

# Double Polarization Observable $E$ in $\eta$ , $\pi^0$ and $2\pi^0$ Photoproduction off Protons and Neutrons

Manuel Dieterle

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Nucleons (NSTAR2015)





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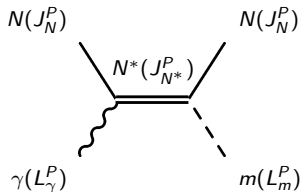
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# Outline

- 1 Motivation
- 2 Experimental Setup
- 3 Total Cross Sections
- 4 Polarization Observable E

# Study Nucleon Resonances $\Leftrightarrow$ Test Hadron Models

## Photoexcitation of Nucleons

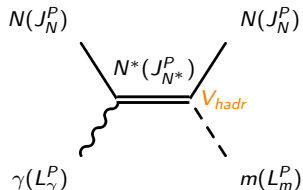


$m(L_m^P)$  : pseudoscalar meson  
 $N^*(J_{N^*}^P)$  : definite  $P, J$

- ▶ Decompose IS, FS into Multipole Components

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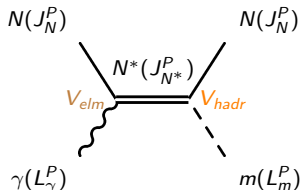


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 $V_{hadr}$  : Isospin Conservation

- ▶ Decompose IS, FS into Multipole Components
- ▶ Isospin Filter:  $\eta$  ( $I = I_3 = 0$ )  $\Rightarrow$  only  $I = 1/2$   $N^*$  possible

# Study Nucleon Resonances $\Leftrightarrow$ Test Hadron Models

## Photoexcitation of Nucleons



$m(L_m^P)$  : pseudoscalar meson  
 $N^*(J_{N^*}^P)$  : definite  $P, J$   
 $V_{hadr}$  : Isospin Conservation  
 $V_{elm}$  : Isospin Violation

- ▶ Decompose IS, FS into Multipole Components
- ▶ Isospin Filter:  $\eta$  ( $I = I_3 = 0$ )  $\Rightarrow$  only  $I = 1/2$   $N^*$  possible
- ▶ Isoscalar ( $\Delta I = 0$ ) and Isovector ( $\Delta I = 0, \pm 1$ ) components of the elm. current
- ▶ Three independent matrix elements  $\langle I_f, I_{f3} | \hat{A} | I_i, I_{i3} \rangle$

$$A^{IS} = \langle \frac{1}{2}, \pm \frac{1}{2} | \hat{S} | \frac{1}{2}, \pm \frac{1}{2} \rangle \mp A^{IV} = \langle \frac{1}{2}, \pm \frac{1}{2} | \hat{V} | \frac{1}{2}, \pm \frac{1}{2} \rangle \quad A^{V3} = \langle \frac{3}{2}, \pm \frac{1}{2} | \hat{V} | \frac{1}{2}, \pm \frac{1}{2} \rangle$$

# Isospin Amplitudes

$\eta$  (Isoscalar):

$$A(\gamma p \rightarrow \eta p) = A^{IS} + A^{IV}$$

$$A(\gamma n \rightarrow \eta n) = A^{IS} - A^{IV}$$

- ▶ Neutron measurement required for complete multipole decomposition

# Isospin Amplitudes

$\eta$  (Isoscalar):

$$A(\gamma p \rightarrow \eta p) = A^{IS} + A^{IV}$$

$$A(\gamma n \rightarrow \eta n) = A^{IS} - A^{IV}$$

$\pi$  (Isovector):

$$A(\gamma p \rightarrow \pi^+ n) = -\sqrt{\frac{1}{3}} A^{V3} + \sqrt{\frac{2}{3}} (A^{IV} - A^{IS})$$

$$A(\gamma p \rightarrow \pi^0 p) = +\sqrt{\frac{2}{3}} A^{V3} + \sqrt{\frac{1}{3}} (A^{IV} - A^{IS})$$

$$A(\gamma n \rightarrow \pi^- p) = +\sqrt{\frac{1}{3}} A^{V3} - \sqrt{\frac{2}{3}} (A^{IV} + A^{IS})$$

$$A(\gamma n \rightarrow \pi^0 n) = +\sqrt{\frac{2}{3}} A^{V3} + \sqrt{\frac{1}{3}} (A^{IV} + A^{IS})$$

- ▶ Neutron measurement required for complete multipole decomposition

# Measurements on the Neutron - Deuterium

▶ no free neutron targets

➤ light nuclei, i.e. deuterium



# Measurements on the Neutron - Deuterium

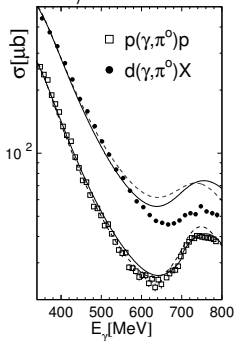
- ▶ no free neutron targets ➤ light nuclei, i.e. deuterium
- ▶ nuclear Fermi motion ➤ kinematical reconstruction

$$W_B^2 = (P_\gamma + P_{N,i})^2 = 2E_\gamma m_N + m_N^2 \Rightarrow W_R^2 = (P_\eta + P_{N,f})^2$$

# Measurements on the Neutron - Deuterium

- ▶ no free neutron targets ➤ light nuclei, i.e. deuterium
- ▶ nuclear Fermi motion ➤ kinematical reconstruction
- ▶ N-N/ $m$ -N Final State Interactions ➤ free  $\Leftrightarrow$  quasi-free protons

$$W_B^2 = (P_\gamma + P_{N,i})^2 = 2E_\gamma m_N + m_N^2 \Rightarrow W_R^2 = (P_\eta + P_{N,f})^2$$



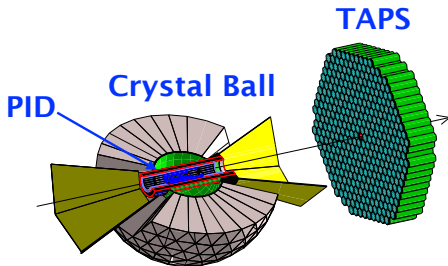
inclusive measurement  $\gamma d \rightarrow X\pi^0$

- ▶ agreement in  $\Delta$  region
- ▶ suppression at higher energies
- ▶  $\gamma d \rightarrow d\pi^0$  negligible at these energies
- ▶ problem with neutron models?
- ▶ large FSI?
- ▶ both?

# Experimental Setup

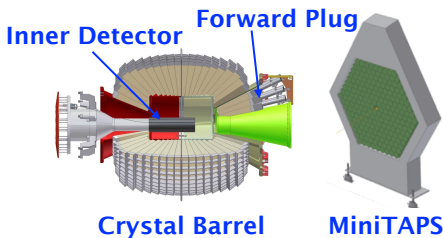
## A2 @ MAMI

- Continuous beam
- $E_\gamma \leq 1.6$  GeV
- CB: 672 NaI
- TAPS: BaF<sub>2</sub> & PbWO<sub>4</sub>
- PID



## CBELSA/TAPS @ ELSA

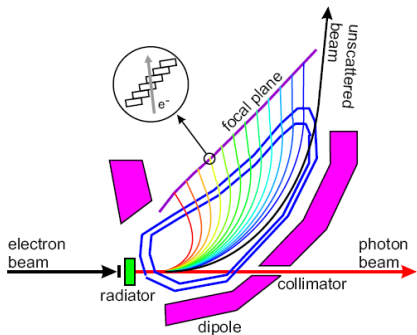
- Quasi-continuous beam
- $E_\gamma \leq 3.2$  GeV
- CBB: 1230 CsI
- MiniTAPS: 216 BaF<sub>2</sub>
- Inner Detector



# Tagger and Targets

## Bremsstrahlungs Tagging

$$E_{\gamma} = E_0 - E'$$

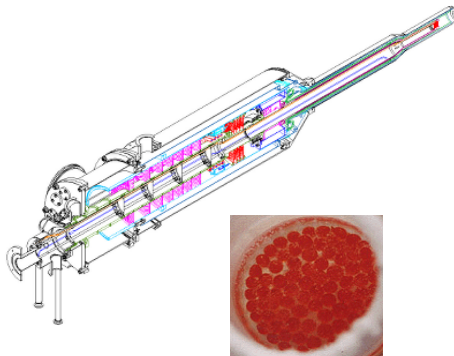


## Unpolarised Target

- ▶ liquid deuterium

## Polarised Target

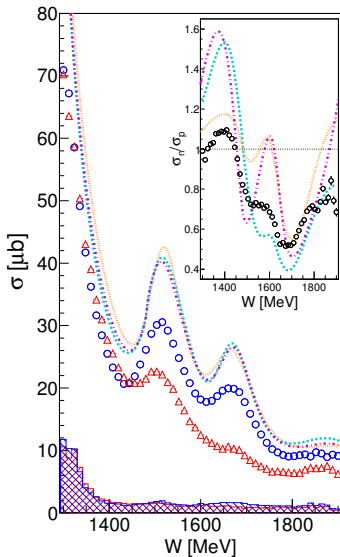
- ▶ deuterated Butanol



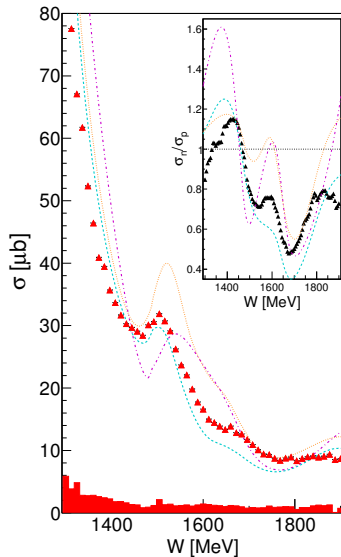
Single  $\pi^0$ 

○  $\gamma d \rightarrow p(n)\pi^0$     $\triangle \gamma d \rightarrow n(p)\pi^0$     $\blacktriangle \gamma n \rightarrow n\pi^0$    — SAID   ··· MAID   - - - BnGa

