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Combined analysis of πN→πN and πN→ππN in chiral effective field theory at one-loop level

- Formal Aspects
- Combined Fit
- Predictions

Motivation and Methodology

Why?

Aim	Theoretical description of $\pi N \to \pi N$ and $\pi N \to \pi\pi N$ above threshold
Problem I	QCD is non-perturbative for low energies
Solution I	Effective Field Theory ⇒ Chiral Perturbation Theory
Problem II	Resonances play an important role
Solution II	Inclusion of the most dominant resonance $\Delta(1232)$ as an explicit degree of freedom

How To

1. Pick Lagrangian
2. Derive Feynman rules
3. Draw all graphs up to specified order
4. Calculate amplitudes in specified decomposition
5. Calculate T-matrix and matrix element squared
6. Calculate observables like cross sections and phase shifts

2 Chiral Approaches

χPT

- EFT of Standard Model
- Relies upon chiral symmetry of QCD
- DOF are mesons and baryons instead of quarks
- Breakdown scale of theory: $\Lambda_\chi \approx 1\ \mathrm{GeV}$

HBχPT

- Non-relativistic limit of χPT
- Inclusion of $1/m_N$ expansion into power counting
- Original motivation: Allows calculations beyond tree level

Formal Aspects

χPT & HBχPT

Effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N}$$

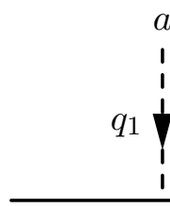
$$Q = \left\{ \frac{q}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi} \right\}$$

Examples

χPT

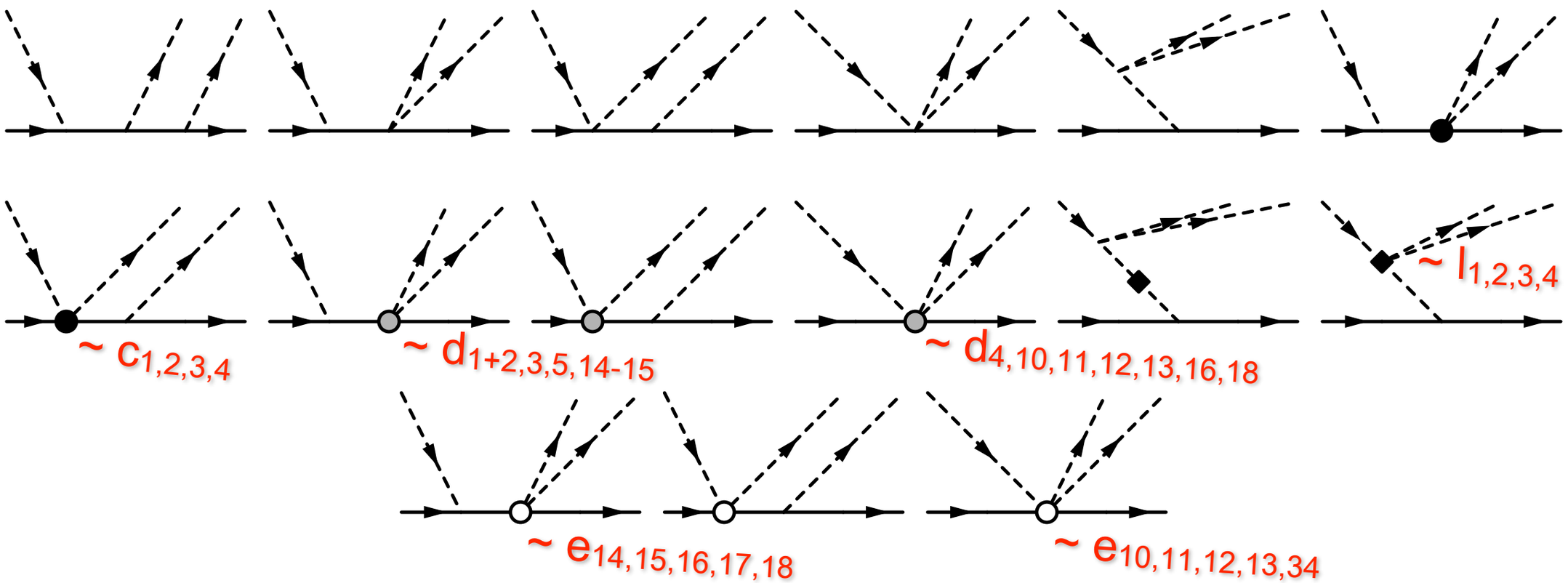
$$-\frac{g_A}{2F_\pi} \not{q}_1 \gamma_5 \tau^a$$

HBχPT



$$-\frac{g_A}{F_\pi} S \cdot q_1 \tau^a$$

Tree Graphs

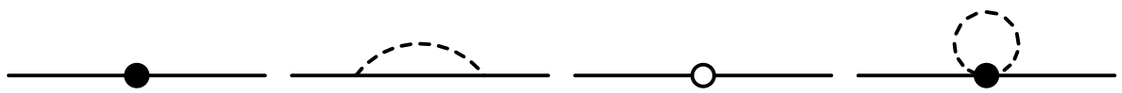


Renormalization I

Meson Sector

$$M^2 = M_\pi^2 + \delta M^{(4)}$$
$$Z_\pi = 1 + \delta Z_\pi^{(4)}$$
$$F = F_\pi + \delta F_\pi^{(4)}$$

