High statistics analysis of nucleon form factor in lattice QCD

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- Lattice result
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  - Isovector form factor
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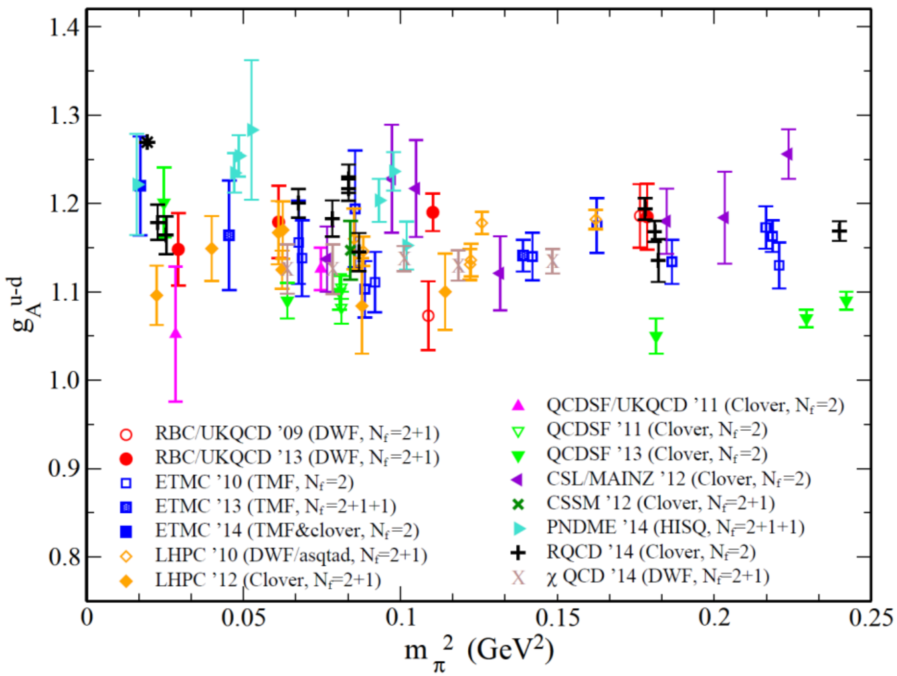
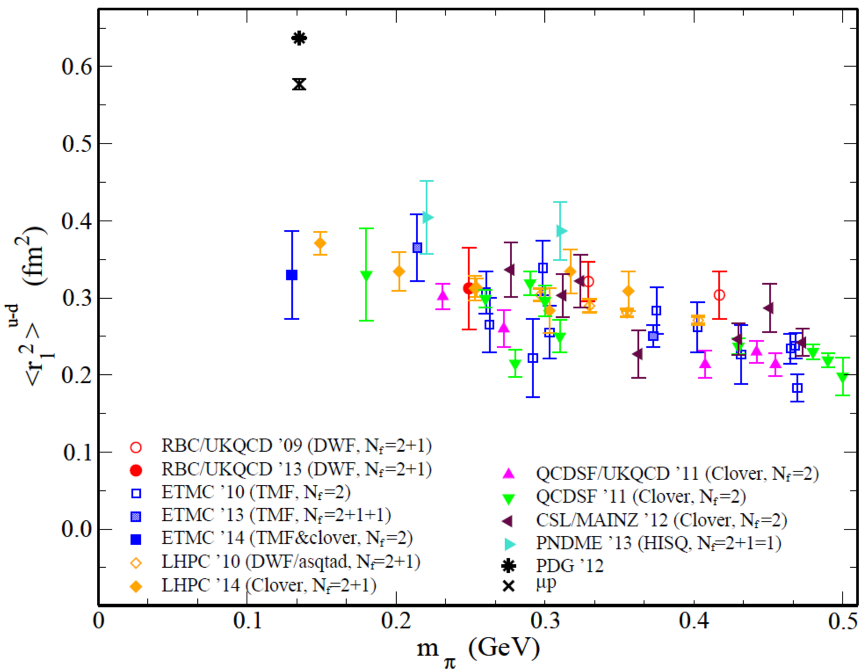
1. Introduction

"Puzzle" of nucleon form factor in LQCD

Constantinou, lattice2014





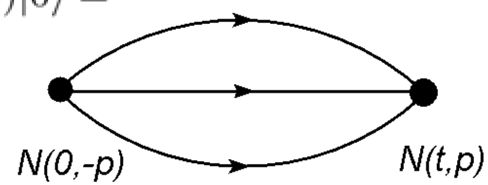
- Many lattice efforts, N_f=2, 2+1(also 2+1+1) with Wilson, Twisted Wilson, DW, …
- There is slight tension from experiment, even between different group

  $\Delta g_A \sim 5$ -- 10%, $\Delta r_E^2 \sim 10$ -- 20%
- Careful estimate of systematic uncertainty should be carried out.

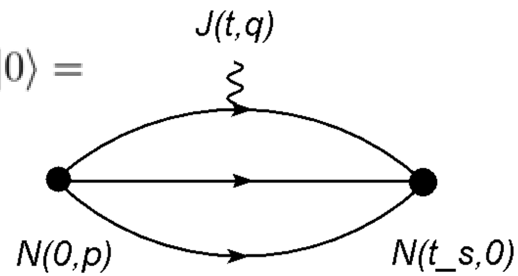
Computation of matrix element

2pt, 3pt function

$\langle 0|\mathcal{N}(t)\mathcal{N}^\dagger(0)|0\rangle =$



$\langle 0|\mathcal{N}(t_s,0)J(t,q)\mathcal{N}^\dagger(0,p)|0\rangle =$



$$\langle 0|\mathcal{N}(t)\mathcal{N}^\dagger(0)|0\rangle = |\langle 0|\mathcal{N}|N\rangle|^2 e^{-m_N t} + \underbrace{|\langle 0|\mathcal{N}|N'\rangle|^2 e^{-m'_N t}}_{\text{First excited state contamination}} + \cdots$$

$$\langle 0|T\{\mathcal{N}(t_s,0)J_\mu(t,q)\mathcal{N}^\dagger(0,p)|0\rangle$$
$$= \langle 0|\mathcal{N}|N\rangle\langle N|J_\mu|N\rangle\langle N|\mathcal{N}^\dagger|0\rangle e^{-E_N t - m_N(t_s-t)} + \langle 0|\mathcal{N}|N'\rangle\langle N'|J_\mu|N'\rangle\langle N'|\mathcal{N}^\dagger|0\rangle e^{-E'_N t - m'_N(t_s-t)} + \cdots$$
$$\simeq Z_N(0)Z_N(p)e^{-E_N t - m_N(t_{\rm sep}-t)} \times \left[\underbrace{\{G_X, g_A\}}_{\text{Matrix element of ground state}} + \underbrace{c_1 e^{-\Delta(t_{\rm sep}-t)} + c_2 e^{-\Delta' t}}_{\text{First excited state contamination}}\right]$$

$\Delta = m'_N - m_N > 0, \Delta' = E'_N - E_N > 0$

- Ground state matrix element is able to be extracted from ratio of 3pt and 2pt function after removing excited state contamination.

What is problem ?

- Signal-to-noise ratio problem
  - Noise of nucleon propagator at time-slice t behaves like

$$\mathrm{S/N} \sim \sqrt{N}\exp[-(m_N - 3m_\pi/2)t]$$

    it means statistics N ~ $\exp[(2m_N-3m_\pi)t]$ are needed for same precision.
- Excited state contamination
  - Excited state exponentially decays at large t, relying on $\Delta = m_{excited} - m_N$
  - To sufficiently isolate the ground state, precision of 2-pt and 3-pt function at large t is needed. It requires large computational cost.

Our strategy:

- To reduce statistical error, the all-mode-averaging (AMA) is applied.
- Systematic study of excited state contamination is performed in light pion mass and large volume, $m_\pi L > 4$.

AMA

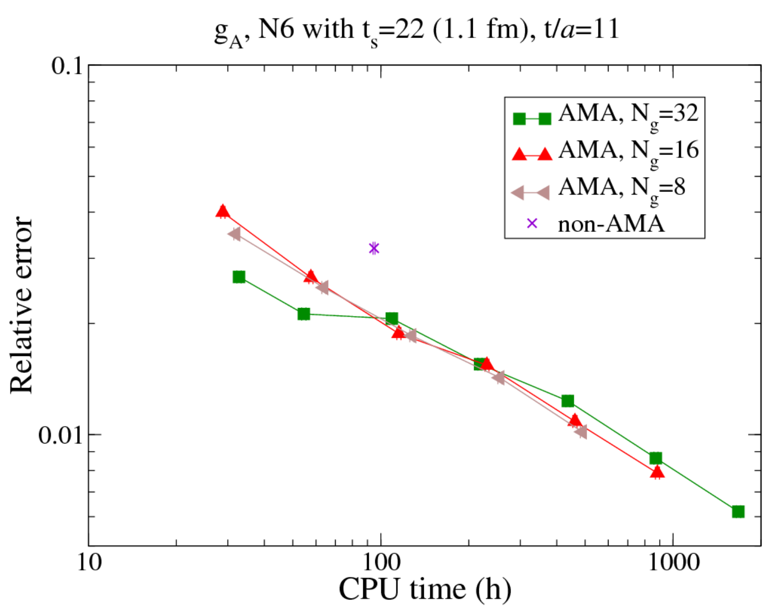
Blum, Izubuchi, ES (2013)

- Reduction of computational cost by using approximation

$$O^{(\mathrm{imp})} = O^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G} O^{(\mathrm{appx}),g}, \quad O^{(\mathrm{rest})} = O - O^{(\mathrm{appx})}$$

  - O: high precision (10^{-10} residue) ⇒ expensive but small number of computation
  - $O^{(\mathrm{appx})}$: low precision ($\sim 10^{-2}$ residue) ⇒ cheap but large number of computation

AMA estimator $O^{(\mathrm{imp})}$ has error reduction depending on quality of $O^{(\mathrm{appx})}$.



