Compositeness of the Δ(1232) resonance in πN scatterings

Takayasu SEKIHARA (RCNP, Osaka Univ.)

in collaboration with

Takashi ARAI (KEK),

Junko YAMAGATA-SEKIHARA (Oshima National Coll. of Maritime Tech.)

and Shigehiro YASUI (Tokyo Inst. Tech.)

1. Introduction
2. Formulation
3. Numerical results
4. Summary

[1] T. S. , T. Arai, J. Yamagata-Sekihara and S. Yasui, in preparation.

1. Introduction

++ The Δ(1232) resonance ++

- **The Δ(1232) resonance is the first excitation of the nucleon.**

--- Status: ** = existence is certain, and properties are at least fairly well explored.**

Δ BARYONS
($S = 0$, $I = 3/2$)

$\Delta^{++} = uuu$, $\Delta^{+} = uud$, $\Delta^{0} = udd$, $\Delta^{-} = ddd$

Δ(1232) $3/2^+$ $\quad I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

Breit-Wigner mass (mixed charges) = 1230 to 1234 (≈ 1232) MeV
Breit-Wigner full width (mixed charges) = 114 to 120 (≈ 117) MeV
$p_{beam} = 0.30$ GeV/c $\quad 4\pi\lambda\!\!\!^{-2} = 94.8$ mb
Re(pole position) = 1209 to 1211 (≈ 1210) MeV
−2Im(pole position) = 98 to 102 (≈ 100) MeV

Δ(1232) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	100 %	229
$N\gamma$	0.55–0.65 %	259
$N\gamma$, helicity=1/2	0.11–0.13 %	259
$N\gamma$, helicity=3/2	0.44–0.52 %	259

Particle Data Group.

1. Introduction

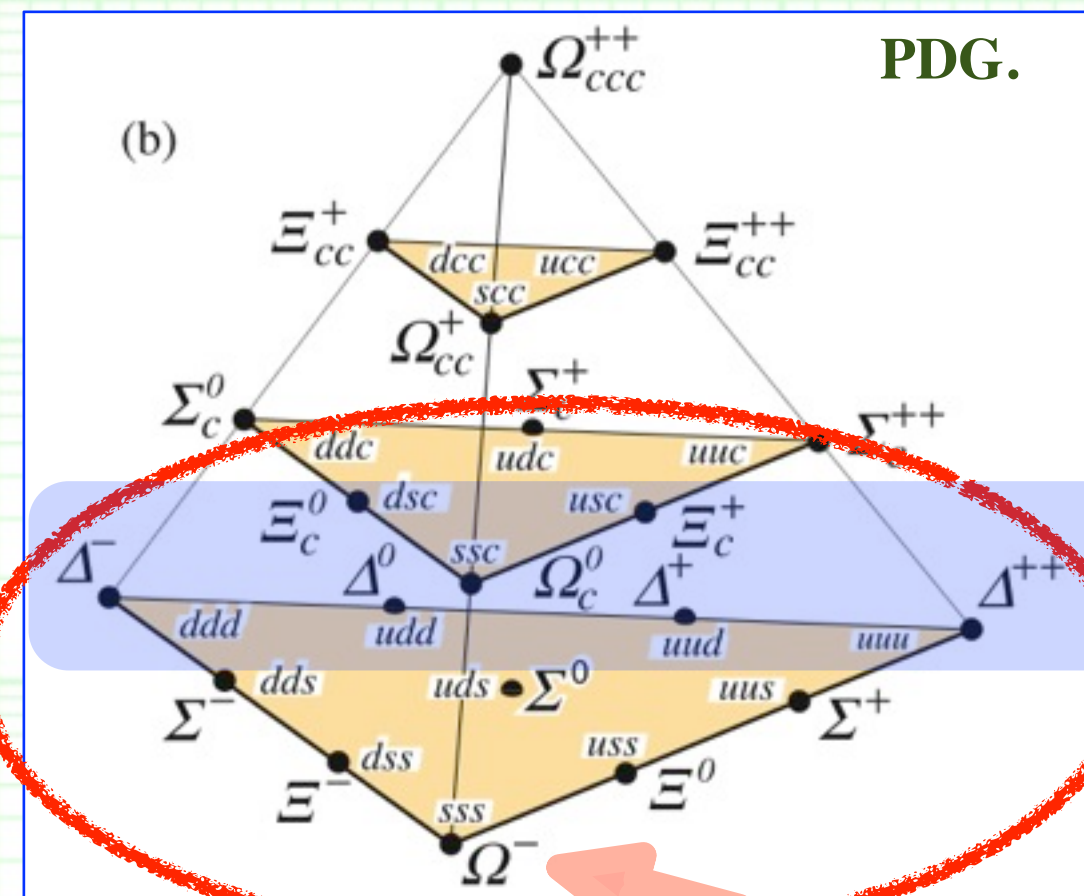
++ The Δ(1232) resonance ++

- **The Δ(1232) resonance** is the first excitation of the nucleon.

- Δ(1232) is **one of the most fundamental states** to understand the underlying theory of strong interaction, QCD.
  - □ Δ(1232)$^{++}$ as a ($u\uparrow$ $u\uparrow$ $u\uparrow$) state leads to an idea that **quarks has color degrees of freedom**. Otherwise, it breaks the Pauli principle with respect to the exchange of quarks. Han and Nambu ('65).
  - □ Δ(1232) belongs to the decuplet in the flavor $SU(3)$ symmetry, together with Σ(1385), Ξ(1530), and Ω, in the quark model.

--- **Prediction** of the existence and properties of **Ω**, and experimental discovery followed it. Barnes ('64).