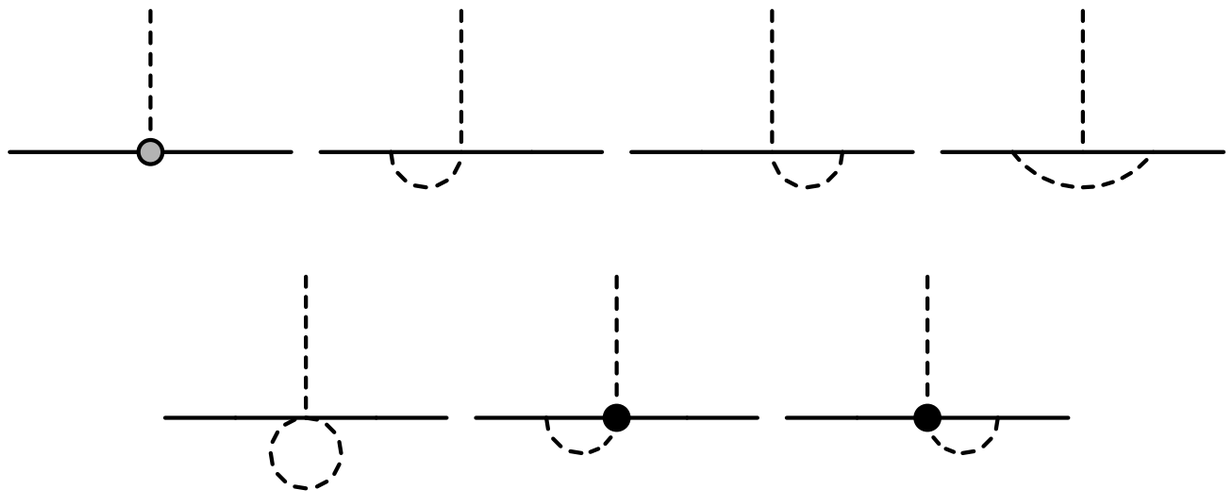
Nucleon Self Energy



$$m = m_N + \delta m^{(2)} + \delta m^{(3)} + \delta m^{(4)}$$

$$Z_N = 1 + \delta Z_N^{(3)} + \delta Z_N^{(4)}$$

Axial-coupling constant



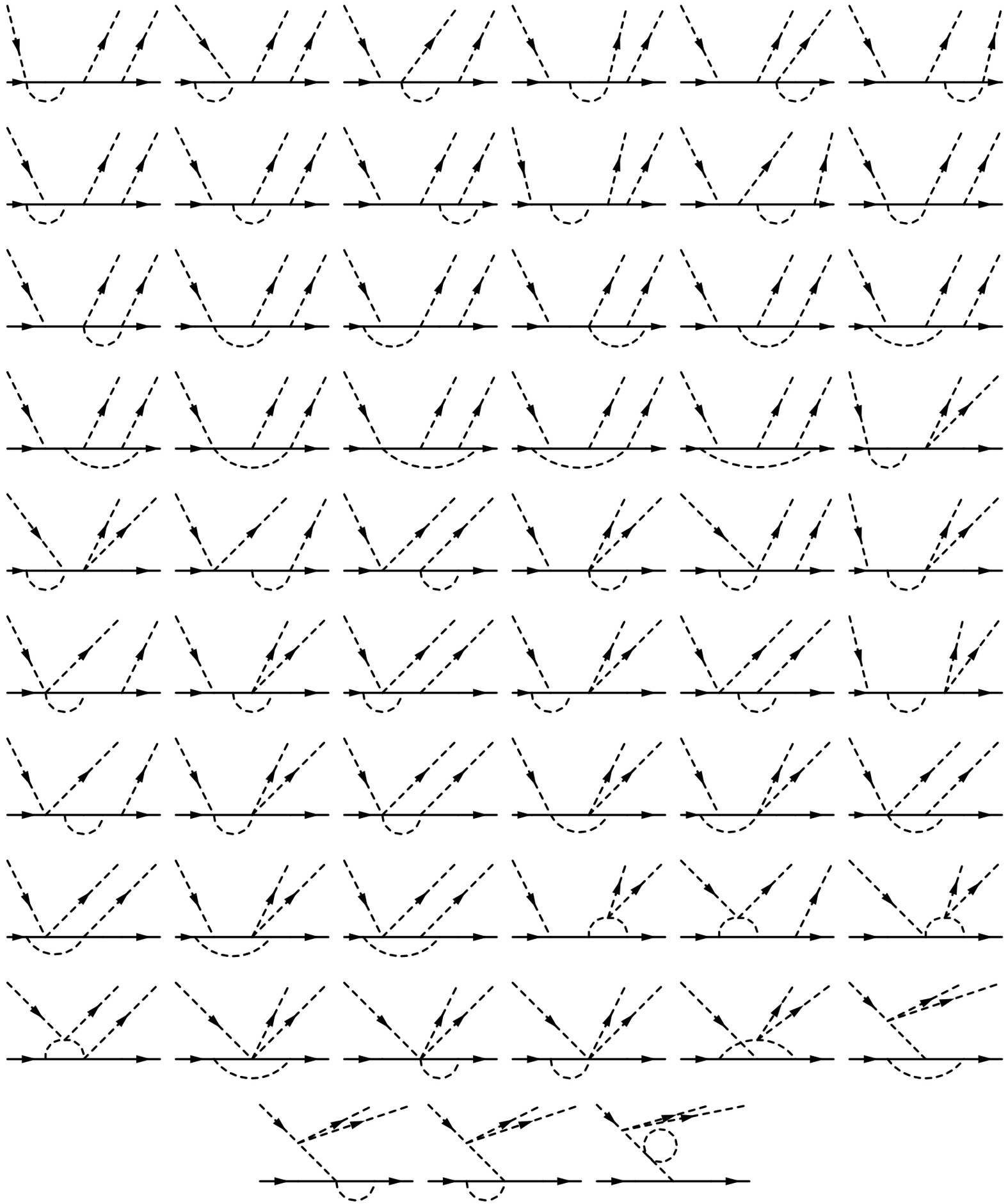
$$g = g_A + \delta g^{(3)} + \delta g^{(4)}$$

