Strange and nonstrange baryon spectra in the interacting qD model

J. Ferretti
Università La Sapienza, Roma, Italy
H. Garcia Tecocoatzi & E. Santopinto
Università di Genova & INFN, Italy

Introduction

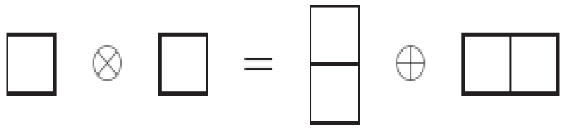
- **Constituent qD models: why?**
- Concept of constituent diquark (D)
- Baryons as qD bound states
- Scalar & axial-vector diquarks
- Problem of the missing resonances

- **Interacting qD model**
- Model Hamiltonian
- Nonstrange spectrum & missing resonances in the model
- Strange spectrum

qD model

▪ Diquark

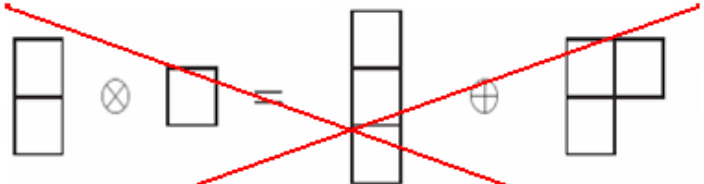
- Two strongly correlated quarks (S wave)
- For simplicity, treated as a point-like object (internal spatial WF neglected)
- Baryon in 1_c color representazion → diquark in bar-3_c
- Diquark WF:



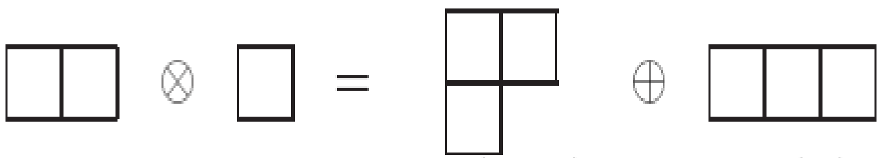
$$6 \otimes 6 = 15 \oplus 21$$

Ψ_D (spin-flavor) antysymmetric
→ 15 (A) repr. not present

• $SU(6)_{sf}$ representations for baryons



$$\cancel{15 \otimes 6 = 20(A) \oplus 70(MA)}$$



$$21 \otimes 6 = 70(MS) \oplus 56(S)$$

- Problem of missing resonances

Scalar & axial-vector diquarks

- **21 $SU(6)_{sf}$ representation**
  - Decomposed in $SU(2)_s$ x $SU(3)_f$
  - [bar-3,0] & [6,1] representations. Notation: [flavor,spin]

- **"Good" & "bad" diquarks**
  - According to OGE-calculations, [bar-3,0] is energetically favored [Wilczek, Jaffe]
  - [bar-3,0]: good (scalar) diquark
  - [6,1]: bad (axial-vector) diquark

Problem of missing resonances

▪ 3-quark QMs

▪ Excessive number of th. states (much more than experimental ones)
▪ Several experiments (CLAS, CB-ELSA, TAPS, GRAAL, SAPHIR, etc.) provided no evidence for these states.
▪ Possible explanation: resonances weakly coupled to the single pion, may decay in 2 (or more) pions/other mesons.

• qD models

- The number of missing resonances decreases notably
- 15 representation for diquark is neglected

Evidences of diquark correlations

- **Regge behavior of hadrons**

  Baryons arranged in rotational Regge trajectories ($J=\alpha+\alpha' M2$) with the same slope of the mesonic ones.

