Nucloen Excited States from Overlap Fermions

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For

(χQCD Collaboration)

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Z. Liu, Y.-B. Yang, et al.

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Outline

I. Introduction

II. N* from overlap fermions in QA

III. N* from full-QCD lattice study

IV. Omega* as a complementary discussion

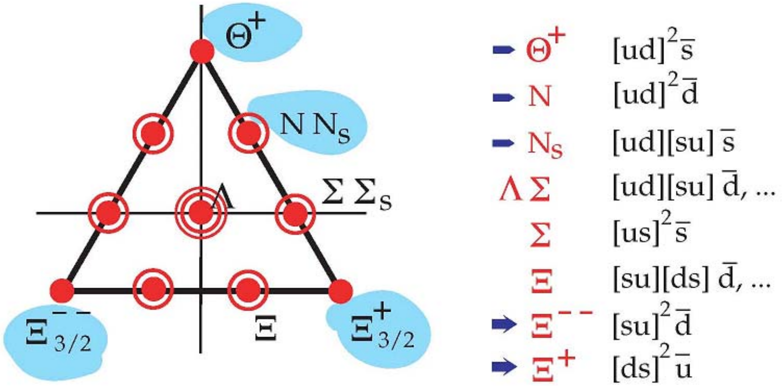
V. Summary and Outlook

I. Introduction

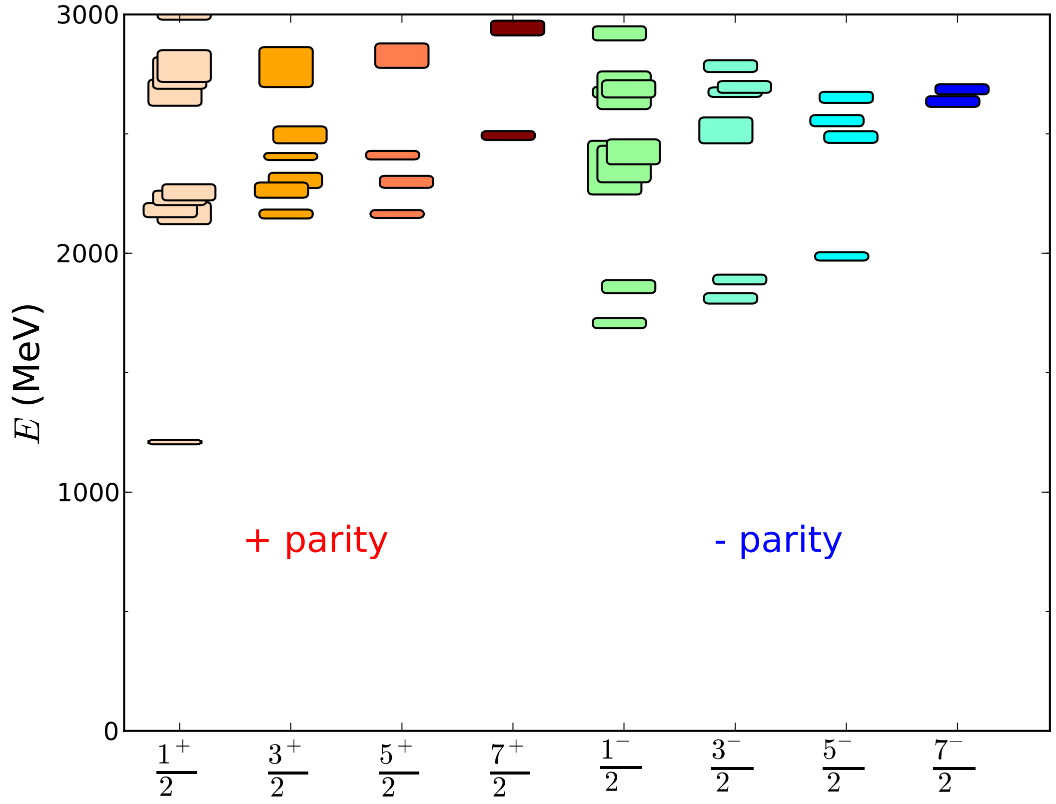
- There are many nucleon excited states predicted by SU(6) quark model.
- Actually many N^* has been observed in experiments, but they are less than the number of states predicted by QM. Where are the missing states?
- The level reversal of the Roper resonance P11(1440) and the S11(1535) resonance. It is still a puzzle.
- However, the lattice results of the mass of the first p-even excited state of N* is very controversial (see Derek' s review this morning)
- The mxing with πN states makes the situation more complicated.
- Confront with Expt.? Still challenging for lattice QCD calculation

As far as the Roper state, there are many phenomenological Considerations:

- Quark potential model prediction is 100-200 MeV too high
  (Liu and Wong, 1983, Capstick and Isgur, 1986)
- Skyrmion can accommodate it as a radial excitation
  (J. Breit and C. Nappi, 1984 , Liu, Zhang, Black, 1984;
  U. Kaulfuss and U. Meissner, 1985)
- Suggestion as a pentaquark (Krewald 2000);
  as a member of the antidecuplet
  (Jaffe, Wilczek, 2003)
- Perhaps a hybrid
  (Barnes, Close, etc. 1983)
- → Lattice calculations



Lattice N* states (m $_{\pi}$ =396MeV)



LQCD finds states as predicted in SU(6)xO(3)

R. Edwards, J. Dudek, D. Richards,
S. Wallace, PRD84, 074508 (2011)

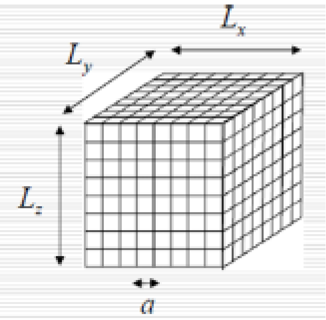
1. The lattice formulation of QCD---Lattice QCD

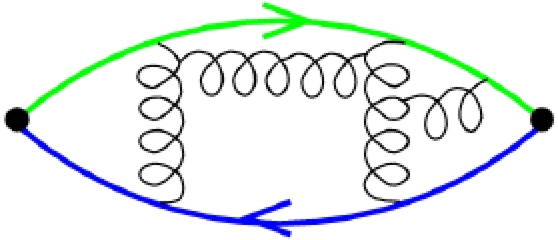
$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \; e^{-S}$$

$$S = S_{gauge} + S_{quarks} = \int d^4x \, \left(\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) - \sum_i \log(\mathrm{Det} M_i)$$

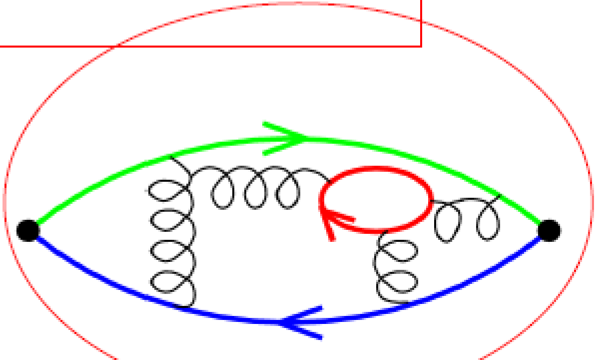
$$Z = \int \mathcal{D}A_\mu \; \det M \; e^{\int d^4x \, \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right)} .$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \; \mathcal{O} \; e^{-S} \; .$$





(A) Quenched QCD: quark loops neglected



(B) Full QCD

Ten years ago

Dominated in the present era

2. Chiral fermions on the lattice

- Chrial symmetry is an important symmetry in QCD. It must play a crucial rule in light hadron physics.