quasi-free proton/neutron



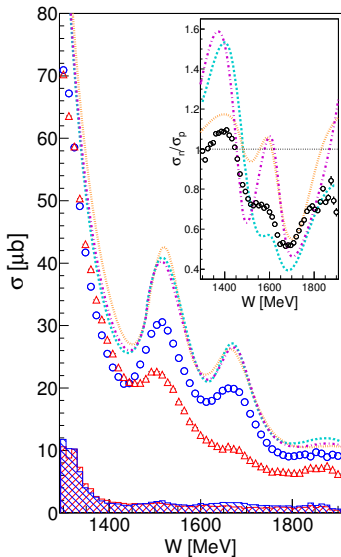
"free" neutron



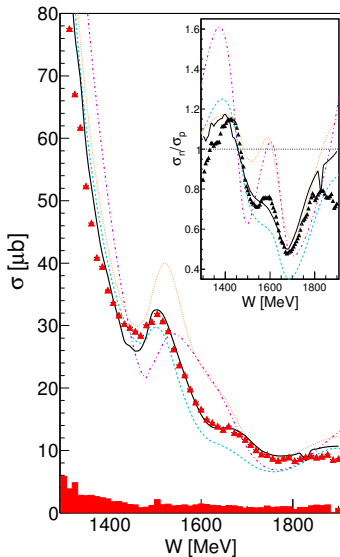
Single  $\pi^0$ 

○  $\gamma d \rightarrow p(n)\pi^0$     $\triangle \gamma d \rightarrow n(p)\pi^0$     $\blacktriangle \gamma n \rightarrow n\pi^0$     $\text{--- SAID}$     $\text{--- MAID}$     $\text{--- BnGa}$     $\text{--- BnGa.refit}$

quasi-free proton/neutron



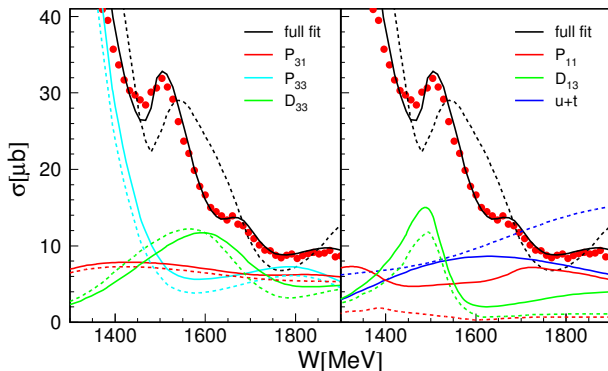
"free" neutron



# Single $\pi^0$ - Impact of the Data

*M. Dieterle et al., Phys. Rev. Lett. 112, 142001*

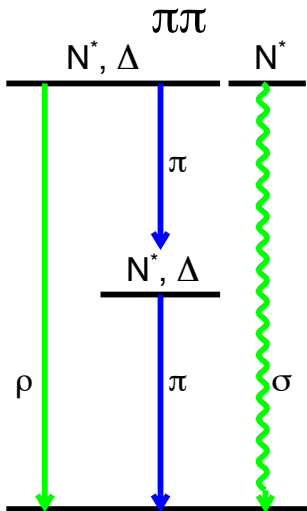
$I = 3/2$   
 $\Delta$



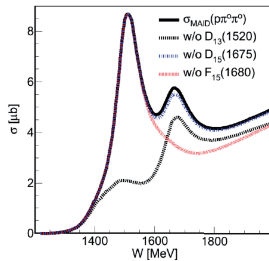
$I = 1/2$   
 $N^*$

- ▶ small changes for  $I = 3/2$  low order resonant partial waves (fixed from  $\gamma p \rightarrow p\pi^0$ )
- ▶ **drastic changes in  $I = 1/2$   $P_{11}(1440)$ ,  $D_{13}(1700)$**  (photon coupling changes sign) and non-resonant background contributions from  $u$ - and  $t$ -channel (mostly  $t$ -channel, i.e. vector-meson exchange)

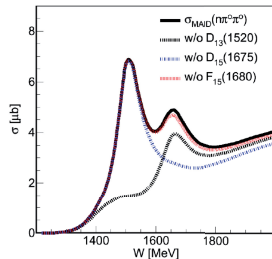
# $2\pi^0$ - Access to higher lying Resonances



## Proton MAID



## Neutron MAID



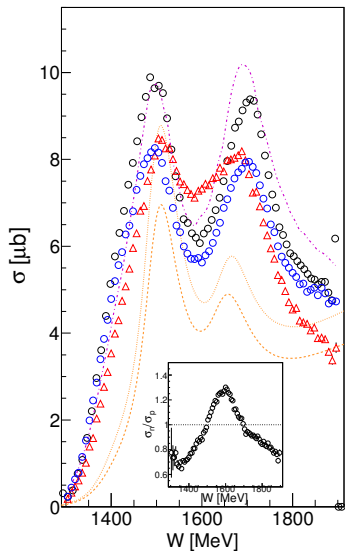
- ▶ Proton:  $D_{13}$ ,  $F_{15}$  dominate
- ▶ Neutron:  $D_{13}$ ,  $D_{15}$  dominate



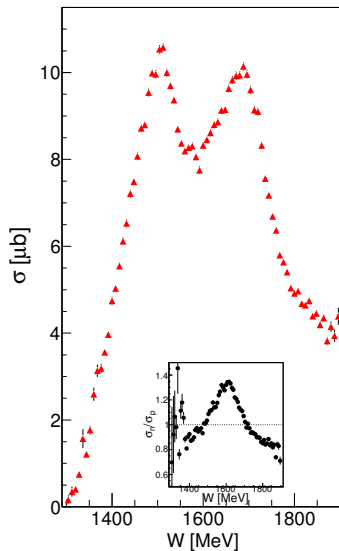
$2\pi^0$ 

○  $\gamma p \rightarrow p\pi^0$  ○  $\gamma d \rightarrow p(n)\pi^0$  △  $\gamma d \rightarrow n(p)\pi^0$  ▲  $\gamma n \rightarrow n\pi^0$  - - MAIDn ' ' MAIDp - - - BnGa

quasi-free proton/neutron

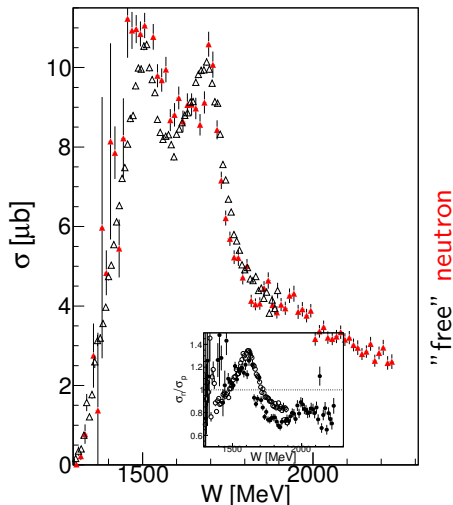
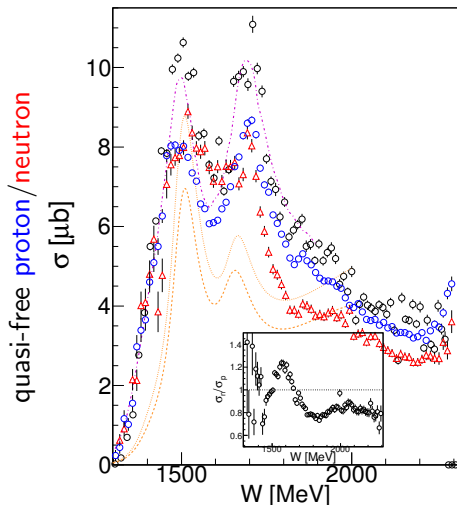


"free" neutron



$2\pi^0$ 

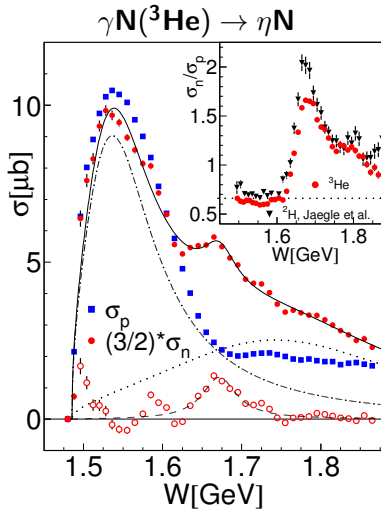
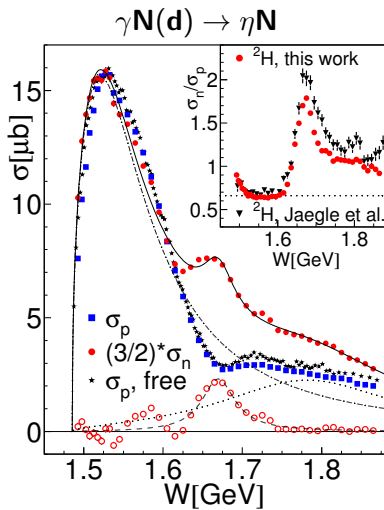
○  $\gamma p \rightarrow p\pi^0$  ○  $\gamma d \rightarrow p(n)\pi^0$  △  $\gamma d \rightarrow n(p)\pi^0$  ▲/△  $\gamma n \rightarrow n\pi^0$  (ELSA/MAMI) - /... MAIDn/p - ··· BnGa



# $\eta$ Photoproduction

D. Werthmüller, L. Witthauer et al., Phys. Rev. Lett. 111, 232001

L. Witthauer et al., Eur. Phys. J. A 49 (2013) 154



$\Rightarrow$  Nuclear Effect (FSI, meson rescattering) unlikely

# Unpolarized Results: Summary

- ▶ Neutron Data increasingly available
  - ▶ Effects from Fermi motion can be handled (experimental resolution remains)
  - ▶ Effects from FSI can be investigated with free to quasi-free proton results
- ⇒ use polarization observables to identify amplitudes and quantum numbers

# Polarization Observables

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = \frac{d\sigma}{d\Omega}(\theta) \cdot \left[ 1 - p_{\gamma}^{\text{lin}} \Sigma(\theta) \cos(2\phi) \right. \\ \left. + p_x \cdot \left( -p_{\gamma}^{\text{lin}} H(\theta) \sin(2\phi) + p_{\gamma}^{\text{circ}} F(\theta) \right) \right. \\ \left. - p_y \cdot \left( +p_{\gamma}^{\text{lin}} P(\theta) \cos(2\phi) - T(\theta) \right) \right. \\ \left. - p_z \cdot \left( -p_{\gamma}^{\text{lin}} G(\theta) \sin(2\phi) + p_{\gamma}^{\text{circ}} E(\theta) \right) \right]$$