- **$\Delta I = ½$ rule in weak nonleptonic decays**

  Neubert and Stech, Phys. Lett. B **231** (1989) 477; Phys. Rev. D **44** (1991) 775

- **Regularities in parton distribution functions and in spin-dependent structure functions**

  Close and Thomas, Phys. Lett. B **212** (1988) 227

- **Regularities in $\Lambda(1116)$ and $\Lambda(1520)$ fragmentation functions**

  Jaffe, Phys. Rept. **409** (2005) 1 [Nucl. Phys. Proc. Suppl. **142** (2005) 343]
  Wilczek, hep-ph/0409168

- **Any interaction that binds π and ρ mesons in the rainbow-ladder approximation of the DSE will produce diquarks**

  Cahill, Roberts and Praschifka, Phys. Rev. D **36** (1987) 2804

- **Indications of diquark confinement**

  Bender, Roberts and Von Smekal, Phys. Lett. B **380** (1996) 7

Non-rel. Interacting qD model

E. Santopinto, PRC72, 022201 (2005)

- **Hamiltonian**

$$H = \frac{p^2}{2m} - \frac{\tau}{r} + \beta r + [B\delta_{S_{12},1} + C\delta_0] +$$

$$+ (-1)^{l+1} 2Ae^{-\alpha r}[(\vec{s}_{12} \cdot \vec{s}_3) + (\vec{t}_{12} \cdot \vec{t}_3) + (\vec{s}_{12} \cdot \vec{s}_3)(\vec{t}_{12} \cdot \vec{t}_3)]$$

- **Non-rel. Kinetic energy + Coulomb + linear confining terms**
- **Splitting between scalar & axial-vector diquarks**
- **Exchange potential**
- **Parameters fitted to nonstrange baryon spectrum**

Rel. Interacting qD model

J. Ferretti, E. Santopinto & A. Vassallo, PRC83, 065204 (2011)

▪ Model

- Relativistic extension of the previous model (point-form formalism).
- Hamiltonian:

$$M = E_0 + \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + M_{\rm dir}(r) + M_{\rm cont}(r) + M_{\rm ex}(r),$$

$$M_{\rm dir}(r) = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r.$$

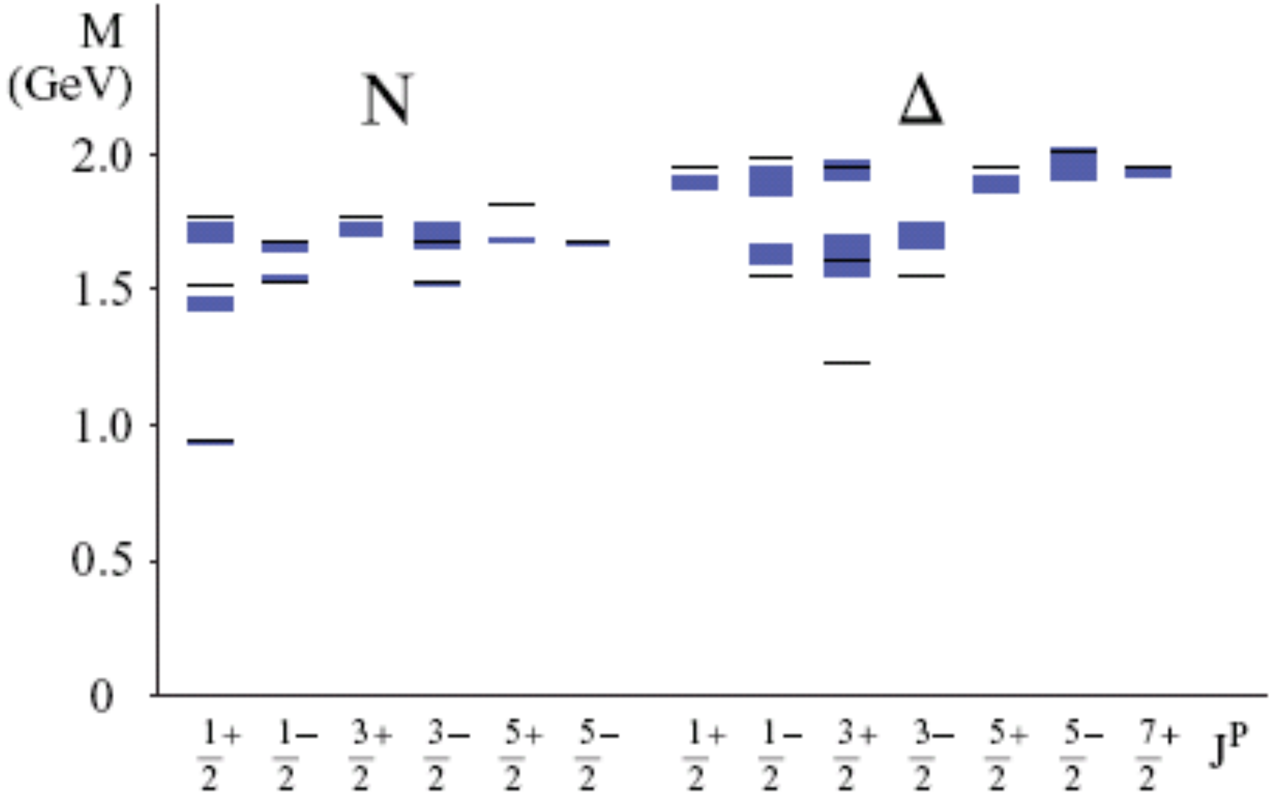
$$M_{\rm ex}(r) = (-1)^{l+1} e^{-\sigma r}[A_S(\vec{s}_1 \cdot \vec{s}_2) + A_I(\vec{t}_1 \cdot \vec{t}_2) + A_{SI}(\vec{s}_1 \cdot \vec{s}_2)(\vec{t}_1 \cdot \vec{t}_2)],$$