- Chiral tranformation $\lfloor D \to \gamma_\mu D_\mu \quad (a \to 0) \rfloor$

$$\psi \to e^{i\theta\gamma_5}\psi; \qquad \psi \to e^{i\theta\gamma_5(1-aD/2)}\psi;$$

$$\overline{\psi} \to \overline{\psi}e^{i\theta\gamma_5} \qquad \overline{\psi} \to \overline{\psi}e^{i\theta(1-aD/2)\gamma_5}$$

In the conitnuum limit

$$\gamma_5 D + D\gamma_5 = 0$$

on the lattice

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

- Ginsburg-Wilson relation---chiral symmetry on lattice

- The lattice action is invariant if GW relation holds for D:

Domain wall fermions
Overlap fermions

1)Chiral symmetry
2) Very low pion mass can be accessed

II. The Roper and S11 from overlap fermion in the quenched lattice QCD

1. The spectrum of the nucleon, the Roper, and S11 (ten years ago)

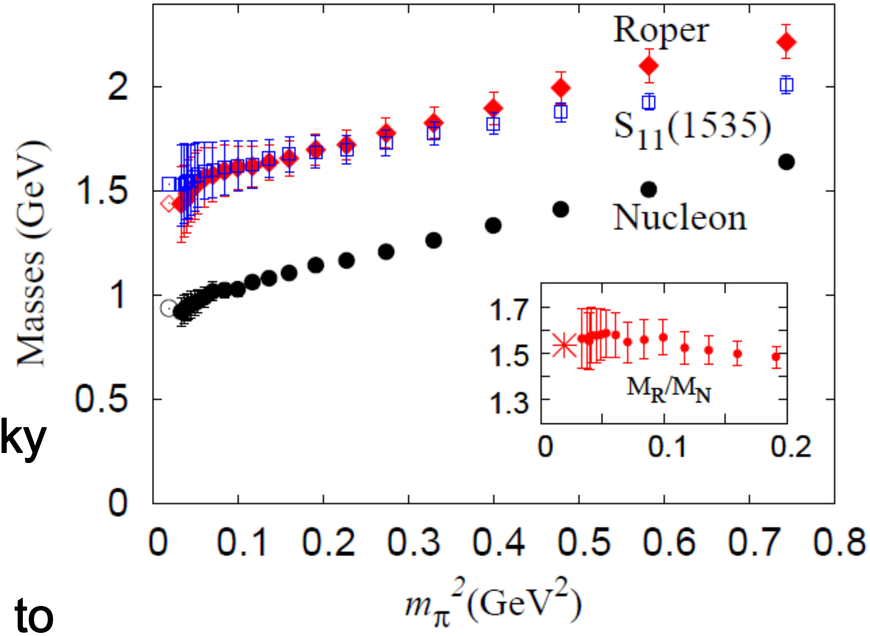
- Lattice

$$L^3 \times T = 16^3 \times 28;$$
$$m_\pi^{\min} = 180 MeV$$

- Nucleon interpolation field operator

$$\eta(x) = \epsilon^{abc} u^{aT}(x) C\gamma_5 d^b(x) u^c(x)$$

- Constrained curve fitting---------
"Sequential Empirical Bayes"(SEB)
(Y.Chen et al., hep-lat/0405001(Kentucky Group))



N. Mathur et al, PLB605(2005)137 (hep-lat/0306199)

- Roper mass and S11(1535) mass tends to cross over as quark mass decreases.

- Chiral symmetry may play a role for this crossover. Flavor-spin interaction takes place due to the Goldstone boson exchange (L.Y. Glozman and D.O. Riska, Phys. Rep. 268, 263(1996))

Sequential Empirical Beyesian Method

prior

Bayes: constrained-curve fitting

$$\chi^2_{\rm aug} = \chi^2 + \chi^2_{\rm prior}, \qquad \chi^2_{\rm prior} = \sum_i \frac{(\rho_i - \tilde{\rho}_i)^2}{\tilde{\sigma}_i^2}$$

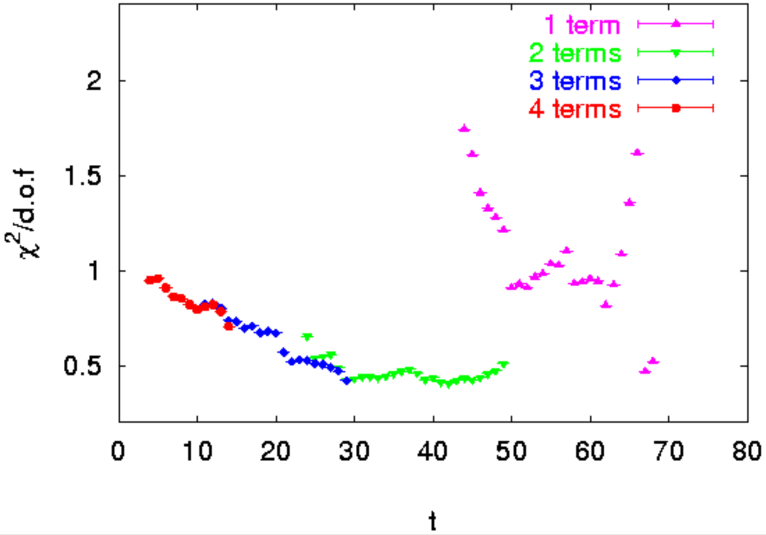
Empirical: priors are derived from part of data

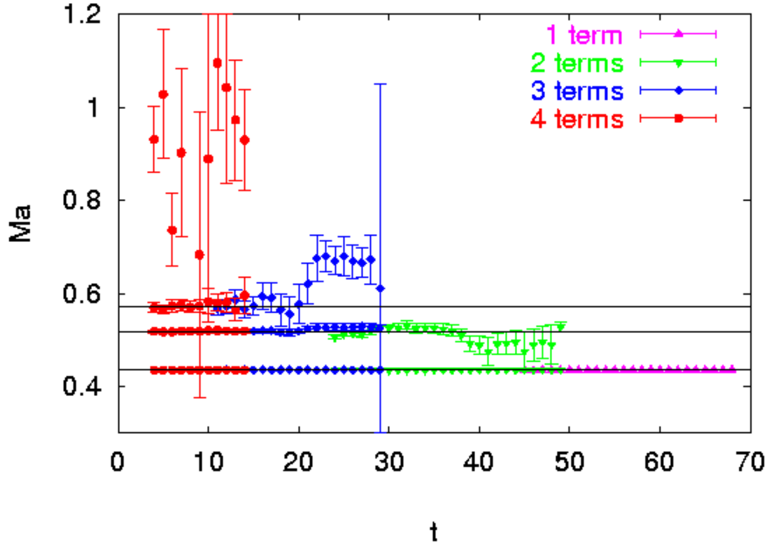
**(‘prior’ means the prior information of parameters)

Sequential: states fitted one by one from low to high.

$$C(t) = \sum_i W_i e^{-m_i t}$$

An Example of SEB





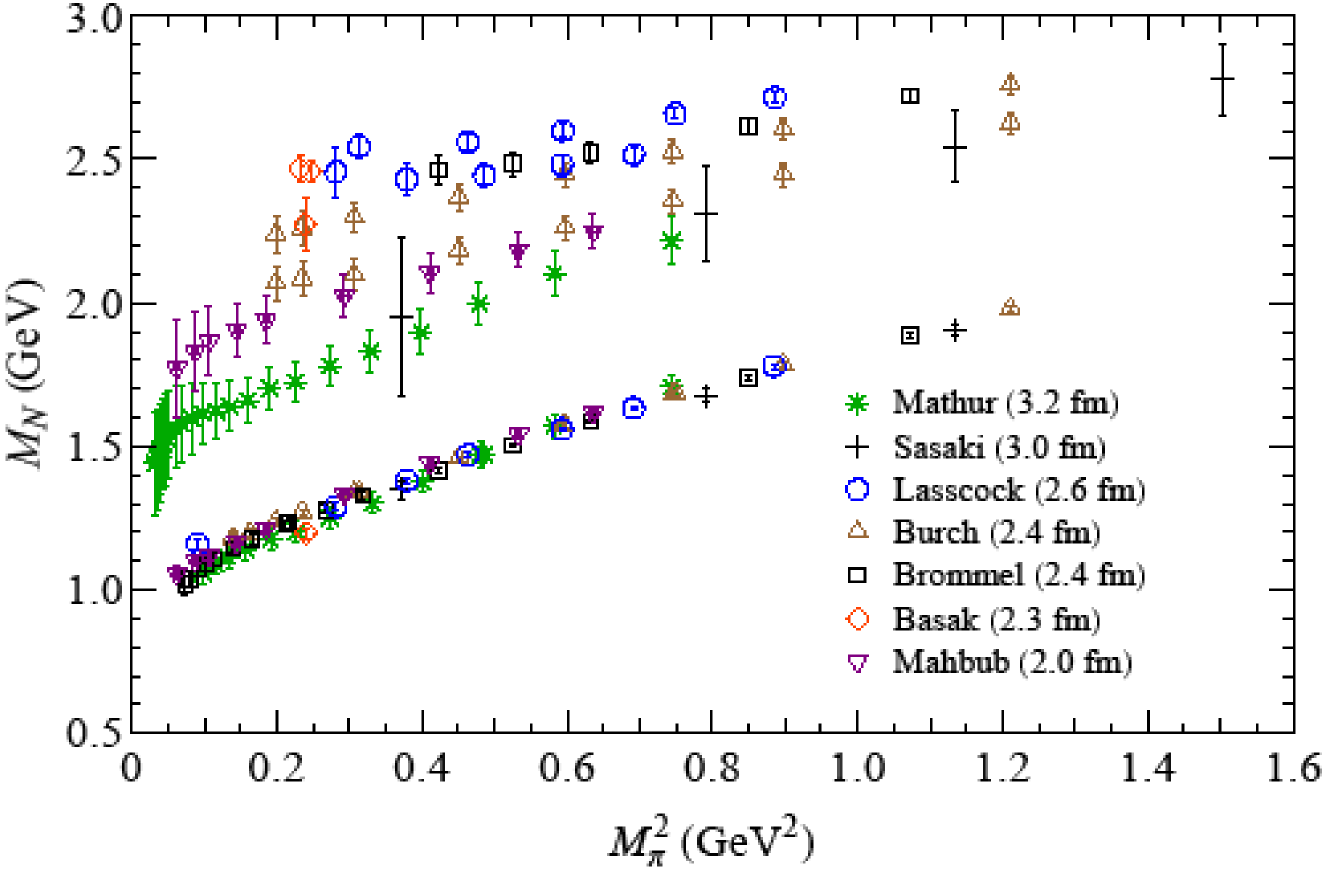
- Here is an example for the fitting procedure in the vector channel (four-mass-term fit), where t_max =68, t_1=50, t_2=30, t_3=15
- Fit model