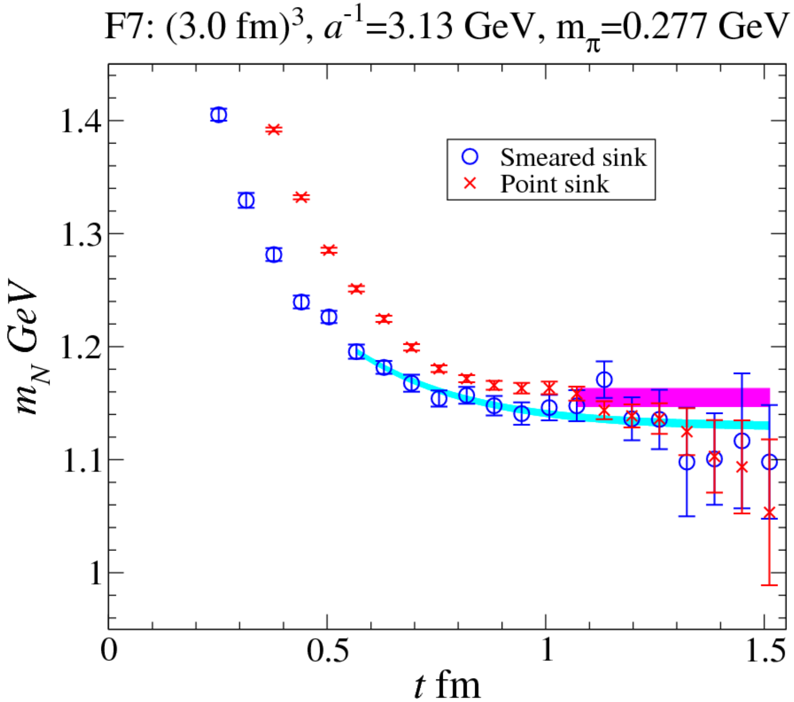
- Parameter tuning of deflation field N_s which is related to performance of iteration algorithm.
- Cost of computing quark propagator is reduced to 1/5 and less.
- Total speed-up is about factor 2 and more. (depending on lattice size and pion mass)

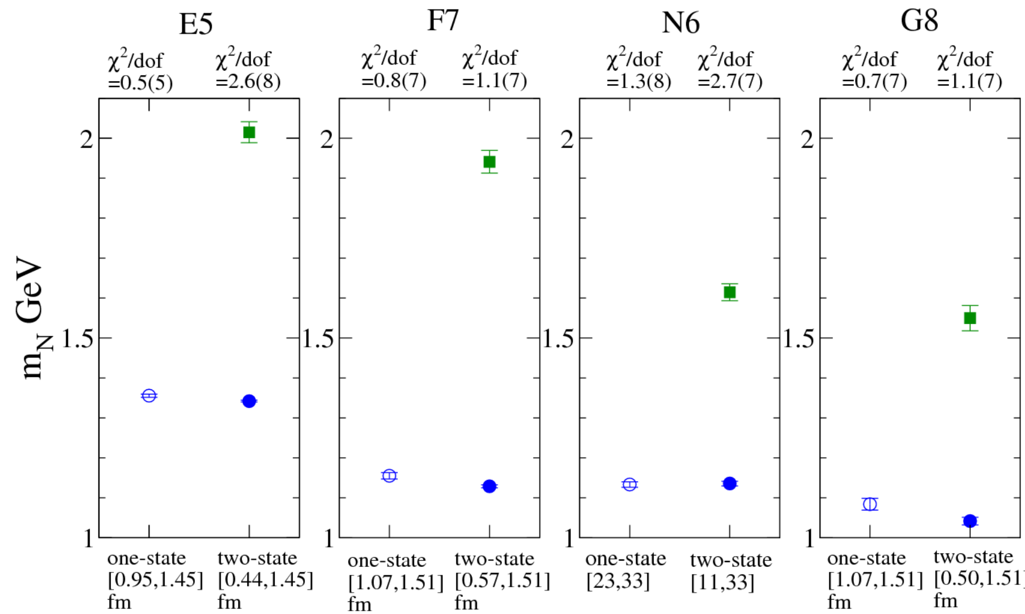
CLS config, N_f = 2 Wilson-clover fermion

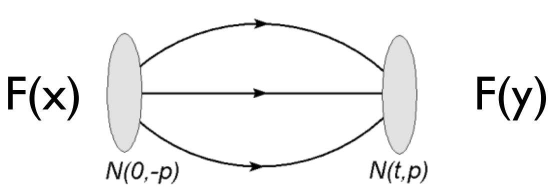
	Lattice	*a* (fm)	m_π (GeV)	N_G	t_s (fm)	#conf	#meas(*)
E5	64×32^3	0.063	0.456	64	0.82, 0.95, 1.13	~480	~30,000
	$(2.0\ \text{fm})^3$		($m_\pi L$=4.7)		1.32	994	63,616
					1.51	1605	102,720
F7	96×48^3	0.063	0.277	64	0.82, 0.95, 1.07	250	16,000
	$(3.0\ \text{fm})^3$		($m_\pi L$=4.2)	128	1.20, 1.32	250	32,000
				192	1.51	250	64,000
N6	96×48^3	0.05	0.332	32	0.9	110	3,520
	$(2.4\ \text{fm})^3$		($m_\pi L$=4.1)	32	1.1,1.3	888	28,416
				32	1.5, 1.7	936	30,272
G8	128×64^3	0.063	0.193	80	0.88	184	14,720
	$(4.0\ \text{fm})^3$		($m_\pi L$=4.0)	112	1.07	94	10,528
				160	1.26	178	28,480
				64	1.51	179	28,640

* Effective statistics : #mes = $N_G \times$ #conf

Nucleon mass and its excited state







F(x): Jacobian function with APE smearing link.

- The ground-state dominant, t = 1--1.5 fm.
- Including the excited state, t = 0.5 -- 1.5 fm
- Fitting function

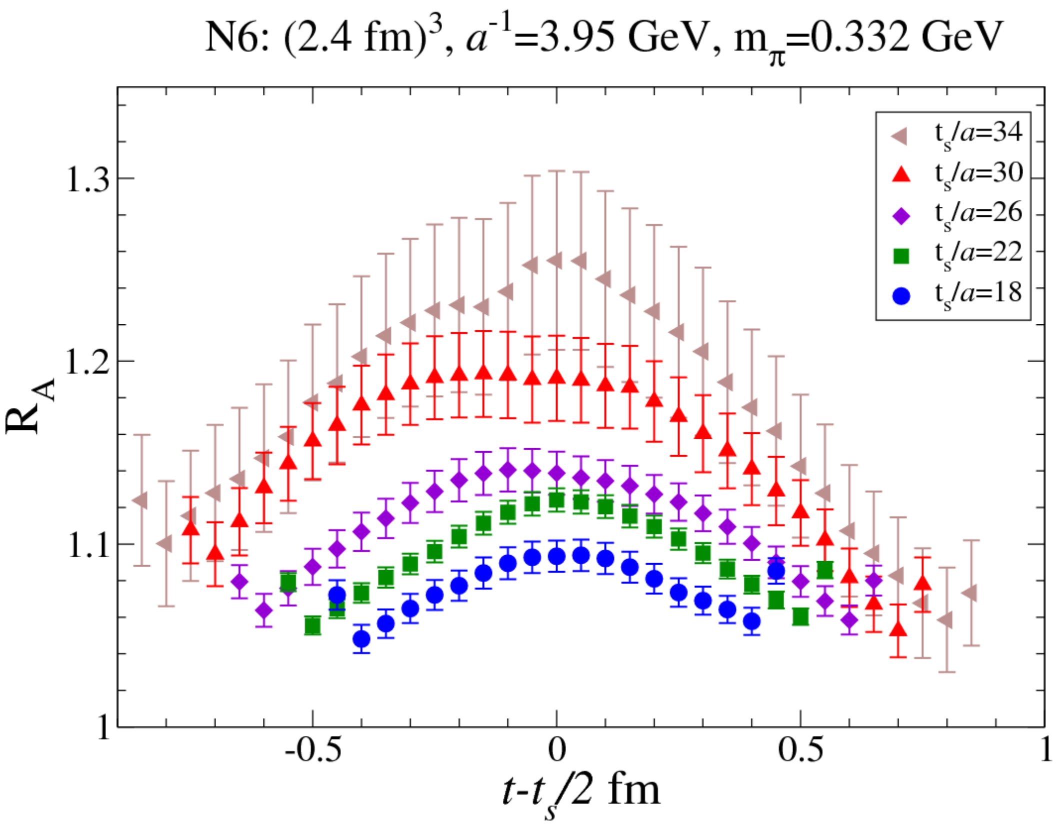
  One-state : Ze^{-mt},

  Two-state : $Z e^{-m t} + Z' e^{-m' t}$
- almost comparable with two fitting results