$$M_{\rm cont} = \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon} \frac{\eta^3 D}{\pi^{3/2}} e^{-\eta^2 r^2} \, \delta_{L,0}\delta_{s_1,1} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon}$$

- Numerical solution with variational program
- Parameters fitted to nonstrange baryon spectrum

Rel. Interacting qD model

J. Ferretti, E. Santopinto & A. Vassallo, PRC83, 065204 (2011)



Resonance	Status	$M^{\rm expt}$ (MeV)	J^P	L^P	S	s_1	n_r	$M^{\rm calc}$ (MeV)
$N(939)\ P_{11}$	****	939	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	0	939
$N(1440)\ P_{11}$	****	1420–1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	1	1513
$N(1520)\ D_{13}$	****	1515–1525	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1535)\ S_{11}$	****	1525–1545	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1650)\ S_{11}$	****	1645–1670	$\frac{1}{2}^-$	1^-	$\frac{1}{2}, \frac{3}{2}$	1	0	1671
$N(1675)\ D_{15}$	****	1670–1680	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1671
$N(1680)\ F_{15}$	****	1680–1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(1700)\ D_{13}$	***	1650–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}, \frac{3}{2}$	1	0	1671
$N(1710)\ P_{11}$	***	1680–1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1768
$N(1720)\ P_{13}$	****	1700–1750	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1768
$\Delta(1232)\ P_{33}$	****	1231–1233	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1233
$\Delta(1600)\ P_{33}$	***	1550–1700	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1602
$\Delta(1620)\ S_{31}$	****	1600–1660	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1700)\ D_{33}$	****	1670–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1900)\ S_{31}$	**	1850–1950	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986
$\Delta(1905)\ F_{35}$	****	1865–1915	$\frac{5}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1910)\ P_{31}$	****	1870–1920	$\frac{1}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1920)\ P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1930)\ D_{35}$	***	1900–2020	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	2005
$\Delta(1950)\ F_{37}$	****	1915–1950	$\frac{7}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$N(2100)\ P_{11}$	*	1855–1915	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	2	1893
$N(2090)\ S_{11}$	*	1869–1987	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$N(1900)\ P_{13}$	**	1820–1974	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(2080)\ D_{13}$	**	1740–1940	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$\Delta(1750)\ P_{31}$	*	1708–1780	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1858
$\Delta(1940)\ D_{33}$	*	1947–2167	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986

0 missing resonances below 2 GeV

Model Parameters

$m_q = 200$ MeV	$m_S = 600$ MeV	$m_{AV} = 950$ MeV
$\tau = 1.25$	$\mu = 75.0\ \mathrm{fm}^{-1}$	$\beta = 2.15\ \mathrm{fm}^{-2}$
$A_S = 375$ MeV	$A_I = 260$ MeV	$A_{SI} = 375$ MeV
$\sigma = 1.71\ \mathrm{fm}^{-1}$	$E_0 = 154$ MeV	$D = 4.66\ \mathrm{fm}^2$
$\eta = 10.0\ \mathrm{fm}^{-1}$	$\epsilon = 0.200$	

Rel. Interacting qD model – strange B.