GTR $$\frac{g_{\pi NN} F_\pi}{m_N} = g_A - 2M_\pi^2 d_{18} + \mathcal{O}(Q^5)$$

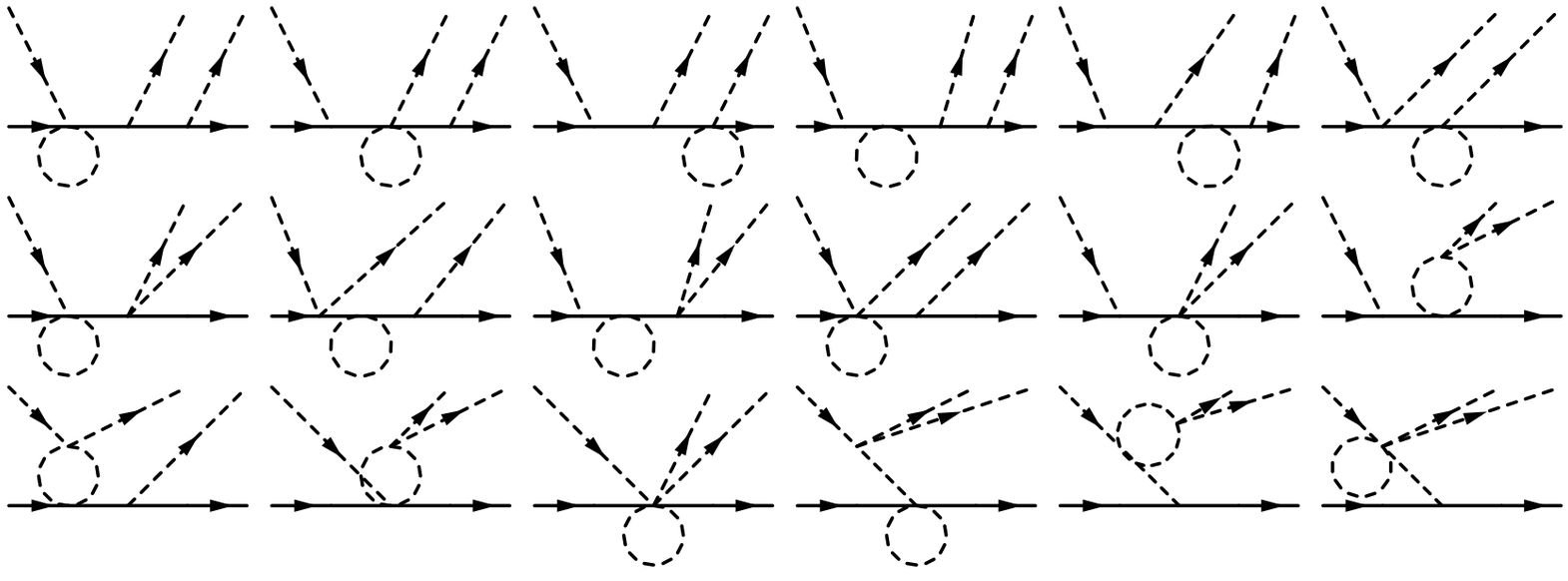
Linear Combinations

$$c_1 \to c_1 + 2M_\pi^2(e_{22} - 4e_{38})$$
$$c_2 \to c_2 - 8M_\pi^2(e_{20} + e_{35})$$
$$c_3 \to c_3 - 4M_\pi^2(2e_{19} - e_{22} - e_{36})$$
$$c_4 \to c_4 - 4M_\pi^2(2e_{21} - e_{37})$$

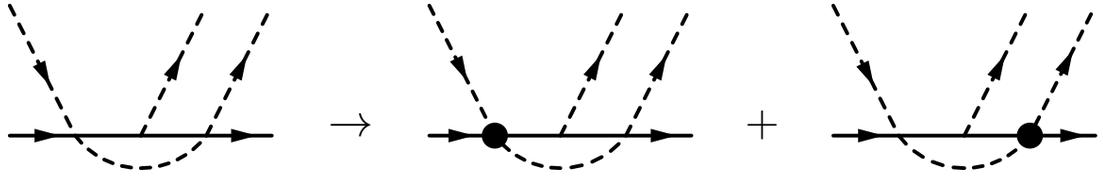
Loop Graphs
Self-Energy type



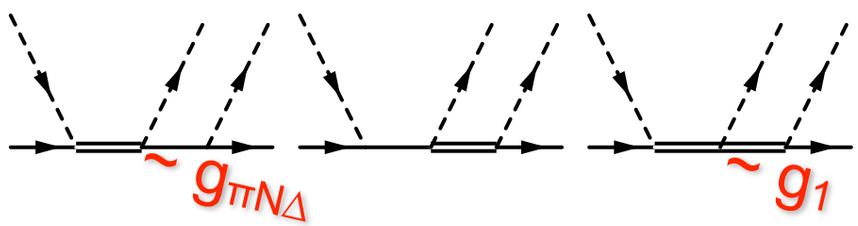
Loop graphs - Tadpole type



Transition from LO loops to NLO loops

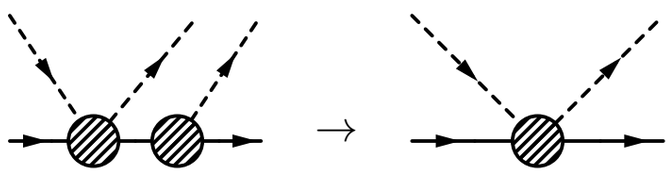


LO Δ graphs



phenomenological

Transition from πN→ππN graphs to πN→πN graphs



Renormalization II

Meson Sector

$$l_i = \frac{\beta_{l_i}}{32\pi^2}\bar{l}_i + \beta_{l_i}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

$$\bar{\lambda} = \frac{1}{16\pi^2}\left(\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi)\right)$$

HB approach

$$d_i = \bar{d}_i + \frac{\beta_{d_i}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

$$e_i = \bar{e}_i + \frac{\beta_{e_i}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

Covariant “Modified“ EOMS

$$c_i = \bar{c}_i + \delta c_i^{(3)} + \delta c_i^{(4)}$$

$$d_i = \bar{d}_i + \delta d_i^{(3)} + \delta d_i^{(4)}$$

$$e_i = \bar{e}_i + \delta e_i^{(4)}$$

$$x \in \{c, d, e\}$$

$$\delta x_i^{(n)} = \bar{x}_{i,f}^{(n)} + \frac{\beta_{x_i,B}^{(n)}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{m_N^2}{\mu^2}\right)\right) + \frac{\beta_{x_i,M}^{(n)}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