<-- **An excellent success of the quark model !**

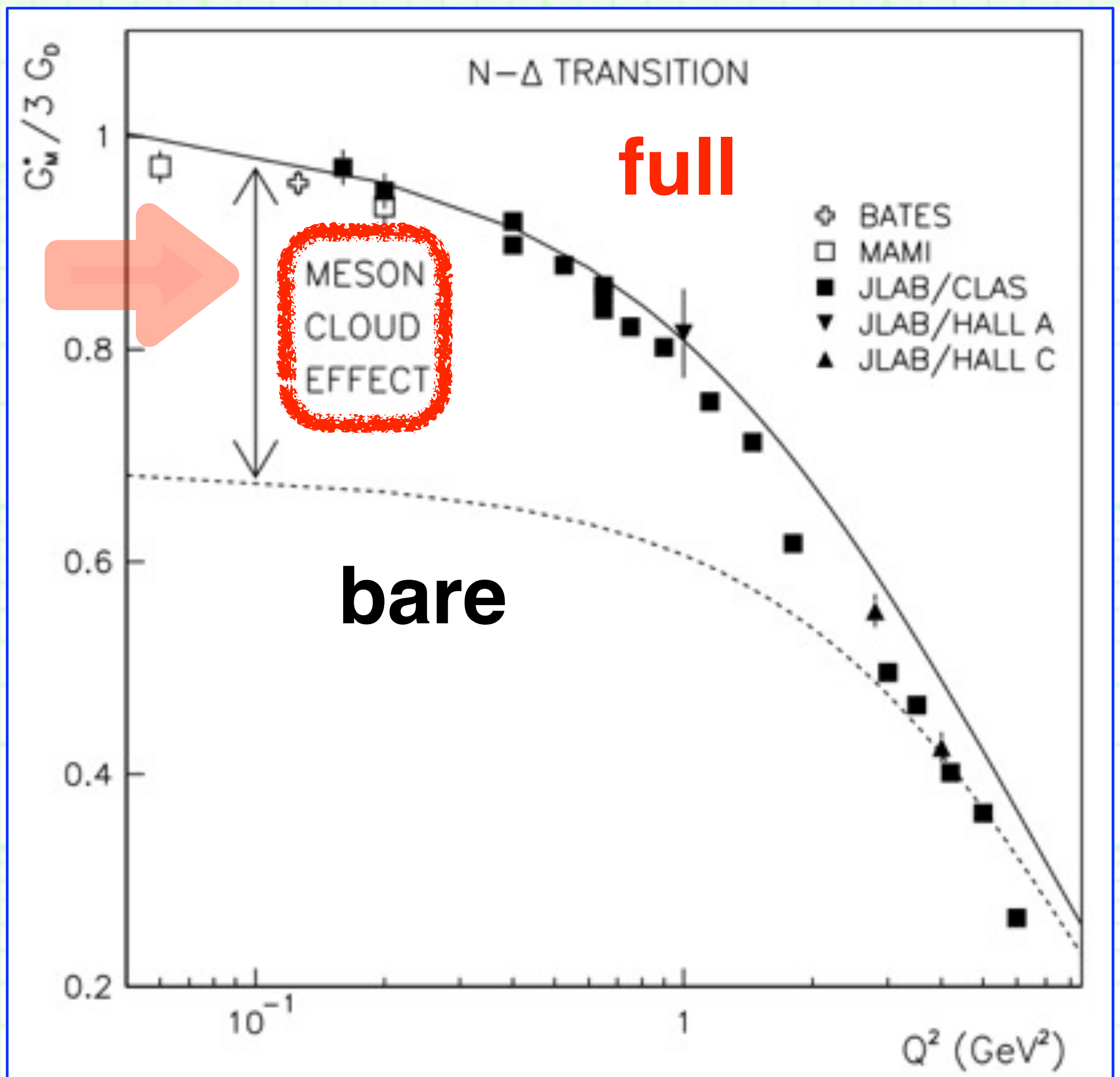


1. Introduction

++ Internal structure of Δ(1232) ++

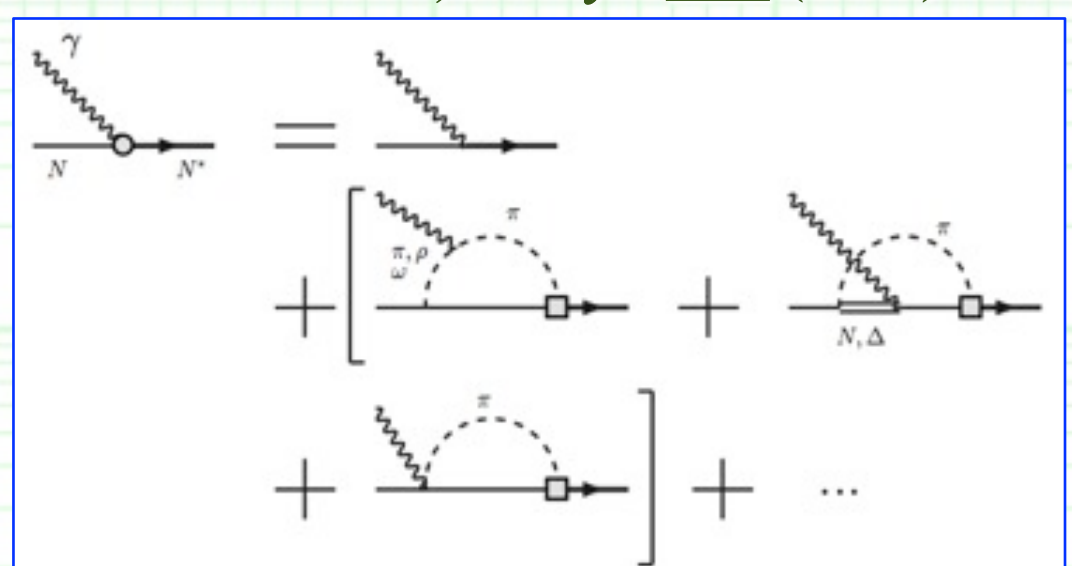
- **The excellent success of the quark model for Δ(1232) and other decuplet states strongly indicates that the decuplet states are described as genuine *qqq* states very well.**

- **However, effect of the meson-nucleon cloud for Δ(1232) seems to be "large".**



- **The magnetic *M*1 form factor of γ *N* --> Δ(1232) shows that the meson cloud effect brings ~ 30 % of the form factor at $Q^2 = 0$.**