E. Santopinto & J. Ferretti, arXiv: 1412.7571

▪ Model

- Model extended to strange sector
- Hamiltonian:

$$\begin{aligned} M &= E_0 + \sqrt{\vec{q}^{\,2} + m_1^2} + \sqrt{\vec{q}^{\,2} + m_2^2} + M_{\text{dir}}(r) \\ &+ M_{\text{ex}}(r) \end{aligned}$$

$$\begin{aligned} M_{\text{ex}}(r) &= (-1)^{L+1}\ e^{-\sigma r}\left[A_S\ \vec{s}_1 \cdot \vec{s}_2 \right. \\ &+ \left. A_F\ \vec{\lambda}_1^f \cdot \vec{\lambda}_2^f\ + A_I\ \vec{t}_1 \cdot \vec{t}_2\right] \end{aligned}$$

$$M_{\text{dir}}(r) = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r.$$

- Gursey-Radicati inspired exchange interaction
- Parameters fitted to strange baryon spectrum

Rel. Interacting qD model – strange B.

E. Santopinto & J. Ferretti, arXiv: 1412.7571

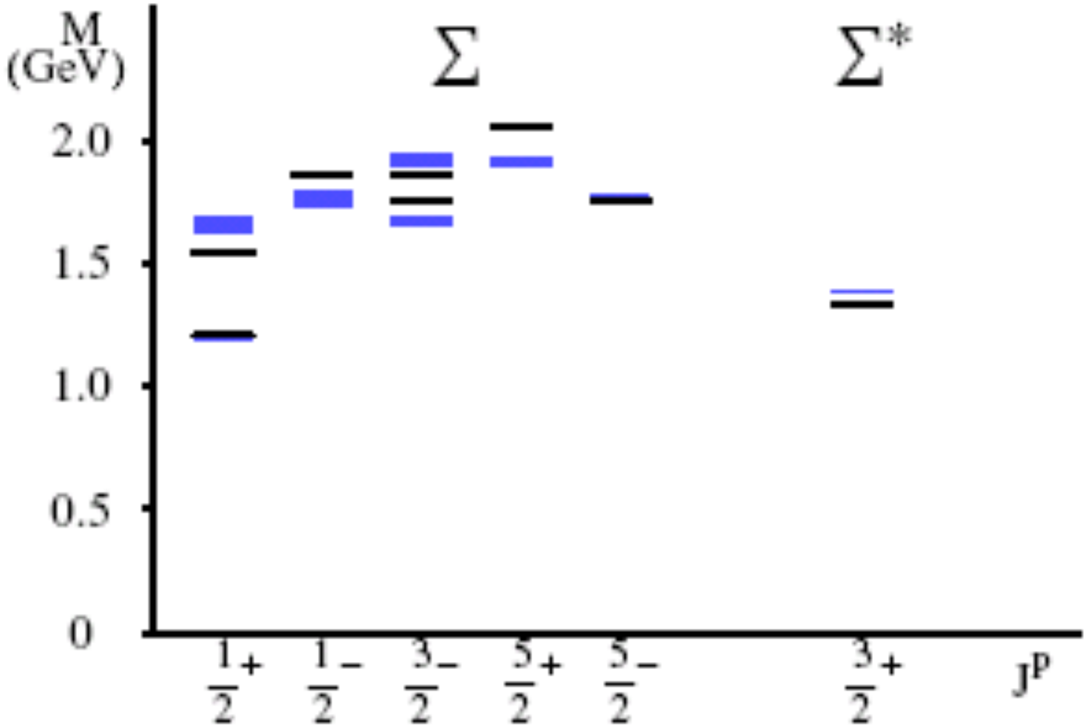
- Parameters

Parameter	Value (Fit 1)	Value (Fit 2)	Parameter	Value (Fit 1)	Value (Fit 2)
m_n	200 MeV	159 MeV	m_s	550 MeV	213 Mev
$m_{[n,n]}$	600 MeV	607 MeV	$m_{[n,s]}$	900 MeV	856 MeV
$m_{\{n,n\}}$	950 MeV	963 MeV	$m_{\{n,s\}}$	1200 MeV	1216 MeV
$m_{\{s,s\}}$	1580 MeV	1352 MeV	τ	1.20	1.02
μ	75.0 fm^{-1}	28.4 fm^{-1}	β	2.15 fm^{-2}	2.36 fm^{-2}
A_S	350 MeV	-436 MeV	A_F	100 MeV	193 MeV
A_I	250 MeV	791 MeV	σ	2.30 fm^{-1}	2.25 fm^{-1}
E_0	141 MeV	150 MeV	ϵ	0.37	–
D	6.13 fm^2	–	η	11.0 fm^{-1}	–

Rel. Interacting qD model – strange B.

E. Santopinto & J. Ferretti, arXiv: 1412.7571

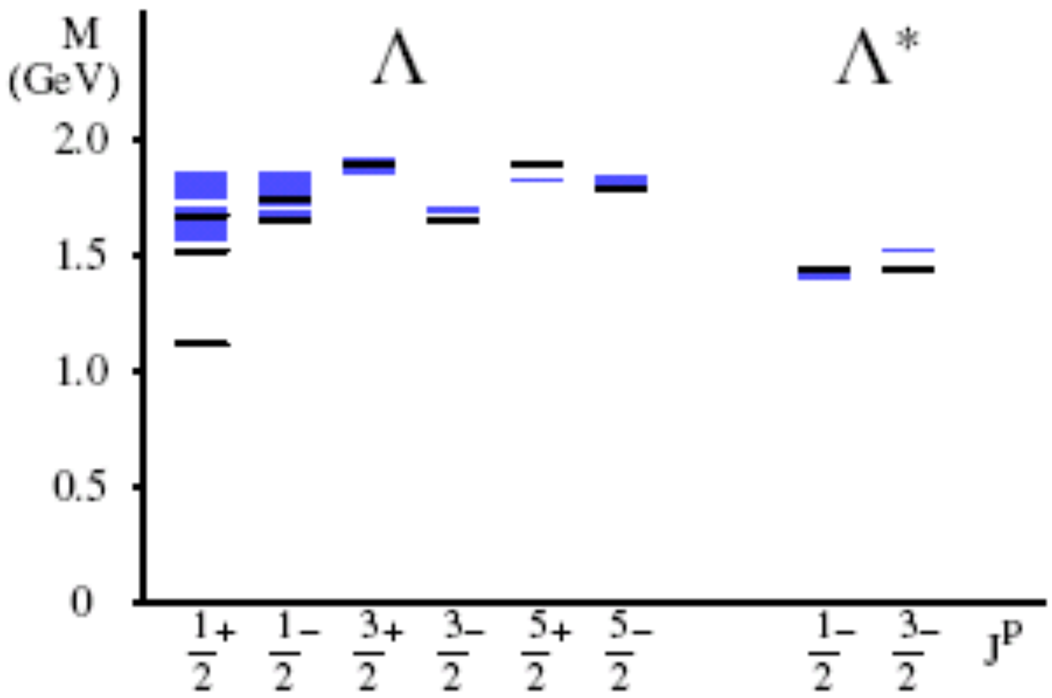
- Sigma & sigma* states



Rel. Interacting qD model – strange B.