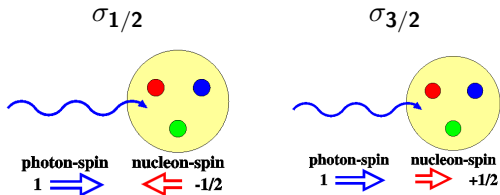
$P_{\gamma}$		$P_T \cdot \vec{e}_i$		
		$x$	$y$	$z$
unpol	$\sigma$	-	$T$	-
linearly	$-\Sigma$	$H$	$-P$	$-G$
circularly	-	$F$	-	$-E$

# Polarization Observables

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = \frac{d\sigma}{d\Omega}(\theta) \cdot \left[ 1 - p_{\gamma}^{\text{lin}} \Sigma(\theta) \cos(2\phi) \right. \\ \left. + p_x \cdot \left( -p_{\gamma}^{\text{lin}} H(\theta) \sin(2\phi) + p_{\gamma}^{\text{circ}} F(\theta) \right) \right. \\ \left. - p_y \cdot \left( +p_{\gamma}^{\text{lin}} P(\theta) \cos(2\phi) - T(\theta) \right) \right. \\ \left. - p_z \cdot \left( -p_{\gamma}^{\text{lin}} G(\theta) \sin(2\phi) + p_{\gamma}^{\text{circ}} E(\theta) \right) \right]$$

$P_{\gamma}$		$P_T \cdot \vec{e}_i$		
		$x$	$y$	$z$
unpol	$\sigma$	-	$T$	-
linearly	$-\Sigma$	$H$	$-P$	$-G$
circularly	-	$F$	-	$-E$

## Double Polarization Observable E



$$E = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}$$

# Double Polarization Observable E

- ▶  $\nexists$  polarized deuterium  $\Rightarrow$  dButanol: **C<sub>4</sub>D<sub>9</sub>OD**
- ▶ 2 ways to measure E:

**w/o carbon  
subtraction:**

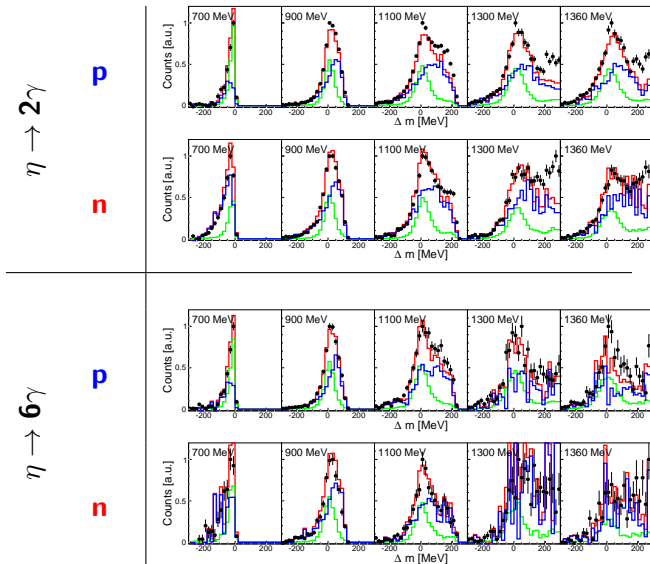
$$E = \frac{\sigma_{1/2} - \sigma_{3/2}}{2\sigma_{tot}}$$

**with carbon  
subtraction:**

$$E = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}$$

# Carbon Subtraction ( $\eta$ )

+ dButanol    — Carbon    — LD2    — LD2 + Carbon





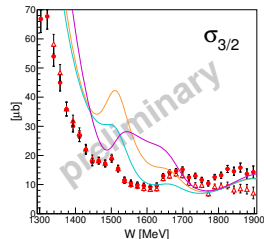
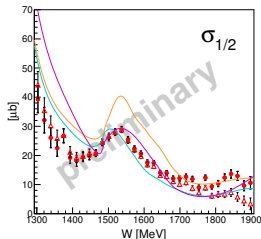
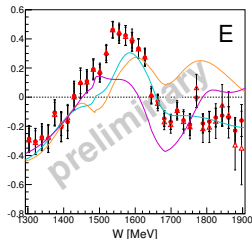
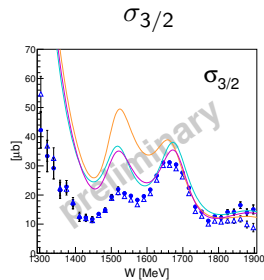
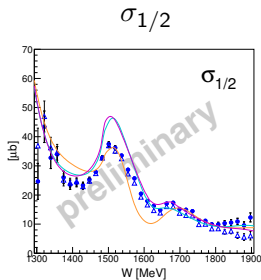
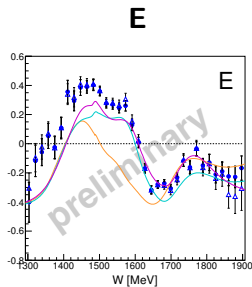
# $\pi^0$ Helicity Asymmetries

● direct    △ sum    — SAID    — MAID    — BnGa

©MAMI

On Proton

On Neutron



M. Dieterle et al., in preparation

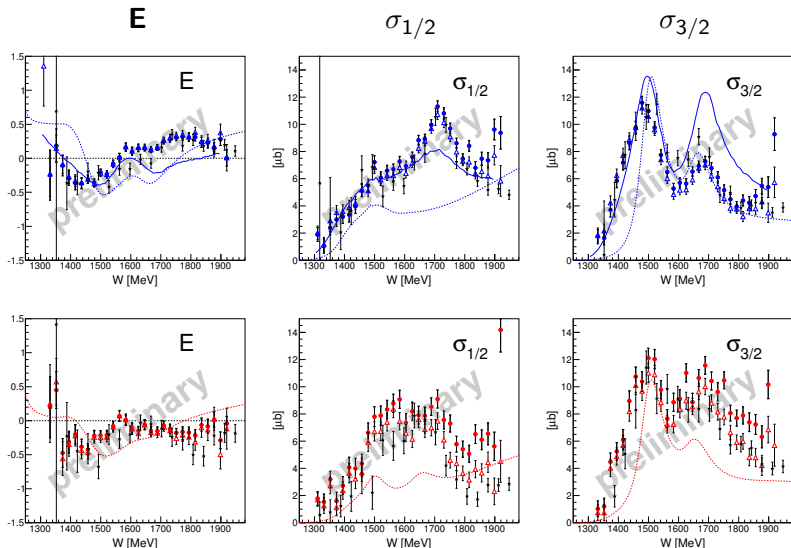
# $2\pi^0$ Helicity Asymmetries

● direct    △ sum    ★ ELSA    — BnGa    - - - MAID

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On Proton

On Neutron



M. Dieterle et al., in preparation

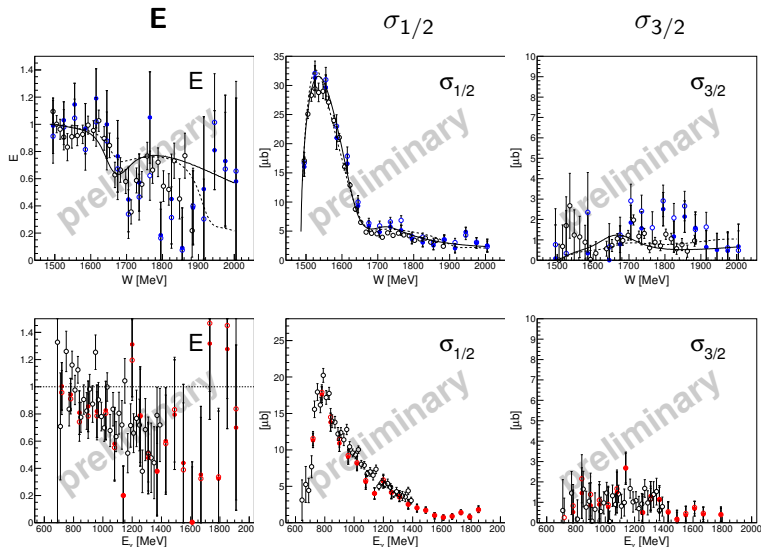
# $\eta$ Helicity Asymmetries

○ direct ● sum — MAID - - - BnGa

©ELSA

On Proton

On Neutron



exclusive

inclusive

L. Witthauer et al., in preparation

# $\eta$ Helicity Asymmetries

○ direct

● sum

— MAID

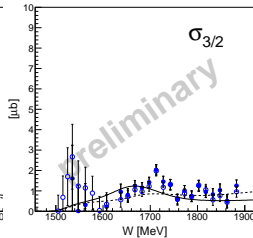
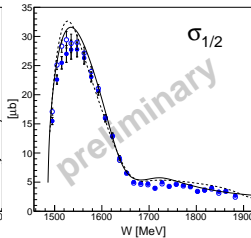
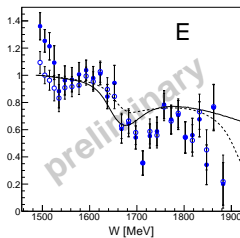
- - - BnGa

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On Proton

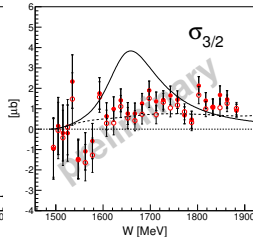
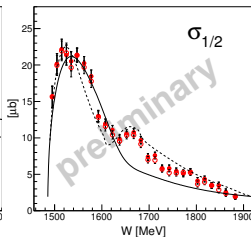
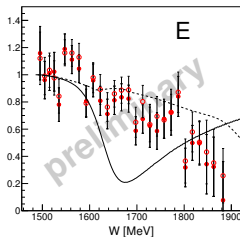
On Neutron

E

 $\sigma_{1/2}$  $\sigma_{3/2}$ 

exclusive

E

 $\sigma_{1/2}$  $\sigma_{3/2}$ 

exclusive

L. Witthauer et al., in preparation

# Summary

- ▶ Neutron Data increasingly available
- ▶ Effects from Fermi motion can be handled (experimental resolution remains)
- ▶ Effects from FSI can be investigated with free to quasi-free proton results
- ▶ measured **E** for:
  - ▶  $\gamma p(n) \rightarrow \eta p(n)$  and  $\gamma n(p) \rightarrow \eta n(p)$
  - ▶  $\gamma p(n) \rightarrow \pi^0 p(n)$  and  $\gamma n(p) \rightarrow \pi^0 p(n)$
  - ▶  $\gamma p(n) \rightarrow 2\pi^0 p(n)$  and  $\gamma n(p) \rightarrow 2\pi^0 n(p)$
- ▶  $\eta$  bump only in  $\sigma_{1/2}$ :  $S_{11}$ ,  $P_{11}$  resonance?
- ▶ new input for theoretical models!