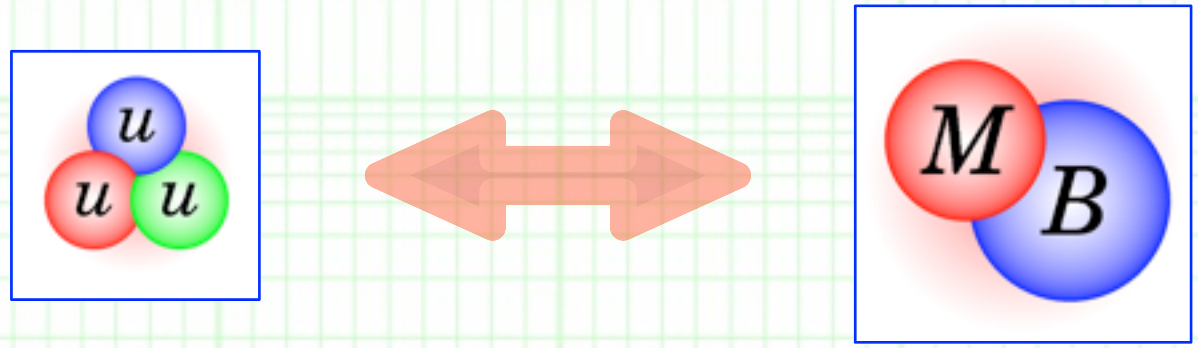
Sato and Lee, *J. Phys.* G36 (2009) 073001.



1. Introduction

++ Large πN component ? ++

- The quark model strongly indicates a genuine qqq state for $\Delta(1232)$, but the transition form factor requires large effect of meson cloud.



- Recently the πN component in $\Delta(1232)$ was studied in terms of the $\Delta(1232)$ wave function from πN Amp. Aceti *et al.*, *Eur. Phys. J.* A50 (2014) 57.

$$\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$

$$X_i = \int \frac{d^3 q}{(2\pi)^3} \langle \Psi^* | \vec{q} \rangle \langle \vec{q} | \Psi \rangle = -g_i^2 \left[\frac{dG_i}{dE} \right]_{E = E_{\text{pole}}}$$

--- Compositeness X as a norm of two-body component, coupling constant g, and πN loop function G.

□ Compositeness X is not observable and hence model dependent. (cf. s- and d-wave p-n components in deuteron) We can evaluate compositeness in appropriate models.

Hyodo, Jido, Hosaka (2012), Aceti-Oset (2012), Hyodo (2013), Nagahiro-Hosaka (2014), ... See also T. S. , Hyodo and Jido, PTEP, in press [arXiv:1411.2308].

1. Introduction

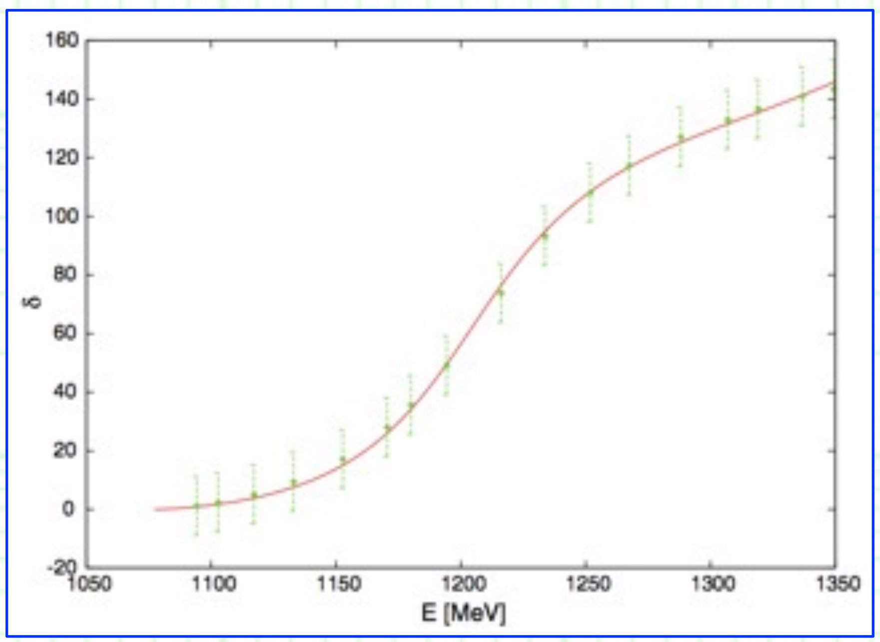
++ πN compositeness ++

- Recently the πN component in Δ(1232) was studied in terms of the Δ(1232) wave function from πN Amp. Aceti *et al.*, *Eur. Phys. J.* A50 (2014) 57.

$$\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$

$$X_i = \int \frac{d^3 q}{(2\pi)^3} \langle \Psi^* | \vec{q} \rangle \langle \vec{q} | \Psi \rangle = -g_i^2 \left[\frac{dG_i}{dE} \right]_{E = E_{\rm pole}}$$

--- Compositeness *X*, coupling constant *g*, and πN loop function *G*.



$$-\tilde{g}_\Delta^2 \left[\frac{\mathrm{d}G^{II}(s)}{\mathrm{d}\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_0}} = (0.62 - i0.41),$$

- □ Large real part of the πN compositeness, but imaginary part is non-negligible.

--- The result implies large πN contribution to, *e.g.*, the transition form factor.

- However, this result was obtained in a very simple model.
  - □ Interaction: constant + bare Δ .

$$v = -\alpha \left(1 + \frac{\beta}{s_R - s} \right),$$

  - □ Loop function: three-dimensional cut-off.

--> Need a more refined model !

1. Introduction

++ Purpose and strategy of this study ++

- In this study we want to **construct a more refined model** and **evaluate the πN compositeness for the $\Delta(1232)$ resonance**.

--> Employ **the chiral unitary approach** for the πN elastic scattering.

$$T = V + VGT = \frac{1}{1/V - G}$$

Meissner and Oller, *Nucl. Phys.* A673 (2000) 311,
Alarcon, Martin Camalich, Oller, *Ann. Phys.* 336 (2013) 413,
… .

- Interaction kernel V from the chiral perturbation theory for πN: Leading order + next-to-leading order + **bare Δ** .

- Loop function G calculated **with the dispersion relation** in a covariant way.