Combined Fit

Phase Shifts - πN→πN

$$T^{ba} = \chi^\dagger_{N'} \left(\delta^{ab} T^+ + \mathrm{i} \epsilon^{bac} \tau_c T^- \right) \chi_N$$

χPT

$$T^\pm = \bar{u}^{(s')} \left(A^\pm + \not{q} B^\pm \right) u^{(s)}$$

$$f^I_{l\pm}(s) = \frac{1}{16\pi\sqrt{s}} \Big((E + m_N) \left(A^I_l(s) + (\sqrt{s} - m_N) B^I_l(s) \right) + (E - m_N) \left(-A^I_{l\pm}(s) + (\sqrt{s} + m_N) B^I_{l\pm} \right) \Big)$$

$$X^I_l(s) = \int_{-1}^{1} \mathrm{d}z \, X^I(s,t) P_l(z)$$

$X \in \{A, B\}$

HBχPT

$$T^\pm = \bar{u}^{(s')}_v \left(g^\pm + 2\mathrm{i}\, S \cdot q \times q' h^\pm \right) u^{(s)}_v$$

$$f^I_{l\pm}(s) = \frac{E + m_N}{16\pi\sqrt{s}} \int_{-1}^{1} \mathrm{d}z \, \left(g^I P_l(z) + \boldsymbol{q}^2 h^I (P_{l\pm}(z) - z P_l(z)) \right)$$

Isospin basis

$$X^{I=1/2} = X^+ + 2X^- \,, \quad X^{I=3/2} = X^+ - X^-$$

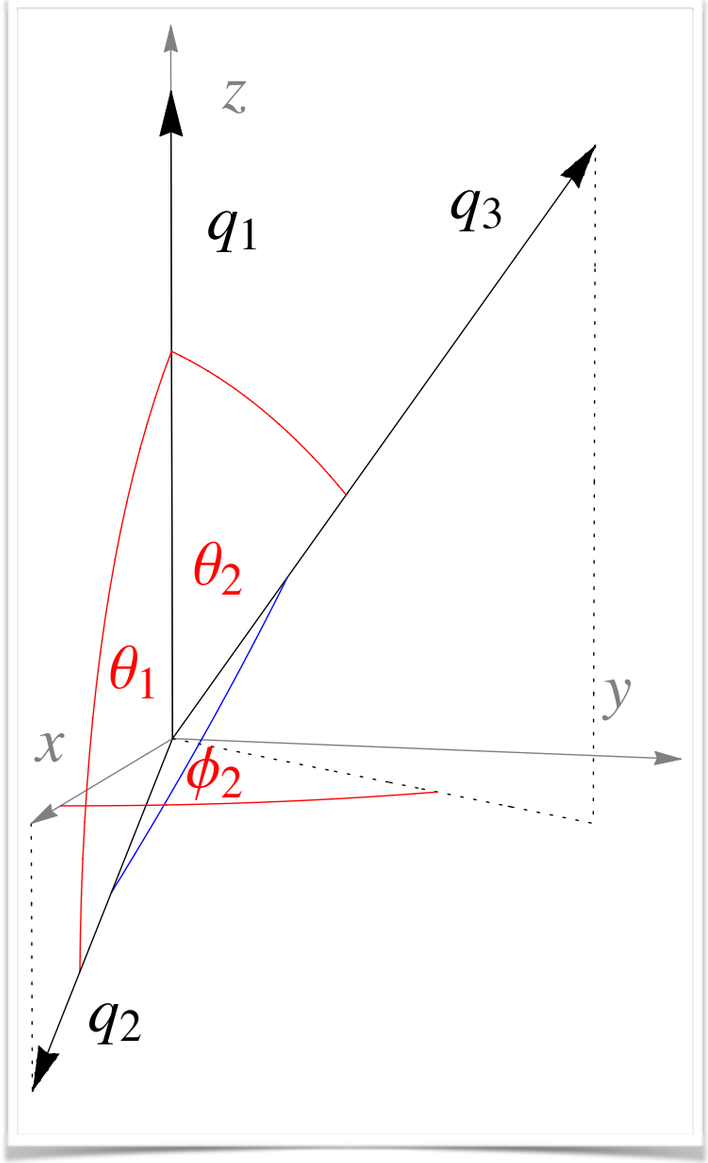
Unitarization prescription

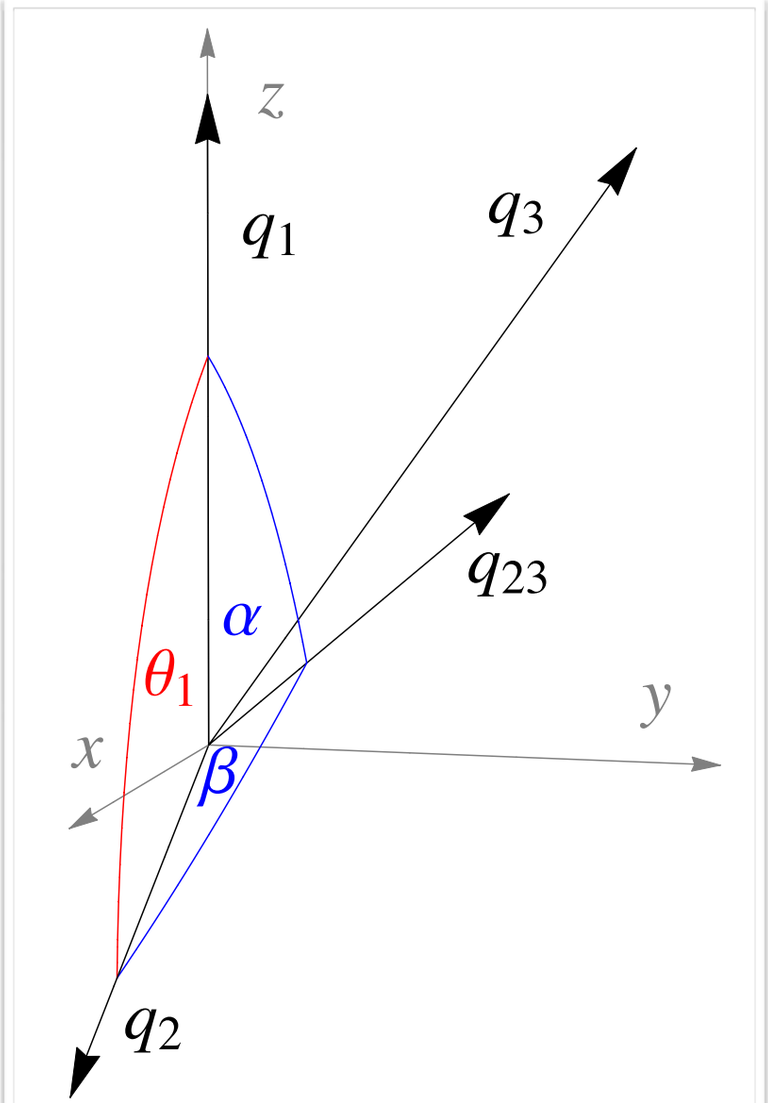
$$\delta^I_{l\pm}(s) = \arctan(|\boldsymbol{q}| \Re\, f^I_{l\pm}(s))$$