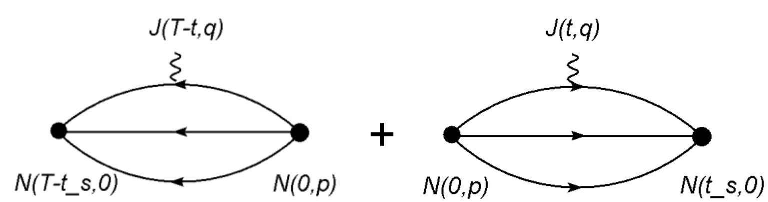
Axial charge

- Single ratio of 2pt and 3pt with fixed t_s

$$R_A(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)J_3(t,q)\mathcal{N}^\dagger(0,0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)\mathcal{N}^\dagger(0,0)|0\rangle} \simeq g_A + c_1 e^{-\Delta t_s} + c_2 e^{-\Delta'(t_s-t)}$$



- Computation of 3pt and 2pt function at zero momentum with spin projection P.
- Signal is regarded as plateau.
- The size of excited state (2nd and 3rd terms) are still unknown ! → significant uncertainty
- Forward and backward averaging



Extraction of g_A

- Ground and excited state ansatz
  - Ground state dominance (plateau method)

$$R_A(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)J_3(t,q)\mathcal{N}^\dagger(0,0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)\mathcal{N}^\dagger(0,0)|0\rangle} \simeq g_A,\ (t_s, t_s - t \gg 1)$$

  - Evaluation from constant fitting for t with fixed t_s.
  - To suppress the excited state contamination, measurement at large t_s is needed.

  - First excited state (two-state) PNDME(2014), RQCD(2014), …

$$R_A(t,t_s) \simeq g_A + c\left(e^{-\Delta t_s} + e^{-\Delta(t_s-t)}\right)$$

  - Δ is mass difference between ground and 1[st] excited state.

- Summation method Capitani et al. PRD86 (2012)

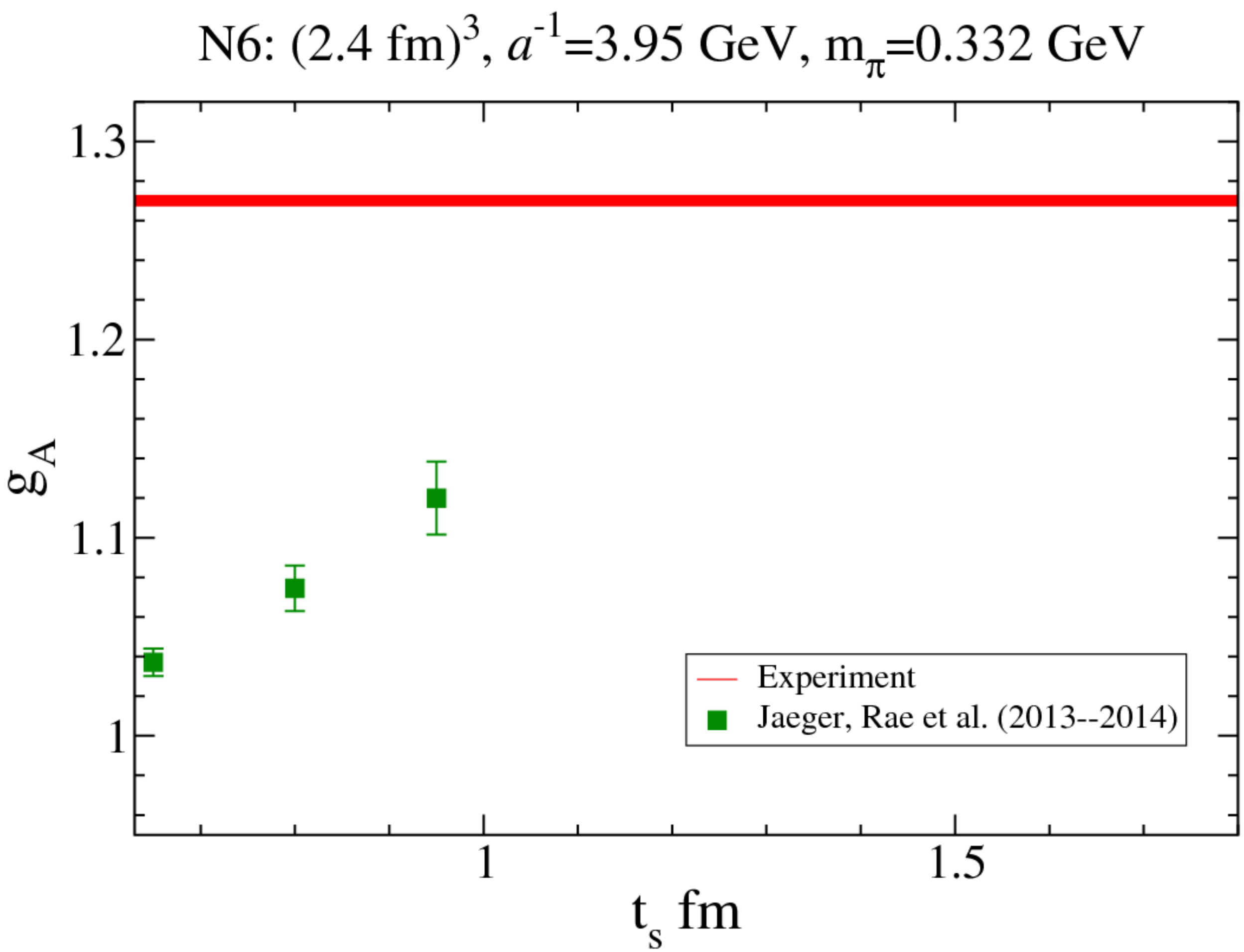
$$R_A^{\mathrm{sum}}(t_s) = \sum_{t=0}^{t_s} R_A(t,t_s) \simeq a_0 + t_s(g_A + O(e^{-\Delta t_s}))$$

  - Using summation in [0,t_s] at fixed t_s , the excited state effect is ~ $O(e^{-\Delta t_s})$
  - g_A is given from t_s linear part at t_s >> 1.