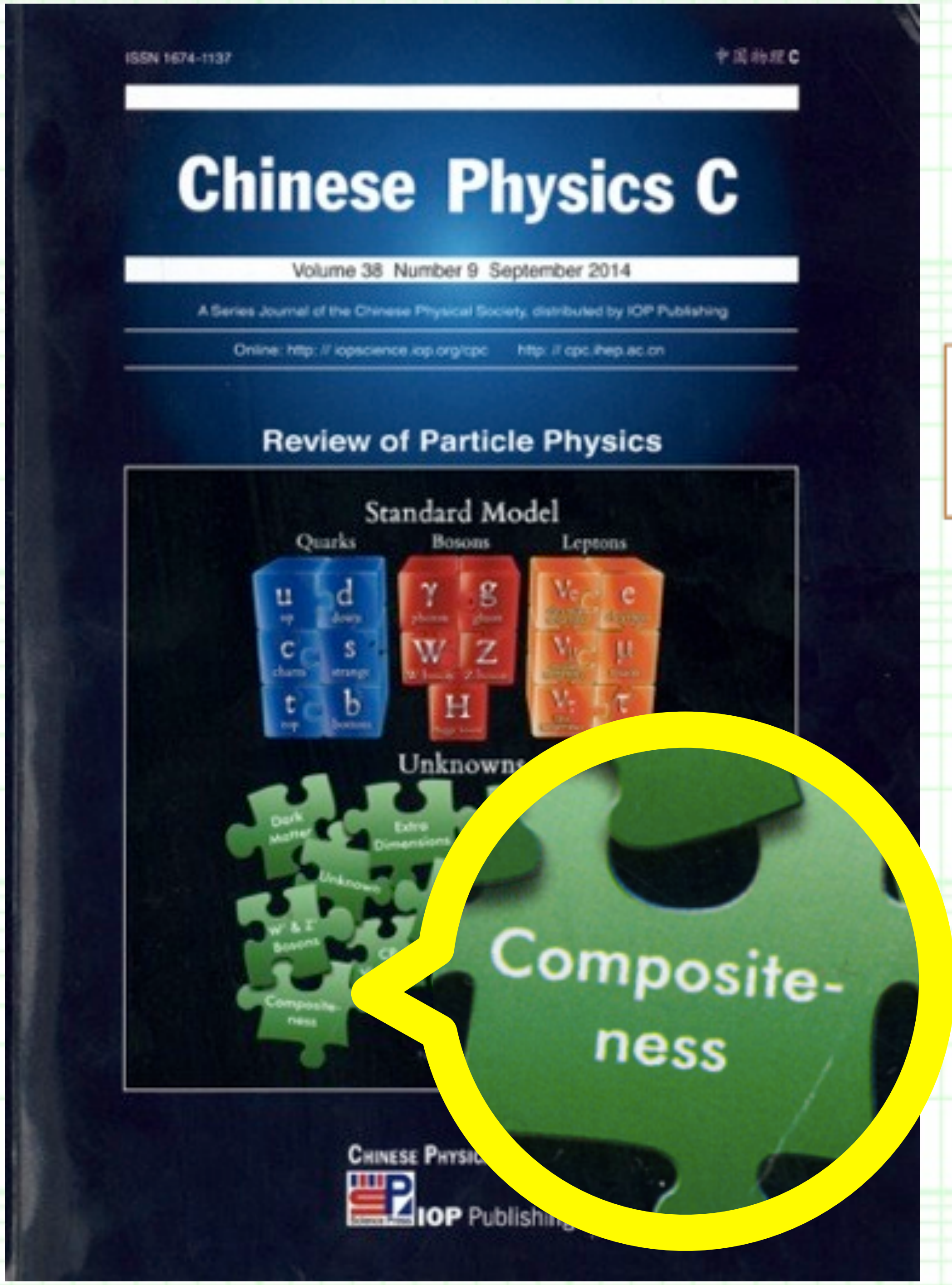
- Fitted to the πN scattering amplitude (S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, and P_{33}) obtained as a PWA solution "WI 08" up to $\sqrt{s}$ = 1.35 GeV.

Workman *et al.*, *Phys. Rev.* D86 (2012) 014012.

- From the pole position and residue for physical $\Delta(1232)$, we evaluate **the πN compositeness for the $\Delta(1232)$ resonance**.

2. Formulation

++ Compositeness for two-body systems ++



Particle Data Group (2014).
(similar but not same as our compositeness)

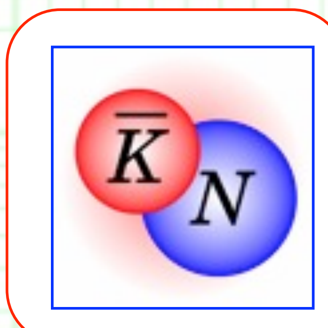
- **Compositeness (X) can be defined as the contribution of the two-body component to the normalization of the total wave function.**

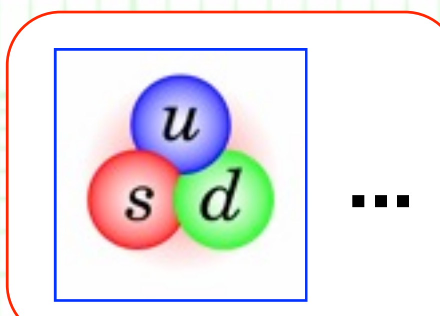
$$\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$

$$X_i = \int \frac{d^3 q}{(2\pi)^3} \langle \Psi^* | \vec{q} \rangle \langle \vec{q} | \Psi \rangle$$

- **Ex.) Λ(1405) in chiral unitary approach.**

T. S. , Hyodo and Jido, PTEP, in press [arXiv:1411.2308].





	Λ(1405) (higher pole)
$\sqrt{s_{\rm pole}}$	$1424 - 26i$ MeV
$X_{\bar{K}N}$	$1.14 + 0.01i$
$X_{\pi\Sigma}$	$-0.19 - 0.22i$
$X_{\eta\Lambda}$	$0.13 + 0.02i$
$X_{K\Xi}$	$0.00 + 0.00i$
Z	$-0.08 + 0.19i$

2. Formulation

++ Compositeness from scattering amplitude ++

- We can extract **the resonance wave function from the solution of the Lippmann-Schwinger equation.**

T. S. , Hyodo and Jido, PTEP, in press [arXiv:1411.2308].

--- Fixing model to describe scattering amplitude.

<=> Fixing analytic form of the amplitude to the resonance pole.

<=> Becomes a "ruler" to measure the compositeness.