$$C(t) = \sum_i W_i e^{-m_i t}$$

- The mass terms are fitted one by one from low to high

1. Choose the t_max.
2. Varying t_1, one-mass -term fitting in the interval [t_1,t_max], until the chi^2/dof blows up ;
3. Add the second mass term at t_2=t_1-1. Varying t_2, two-mass-term fitting in the interval [t_2,t_max] with the first state constrained by the fitted parameters in Step 2.
4. When chi^2/dof blows up again at the last t_2, and add the third state at t_3=t_2-1.
5. Repeat.
6. The latest state is generally taken as a garbage can and therefore not a realistic state.

However, when comparing with other quenched results at that time,



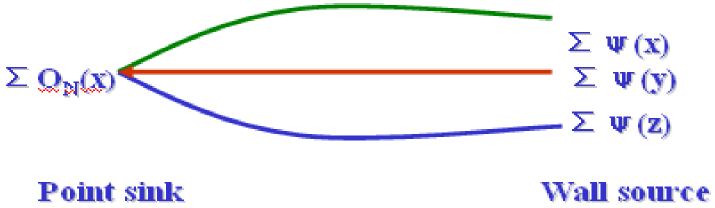
The discrepancy can be as large as 1GeV for the mass of Roper

2. The Bethe-Salpeter wave functions of nucleon and the Roper

- Coulomb gauge fixed
- Wall-source point sink correlation functions are calculated and analyzed.
- The spectrum is similar to that of point source.
- The spectral weights of Roper are negative for both heavy and light quark masses.

$$\langle 0|\eta(\vec{x},0)|N\rangle \propto \Phi_N(\vec{x})$$

- Wall-source means, qualitatively

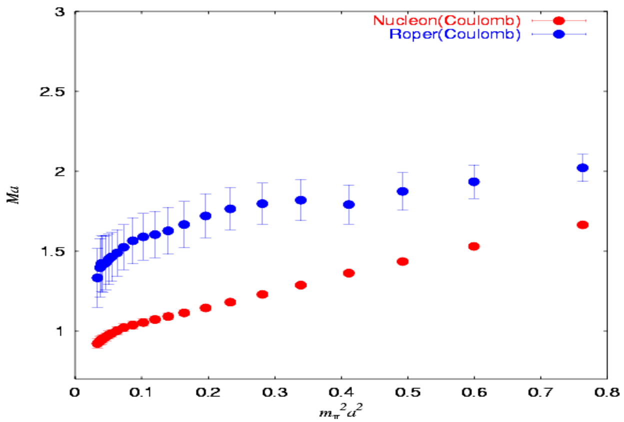


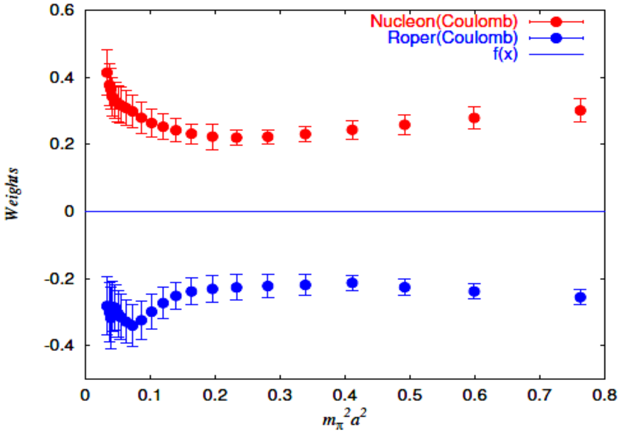
$$<0|O_N(0)|N><N|\ \Sigma\ \psi(x)\ \Sigma\ \psi(y)\ \ \Sigma\ \psi(z)|0> \ > 0$$

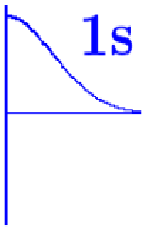
$$<0|O_N(0)|R><R|\ \Sigma\ \psi(x)\ \ \Sigma\ \psi(y)\ |\ \Sigma\ \psi(z)|0> \ < 0$$

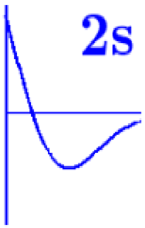
$$\int d^3\vec{x}\Phi_{N^*}(\vec{x}) = 4\pi\int drr^2\Phi_{N^*}(r) < 0$$

- This hints that $\Phi_{N^*}(r)$ a radial node.

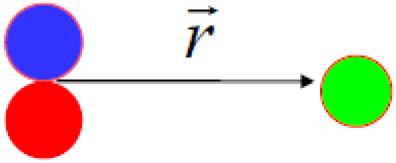








- Coulomb gauge fixed
- Spatially extended sink operator



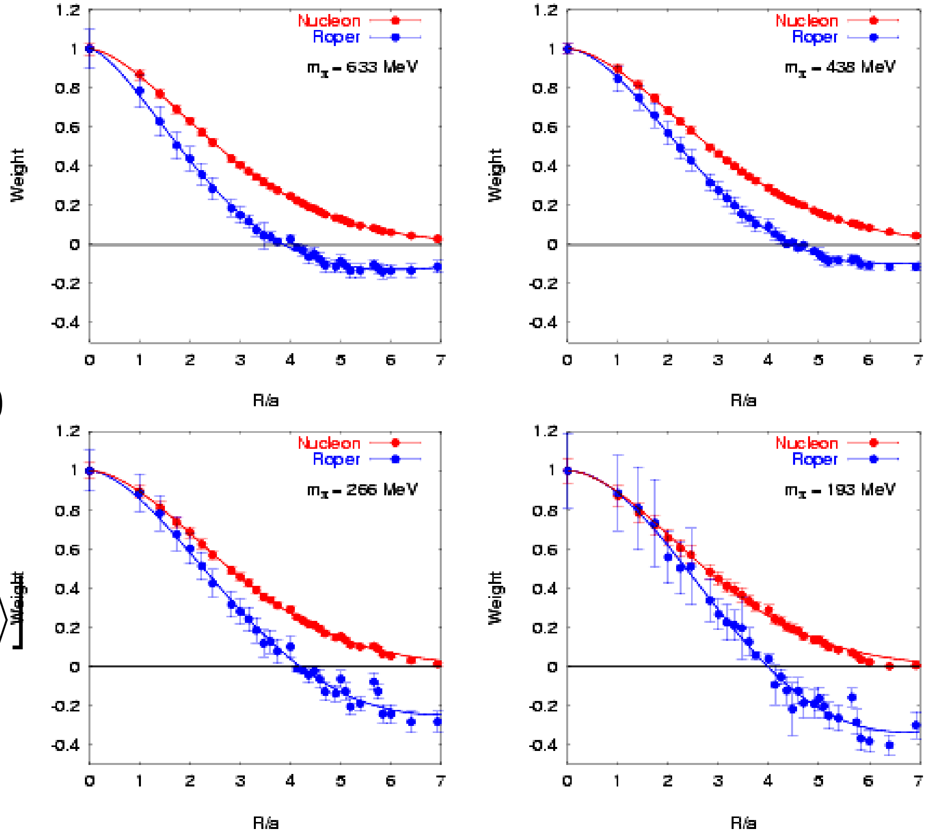
$$\eta(\vec{x},t;\vec{r}) = \varepsilon^{abc}\left(u^{aT}\gamma_5 d^b\right)(\vec{x},t)u^c(\vec{x}+\vec{r},t)$$