E. Santopinto & J. Ferretti, arXiv: 1412.7571

- **Lambda & Lambda* states**



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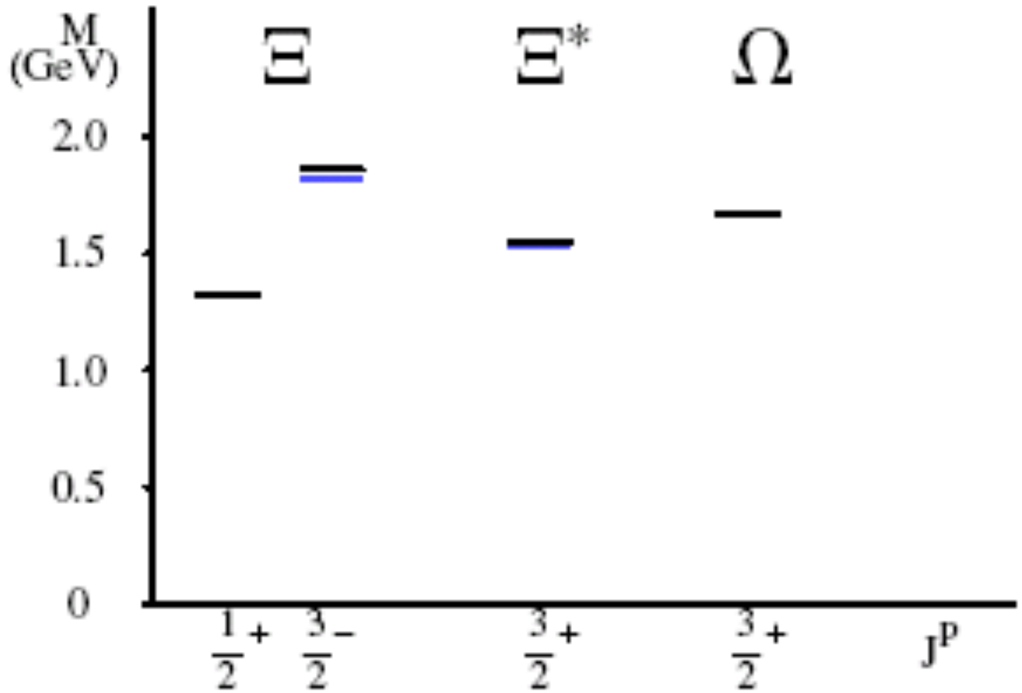
- Xi, Xi* & Omega states