- Lippmann-Schwinger equation in the NR framework.

$$\hat{T}(E) = \hat{V} + \hat{V}\frac{1}{E - \hat{H}_0}\hat{T} = \hat{V} + \hat{V}\frac{1}{E - \hat{H}}\hat{V}$$

$$\langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle = T(E; \vec{q}, \vec{q}\,')$$

- Near the resonance pole position E_{pole}, amplitude is dominated by the pole term in the expansion by the eigenstates of H as

$$\langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \langle \vec{q}\,' | \hat{V} | \Psi \rangle \frac{1}{E - E_{\mathrm{pole}}} \langle \Psi^{*} | \hat{V} | \vec{q} \rangle$$

- For an s-wave separable interaction: $\langle \vec{q} | \hat{V} | \Psi \rangle = \langle \Psi^{*} | \hat{V} | \vec{q} \rangle = g$

T. S. , Hyodo and Jido, PTEP, in press [arXiv:1411.2308].

2. Formulation

++ Compositeness from scattering amplitude ++

- Near **the resonance pole position E_{pole}**, amplitude is dominated by the pole term in the expansion by the eigenstates of H as

$$\langle \vec{q}\,' | \hat{T}(E) | \vec{q} \rangle \approx \langle \vec{q}\,' | \hat{V} | \Psi \rangle \frac{1}{E - E_{\text{pole}}} \langle \Psi^* | \hat{V} | \vec{q} \rangle$$

- For *L*-wave separable interaction: $\langle \vec{q} | \hat{V} | \Psi \rangle = \langle \Psi^* | \hat{V} | \vec{q} \rangle = g \times |\vec{q}|^L$

Aceti and Oset, *Phys. Rev.* D86 (2012) 014012.

--- This form is **necessary for the correct behavior** of the amplitude near the threshold: $T_{L\text{-wave}} = |\vec{q}|^{2L} \times T'(E)$

- As a result, **the norm of the two-body wave function** is written as

$$X = \int \frac{d^3 q}{(2\pi)^3} \langle \Psi^* | \vec{q} \rangle \langle \vec{q} | \Psi \rangle = g^2 \int \frac{d^3 q}{(2\pi)^3} \frac{|\vec{q}|^{2L}}{[E - (M_{\text{th}} + q^2/(2\mu))]^2} = -g^2 \left[\frac{dG_L}{dE} \right]_{E = E_{\text{pole}}}$$

--- G is the loop function: $G_L = \int \frac{d^3 q}{(2\pi)^3} \frac{|\vec{q}|^{2L}}{E - (M_{\text{th}} + q^2/(2\mu))}$

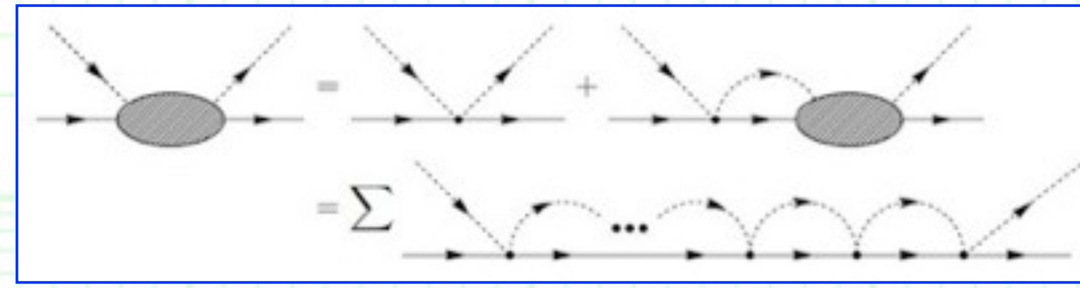
--> **We must keep | q |^{2L} dep.** without "on-shell factorization" on it !

2. Formulation

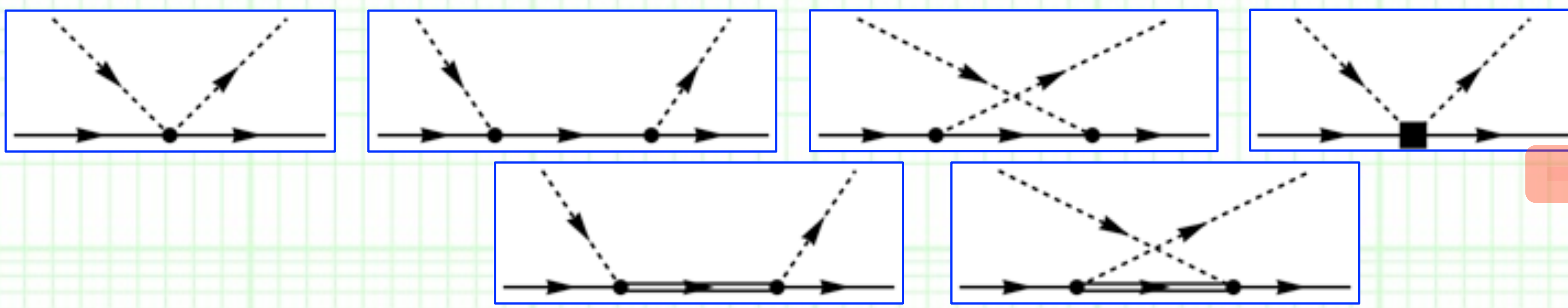
++ Chiral unitary approach for πN scattering ++

- We employ **chiral unitary approach for the πN elastic scattering**.

$$T'^{\pm}_{IL} = V'^{\pm}_{IL} + V'^{\pm}_{IL} G_L T'^{\pm}_{IL} = \frac{1}{1/V'^{\pm}_{IL} - G_L}$$



□ For the interaction kernel V we take LO + NLO + bare Δ of πN chiral perturbation theory and project it to $(I, L, J = L \pm 1/2)$ with "off-shell" momentum $|q|^{2L}$ as a prefactor. --> V^{prime}.



$$V^{\pm}_{IL} = |\vec{q}|^{2L} \times V'^{\pm}_{IL}$$

□ The loop function G_L is obtained with the dispersion relation:

$$G_L = \int_{s_{\mathrm{th}}}^{\infty} \frac{ds'}{2\pi} \frac{\rho(s') q(s')^{2L}}{s' - s - i0} = i \int \frac{d^4 q}{(2\pi)^4} \frac{|\vec{q}|^{2L}}{[(P-q)^2 - m_\pi^2](q^2 - M_N^2)}$$