3. Lattice results (preliminary): axial charge

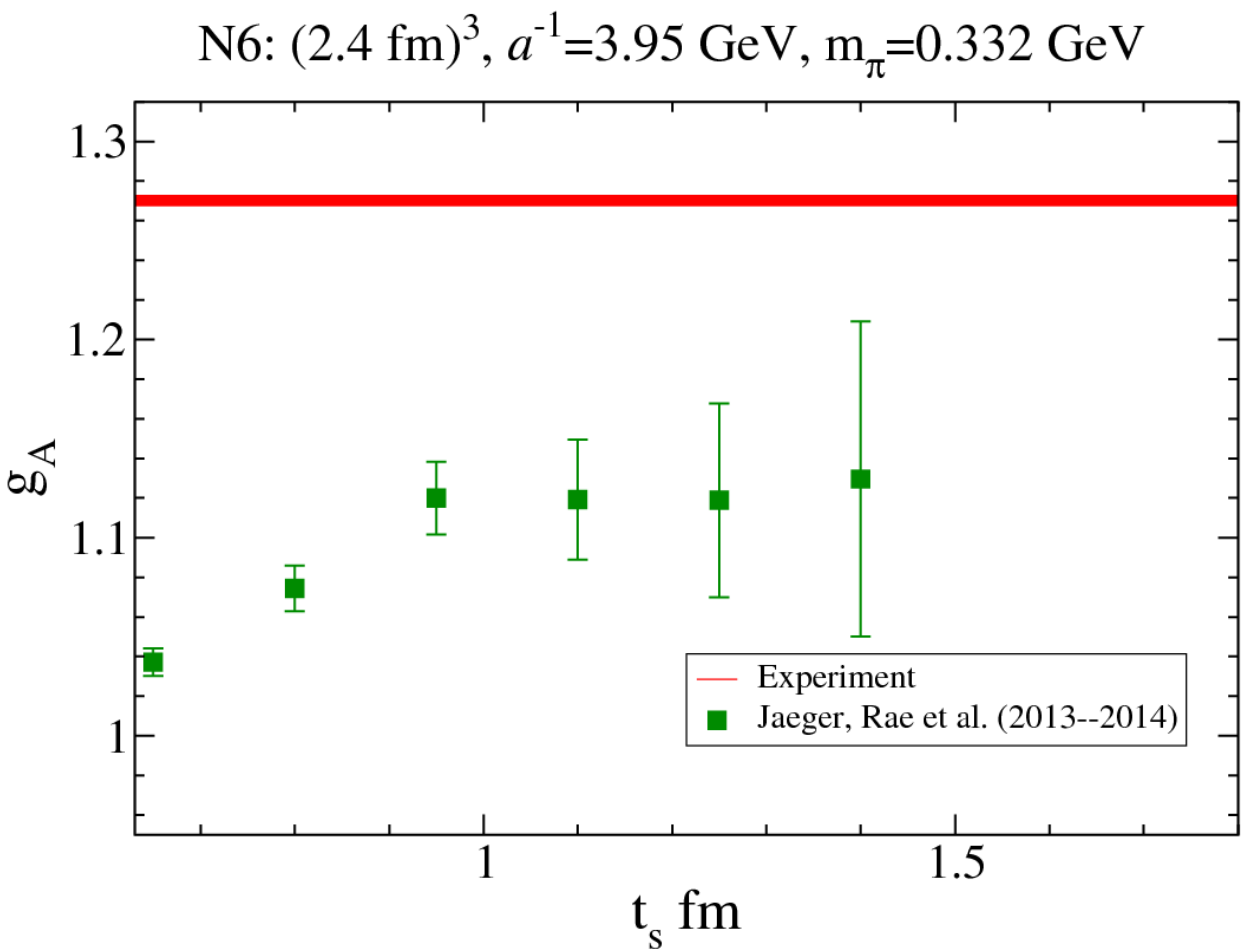
Plateau method

- Non-AMA results at $t_s < 1$ fm



Plateau method

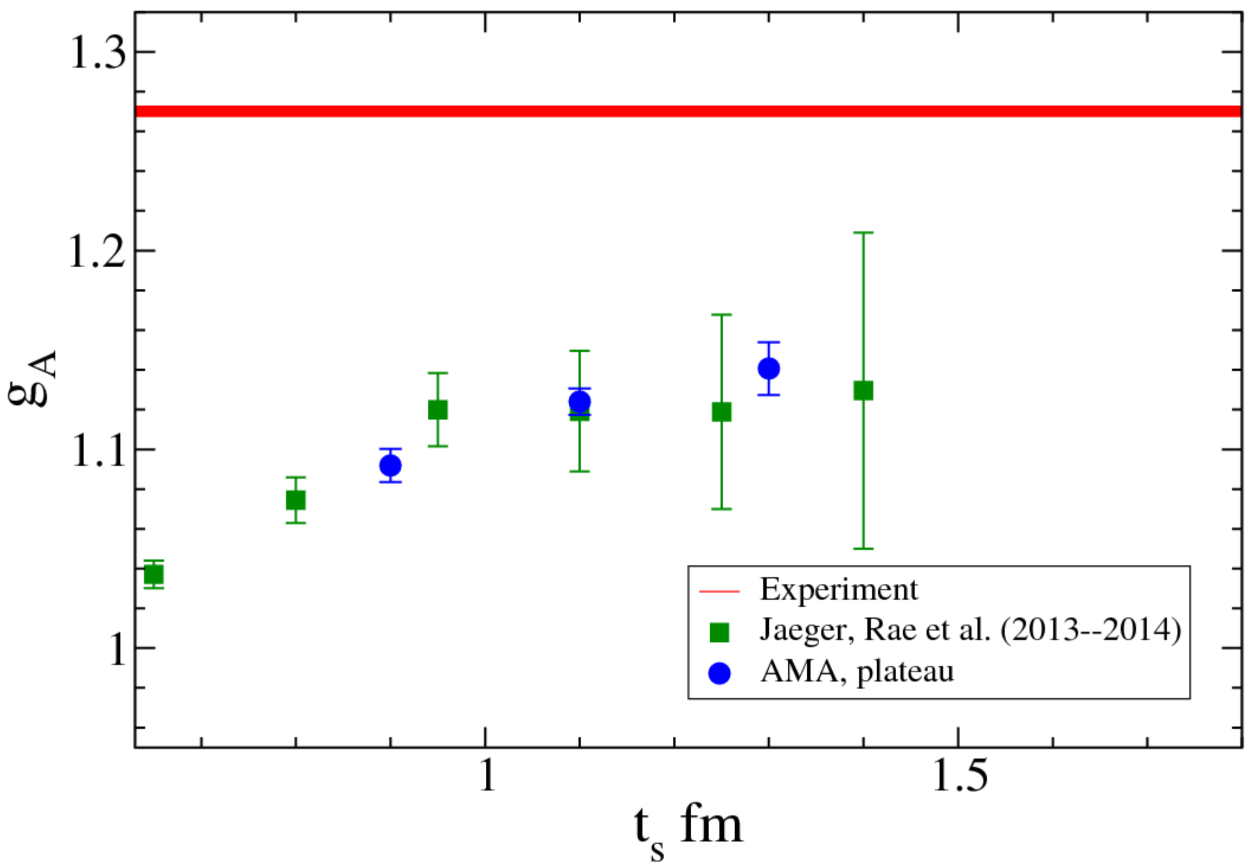
- Non-AMA results at t_s <1.5 fm



Plateau method

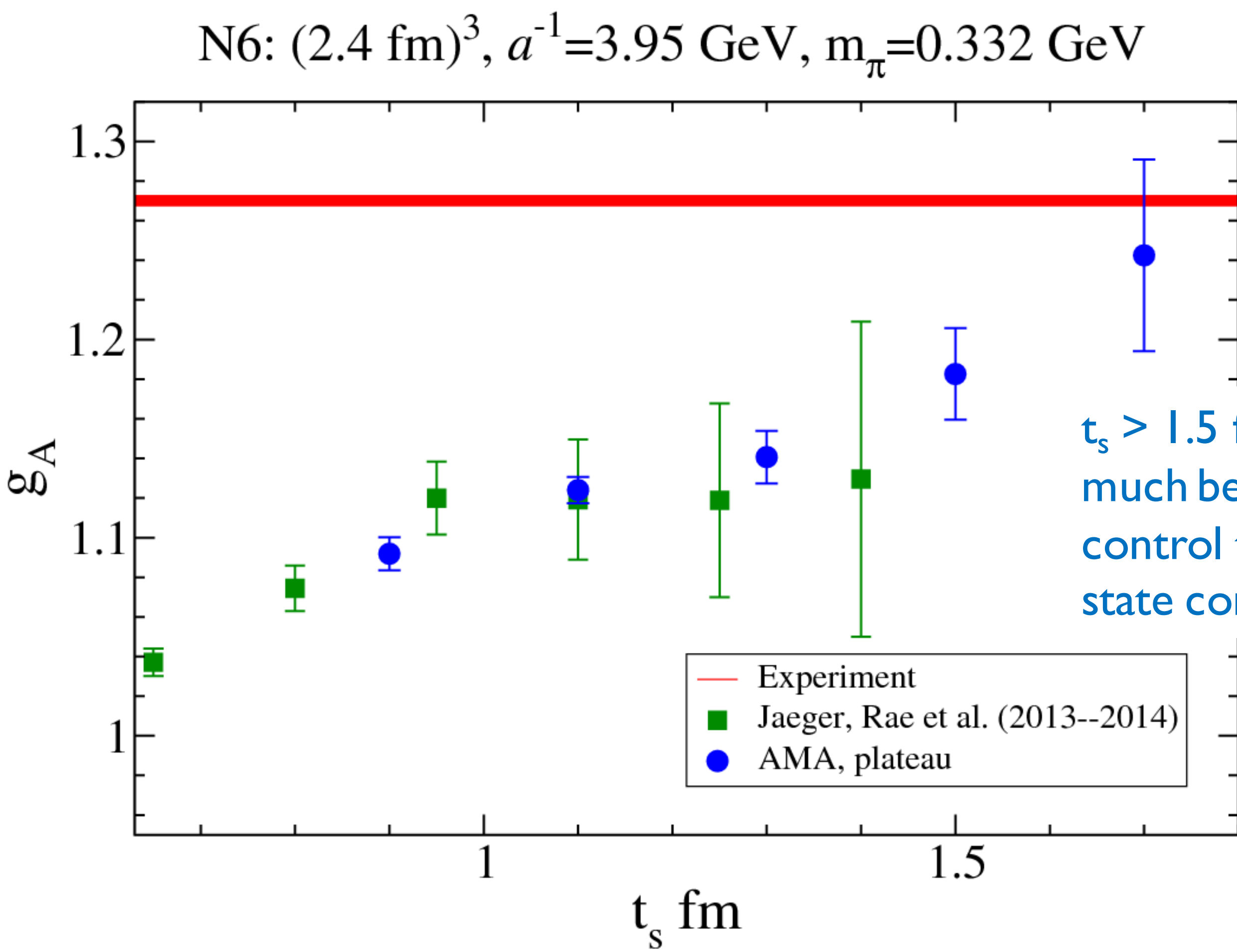
- AMA results at t_s <1.5 fm

N6: $(2.4\ \text{fm})^3$, a^{-1}=3.95 GeV, m_π=0.332 GeV



Plateau method

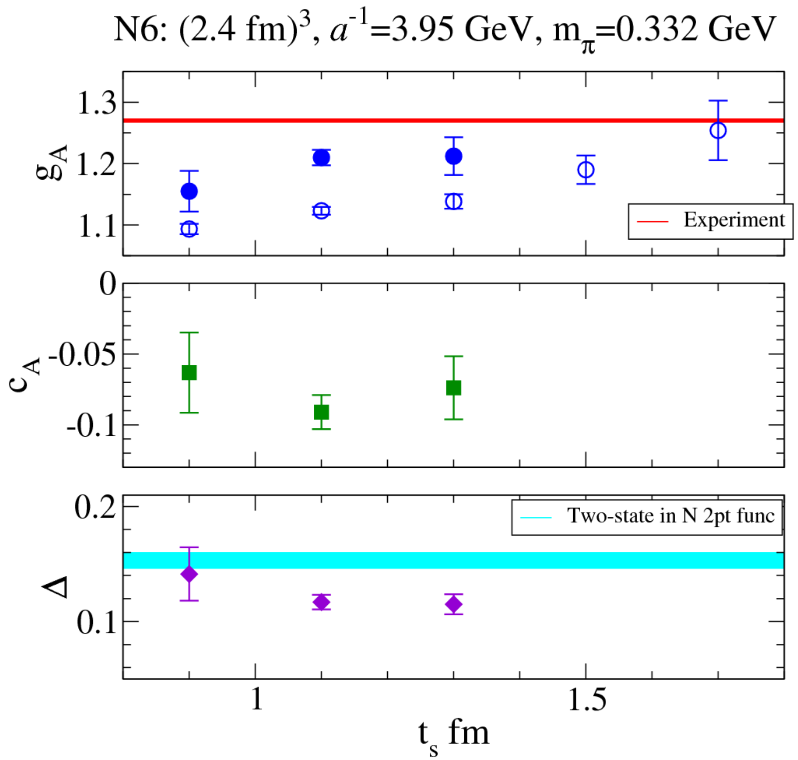