# Thanks for your attention

This work is supported by:

**FNSNF**

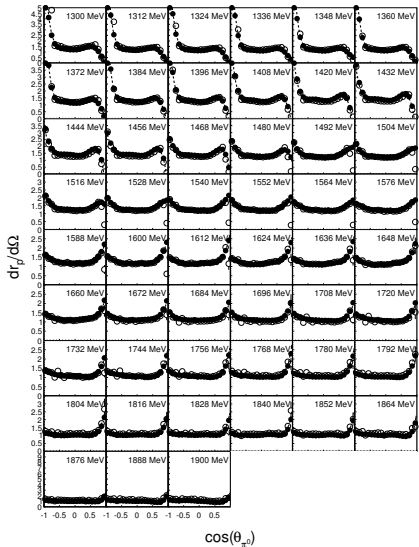
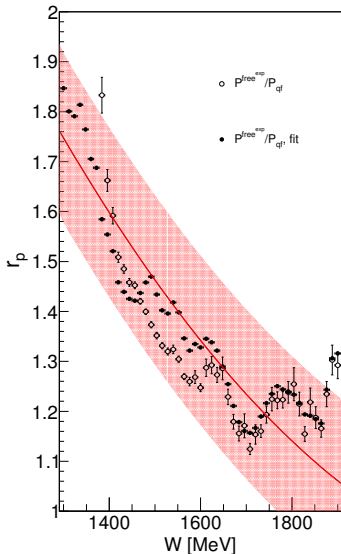
FONDS NATIONAL SUISSE  
SCHWEIZERISCHER NATIONALFONDS  
FONDO NAZIONALE SVIZZERO  
SWISS NATIONAL SCIENCE FOUNDATION

Deutsche  
Forschungsgemeinschaft

**DFG**

# Correct Final State Effects

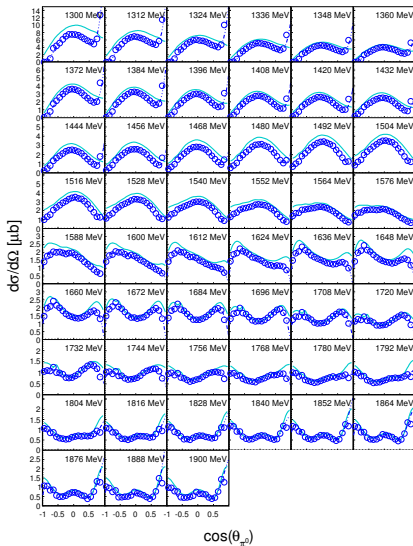
○ free/quasi-free ● correction factor



# Correct Final State Effects

 $\bigcirc \gamma p \rightarrow p\pi^0$  — SAID

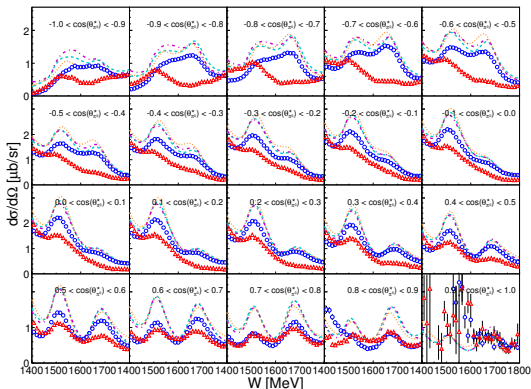
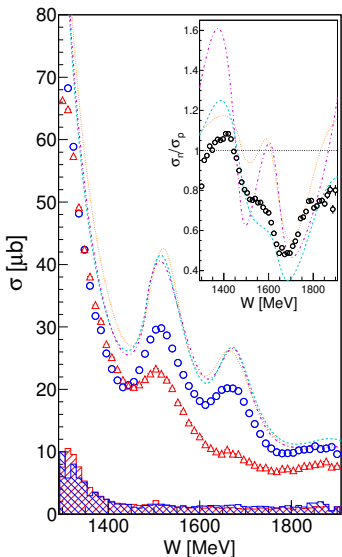
- ▶ assume similar effects for  $\gamma p(n) \rightarrow p(n)\pi^0$  as for  $\gamma n(p) \rightarrow n(p)\pi^0$
- ▶ correction factor from  $(\gamma p \rightarrow p\pi^0) / (\gamma p(n) \rightarrow p(n)\pi^0)$
- ▶ apply to quasi-free neutron data





# $\pi^0$ Cross Sections

○  $\gamma p \rightarrow p\pi^0$     △  $\gamma n \rightarrow n\pi^0$     - - - SAID-p    ····· MAID-p    - · - · BnGa-p



# $\pi^0$ Cross Sections

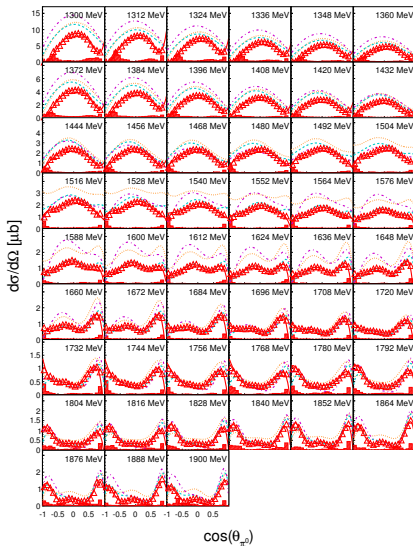
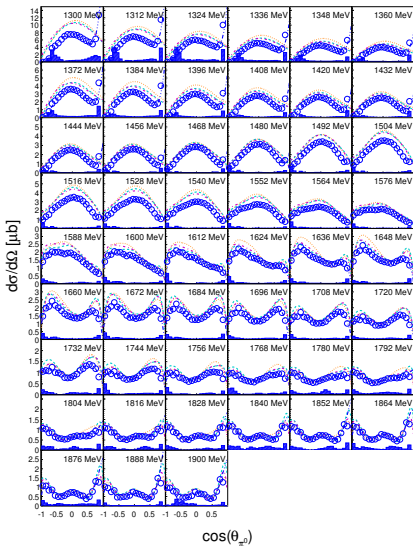
○  $\gamma p \rightarrow p\pi^0$

△  $\gamma n \rightarrow n\pi^0$

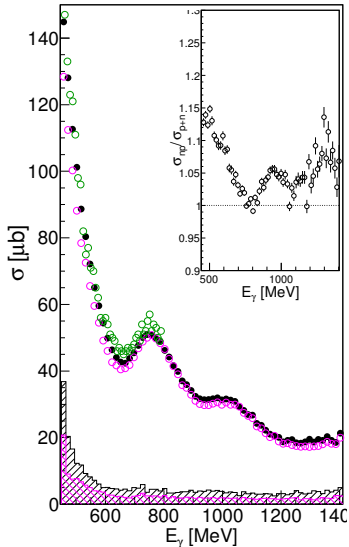
--- SAID-p

⋯ MAID-p

- - - BnGa-p



# Analysis Cross Check - QF-Inclusive

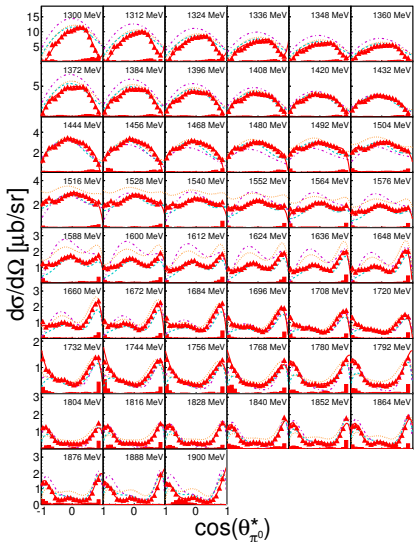
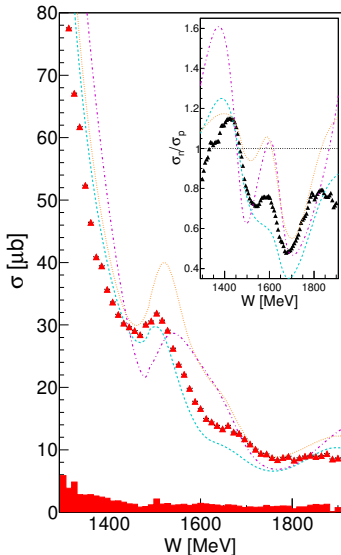


- ▶ compare qf-inclusive cross section with sum of proton and neutron cross sections  
 $(\gamma n \rightarrow n\pi^0) + (\gamma p \rightarrow p\pi^0) \approx \gamma N \rightarrow (N)\pi^0$
- ▶ good agreement between two reconstructions
- ▶ good agreement with former data
- ▶ nucleon identification/detection under control

# Free Cross Sections

$\blacktriangle \gamma n \rightarrow n\pi^0$

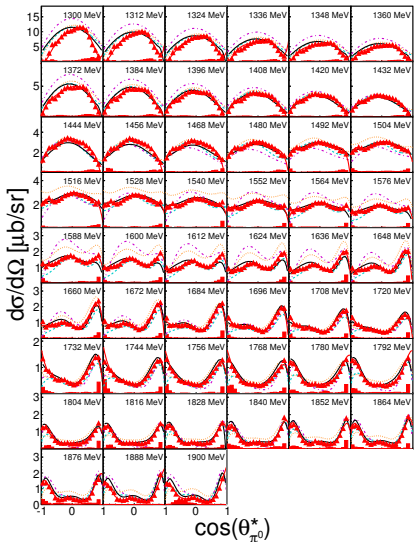
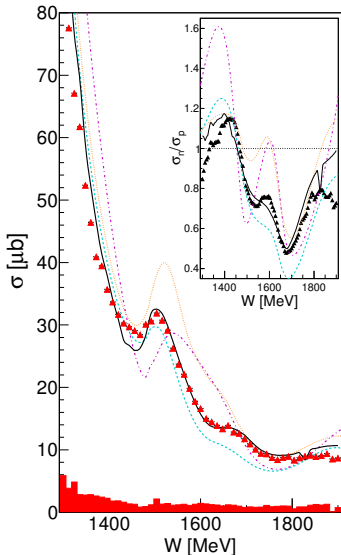
--- SAID --- MAID --- BnGa