$\rho(s)$: phase space

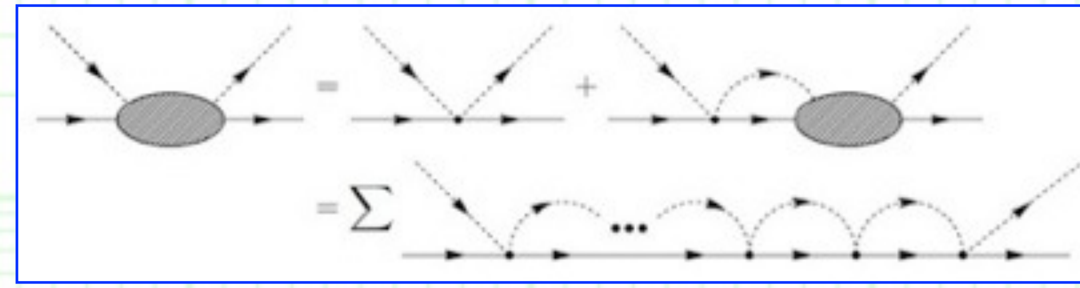
--- We need two subtractions for p wave: one subtraction const. is fixed so as to obtain the physical nucleon mass: $G_L(\sqrt{s} = M_N) = 0$

2. Formulation

++ Chiral unitary approach for πN scattering ++

- We employ **chiral unitary approach for the πN elastic scattering**.

$$T'^{\pm}_{IL} = V'^{\pm}_{IL} + V'^{\pm}_{IL} G_L T'^{\pm}_{IL} = \frac{1}{1/V'^{\pm}_{IL} - G_L}$$



- □ We have 7 model parameters: 4 LECs, bare Δ mass and coupling constant to πN, and 1 subtraction const.

--> Fitted to the πN scattering amplitude ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$) obtained as a PWA solution "WI 08" up to √s = 1.35 GeV.

Workman *et al.*, *Phys. Rev.* D86 (2012) 014012.

- □ **The P_{11} and P_{33} amplitude** contain poles corresponding to **the physical $N(940)$ and $\Delta(1232)$**, respectively:

$$T'^{\pm}_{IL}(\sqrt{s}) = \frac{g^2}{\sqrt{s} - \sqrt{s_{\mathrm{pole}}}} + (\mathrm{regular})$$

Elementariness

$$Z = -g^2 \left[G_L^2 \frac{dV'^{\pm}_{IL}}{d\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_{\mathrm{pole}}}}$$

Compositeness

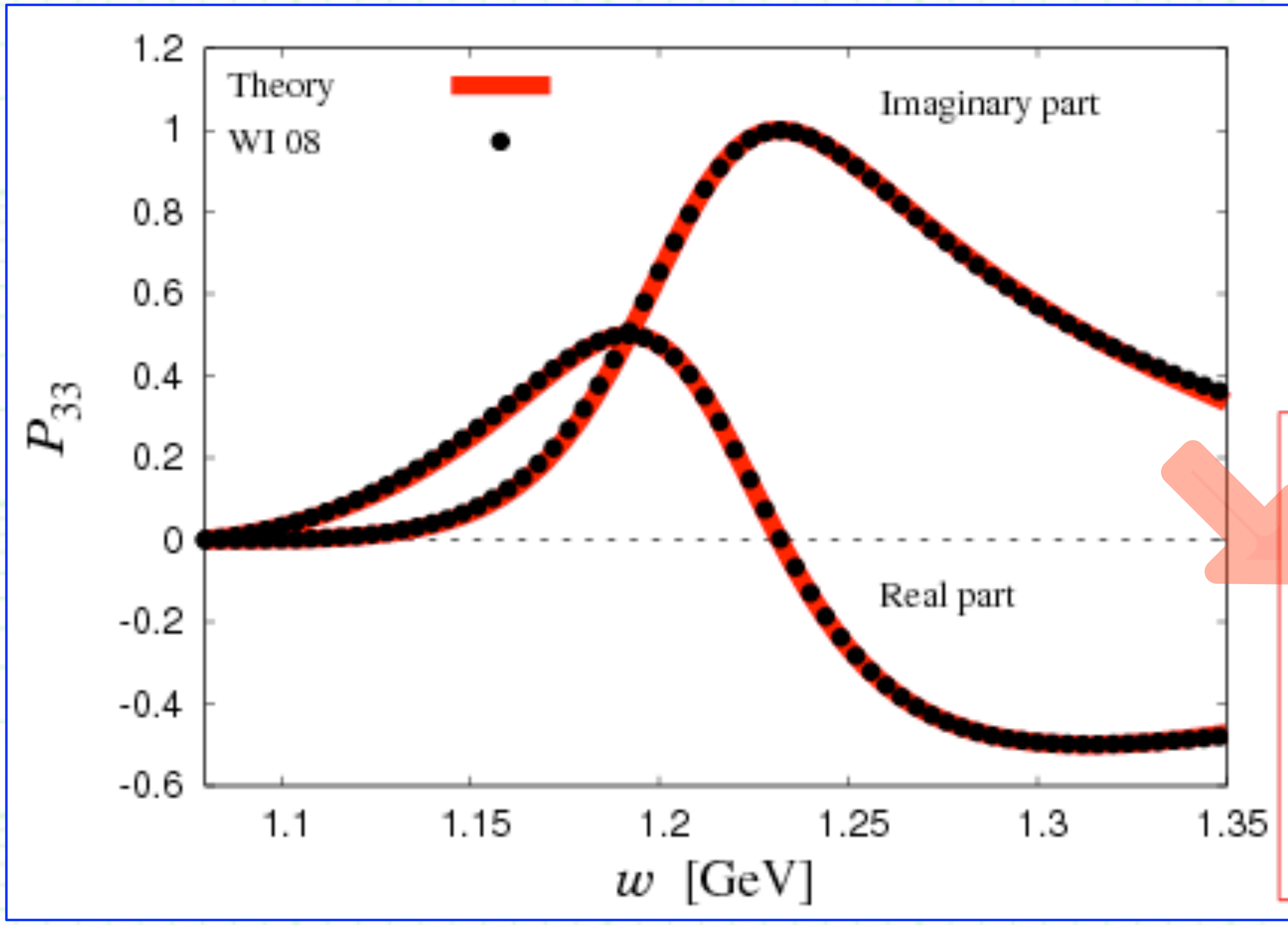
$$X_{\pi N} = -g^2 \left[\frac{dG_L}{d\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_{\mathrm{pole}}}}$$

$$X_{\pi N} + Z = 1$$

3. Numerical results

++ Compositeness from fitted amplitude ++

- **Fitted to the πN amplitude** WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).