Observables - πN→ππN

Unpolarized Observables

- σ_{tot}
- $\frac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\frac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\frac{\mathrm{d}\sigma}{dM^2_{\pi\pi}}$, $\frac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M^2_{\pi\pi}}$ and $\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$





Observables - πN→ππN

Unpolarized Observables

- σ_{tot}
- $\frac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\frac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\frac{\mathrm{d}\sigma}{dM_{\pi\pi}^2}$, $\frac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M_{\pi\pi}^2}$ and $\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$

Unpolarized Matrix Element Squared

$$|\mathcal{M}|^2 = \frac{1}{2}\sum_{s,s'} T^{\dagger}_{ss'} T_{ss'}$$

χPT

$$T^{abc}_{ss'} = \mathrm{i}\bar{u}^{(s')}\gamma_5\left(F_1^{abc} + (\not{q}_2 + \not{q}_3)F_2^{abc} + (\not{q}_2 - \not{q}_3)F_3^{abc} + (\not{q}_2\not{q}_3 - \not{q}_3\not{q}_2)F_4^{abc}\right)u^{(s)}$$

HBχPT

$$T^{abc}_{ss'} = \bar{u}_v^{(s')}\left(S\cdot q_1\, A^{abc} + S\cdot q_2\, B^{abc} + S\cdot q_3\, C^{abc} + \mathrm{i}\epsilon_{\mu\nu\alpha\beta}\, q_1^{\mu} q_2^{\nu} q_3^{\alpha} v^{\beta}\, D^{abc}\right)u_v^{(s)}$$

$$X^{abc} = \chi^{\dagger}_{N'}\left(\tau^a\delta^{bc}X_1 + \tau^b\delta^{ac}X_2 + \tau^c\delta^{ab}X_3 + \mathrm{i}\epsilon^{abc}X_4\right)\chi_N$$

Physically Accessible Channels

I. $\pi^- p \to \pi^0\pi^0 n$

II. $\pi^- p \to \pi^+\pi^- n$

III. $\pi^+ p \to \pi^+\pi^+ n$

IV. $\pi^+ p \to \pi^+\pi^0 p$

V. $\pi^- p \to \pi^0\pi^- p$

T-matrix Reduction

I. $X = \sqrt{2}X_1$

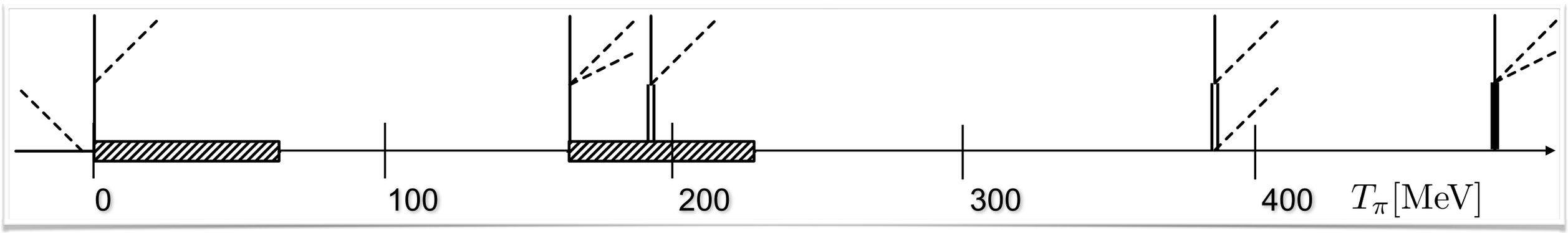
II. $X = \sqrt{2}(X_1 + X_2)$

III. $X = \sqrt{2}(X_2 + X_3)$

IV. $X = X_3 + X_4$

V. $X = X_2 + X_4$

Fitting Procedure



Combined Fit

$$\chi^2 = \chi^2_{\pi N} + \chi^2_{\pi\pi N} + \chi^2_c$$

$$\chi^2_{\pi N} = \sum_i \frac{(\delta_i^{exp} - \delta_i^{th})^2}{\Delta\delta_i^2}$$

$\Delta\delta_{S,P} = 5\%$ $\Delta\delta_D = 20\%$

$$\chi^2_{\pi\pi N} = \sum_i \frac{(\sigma_i^{exp} - \sigma_i^{th})^2}{\Delta\sigma_i^2}$$

$\Delta\sigma = \Delta\sigma^{exp}$

$$\chi^2_c = \sum_i \frac{(x_i - \bar{x}_i)^2}{R_i^2}$$

$R_{d_i} = 3$ $R_{e_i} = 5$

Naturalness Condition

$$|x_i| = \frac{|\tilde{x}_i|}{\Lambda^n} \sim \frac{Q^0}{\Lambda^n} < \frac{Q^{-1}}{\Lambda^n} = \frac{1}{q\Lambda^{n-1}}$$