- For ½+ states,

$$C(r,t) = \sum_{\vec{x}} Tr\left[(1+\gamma_4)\left\langle \eta(\vec{x},t;\vec{r})\overline{\eta}^{(W)}(0)\right\rangle\right]$$

$$= \sum_n W_i(r)e^{-m_i t}$$

$$W_i(r) \propto \left\langle 0\middle|\eta(0,0;r)\middle|i\right\rangle$$



$$\phi_0(r) = e^{-\left(\frac{r}{\beta_0}\right)^{\alpha_0}}$$

$$\phi_1(r) = \left(1+\kappa r^{\alpha_1}\right) e^{-\left(\frac{r}{\beta_1}\right)^{\alpha_1}}$$

$$
\begin{aligned}
\phi_0(r) &= e^{-\left(\frac{r}{\beta_0}\right)^{\alpha_0}} \\
\phi_1(r) &= \left(1+\kappa r^{\alpha_1}\right) e^{-\left(\frac{r}{\beta_1}\right)^{\alpha_1}}
\end{aligned}
$$

$$
\langle r^2 \rangle = \frac{\int\limits_0^\infty dr r^4 \phi_0(r)^2}{\int\limits_0^\infty dr r^2 \phi_0(r)^2}.
$$

$m_q a$	$M_\pi a$	β_0	α_0	r_{RMS} (fm)
0.22633	0.6416	3.21(1)	1.62(1)	0.362
0.10850	0.4438	3.51(1)	1.71(1)	0.404
0.03617	0.2693	3.44(1)	1.77(2)	0.334
0.01633	0.1953	3.36(2)	1.78(3)	0.334

κ	β_1	α_1
-0.130(6)	3.79(8)	1.51(4)
-0.073(3)	4.09(7)	1.75(3)
-0.088(9)	5.07(25)	1.70(8)
-0.073(10)	5.21(32)	1.88(11)

III. N* from a full-QCD lattice study with overlap fermions

Lattice formulation:

- overlap fermions as valence quarks;
- 2+1-flavor domain-wall fermion configurations (generated by RBC/UKQCD Collaboration) (Y. Aoki et al, PRD83(2011)074508.

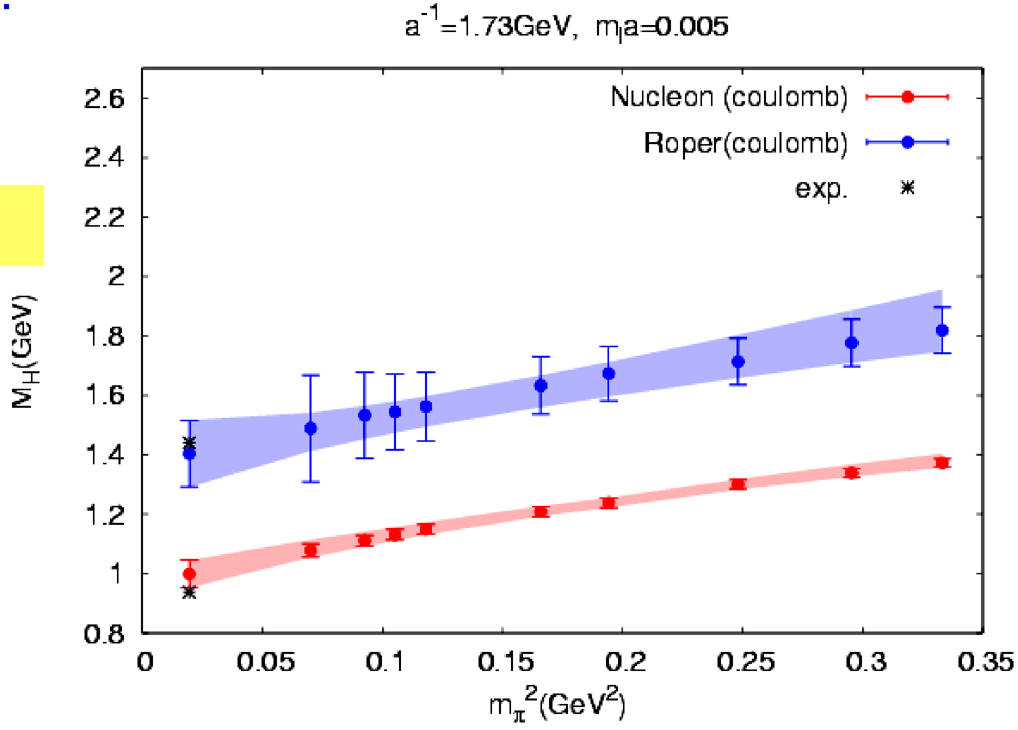
$L^3 \times T$	r_0/a(GeV)	$m^{res}a$	$m^l_{sea}a$	$m^s_{sea}a$
$24^3 \times 64$	4.126(11)	0.00315	0.005	0.04

r_0(fm)	a^{-1}(GeV)	Z_m	Z_A	N_{conf}
0.458(11)	1.77(5)	0.884(7)	1.110(1)	77

$$m_\pi^{(ss)} = 331MeV$$

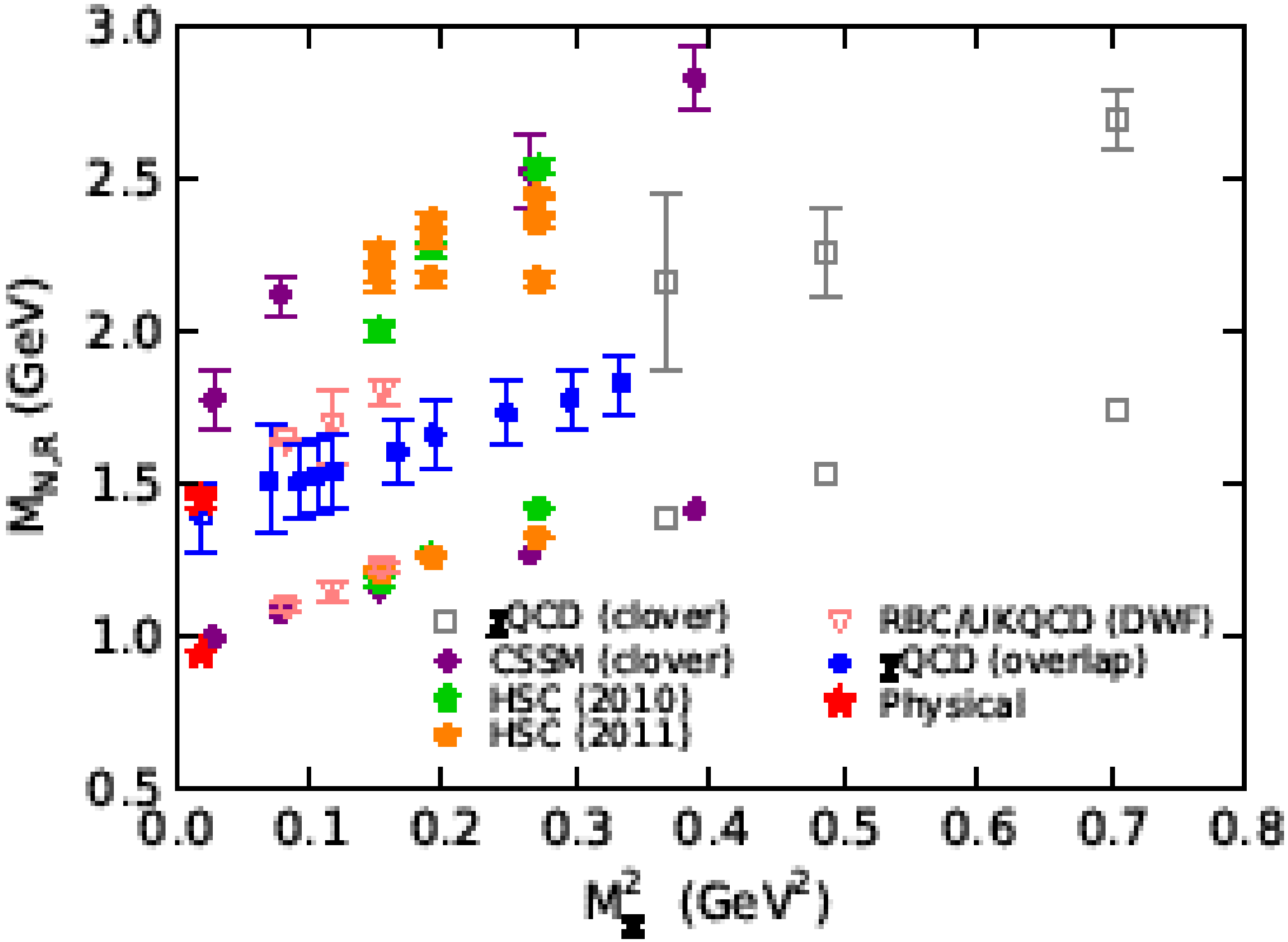
1. The spectrum of p-parity states.