# Free Cross Sections

$\blacktriangle \gamma n \rightarrow n\pi^0$

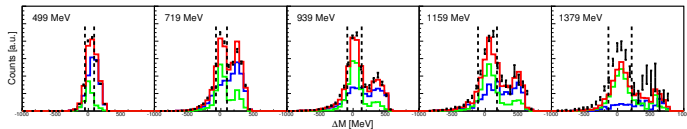
- - - SAID ⋯ MAID - · - · - BnGa — BnGa\_refit



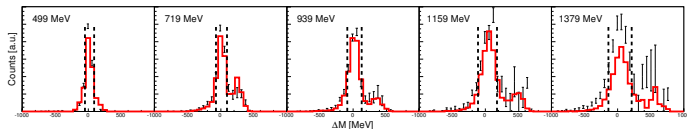
# $\pi^0$ Carbon Subtraction

+ dButanol    — Carbon    — LD2    — LD2 + Carbon

p

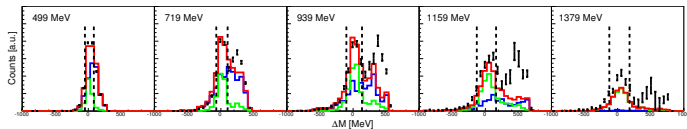


$-0.8 < \cos(\theta_{2\pi^0}^*) < -0.6$

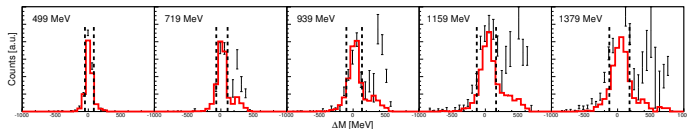


$-0.8 < \cos(\theta_{2\pi^0}^*) < 0.6$

n



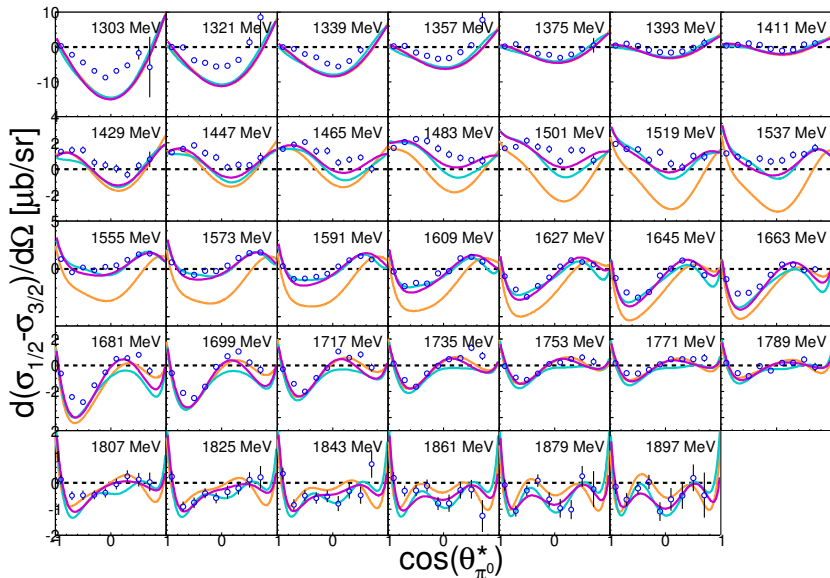
$-0.8 < \cos(\theta_{2\pi^0}^*) < -0.6$



$-0.8 < \cos(\theta_{2\pi^0}^*) < 0.6$

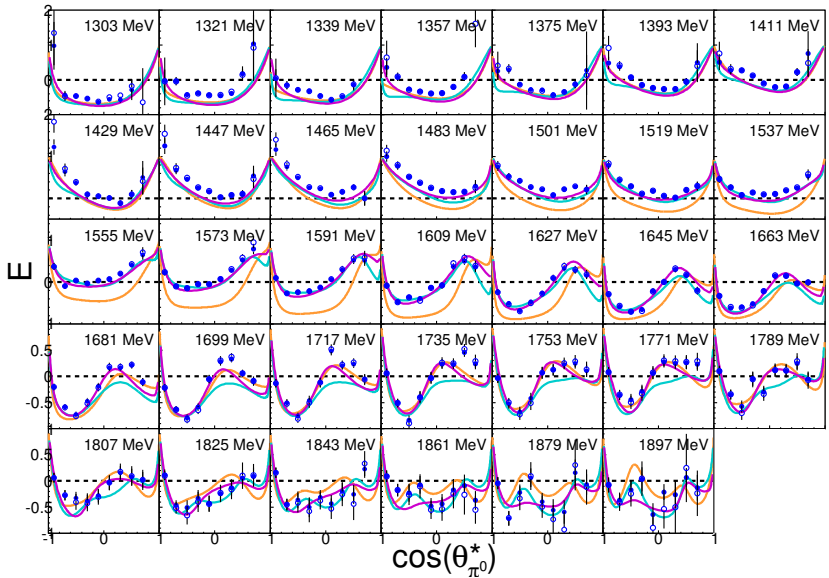
$$\pi^0 p - d(\sigma_{1/2} - \sigma_{3/2})/d\Omega$$

● direct    △ sum    — SAID    — MAID    — BnGa



# $\pi^0 p - dE/d\Omega$

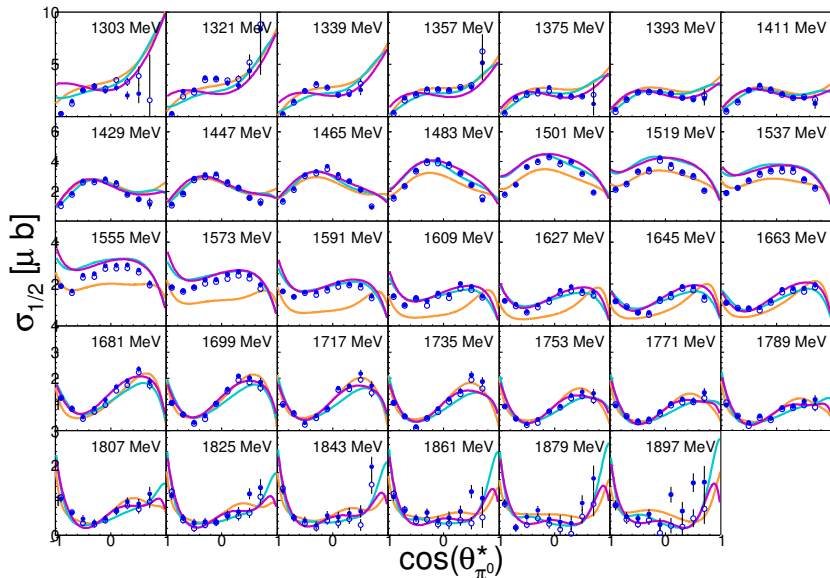
● direct    △ sum    — SAID    — MAID    — BnGa