--> $\chi^2 / N_{d.o.f.} = 486 / 809 \approx 0.6$.

- Chiral unitary approach reproduces the amplitude of PWA very well.

	$\Delta(1232)$	$N(940)$
$\sqrt{s_{pole}}$ [MeV]	$1209.8 - 47.6i$	938.9
g [MeV$^{-1/2}$]	$0.383 - 0.053i$	0.560
$X_{\pi N}$	$0.69 + 0.39i$	-0.18
Z	$0.31 - 0.39i$	1.18

- For $\Delta(1232)$, its pole position is very similar to the PDG value.

Re(pole position) = 1209 to 1211 (≈ 1210) MeV
−2Im(pole position) = 98 to 102 (≈ 100) MeV

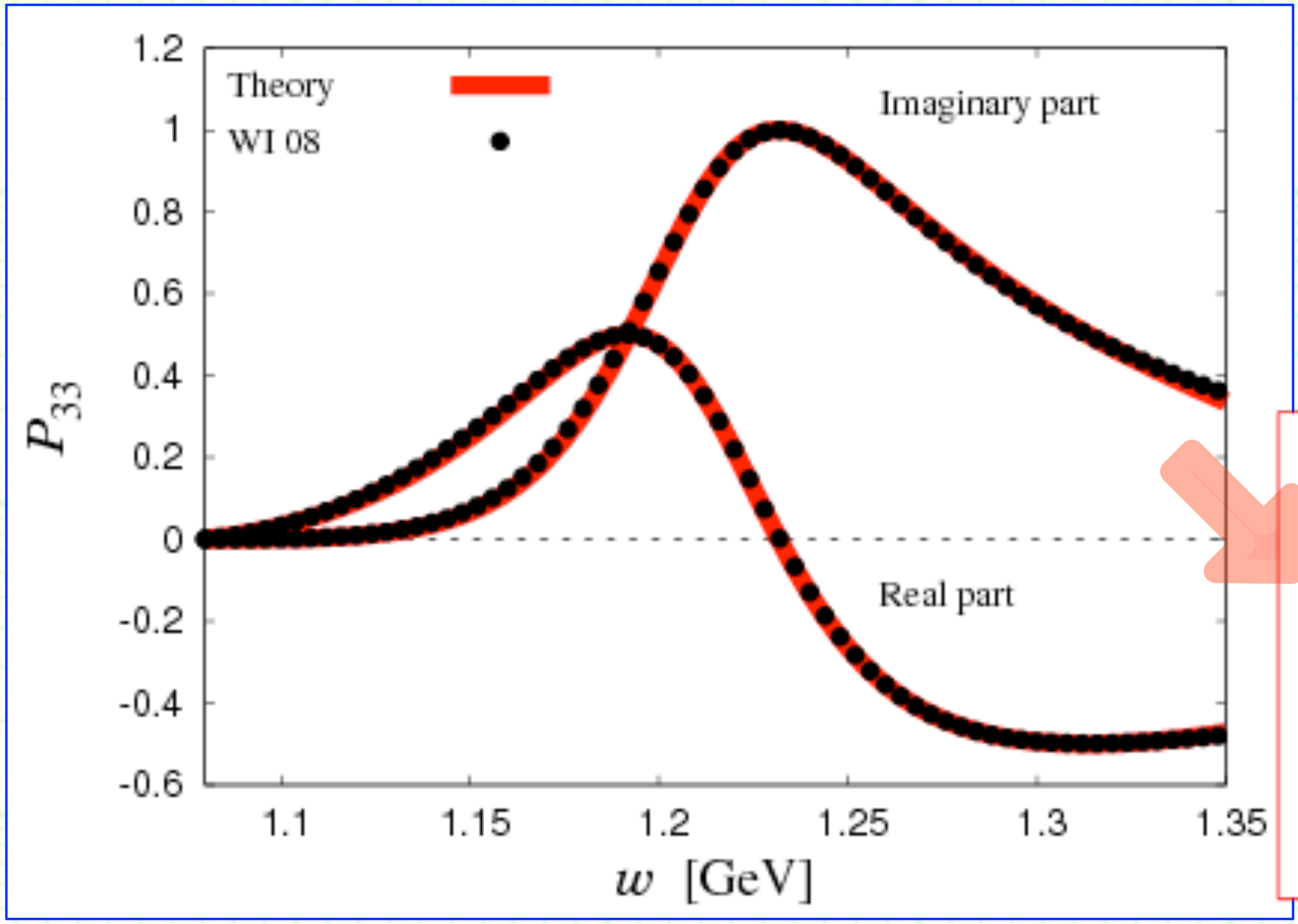
- The πN compositeness $X_{\pi N}$ takes large real part ! But non-negligible imaginary part as well.

--> Our refined model reconfirms the result in the previous study.

3. Numerical results

++ Compositeness from fitted amplitude ++

- **Fitted to the πN amplitude** WI 08 ($S_{11}, S_{31}, P_{11}, P_{31}, P_{13}, P_{33}$).



--> $\chi^2 / N_{\text{d.o.f.}} = 486 / 809 \approx 0.6$.

□ Chiral unitary approach <u>reproduces the amplitude of PWA very well</u>.

	$\Delta(1232)$	$N(940)$
$\sqrt{s_{\text{pole}}}$ [MeV]	$1209.8 - 47.6i$	938.9
g [MeV$^{-1/2}$]	$0.383 - 0.053i$	0.560
$X_{\pi N}$	$0.69 + 0.39i$	-0.18
Z	$0.31 - 0.39i$	1.18

□ However, for $N(940)$, <u>$X_{\pi N}$ is negative</u> because <u>$dG_L / d\sqrt{s}$ is positive</u> !