$\Lambda \sim m_\rho \sim 770$ MeV

$\omega_{\pi N}^{CMS} < 190$ MeV

$\omega_{\pi\pi N}^{CMS} < 280$ MeV

$|c_i| \sim 1.0 < 3.0 < 5.5$ GeV^{-1}

$|d_i| \sim 1.5 < 4.0 < 7.0$ GeV^{-2}

$|e_i| \sim 2.0 < 5.5 < 9.0$ GeV^{-3}

Fits

Input

m_N	M_π	F_π	g_A	l_1	l_2	l_3	l_4
938.27	139.57	92.4	1.27	-0.4 ± 0.6	4.3 ± 0.1	2.9 ± 2.4	4.4 ± 0.2

LECs	HB KH			HB GW			Cov KH			Cov GW		
c_1	-1.27	$\pm$	0.08	-1.60	$\pm$	0.07	-1.12	$\pm$	0.08	-1.43	$\pm$	0.07
c_2	3.55	$\pm$	0.12	3.35	$\pm$	0.11	3.49	$\pm$	0.11	3.38	$\pm$	0.10
c_3	-6.28	$\pm$	0.08	-6.42	$\pm$	0.07	-5.94	$\pm$	0.08	-6.15	$\pm$	0.07
c_4	3.59	$\pm$	0.04	3.64	$\pm$	0.04	3.35	$\pm$	0.04	3.44	$\pm$	0.04
$d_1 + d_2$	3.65	$\pm$	0.15	3.32	$\pm$	0.13	3.05	$\pm$	0.12	2.98	$\pm$	0.11
d_3	-4.18	$\pm$	0.29	-3.14	$\pm$	0.28	-2.47	$\pm$	0.18	-1.98	$\pm$	0.17
d_4	-0.74	$\pm$	2.12	-0.89	$\pm$	2.11	4.41	$\pm$	1.70	4.40	$\pm$	1.70
d_5	0.69	$\pm$	0.18	0.01	$\pm$	0.17	0.01	$\pm$	0.15	-0.49	$\pm$	0.14
d_{10}	-1.09	$\pm$	1.83	-0.71	$\pm$	1.85	-1.86	$\pm$	1.91	-1.23	$\pm$	1.93
d_{11}	-2.35	$\pm$	1.98	-2.01	$\pm$	1.99	-2.27	$\pm$	2.07	-2.02	$\pm$	2.07
d_{12}	3.61	$\pm$	1.95	3.18	$\pm$	1.99	5.41	$\pm$	1.80	4.72	$\pm$	1.82
d_{13}	1.55	$\pm$	2.05	1.42	$\pm$	2.05	-0.73	$\pm$	2.02	-0.75	$\pm$	2.02
$d_{14} - d_{15}$	-6.88	$\pm$	0.27	-5.92	$\pm$	0.25	-5.02	$\pm$	0.21	-4.50	$\pm$	0.19
d_{16}	2.73	$\pm$	0.76	2.42	$\pm$	0.76	1.86	$\pm$	0.72	1.74	$\pm$	0.73
$\chi^2_{\pi N}$	171			132			242			98		
$\chi^2_{\pi\pi N}$	172			169			176			170		

$$|c_i| \sim 1.0 < 3.0 < 5.5 \qquad |d_i| \sim 1.5 < 4.0 < 7.0$$

Fits

Input

m_N	M_π	F_π	g_A	l_1	l_2	l_3	l_4	$g_{\pi N\Delta}$	g_1
938.27	139.57	92.4	1.27	-0.4 ± 0.6	4.3 ± 0.1	2.9 ± 2.4	4.4 ± 0.2	1.35	2.29

large N_c

$Q^3+\delta^1$

LECs	HB KH			HB GW			Cov KH			Cov GW		
c_1	-1.29	$\pm$	0.08	-1.61	$\pm$	0.07	-0.93	$\pm$	0.08	-1.26	$\pm$	0.07
c_2	1.50	$\pm$	0.12	1.34	$\pm$	0.11	1.44	$\pm$	0.11	1.39	$\pm$	0.10
c_3	-2.52	$\pm$	0.08	-2.70	$\pm$	0.08	-2.34	$\pm$	0.08	-2.65	$\pm$	0.08
c_4	1.83	$\pm$	0.04	1.90	$\pm$	0.04	1.62	$\pm$	0.04	1.74	$\pm$	0.04
$d_1 + d_2$	0.57	$\pm$	0.15	0.32	$\pm$	0.14	0.42	$\pm$	0.13	0.46	$\pm$	0.12
d_3	-1.66	$\pm$	0.29	-0.76	$\pm$	0.27	-1.16	$\pm$	0.18	-0.79	$\pm$	0.17
d_4	-1.11	$\pm$	2.37	-1.13	$\pm$	2.37	0.04	$\pm$	2.21	0.24	$\pm$	2.12
d_5	0.92	$\pm$	0.18	0.28	$\pm$	0.17	0.66	$\pm$	0.15	0.18	$\pm$	0.14
d_{10}	-0.58	$\pm$	1.93	-0.31	$\pm$	1.93	0.29	$\pm$	2.09	0.62	$\pm$	2.08
d_{11}	-3.10	$\pm$	2.00	-2.87	$\pm$	2.00	-0.20	$\pm$	2.06	-0.09	$\pm$	2.05
d_{12}	1.06	$\pm$	2.05	0.72	$\pm$	2.06	0.66	$\pm$	1.95	0.44	$\pm$	1.94
d_{13}	-2.49	$\pm$	2.05	-2.59	$\pm$	2.05	-2.53	$\pm$	1.99	-2.56	$\pm$	1.98
$d_{14} - d_{15}$	-1.66	$\pm$	0.28	-0.81	$\pm$	0.26	-0.89	$\pm$	0.22	-0.59	$\pm$	0.20
d_{16}	-0.51	$\pm$	0.70	-0.63	$\pm$	0.72	0.97	$\pm$	0.70	0.82	$\pm$	0.70
$\chi^2_{\pi N}$	123			205			126			154		
$\chi^2_{\pi\pi N}$	180			178			189			186		