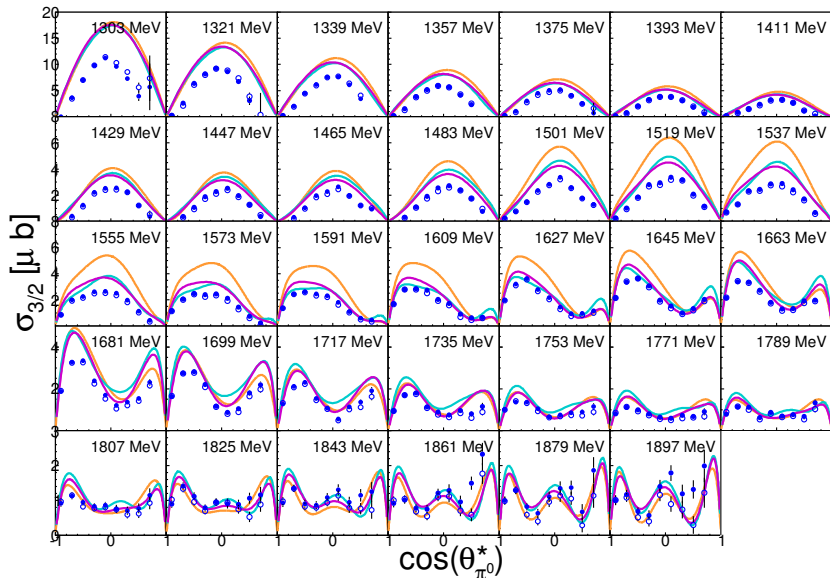
# $\pi^0 p - d\sigma_{1/2}/d\Omega$

● direct    △ sum    — SAID    — MAID    — BnGa



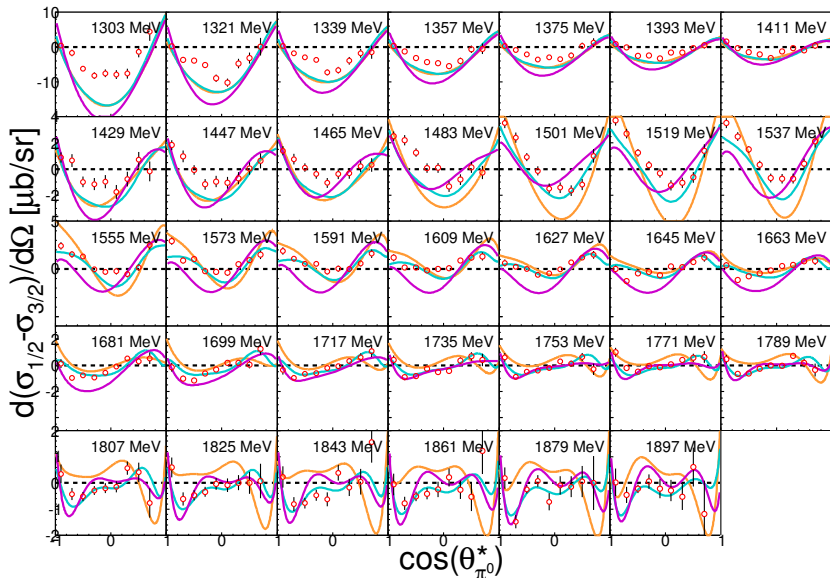
# $\pi^0 p - d\sigma_{3/2}/d\Omega$

● direct    △ sum    — SAID    — MAID    — BnGa



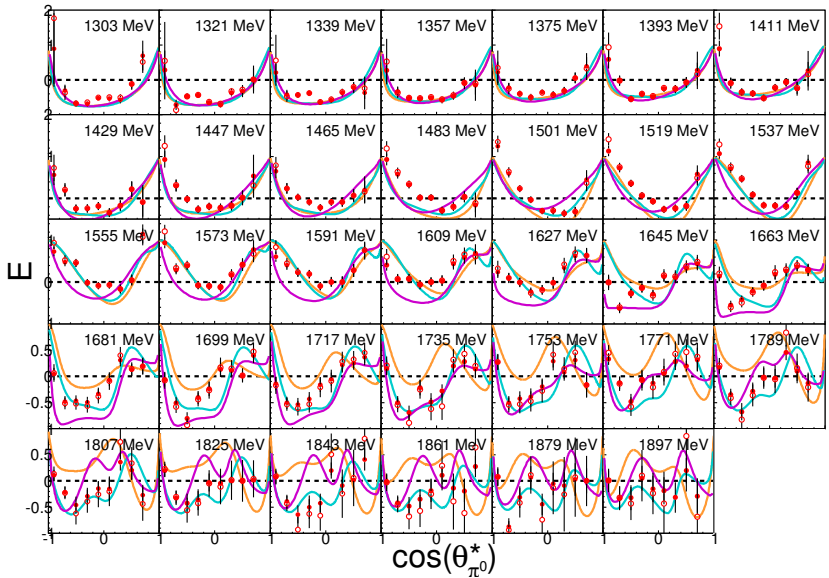
$$\pi^0 n - d(\sigma_{1/2} - \sigma_{3/2})/d\Omega$$

● direct    △ sum    — SAID    — MAID    — BnGa



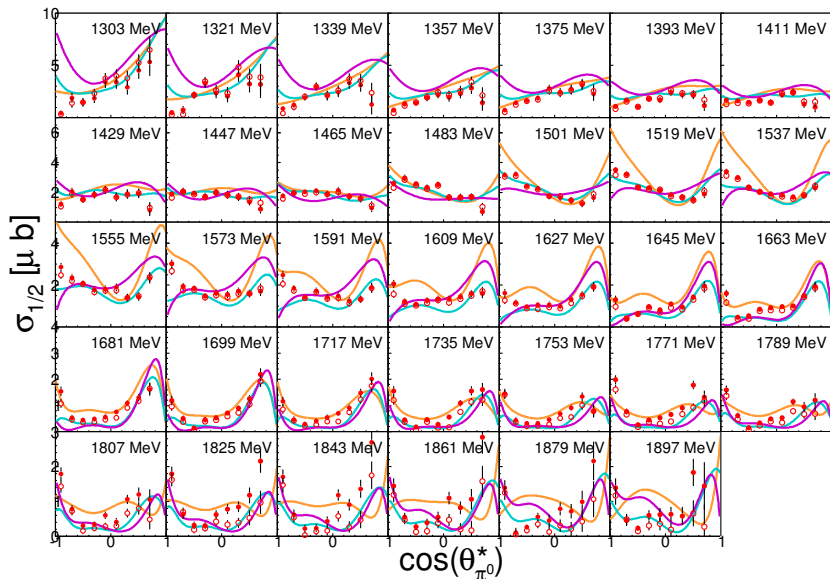
# $\pi^0 n - dE/d\Omega$

● direct    △ sum    — SAID    — MAID    — BnGa



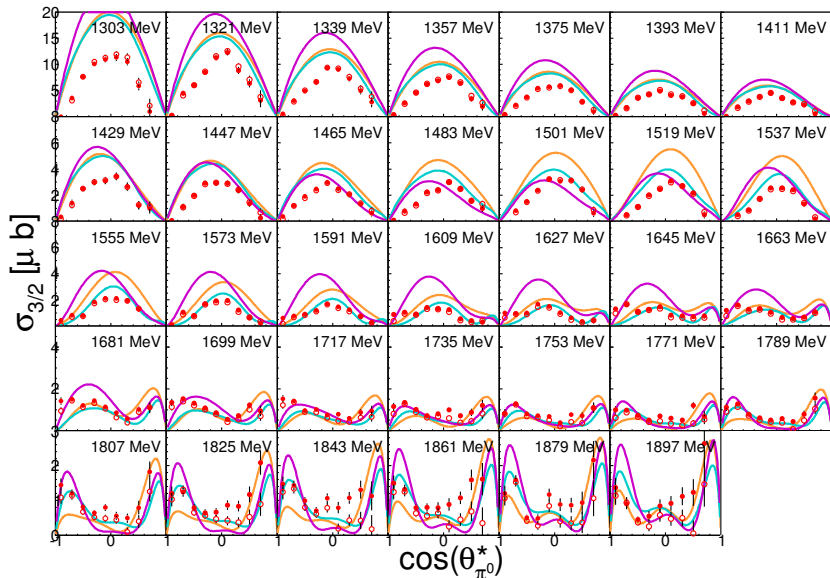
# $\pi^0 n - d\sigma_{1/2}/d\Omega$

● direct    △ sum    — SAID    — MAID    — BnGa

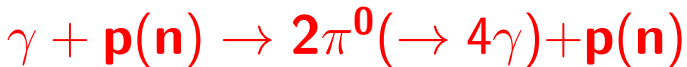


# $\pi^0 n - d\sigma_{3/2}/d\Omega$

● direct    △ sum    — SAID    — MAID    — BnGa



## Reaction Identification



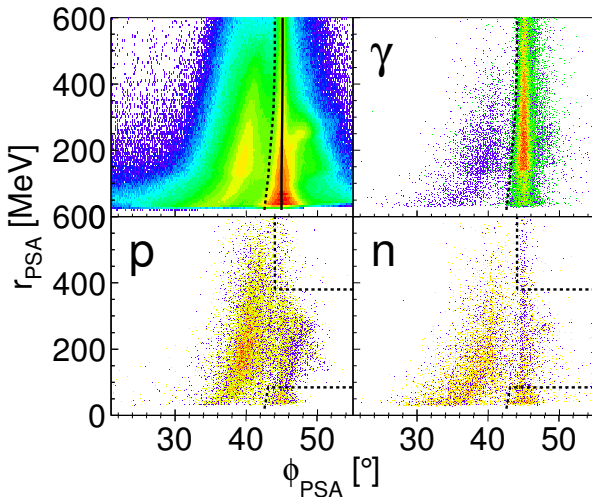
Reaction	Requirement: exclusive	inclusive
on Proton	4 neutral ( $2\pi^0$ ) 1 charged ( $p$ )	4 neutral 1(0) charged
on Neutron	5 neutral ( $2\pi^0 + n$ ) 0 charged	5(4) neutral 0 charged

$\chi^2$ -**test**: Reconstruct the  $2\pi^0$  out of the neutral particles  
(on Neutron: Remaining neutral hit is the Neutron candidate)

# Pulse Shape Analysis (PSA) (Remove wrong candidates)

**Photons:**  $3\sigma$  cut

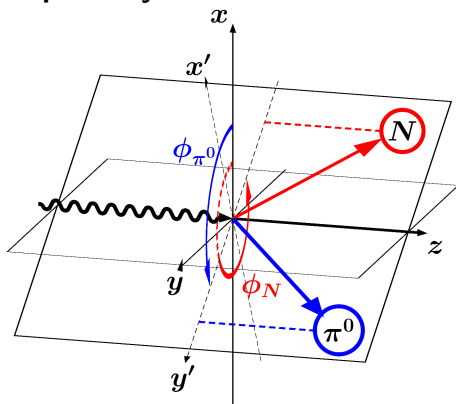
**Nucleons:** exclusion  
zones 85 – 380 MeV