<--> $dG_L / d\sqrt{s}$ should be negative:

$$X_{\pi N} = -g^2 \left[\frac{dG_L}{d\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_{\text{pole}}}}$$

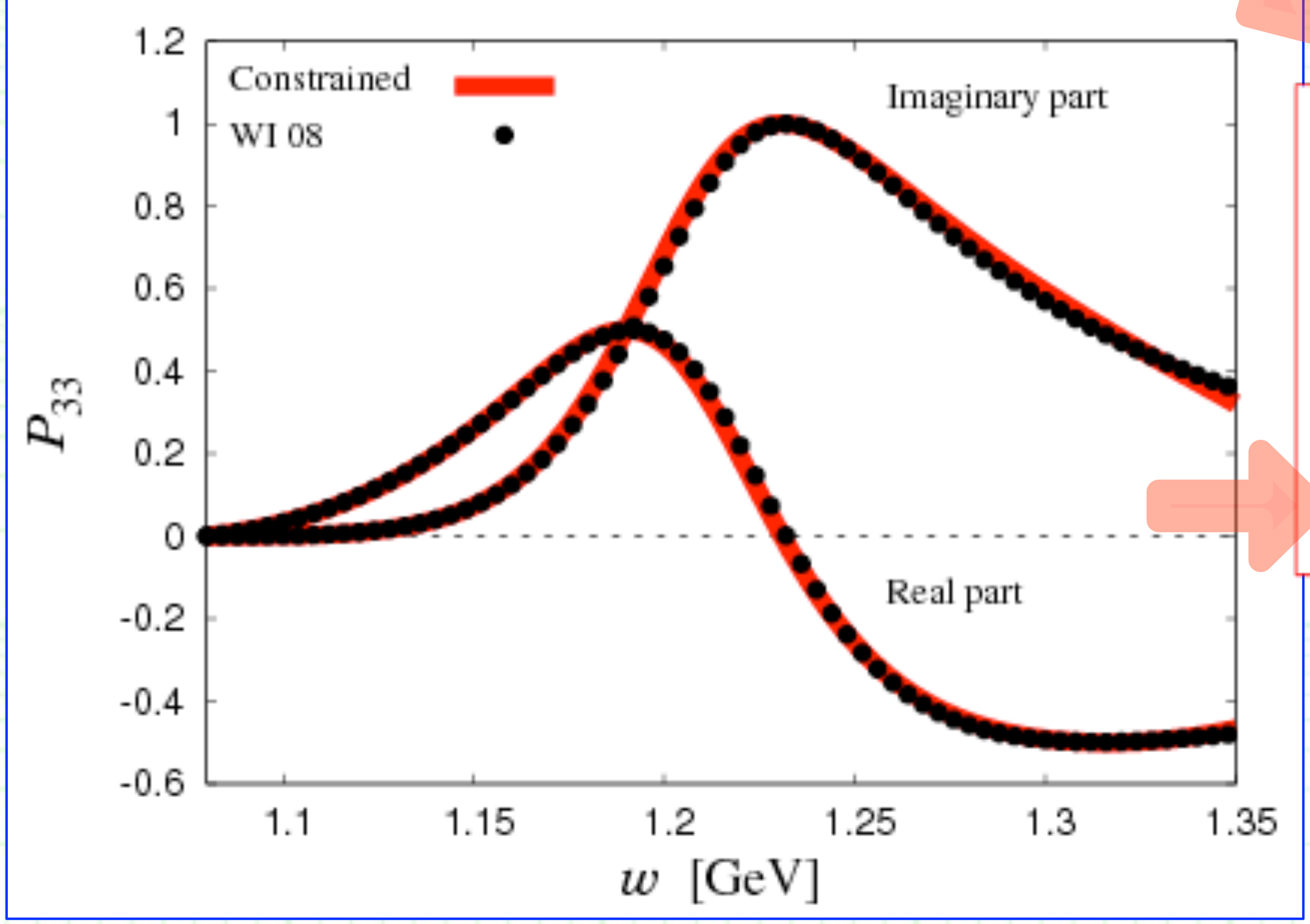
$$\frac{dG_L}{d\sqrt{s}} = -2\sqrt{s} \int_{s_{\text{th}}}^{\infty} \frac{ds'}{2\pi} \frac{\rho(s') q(s')^{2L}}{(s' - s - i0)^2}$$

--> Constrain subtraction const. !

3. Numerical results

++ Compositeness from fitted amplitude ++

- **The subtraction const. is constrained** so that $dG_L / d\sqrt{s}$ is non-positive at the nucleon pole position: $\frac{dG_L}{d\sqrt{s}}(M_N) \le 0$



--> $\chi^2 / N_{\mathrm{d.o.f.}} = 1240 / 809 \approx 1.5$.

Modified	$\Delta(1232)$	$N(940)$
$\sqrt{s_{\mathrm{pole}}}$ [MeV]	$1206.9 - 49.6i$	938.9
g [MeV$^{-1/2}$]	$0.395 - 0.061i$	0.516
$X_{\pi N}$	$0.87 + 0.35i$	0.00
Z	$0.13 - 0.35i$	1.00

□ For $\Delta(1232)$, values shift only slightly.

--- Large real part of $X_{\pi N}$ and non-negligible imag. part.

□ For $N(940)$, $X_{\pi N}$ is now non-negative, but $X_{\pi N}$ becomes zero.

--> Implies no πN cloud on $N(940)$ in the dispersive approach ???

4. Summary

- We have investigated internal structure of Δ(1232) and *N*(940) in terms of the π*N* component by using the compositeness.

- The π*N* compositeness was extracted from the π*N* elastic scattering amplitude in the chiral unitary approach.
  - Interaction from LO + NLO + bare Δ of chiral perturbation theory.
  - Amplitude is unitarized in the dispersive approach.

$$T'^{\pm}_{IL}(\sqrt{s}) = \frac{g^2}{\sqrt{s} - \sqrt{s_{\text{pole}}}} + (\text{regular})$$

→

$$X_{\pi N} = -g^2 \left[\frac{dG_L}{d\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_{\text{pole}}}}$$

- Fitting the π*N* amplitude to the solution of PWA, we have obtained large real part of π*N* compositeness for Δ(1232) and non-negligible imaginary part as well.

--> Our refined model reconfirms the result in the previous study.

- Open question: why physical Δ(1232) has large coupling to π*N* ?

--- This leads to large π*N* compositeness and π*N* cloud for Δ(1232).

Can we answer this question by using QCD ?

Thank you very much for your kind attention !