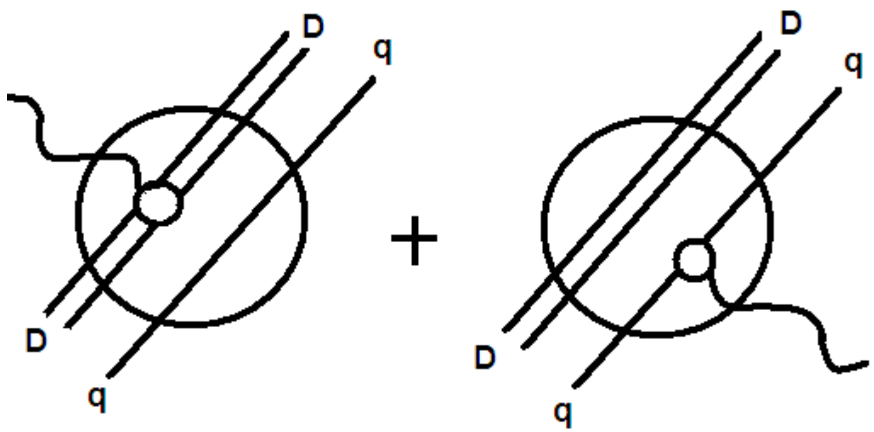


e.m. nucleon form factors

*De Sanctis, Ferretti, Santopinto, Vassallo, Phys. Rev. C **84**, 055201(2011*

Nucleon e.m. form factors impulse approximation

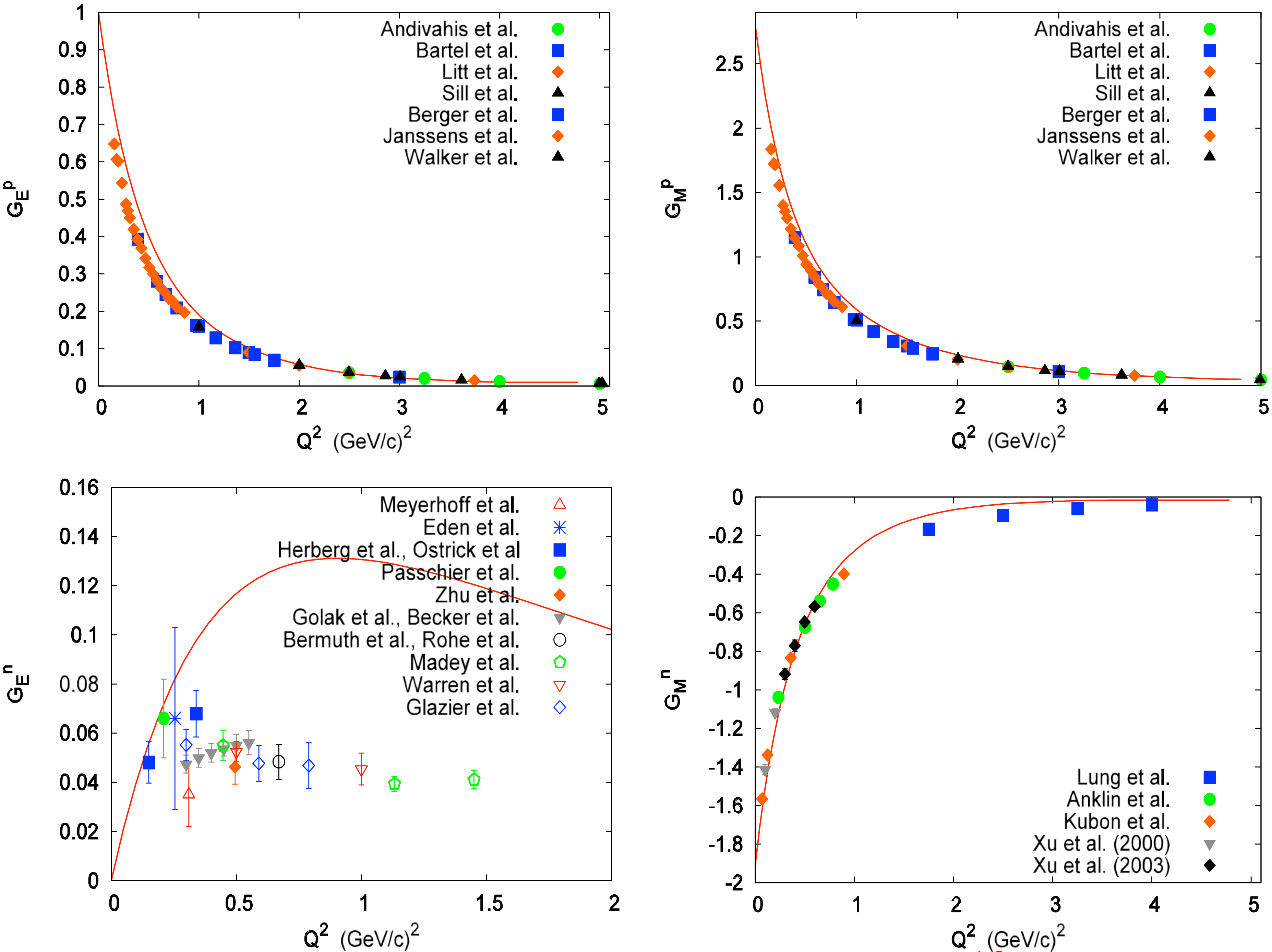
Klink, Phys. Rev. C **58**, 3587 (1998)



Schematic representation of the e.m. current in the impulse approx.

Nucleon state a s eigenvalue problem of mass operator. Quark-(scalar) diquark state.