- Coulomb gauge fixed
- Wall source propagator, similar to the case of QA.
- also SEB fit.
- In agreement with QA results.
- However, ……



$$m_N = m_0 + c_1 m_\pi^2 + c_2 m_\pi^{(vs),2}$$

When comparing with other full-QCD results, the discrepancy is still very large!



What are the reasons for this discrepancy?

- Different data analysis procedures
  Variational method(VM) vs. SEB

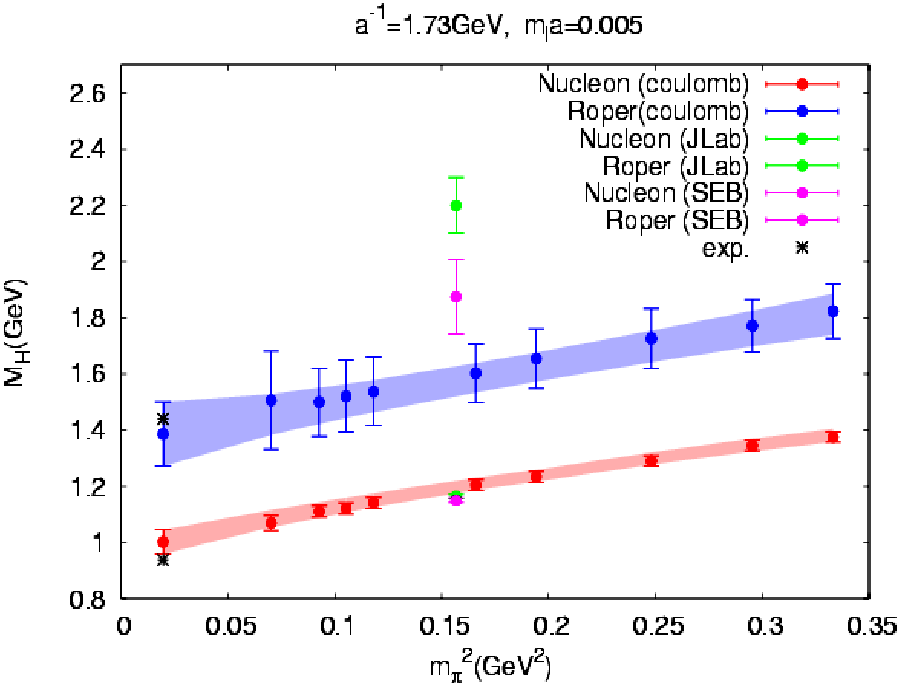
Variational method:

$$\{O_\alpha, \alpha = 1,2,\ldots n\};$$

$$C_{\alpha\beta}(t) = \langle O_\alpha(t) O_\beta(0) \rangle$$

$$C(t)V = \lambda_i(t_0)C(t_0)V_i$$

$$O_i = \sum V_{i,\alpha} O_\alpha$$



Caveat of VM: The number of the operators is usually much smaller that the number of states, so the resultant state can be an admixture of several physical states.

- Different source operators

  Gaussian smeared source vs. Coulomb gauge fixed wall source

  Does the hadonic correlation functions calculated through coulomb wall source propagators have less contribution from scattering states (and higher states)?

We check this using the new gauge configurations with 2+1 flavor domain-wall sea quarks on a large lattice.

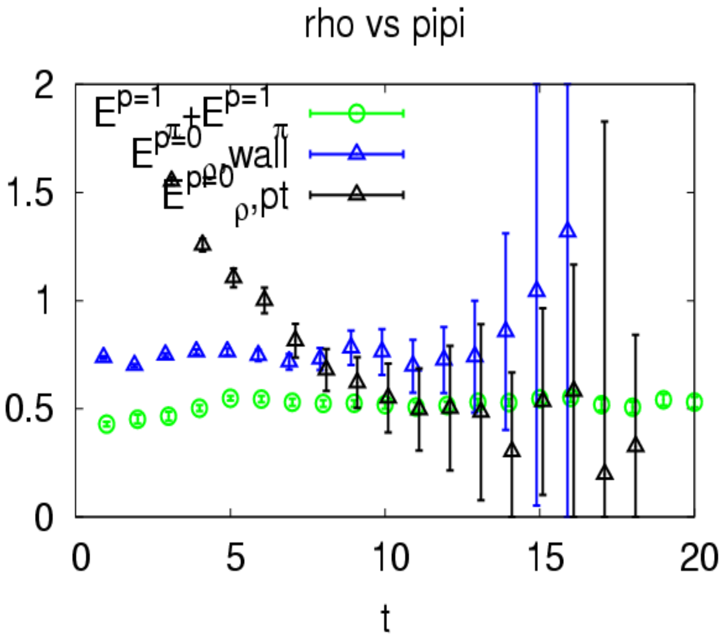
$1/a$(GeV)	label	$am^{(s)}$	$L^3 \times T$	$N_{\rm conf}$
1.76(1)	48I	0.00078/0.0362	$48^3 \times 96$	45

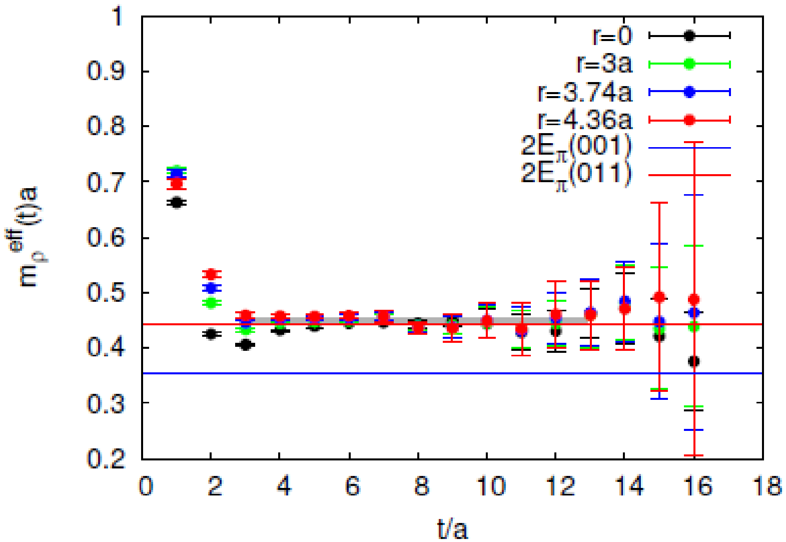
$$m_\pi^{(ss)} \approx 140 MeV, \quad m_\pi^{(vv)} \approx 117{,}140{,}150{,}170{,}210 MeV$$

$$La = 5.5\,fm \qquad\qquad p_{\min} \approx 230 MeV$$

We take the the rho-channel for example, because

$$m_\rho > 2E_\pi(p_{\min}) = 2\sqrt{m_\pi^2 + p_{\min}^2}$$





Left panel-Point source

- green points: $2E_\pi(p_{\min}) = 540MeV$
- blue triangle:

$$m_\rho = 780MeV$$

- black triangle: effetive masses of the point-source correlation functions.