- AMA results at t_s >1.5 fm

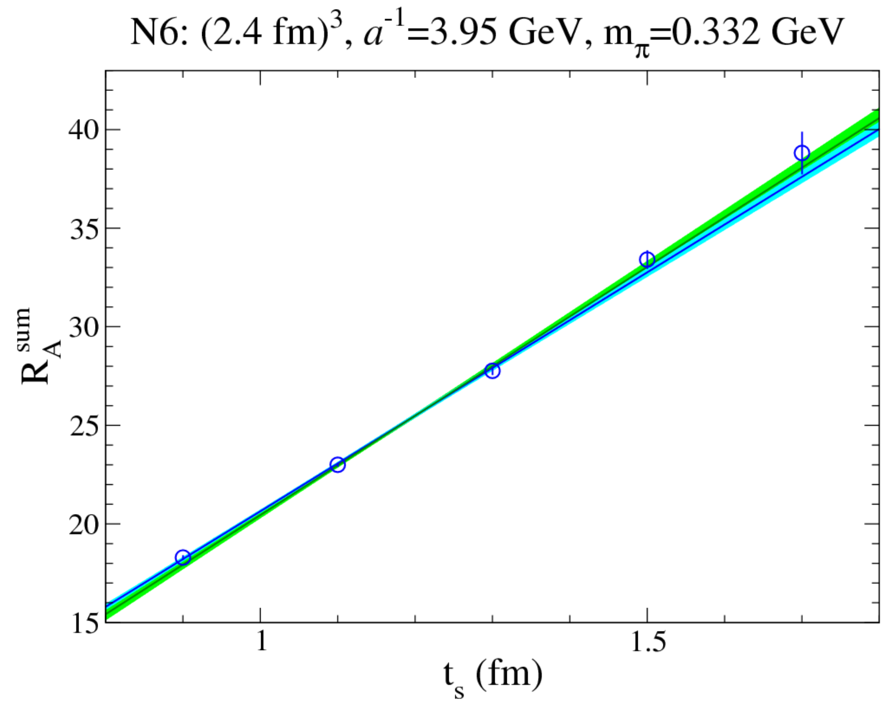


t_s > 1.5 fm region is much better to control the excited state contribution.

3. Lattice results (preliminary): axial charge

Two state and summation method



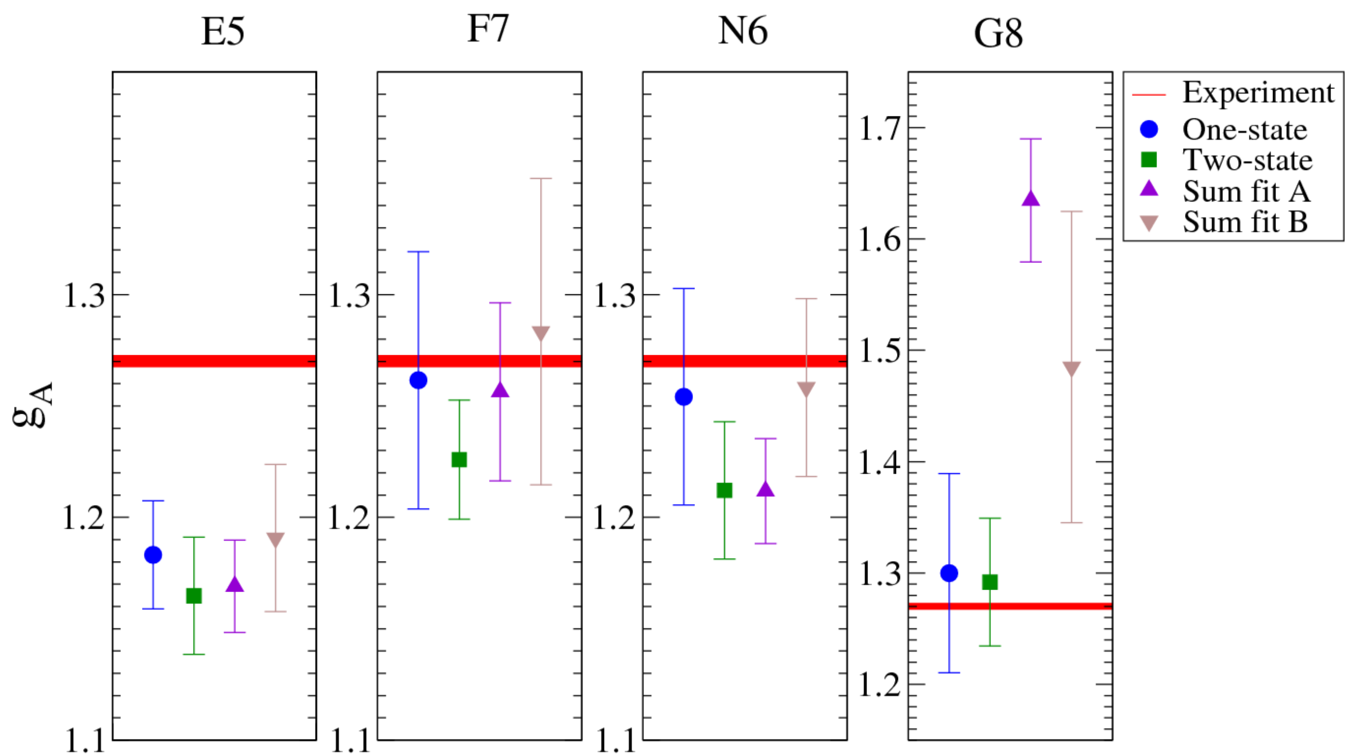


- After correction to excited state, g_A increases, and in agreement with plateau method in t_s > 1.5 fm.
- Mass difference Δ is compatible with two state fit of 2pt function.