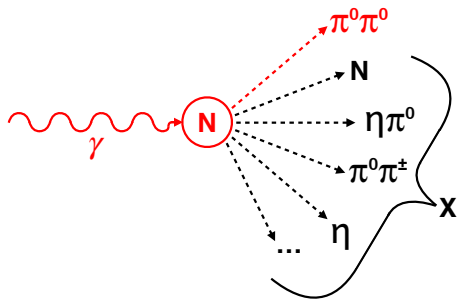
# Reaction Identification

## Coplanarity



$$\Delta\phi = 360^\circ - |\phi_{2\pi^0} - \phi_N|$$

## Missing Mass

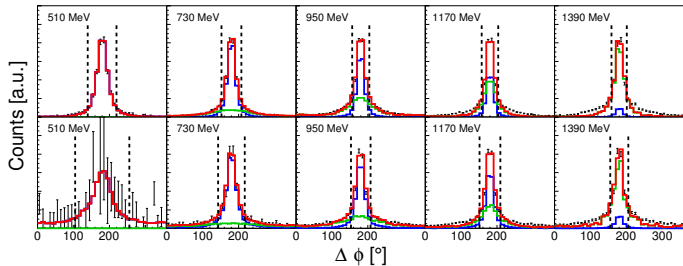


$$\Delta M = |P_{\text{Beam}} + P_N - P_{2\pi^0}| - m_N$$

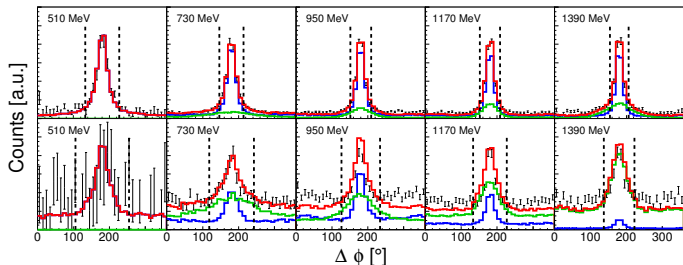
# Coplanarity Cut

+ LD2    — MC signal    — MC bg    — MC total    —  $2\sigma$  cut

**p**



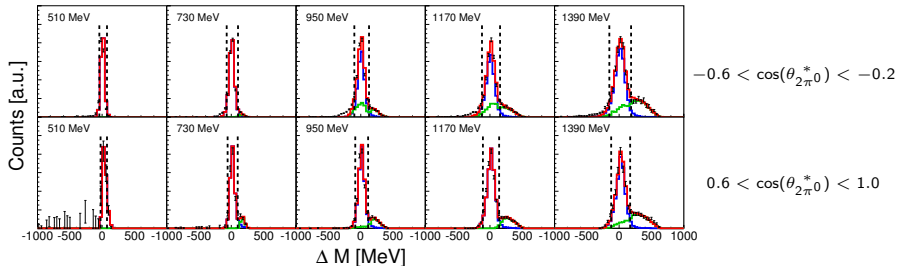
**n**



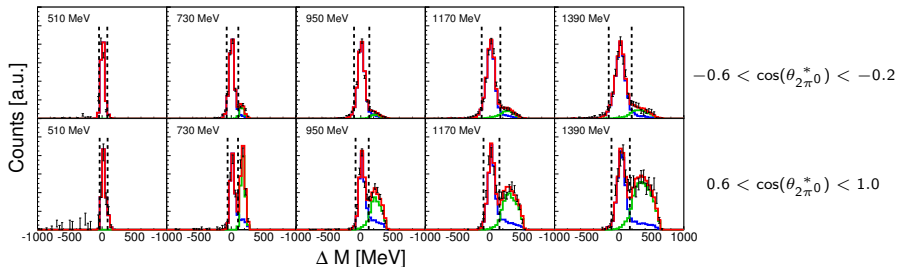
# Missing Mass

+ LD2    — MC signal    — MC bg    — MC total    —  $2\sigma$  cut

p



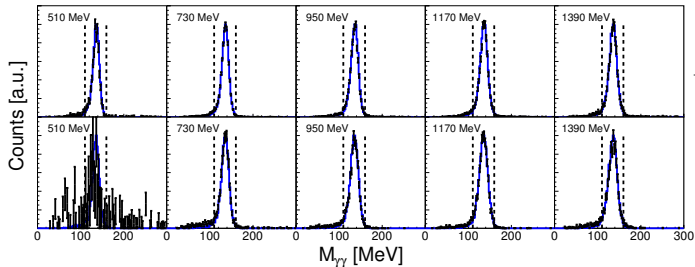
n



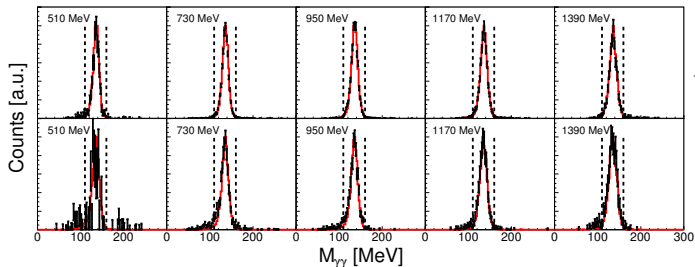
# Final Invariant Mass

+LD2 — MC proton — MC neutron — [110, 160] MeV cut

p



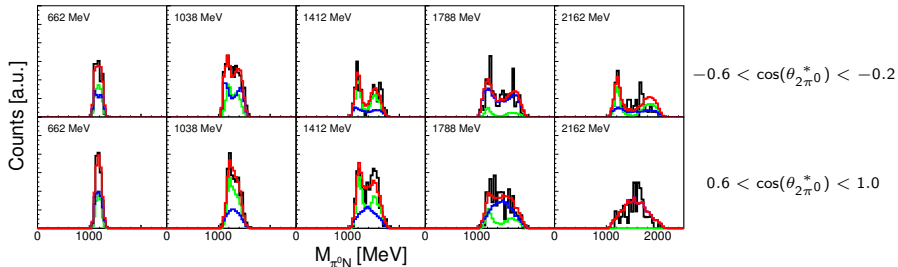
n



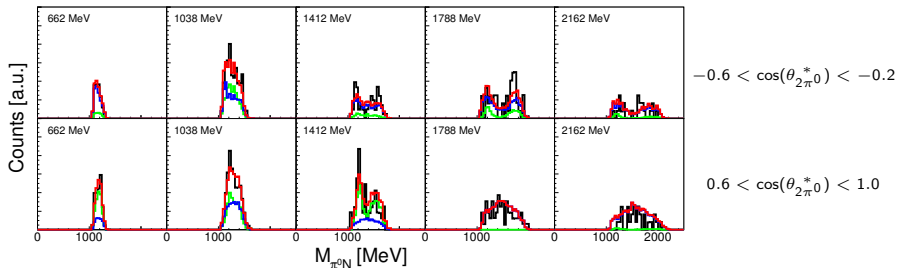
# Sequential Decays

+ Data    — Phase Space    —  $\Delta^0(1232)$     — Sum

p

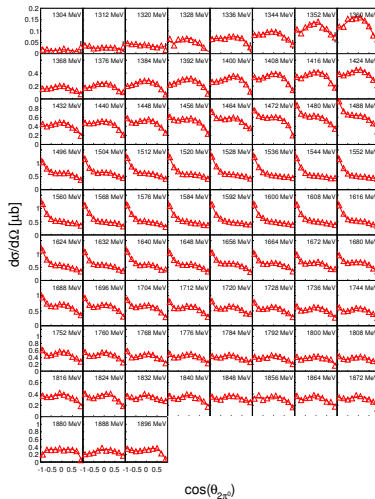
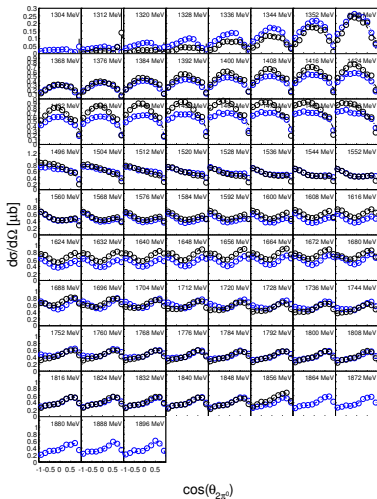


n



# $2\pi^0$ Photoproduction

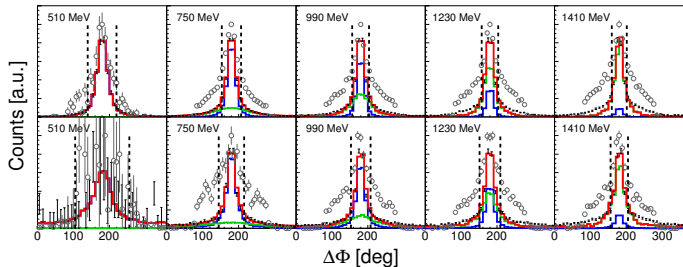
○ Proton   ○ Neutron



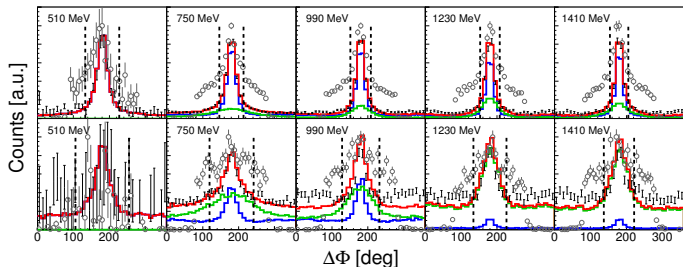
# Coplanarity Cut

○ dButanol    + LD2    — MC signal    — MC bg    — MC total    —  $2\sigma$  cut

**p**



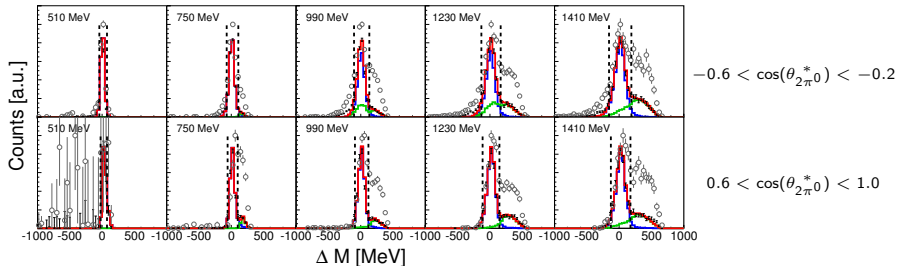
**n**



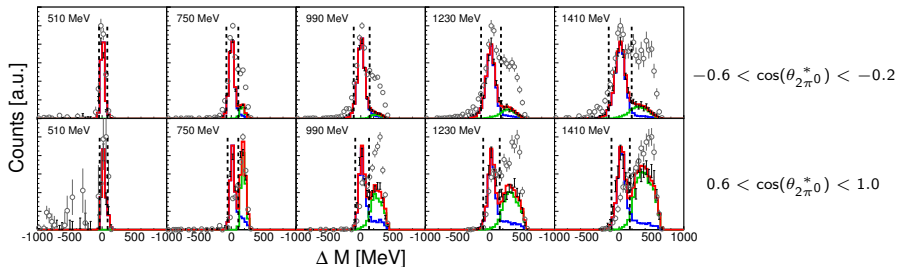
# Missing Mass

○ dButanol    + LD2    — MC signal    — MC bg    — MC total    —  $2\sigma$  cut

**p**



**n**

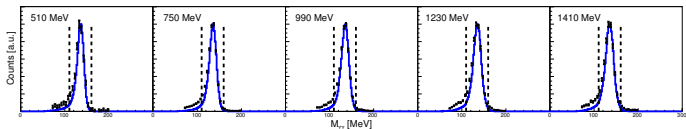




# Final Invariant Mass

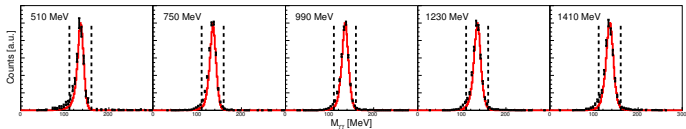
+  $C_4D_9OD$  — MC proton — MC neutron — [110, 160] MeV cut