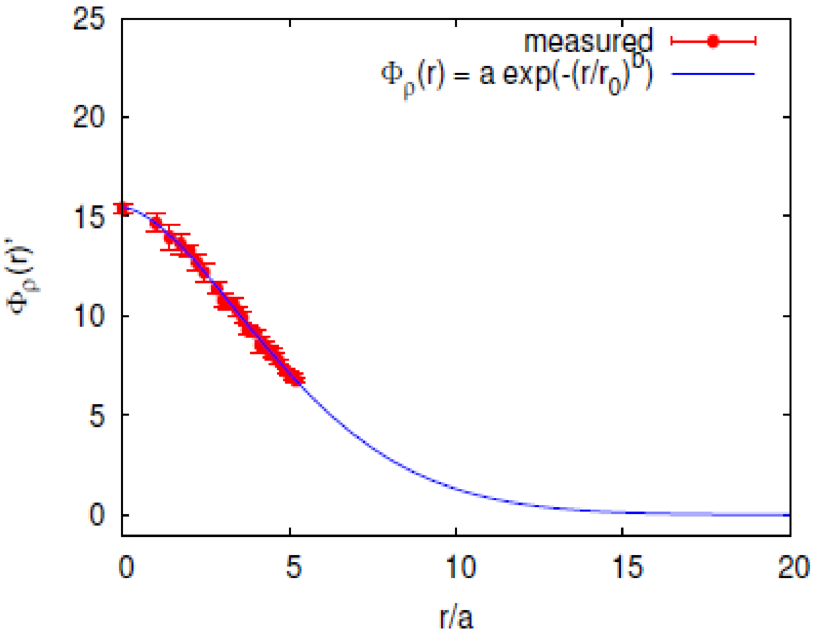
Right panel-Coulomb wall source

- points: effective masses of wall-source correlation functions.

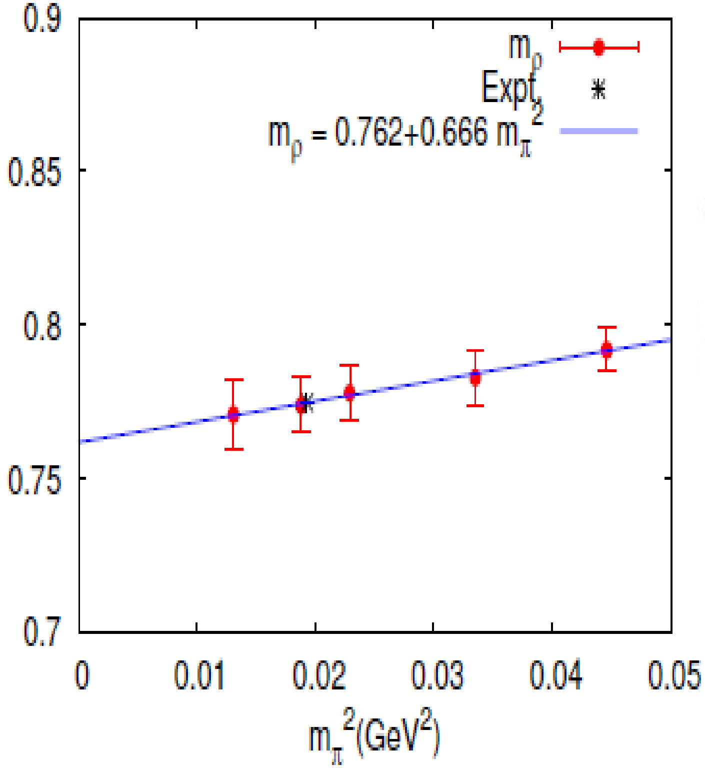
$$m_\rho \approx 780MeV$$

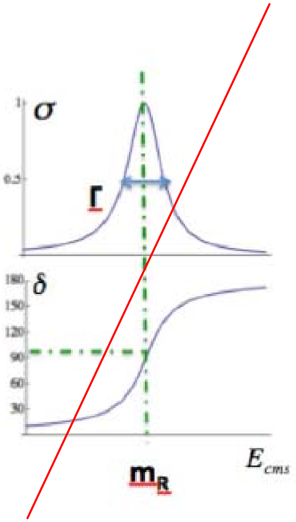
- blue line: $2E_\pi(p_{\min})$
- red line : two-pion threshold with the next smallest lattice momentum.

Some preliminary results for the rho-meson near the physical point



The radial profile of "rho". It is spatially compact.





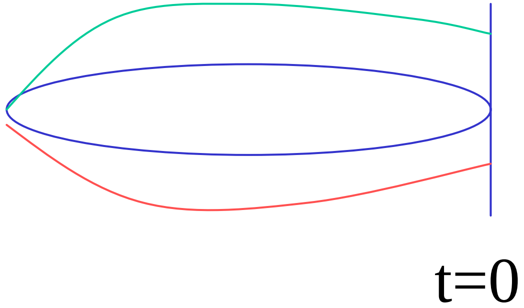
$$m_\rho(m_\pi) = m_\rho(0) + c_1 m_\pi^2,$$

Hints from the prvious plots:

- pion-pion p-wave scattering states do show up in the the I=1 point-source vector correlation function.

- the lowest pion-pion scattering state does not appear in the wall-source I=1 vector correlation function.

- the possible reason for the suppressior of scattering states by Coulomb wall source



$$
\begin{aligned}
&\left\langle \sum_{\mathbf{y},\mathbf{z}} \mathrm{Tr}\left[S_F(\mathbf{x},t;\mathbf{y},0)\gamma_5\gamma_i S_F^\dagger((x),t;\mathbf{z},0)\gamma_5\gamma_i\right]\right.\\
\times &\left.\sum_{\mathbf{u},\mathbf{v}} \mathrm{Tr}\left[S_F^{(s)}(\mathbf{u},t;\mathbf{v},0)S_F^{(s)}(\mathbf{v},0;\mathbf{u},t)\right]\right\rangle\\
\sim &\sum_{\mathbf{p}} \langle 0|O_{V,i}(\mathbf{x},t)\sum_{\mathbf{u}}(\bar{q}^{(s)}q^{(s)})(\mathbf{u},t)|\pi(\mathbf{p})\pi(-\mathbf{p})\rangle\\
\times &\langle \pi(\mathbf{p})\pi(-\mathbf{p})|O_{V,i}^{(w),\dagger}(0)\sum_{\mathbf{v}}(\bar{q}^{(s)}q^{(s)})(\mathbf{v},0)|0\rangle + \ldots\\
&\langle 0|O_{V,i}^{(w)}(0)\sum_{\mathbf{v}}(\bar{q}^{(s)}q^{(s)})(\mathbf{v},0)|\pi(\mathbf{p})\pi(-\mathbf{p})\rangle\\
\equiv &\sum_{\mathbf{y},\mathbf{z},\mathbf{v}} \langle 0|\bar{u}(\mathbf{y},0)\gamma_i d(\mathbf{z},0)(\bar{q}^{(s)}q^{(s)})(\mathbf{v},0)|\pi(\mathbf{p})\pi(-\mathbf{p})\rangle\\
\sim &\sum_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{R}} \Phi_\pi(\mathbf{r}_1)\Phi_\pi(\mathbf{r}_2)\Psi_{\pi\pi}(\mathbf{R},\mathbf{p}) \qquad (8)
\end{aligned}
$$

$$
\sum_{\mathbf{R}} \Psi_{\pi\pi}(\mathbf{R},\mathbf{p}) \approx \sum_{\mathbf{R}} e^{i\mathbf{p}\cdot\mathbf{R}} = L^3\delta_{\mathbf{p},\mathbf{0}}
$$

- Different femion actions

  Wilson type   vs. Chiral fermion???

