \left(\sqrt{k^2_\perp} \sqrt{q^2_\perp} \right)^L P_L(z) \, ,$$

$$z = \frac{(k^\perp q^\perp)}{\sqrt{k^2_\perp}\sqrt{q^2_\perp}} \qquad \alpha(L) = \prod_{n=1}^{L} \frac{2n-1}{n}$$

πN interaction

States with $J=L-1/2$ are called '-' states ($1/2^+$, $3/2^-$,$5/2^+$,...) and states with $J=L+1/2$ are called '+' states ($1/2^-$, $3/2^+$,$5/2^-$,...).

$$\tilde{N}^{+}_{\mu_1\ldots\mu_n} = X^{(n)}_{\mu_1\ldots\mu_n} \qquad \tilde{N}^{-}_{\mu_1\ldots\mu_n} = i\gamma_\nu\gamma_5 X^{(n+1)}_{\nu\mu_1\ldots\mu_n}$$

$$A = \bar{u}(k_1) N^{\pm}_{\mu_1\ldots\mu_L} F^{\mu_1\ldots\mu_{L-1}}_{\nu_1\ldots\nu_{L-1}} N^{\pm}_{\nu_1\ldots\nu_L} u(q_1) BW^{\pm}_L(s) \xrightarrow[c.m.s.]{} \omega^* \left[G(s,t) + H(s,t) i(\vec{\sigma}\vec{n})\right] \omega'$$

$$G(s,t) = \sum_L \left[(L+1)F^{+}_L(s) - LF^{-}_L(s)\right] P_L(z) \; ,$$

$$H(s,t) = \sum_L \left[F^{+}_L(s) + F^{-}_L(s)\right] P'_L(z) \; .$$

$$F^{+}_L \quad = (-1)^{L+1}(|\vec{k}||\vec{q}|)^L \sqrt{\chi_i\chi_f}\, \frac{\alpha(L)}{2L+1} BW^{+}_L(s) \; ,$$

$$F^{-}_L \quad = (-1)^{L}(|\vec{k}||\vec{q}|)^L \sqrt{\chi_i\chi_f}\, \frac{\alpha(L)}{L} BW^{-}_L(s) \; .$$

$$\chi_i = m_i + k_{i0} \qquad\qquad \alpha(L) = \prod_{l=1}^{L} \frac{2l-1}{l} = \frac{(2L-1)!!}{L!} \; .$$

γN interaction

Photon has quantum numbers $J^{PC}=1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$.

Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.

In general case: $1/2^-$, $1/2^+$ described by two amplitudes and higher states by three amplitudes.

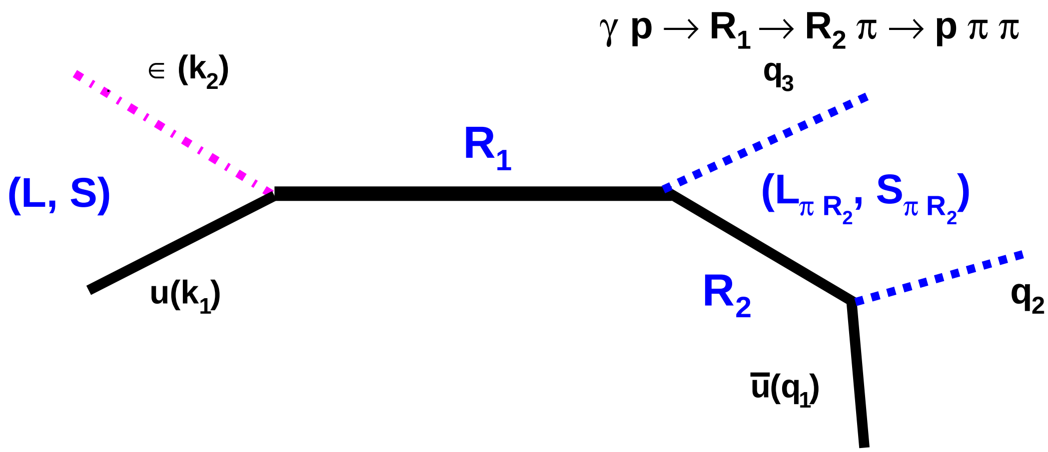
$$
\begin{aligned}
V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\mu i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n}\ , & V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\xi\gamma_\mu X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n}\ ,\\
V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1\ldots\alpha_n}\ , & V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n+1)}_{\mu\alpha_1\ldots\alpha_n}\ ,\\
V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^\perp_{\mu\alpha_n}\ , & V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^\perp_{\alpha_1\mu}\ .
\end{aligned}
$$

Gauge invariance: $\varepsilon_\mu q_{1\mu}=0$ where q_1-photon momentum.

$$\varepsilon_\mu V^{(2\pm)\mu}_{\alpha_1\ldots\alpha_n} = C^\pm \varepsilon_\mu V^{(3\pm)\mu}_{\alpha_1\ldots\alpha_n}$$

where $C^\pm$ do not depend on angles.

Resonance amplitudes for meson photoproduction



General form of the angular dependent part of the amplitude:

$$\bar{u}(q_1)\tilde{N}_{\alpha_1\ldots\alpha_n}(R_2\to\mu N)F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}(q_1+q_2)\tilde{N}^{(j)\beta_1\ldots\beta_n}_{\gamma_1\ldots\gamma_m}(R_1\to\mu R_2)$$
$$F^{\gamma_1\ldots\gamma_m}_{\xi_1\ldots\xi_m}(P)V^{(i)\mu}_{\xi_1\ldots\xi_m}(R_1\to\gamma N)u(k_1)\varepsilon_\mu$$

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat{p})O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\left(g^\perp_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

$$\sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$