- Linear behavior which is consistent with linear ansatz as expected.
- Comparison between two fitting range: t_s = (fit A)[0.9, 1.7], (fit B)[1.1, 1.7] ⇒ estimate of systematic uncertainty

3. Lattice results (preliminary)): axial charge

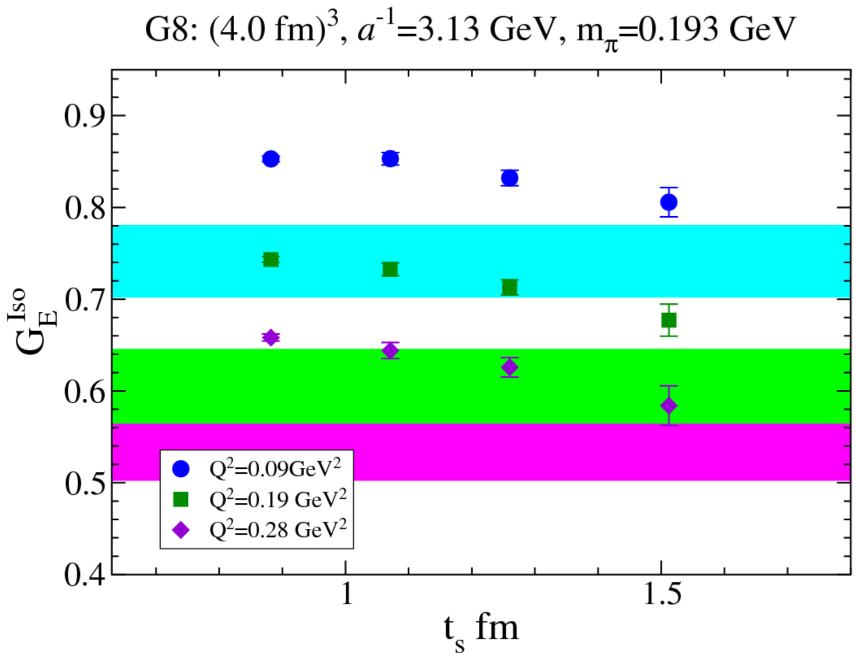
Comparison

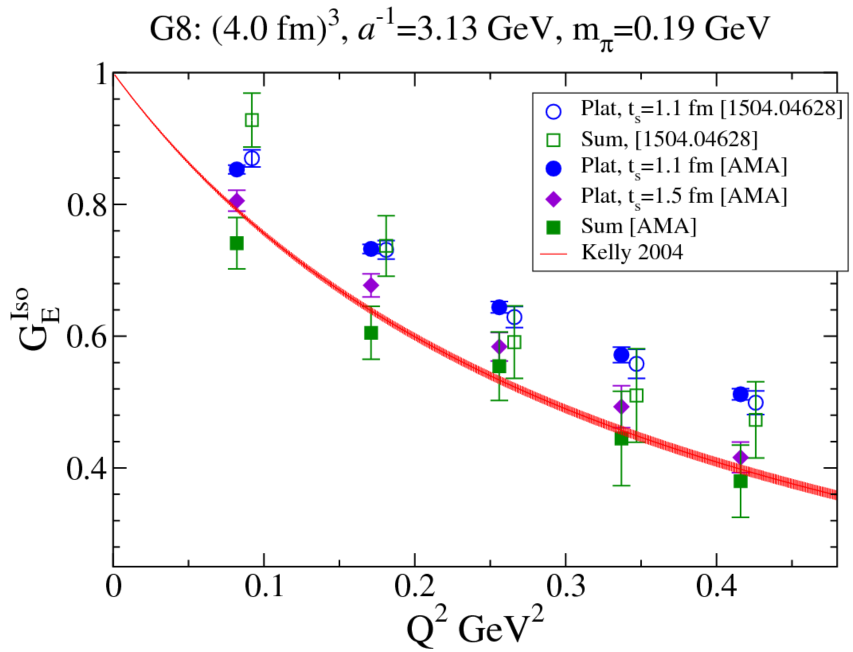


- Four methods provide comparable result except for G8 ensemble at m_π = 0.19 GeV .
- Excited state contamination in G8 at t_s < 1.0 fm significantly appears.
- Finite pion mass effect of g_A is rather mild.

3. Preliminary results: Isovector form factor

Analysis at large t_s



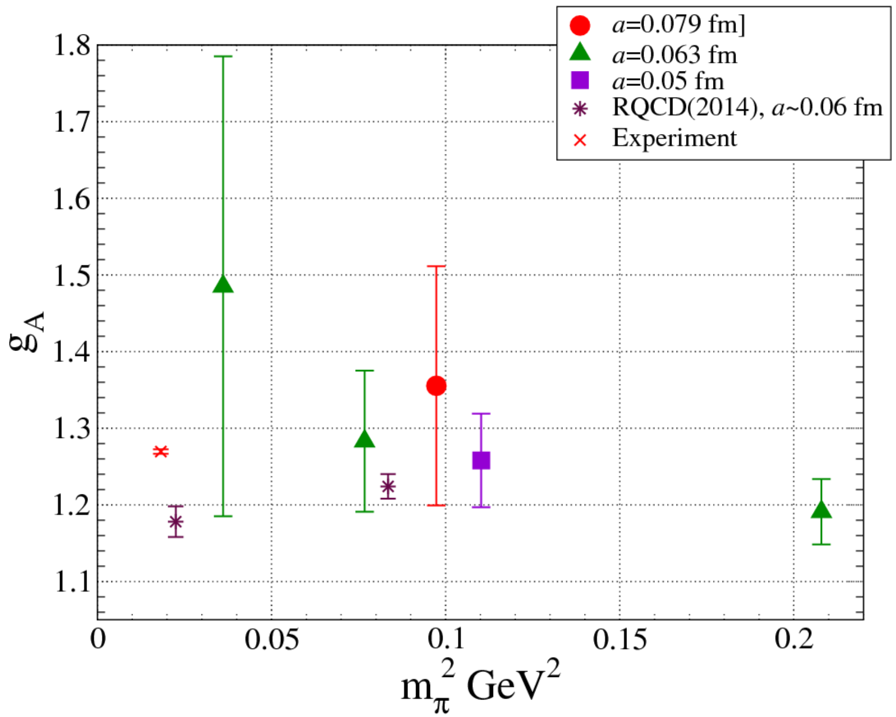


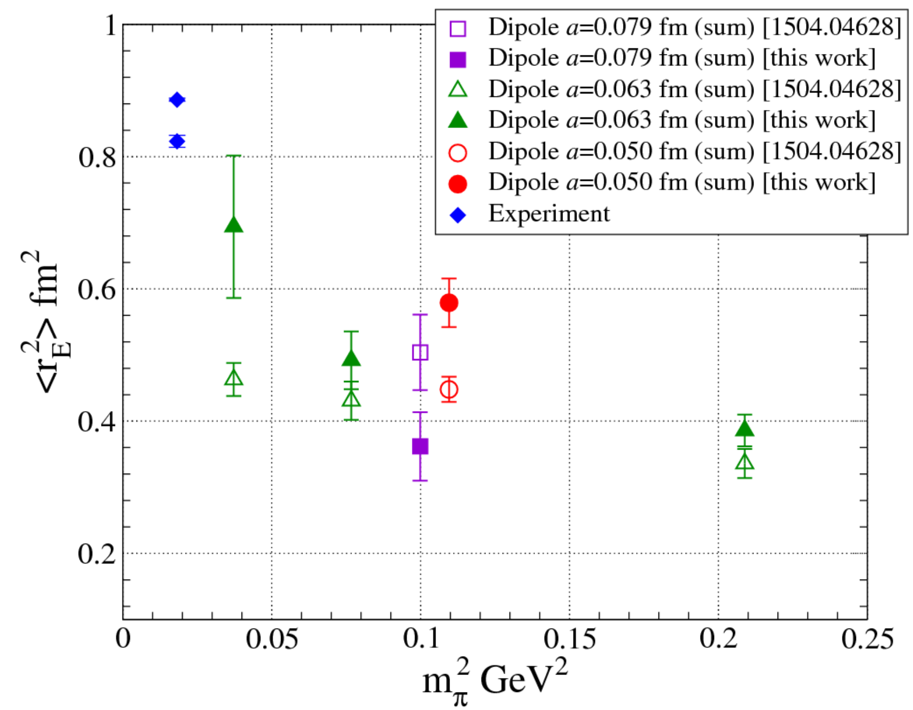
- From t_s > 1 fm, there is still tendency to decrease by ~10%.
- Summation method and plateau method at t_s > 1.5 fm are compatible.

- Comparison with previous work on the same ensemble.
- Discrepancy between plateau method at t_s = 1.1 fm and 1.5 fm, due to excited state contamination.
- Approach to experimental value.

3. Preliminary results

Axial charge and charge radius





- Analysis of axial charge and charge radius with large t_s by 1.7 fm.
- Light pion around 0.2 GeV has still large statistical error.
- In t_s = 1.1 fm, there is still unsuppressed excited state effect, which may be one of the reason for large discrepancy from experiment ⇒ need more than 1.5 fm.
- Axial charge may not have strong m_π dependence, but $\langle r_E \rangle$ may have.

Summary

- High statistics calculation of nucleon form factor is performed in N_f=2 Wilson-clover at $Lm_\pi > 4$ with m_π = 0.19--0.46 GeV.
- All-mode-averaging technique is working well for reduction of statistical error.
- t_s > 1.5 fm is required for small contribution of excited state contamination in axial charge and (iso)vector form factor.
- Axial charge and charge radius are approaching to experimental value.
- Feasible study for application to N_f = 2+1 CLS configurations with open boundary condition.