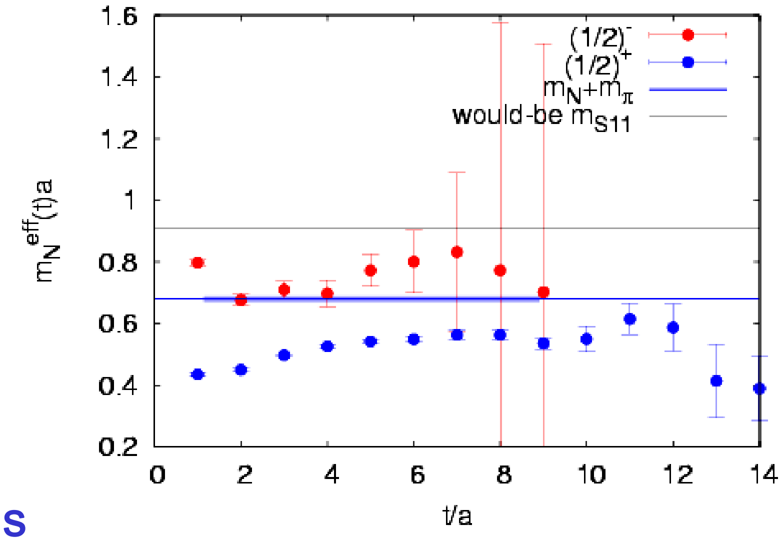
2. Pion-N scattering states in S11 channel

- Pion-N S-wave scattering states.
- The enengy of the lowest pion-N state is

$$E_{\pi N}^{\min}(L=0)\approx m_{\pi}+m_{N}$$

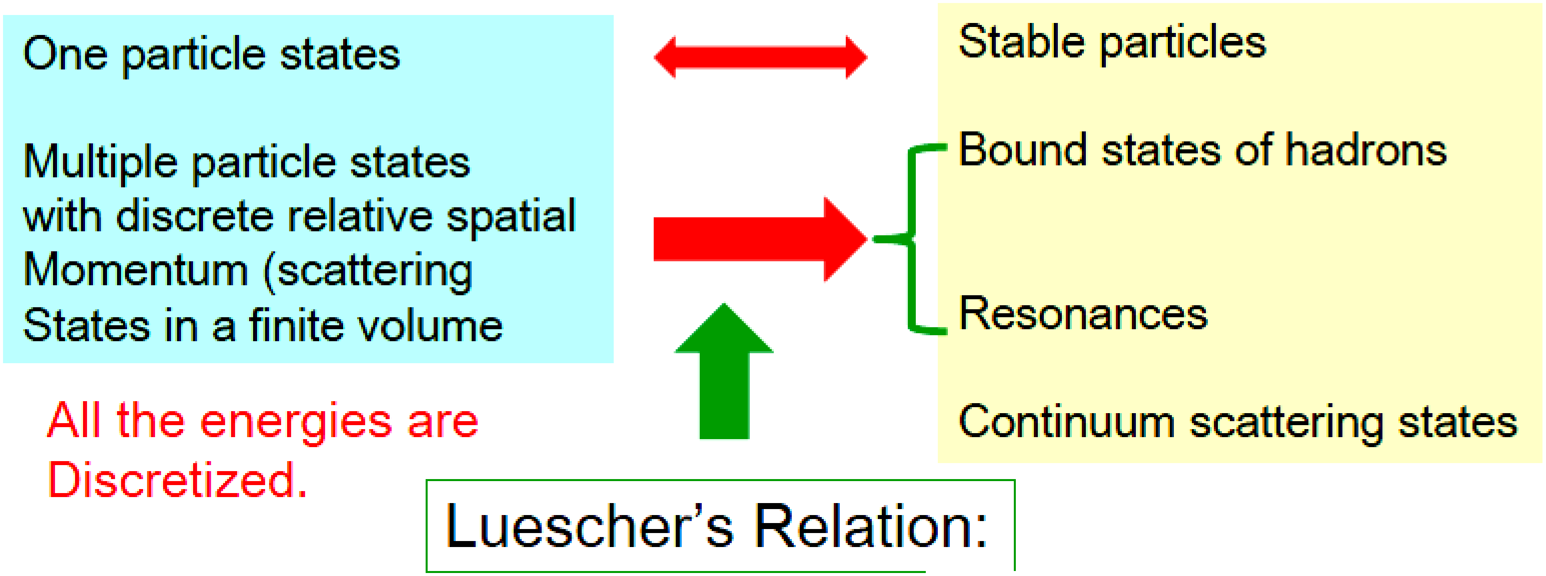
- we observe it, but with large errorbars.

. No hope for the S11 resonance with this kind of correlators.



3. Comments on the present status of LQCD study on N* spectroscopy

Comparison of the hadron spectra



Euclidean spacetime lattice | **Minkowski continuum spacetime**

One particle states

Multiple particle states with discrete relative spatial Momentum (scattering States in a finite volume

Stable particles

Bound states of hadrons

Resonances

Continuum scattering states

All the energies are Discretized.

Luescher's Relation:

$$E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$$

$$\tan\delta(p) = \frac{\sqrt{\pi}\ p\ L}{2\ \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

Resonances

$$T(p) = \frac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\,\Gamma(p)} = \frac{1}{\cot\delta(p) - i}$$

$$\Gamma(p) = g^2\,\frac{p^{2l+1}}{s}\,, \quad \frac{p^{2l+1}}{\sqrt{s}}\cot\delta(p) = \frac{1}{g^2}(m_R^2 - s)$$

Bound states

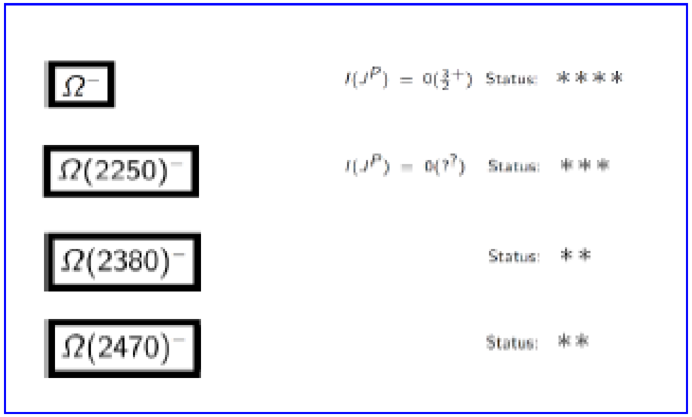
$$p\cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2}r_0\,p^2\,, \quad -|p_B| = \frac{1}{a_0} - \frac{1}{2}r_0\,|p_B|^2$$

$$T = \frac{}{\cot(\delta_l(p_B)) - i} = \infty$$

$$m_B = E_{H_1}(p_B) + E_{H_2}(p_B)\,,\ p_B = i|p_B|$$

IV. Omega* as a complementary discussion

- There are only four Ω baryons in the PDG.
- Apart from the $\Omega(1672)$ baryons, the J^P quantum numbers of the other three states are not determined yet.



Ω^-	$I(J^P) = 0(\frac{3}{2}^+)$ Status: ∗∗∗∗
$\Omega(2250)^-$	$I(J^P) = 0(?^?)$ Status: ∗∗∗
$\Omega(2380)^-$	Status: ∗∗
$\Omega(2470)^-$	Status: ∗∗

- In the

$$SU(6)\otimes O(3) \supset SU(3)_F \otimes SU(2)_{spin} \otimes O(3)$$

quark model, the level ordering should be

$$M(\tfrac{3}{2}^+) < M(\tfrac{3}{2}^-) \approx M(\tfrac{1}{2}^-) < M(\tfrac{3}{2}^{+*}) \approx M(\tfrac{1}{2}^+)$$

- Phenomenologicalstudies shows that the the mixing of the three-quark and five quarrk Fock space may drag the $M(\frac{3}{2}^-)$ lower than $M(\frac{1}{2}^-)$ (An & Zou, PRC87(2013)065207, PRC89(2014)055209)).

$$\underset{\text{antisymmetric}}{\Phi_{baryon}} = \underbrace{\phi_{space} \otimes \xi_{spin}}_{\text{symmetric}} \otimes \underset{\text{symmetric}}{\chi_{flavor}} \otimes \underset{\text{antisymmetric}}{\psi_{color}}$$

In a harmonious oscillator potential

$E = (2n_r + l + 3/2)\omega$

$J = \frac{3}{2}^{+}$:	$1s1s1s$		$1s1s2s$
$J = \frac{3}{2}^{-}$:		$1s1s1p$	
$J = \frac{1}{2}^{+}$:			$1s1s2s$
$J = \frac{1}{2}^{-}$:		$1s1s1p$	

Energy increases.