**p**



$$-1.0 < \cos(\theta_{2\pi^0}^*) < 1.0$$

**n**

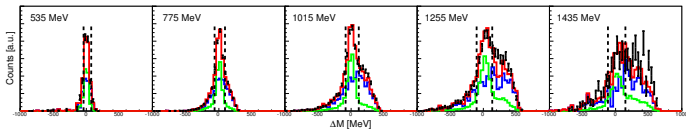


$$-1.0 < \cos(\theta_{2\pi^0}^*) < 1.0$$

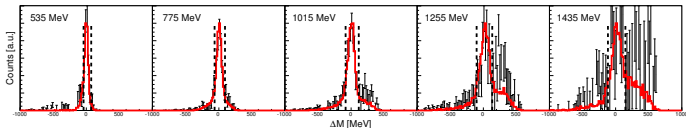
# Carbon Subtraction

● dButanol    — Carbon    — LD2    — LD2 + Carbon

p

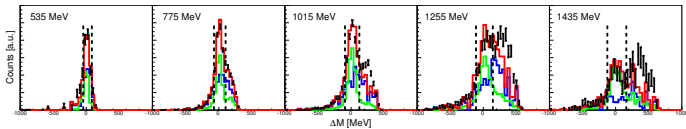


$-0.6 < \cos(\theta_{2\pi^0}^*) < -0.2$

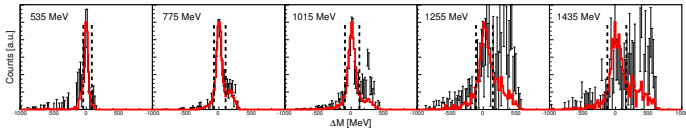


$0.6 < \cos(\theta_{2\pi^0}^*) < 1.0$

n



$-0.6 < \cos(\theta_{2\pi^0}^*) < -0.2$

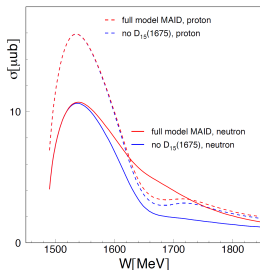


$0.6 < \cos(\theta_{2\pi^0}^*) < 1.0$

# $\eta$ Photoproduction

## 1. etaMAID:

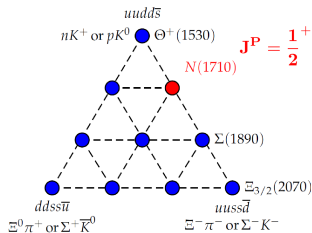
Large contribution of the  $D_{15}(1675)$   
 ➤ high value for the branching ratio  
 of  $\Gamma_{\eta N}/\Gamma_{tot} = 17\%$   
 (PDG:  $\Gamma_{\eta N}/\Gamma \simeq 0 - 1\%$ )  
 (L.Tiator, NSTAR2005)



## 2. Chiral Soliton model:

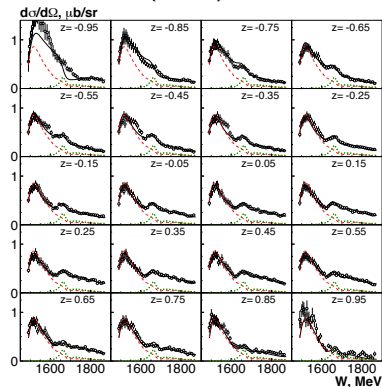
non-strange member of the baryon  
 antidecuplet:  $P_{11}(1680)$ .

bigger coupling to the neutron than  
 to the proton  
 (D.Diakonov et al., arXiv:hep-ph/9703373v2)

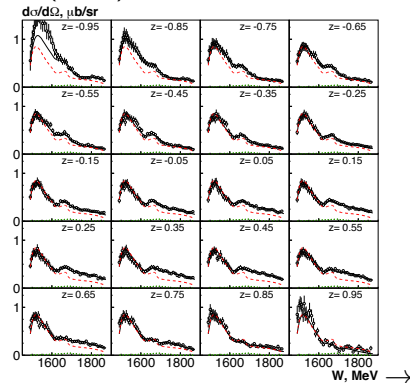


# Narrow Structure: Fit with BnGa

Narrow  $P_{11}(1685)$ :



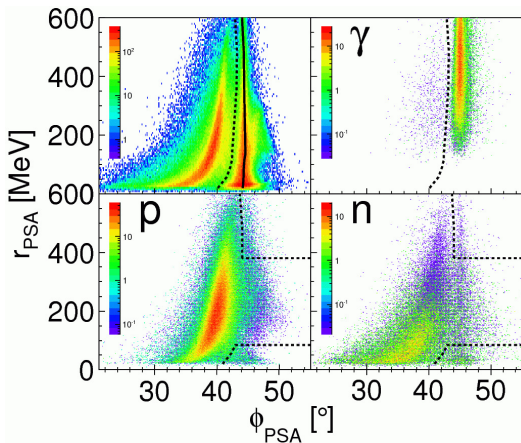
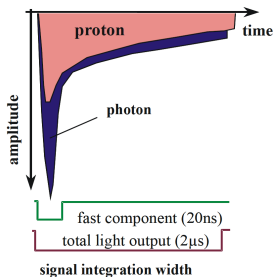
$S_{11}(1650)$  Interference :



sign change of elm.

$A_{1/2}$  coupling of  $S_{11}(1650)$

# Pulse Shape Analysis (TAPS)

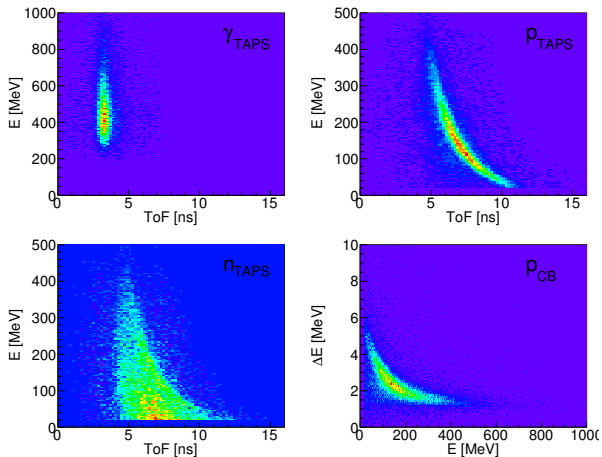


Photon :  $3\sigma$

nucleon: exclusion zone ]85,380[ MeV

# Other identification possibilities

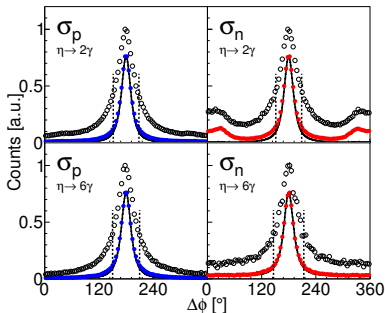
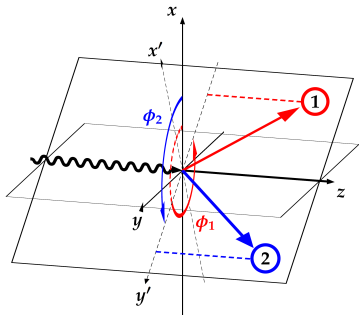
energy versus time of flight and  $\Delta E$  versus  $E$



# Background Suppression

Coplanarity:

$$\Delta\phi = \phi_N - \phi_\eta$$

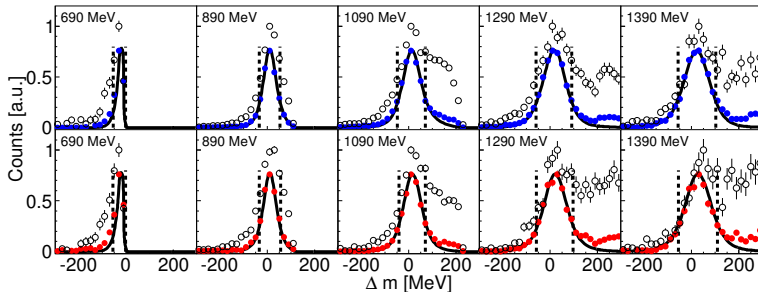


cut on  $\pm 2\sigma$

# Background Suppression

Missing Mass:

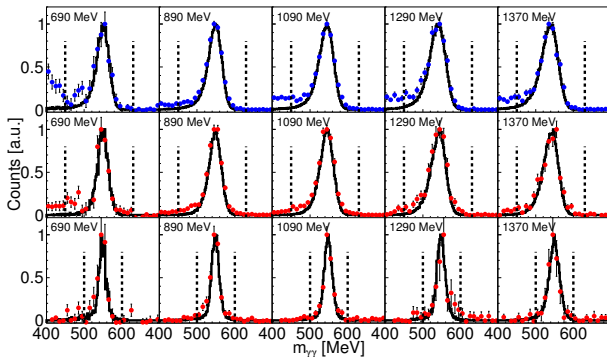
$$\Delta M = |P_{Beam} + P_N - P_\eta| - m_N$$



cut on  $\pm 1.5 \sigma$

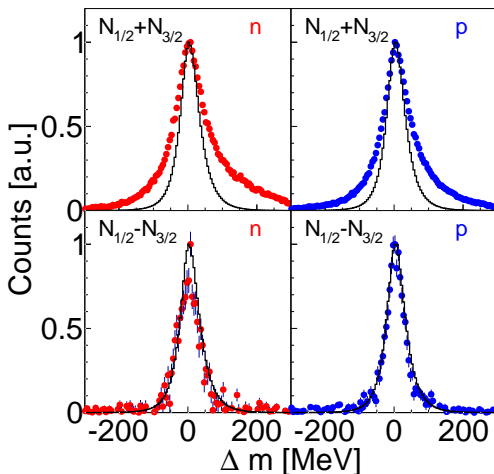


# Invariant Mass Distributions



- ▶ integrate  $m_{\gamma\gamma}(E, \cos(\theta))$   $2\gamma$ : 450-630 MeV  $6\gamma$ : 500-600 MeV

# Carbon Subtraction



- ▶  $N_{1/2} + N_{3/2}$ :  
carbon contribution  
→ carbon subtraction  
needed!
- ▶  $N_{1/2} - N_{3/2}$ :  
no carbon left!