Thank you for your attention.

Isovector form factor

- Ratio with momentum transition

$$R_G(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,p_1)J_\mu(t,q)\mathcal{N}^\dagger(0,p_0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,p_0)\mathcal{N}^\dagger(0,p_0)|0\rangle}K(p_1,p_0) \simeq G_X + d_1 e^{-\Delta t_s} + d_2 e^{-\Delta'(t_s-t)}$$

$$K(p_1,p_0) = \sqrt{\frac{C_{\rm 2pt}^{\rm lc}(p_1,t_s-t)C_{\rm 2pt}^{\rm sm}(p_0,t)C_{\rm 2pt}^{\rm lc}(p_0,t_s)}{C_{\rm 2pt}^{\rm lc}(p_0,t_s-t)C_{\rm 2pt}^{\rm sm}(p_1,t)C_{\rm 2pt}^{\rm lc}(p_1,t_s)}},$$

- The ratio consists of 3pt and 2pt, with combination of local "lc" and smeared "sm" sink.
- Matrix element with Sachs form factor

$$\langle N(\vec{p}_1)|J_\mu|N(\vec{p}_0)\rangle = \bar{u}(p_1)\left[F_1^v(q^2)\gamma_\mu + F_2 q_\nu \sigma_{\mu\nu}/2m_N\right]u_N(p_0)$$

$$G_E = F_1 - \frac{q^2}{4m_N^2}F_2,\ G_M = F_1 + F_2$$

- Form factor G_X as a function of q^2, $q = p_1 - p_0$, in which $p_1 = (0,m_N)$ $p_0 = (p,E)$ are used.
- Systematic study of excited state contamination with plateau and summation method is necessary.

AMA

Blum, Izubuchi, ES (2013)

Improvement of standard deviation:

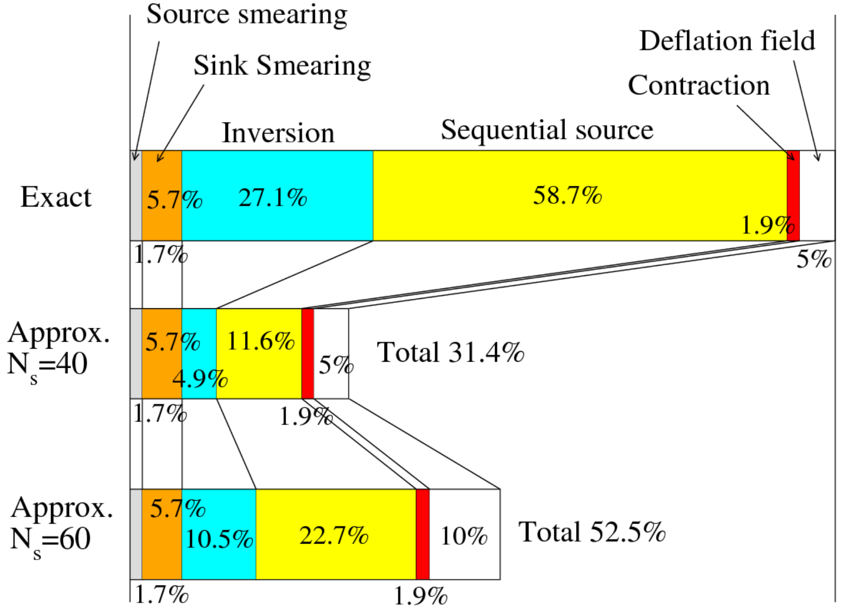
$$\frac{\sigma^{\mathrm{imp}}}{\sigma} \simeq \sqrt{\frac{1}{N_G} + 2(1-r) + \frac{1}{N_g^2}\sum_{g\neq g'} r_{gg'}}$$

$$r = \frac{\langle \Delta O \Delta O^{(\mathrm{appx})}\rangle}{\sigma \sigma^{(\mathrm{appx})}} \qquad r_{gg'} = \frac{\langle \Delta O^{(\mathrm{appx}),g} \Delta O^{(\mathrm{appx}),g'}\rangle}{\sigma^{(\mathrm{appx}),g}\sigma^{(\mathrm{appx}),g'}}$$

r: correlation between O and $O^{(\mathrm{appx})}$

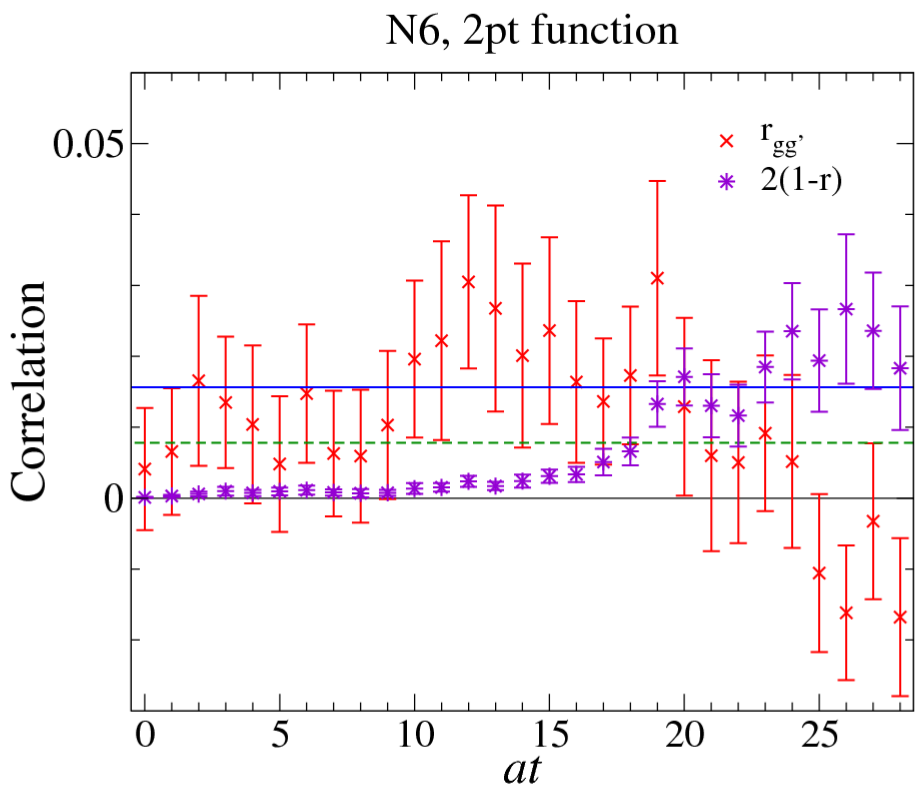
$r_{gg'}$: correlation between $O^{(\mathrm{appx}),g}$ and $O^{(\mathrm{appx}),g'}$

- $O^{(\mathrm{appx})}$ has several tuning parameters to control of r and $r_{gg'}$
  e.g. stopping condition, deflation field, source location



Performance test of AMA

- Correlation



Expected error reduction in AMA:

$$\frac{\sigma^{\mathrm{imp}}}{\sigma} \simeq \sqrt{\frac{1}{N_G} + 2(1-r) + \frac{1}{N_g^2}\sum_{g \neq g'} r_{gg'}}$$