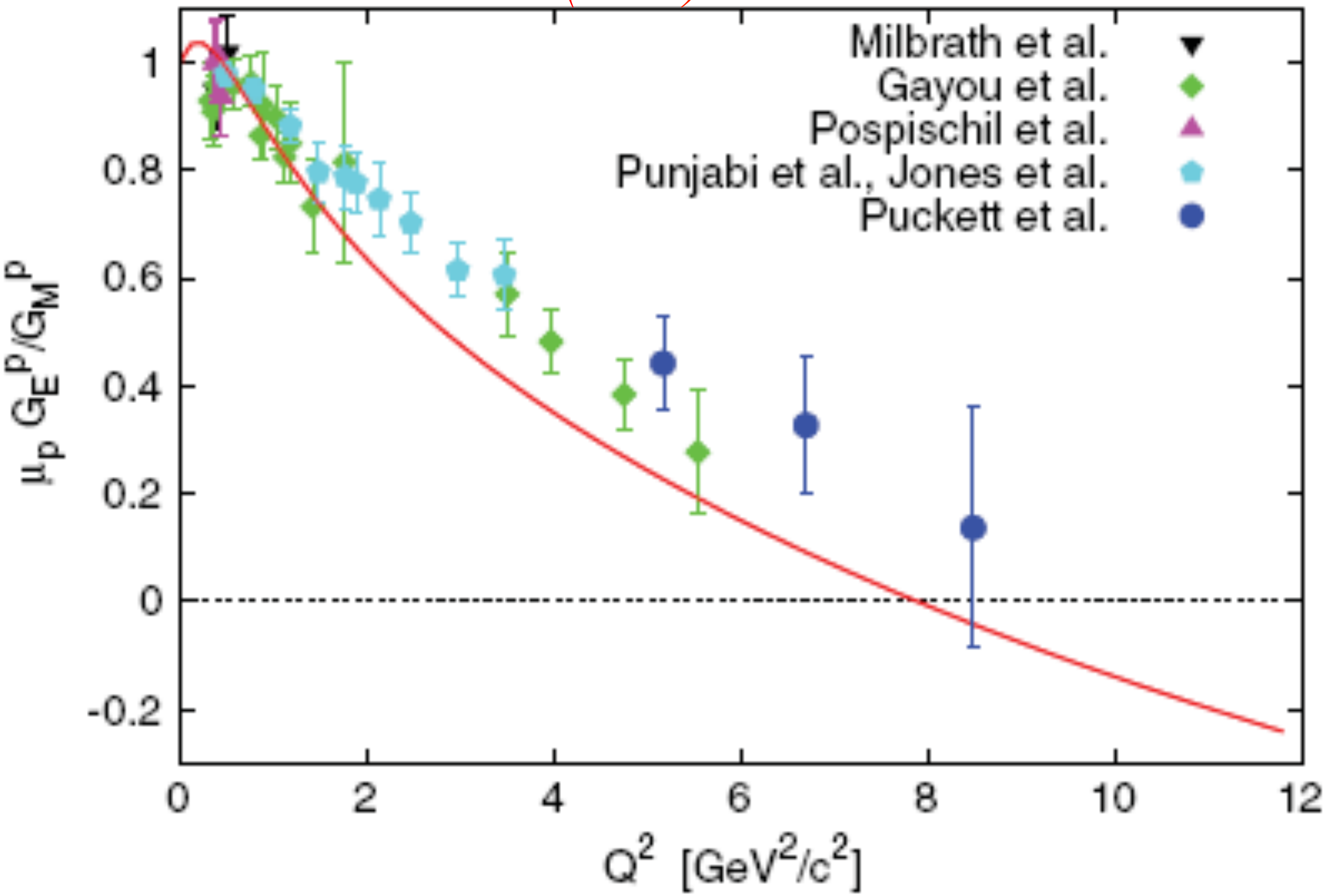
- **Matrix elements of e.m. current operator with quark and diquark form factors.**



Andivahis et al.
Bartel et al.
Litt et al.
Sill et al.
Berger et al.
Janssens et al.
Walker et al.
G_E^p
G_M^p
Q^2 (GeV/c)^2
Meyerhoff et al.
Eden et al.
Herberg et al., Ostrick et al
Passchier et al.
Zhu et al.
Golak et al., Becker et al.
Bermuth et al., Rohe et al.
Madey et al.
Warren et al.
Glazier et al.
G_E^n
Lung et al.
Anklin et al.
Kubon et al.
Xu et al. (2000)
Xu et al. (2003)
G_M^n

Ratio μp GEp/GMp

*De Sanctis, Ferretti, Santopinto, Vassallo, Phys. Rev. C **84**, 055201 (2011)*



Conclusion

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Thank you for your attention