$$|c_i| \sim 1.0 < 3.0 < 5.5 \qquad |d_i| \sim 1.5 < 4.0 < 7.0$$

LECs	HB						Cov					
	KH			GW			KH			GW		
c_1	-0.70	$\pm$	0.11	-0.89	$\pm$	0.11	-0.90	$\pm$	0.14	-1.18	$\pm$	0.13
c_2	3.20	$\pm$	0.33	4.19	$\pm$	0.32	3.52	$\pm$	0.32	3.73	$\pm$	0.31
c_3	-3.99	$\pm$	0.10	-4.90	$\pm$	0.08	-5.26	$\pm$	0.12	-6.00	$\pm$	0.11
c_4	2.91	$\pm$	0.09	3.42	$\pm$	0.07	3.48	$\pm$	0.08	3.83	$\pm$	0.06
$d_1 + d_2$	4.51	$\pm$	0.14	4.29	$\pm$	0.13	5.18	$\pm$	0.15	4.94	$\pm$	0.14
d_3	-4.03	$\pm$	0.21	-3.03	$\pm$	0.20	-5.65	$\pm$	0.28	-5.13	$\pm$	0.25
d_4	1.01	$\pm$	2.10	0.59	$\pm$	2.04	-2.26	$\pm$	1.88	-2.87	$\pm$	1.76
d_5	0.18	$\pm$	0.16	-0.57	$\pm$	0.15	0.69	$\pm$	0.18	0.24	$\pm$	0.16
d_{10}	-4.79	$\pm$	1.72	-2.80	$\pm$	1.76	-7.19	$\pm$	1.79	-5.65	$\pm$	1.81
d_{11}	-2.48	$\pm$	1.97	-2.60	$\pm$	1.98	-2.47	$\pm$	2.00	-1.34	$\pm$	1.99
d_{12}	6.12	$\pm$	1.72	6.10	$\pm$	1.74	8.82	$\pm$	1.78	7.28	$\pm$	1.76
d_{13}	-2.16	$\pm$	2.05	-3.41	$\pm$	2.05	-1.14	$\pm$	1.97	-1.32	$\pm$	1.92
$d_{14} - d_{15}$	-8.09	$\pm$	0.25	-6.99	$\pm$	0.23	-9.54	$\pm$	0.26	-8.77	$\pm$	0.24
d_{16}	4.98	$\pm$	0.70	5.66	$\pm$	0.73	-0.70	$\pm$	0.65	-0.89	$\pm$	0.63
e_{10}	-3.60	$\pm$	4.59	-4.12	$\pm$	4.56	-3.73	$\pm$	4.42	-4.91	$\pm$	4.33
e_{11}	0.36	$\pm$	4.77	0.38	$\pm$	4.76	2.58	$\pm$	4.10	3.30	$\pm$	3.92
e_{12}	2.09	$\pm$	3.71	1.12	$\pm$	3.81	1.80	$\pm$	3.52	2.27	$\pm$	3.51
e_{13}	-0.63	$\pm$	3.76	-0.91	$\pm$	3.81	-2.21	$\pm$	3.36	-3.20	$\pm$	3.27
e_{14}	1.41	$\pm$	0.11	1.42	$\pm$	0.10	0.32	$\pm$	0.12	1.09	$\pm$	0.11
e_{15}	-12.59	$\pm$	0.65	-6.33	$\pm$	0.56	-5.36	$\pm$	0.39	-3.37	$\pm$	0.36
e_{16}	5.92	$\pm$	1.31	-1.56	$\pm$	1.26	0.92	$\pm$	0.60	-1.48	$\pm$	0.55
e_{17}	-0.49	$\pm$	0.11	-0.44	$\pm$	0.11	0.47	$\pm$	0.09	0.02	$\pm$	0.09
e_{18}	4.97	$\pm$	0.49	1.93	$\pm$	0.39	1.15	$\pm$	0.28	0.08	$\pm$	0.22
e_{34}	0.18	$\pm$	4.83	0.31	$\pm$	4.84	0.86	$\pm$	4.77	1.22	$\pm$	4.75
$\chi^2_{\pi N}$	$181 + 163$			$119 + 171$			$147 + 7$			$79 + 55$		
$\chi^2_{\pi\pi N}$	233			232			234			238		

$$|c_i| \sim 1.0 < 3.0 < 5.5 \qquad |d_i| \sim 1.5 < 4.0 < 7.0 \qquad |e_i| \sim 2.0 < 5.5 < 9.0$$