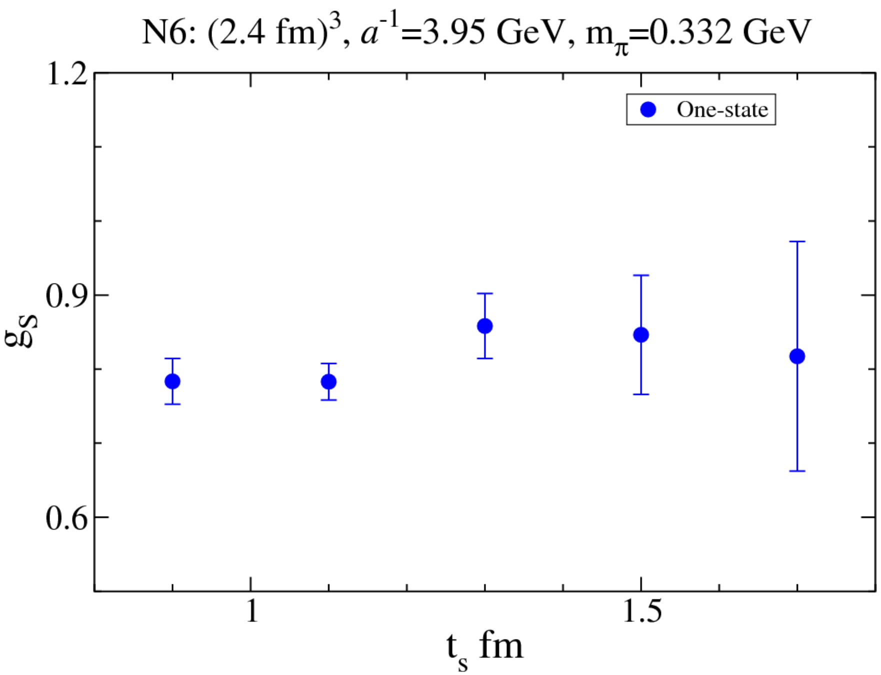
$1/N_G = 1/64$

$1/N_G = 1/128$

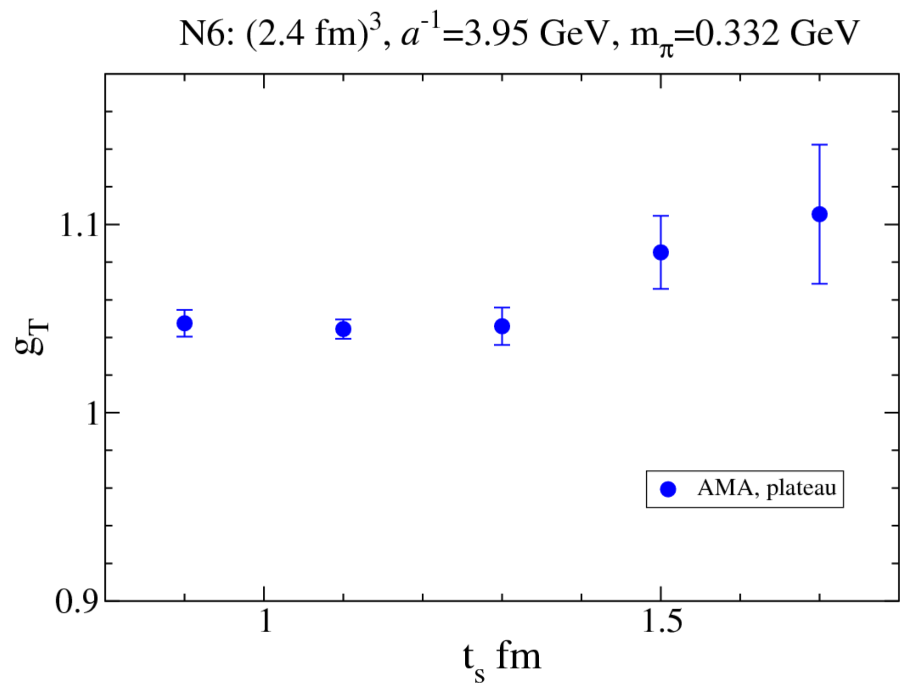
- $r_{gg'}$: correlation between $O^{(appx)}$ with g and g' transformation.
- 2(1-r) : correlation between $O^{(appx)}$ and O.
- At t ~ 24, size of correlation is similar to $1/N_G$, ⇒maximum point to reduce error

Scalar and tensor charge

Scalar (lattice)

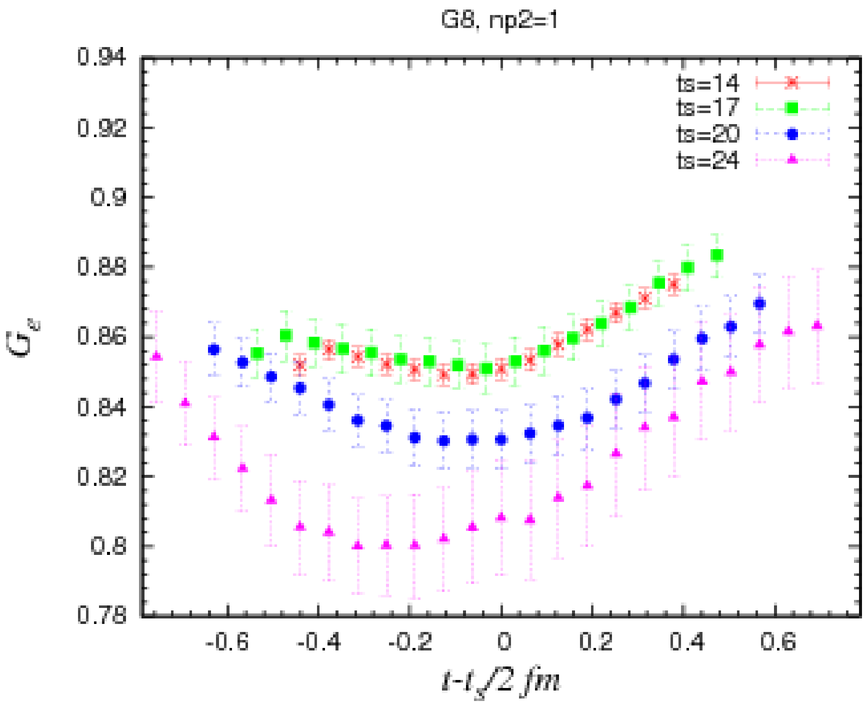


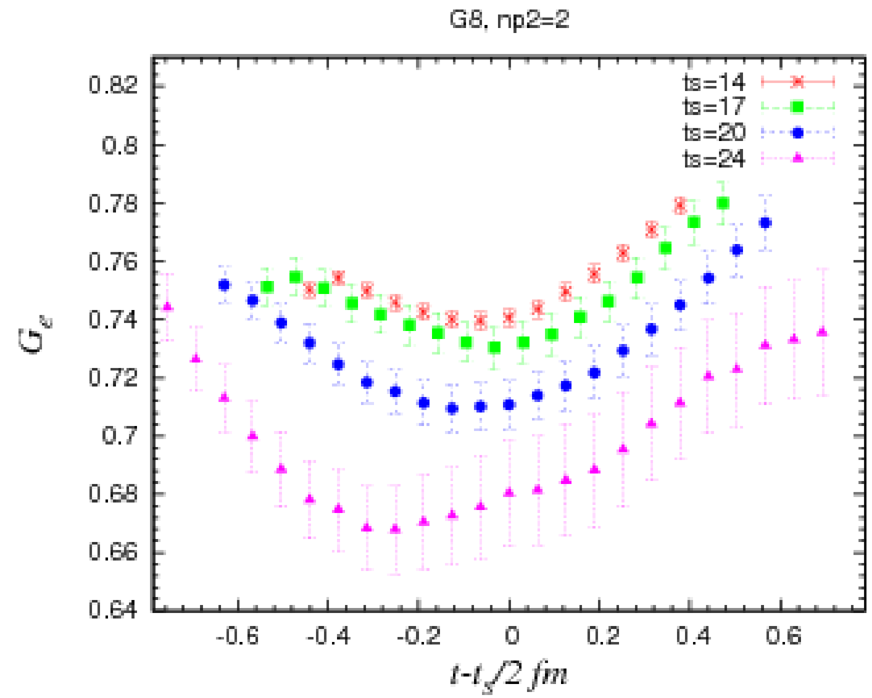
Tensor (lattice)



- There does not appear significant effect of excited state.

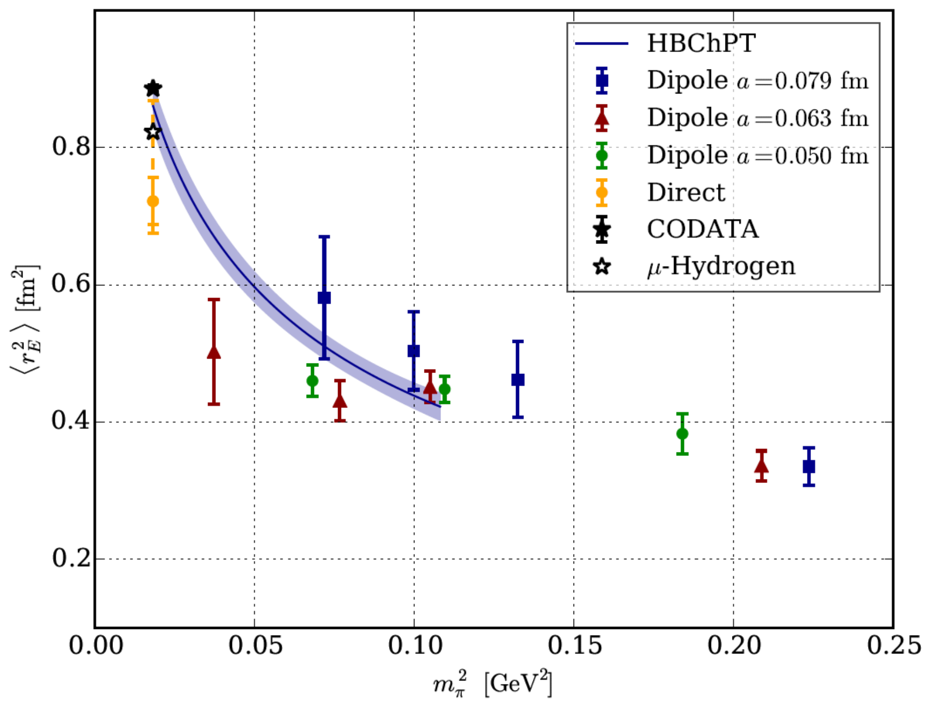
t dependence of G_E





Chiral behavior of $\langle r_E \rangle$

1504.04628



This work