- Interpolation field operator for Ω baryons

$$\mathcal{O}^{\mu}_{\Omega} = \epsilon^{abc}(s_a^T \mathcal{C}\gamma^{\mu} s_b)s_c.$$

- This operator has both the spin-1/2 and spin-3/2 components

- Introducing the projection operators from the quantum field theory

$$\mathcal{P}^{\mu\nu}_{3/2} = \delta^{\mu\nu} - \frac{1}{3}\gamma^{\mu}\gamma^{\nu} - \frac{1}{3p^2}(\not{p}\gamma^{\mu}p^{\nu} + p^{\mu}\gamma^{\nu}\not{p})$$

$$\mathcal{P}^{\mu\nu}_{1/2} = \delta^{\mu\nu} - \mathcal{P}^{\mu\nu}_{3/2}$$

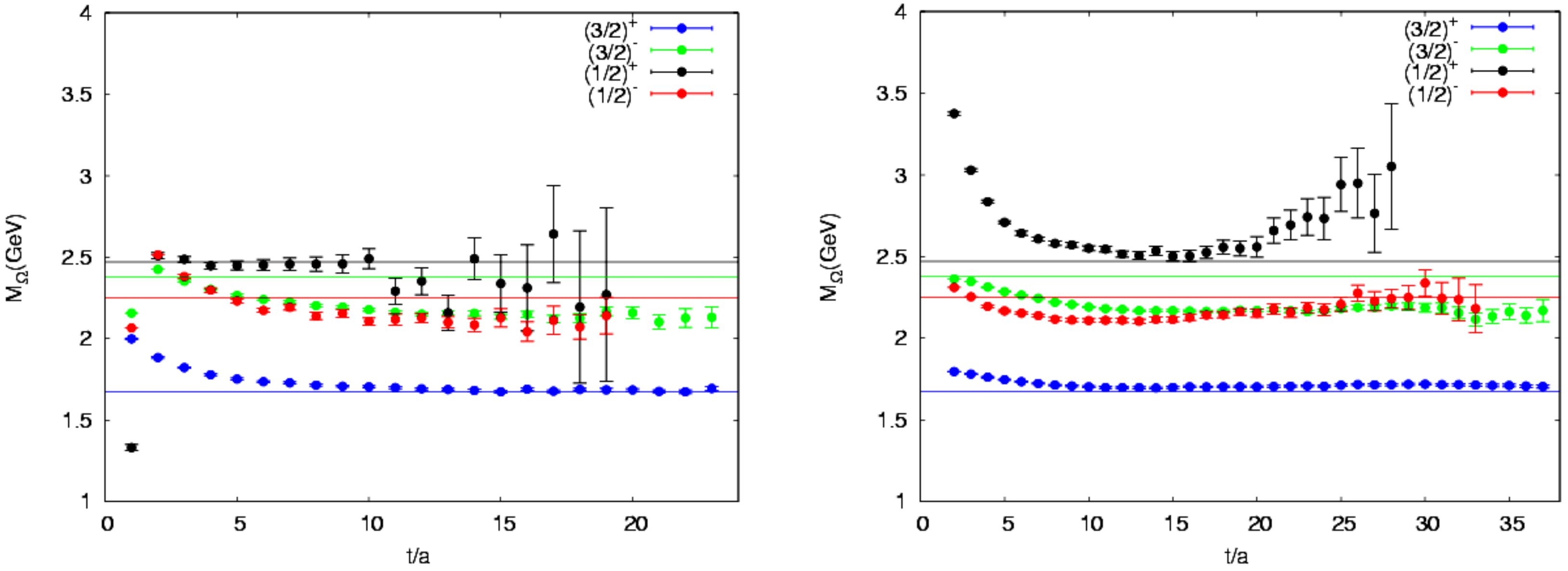
- We obtain the following operators with definite spin quantum number

$$\mathcal{O}^{\mu}_{3/2} = \sum_{\nu} \mathcal{P}^{\mu\nu}_{3/2}\mathcal{O}^{\nu}_{\Omega}$$

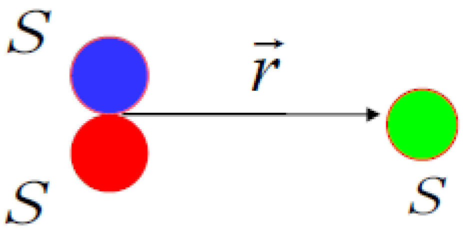
$$\mathcal{O}^{\mu}_{1/2} = \sum_{\nu} \mathcal{P}^{\mu\nu}_{1/2}\mathcal{O}^{\nu}_{\Omega}$$

- Then we can calculate the two-point functions of these operators

$$C_J(t) = \sum_{\vec{x}} Tr(1 \pm \gamma_4)\langle O_J(\overrightarrow{x}, t)\bar{O}_J(0)\rangle = \sum_{n} Z_J e^{-m_J t}$$



J^P	$16^3 \times 96$ M_Ω(GeV)	$24^3 \times 128$ M_Ω(GeV)	PDG
$(3/2)^+$	1.680(7)	1.704(7)	Ω^-
$(3/2)^-$	2.155(12)	2.169(10)	$\Omega^-(2250)$ or $\Omega^-(2380)$???
$(1/2)^-$	2.155(20)	2.140(19)	$\Omega^-(2380)$ or $\Omega^-(2250)$???
$(1/2)^+$	2.452(31)	2.490(25)	$\Omega^-(2470)$???

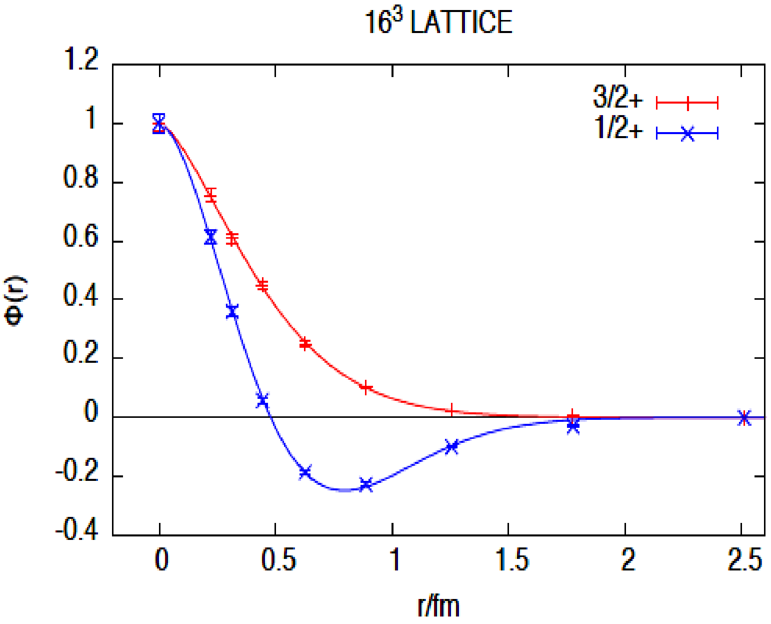


$$\mathcal{O}_1(r) = \sum_{|\vec{r}|} \epsilon^{abc}[s_a^T(x+\vec{r})\mathcal{C}\gamma^\mu s_b(x)]s_c(x)$$

$$\mathcal{O}_2(r) = \sum_{|\vec{r}|} \epsilon^{abc}[s_a^T(x)\mathcal{C}\gamma^\mu s_b(x+\vec{r})]s_c(x)$$

$$\mathcal{O}_3(r) = \sum_{|\vec{r}|} \epsilon^{abc}[s_a^T(x)\mathcal{C}\gamma^\mu s_b(x)]s_c(x+\vec{r})$$

$$C(r,t) = \left\langle O(\vec{r},t)\bar{O}^{(W)}(0)\right\rangle = \sum_i W_i(r)e^{-m_i t}$$



- $\frac{3}{2}^+ \rightarrow (1s1s1s)$
- $\frac{1}{2}^+ \rightarrow (1s1s2s)$

$$\phi_0(r) = e^{-\left(\frac{r}{\beta_0}\right)^{\alpha_0}}$$

$$\phi_1(r) = \left(1+\kappa r^{\alpha_1}\right) e^{-\left(\frac{r}{\beta_1}\right)^{\alpha_1}}$$

$$\alpha_0 \sim \alpha_1 \sim 1.75$$

V. Summary and Outlook

- By using overlap fermions, we can access the physics at very low pion masses.
- In QA, we reproduce the experimental level ordering of the Roper and S11 resonances in the low pion mass region.
- It signals that the underlying dynamics in the the low pion mass region may differ from that in the heavy quark region.
- Our preliminary full-QCD result of the Roper mass is compatible with our previous results in QA.
- However, our results are very different from other group's, and the reson is under investigation.
- Nucleon-meson scattering should also be investigated on the lattice, especially for wide baryon resonance.

- We also study the spectrum and the BS wave functions of Omega and its excited states, and the result is qualitative agreement with the quark model picture.

Thanks!

The latest lattice calculation of Ω baryon spectrum

Bulava et al. (Hadron Spectrum Collaboration),
Phys. Rev. D 82, 014507 (2010)