LECs	HB						Cov					
	KH			GW			KH			GW		
c_1	-1.49	$\pm$	0.20	-1.62	$\pm$	0.24	-1.02	$\pm$	0.19	-1.63	$\pm$	0.20
c_2	0.10	$\pm$	0.59	1.22	$\pm$	0.74	1.54	$\pm$	0.40	1.15	$\pm$	0.39
c_3	-1.77	$\pm$	0.11	-2.63	$\pm$	0.10	-2.52	$\pm$	0.16	-3.45	$\pm$	0.15
c_4	1.79	$\pm$	0.09	2.25	$\pm$	0.07	2.08	$\pm$	0.08	2.41	$\pm$	0.06
$d_1 + d_2$	1.27	$\pm$	0.15	1.00	$\pm$	0.14	1.48	$\pm$	0.16	1.26	$\pm$	0.15
d_3	-1.95	$\pm$	0.22	-0.82	$\pm$	0.21	-2.46	$\pm$	0.31	-2.08	$\pm$	0.27
d_4	2.75	$\pm$	2.22	2.55	$\pm$	2.14	-0.15	$\pm$	2.19	-1.70	$\pm$	2.14
d_5	0.76	$\pm$	0.17	-0.05	$\pm$	0.16	0.83	$\pm$	0.19	0.43	$\pm$	0.17
d_{10}	-2.65	$\pm$	2.17	-0.59	$\pm$	2.39	-1.87	$\pm$	2.26	-1.21	$\pm$	2.23
d_{11}	-0.55	$\pm$	2.28	-0.57	$\pm$	2.29	-1.12	$\pm$	2.21	0.63	$\pm$	2.21
d_{12}	-2.06	$\pm$	2.12	-1.90	$\pm$	2.17	0.44	$\pm$	2.06	-0.78	$\pm$	2.05
d_{13}	-0.52	$\pm$	2.48	-2.52	$\pm$	2.80	-0.1	$\pm$	2.27	0.44	$\pm$	2.07
$d_{14} - d_{15}$	-2.49	$\pm$	0.27	-1.29	$\pm$	0.25	-3.11	$\pm$	0.28	-2.30	$\pm$	0.26
d_{16}	3.96	$\pm$	0.78	4.51	$\pm$	0.78	3.05	$\pm$	0.70	1.93	$\pm$	0.74
e_{10}	0.52	$\pm$	5.07	0.69	$\pm$	4.97	0.83	$\pm$	5.15	0.15	$\pm$	5.00
e_{11}	-1.24	$\pm$	5.22	-1.62	$\pm$	5.09	-0.08	$\pm$	4.73	1.24	$\pm$	4.41
e_{12}	-1.90	$\pm$	3.75	-3.17	$\pm$	3.91	-1.40	$\pm$	3.85	-0.61	$\pm$	3.79
e_{13}	-1.57	$\pm$	4.29	-2.95	$\pm$	4.41	-1.58	$\pm$	3.58	-1.76	$\pm$	3.42
e_{14}	0.63	$\pm$	0.11	0.75	$\pm$	0.10	0.37	$\pm$	0.14	1.28	$\pm$	0.13
e_{15}	-5.66	$\pm$	0.72	0.36	$\pm$	0.70	-1.55	$\pm$	0.47	1.14	$\pm$	0.47
e_{16}	6.81	$\pm$	2.21	-1.04	$\pm$	2.69	-0.55	$\pm$	0.80	-1.75	$\pm$	0.80
e_{17}	-0.42	$\pm$	0.11	-0.50	$\pm$	0.11	-0.11	$\pm$	0.09	-0.55	$\pm$	0.09
e_{18}	1.47	$\pm$	0.50	-1.22	$\pm$	0.40	-0.23	$\pm$	0.28	-1.32	$\pm$	0.22
e_{34}	0.69	$\pm$	4.92	1.28	$\pm$	4.95	0.72	$\pm$	4.82	0.89	$\pm$	4.78
$\chi^2_{\pi N}$	$131 + 69$			$79 + 73$			$128 + 6$			$69 + 47$		
$\chi^2_{\pi\pi N}$	176			178			180			181		

$Q^4+\delta^1$

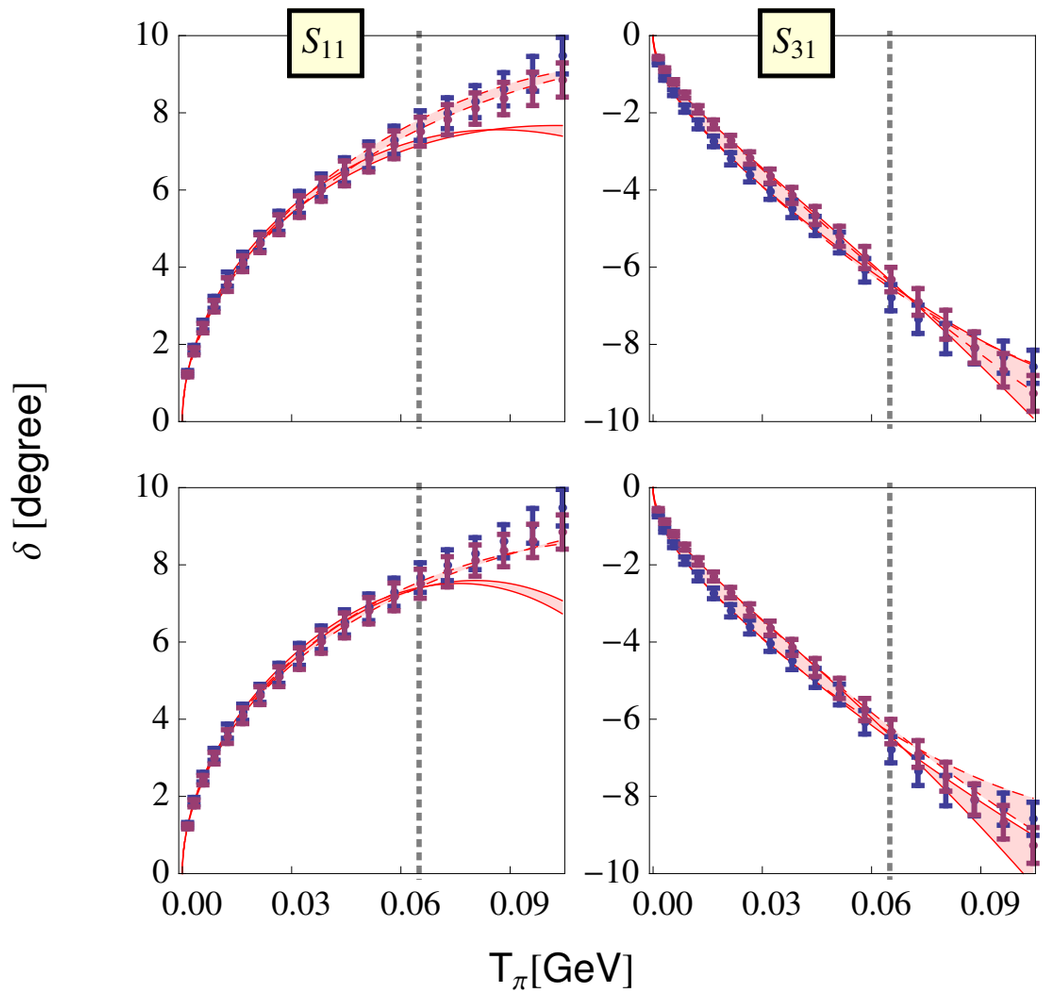
$|c_i| \sim 1.0 < 3.0 < 5.5$ $\quad$ $|d_i| \sim 1.5 < 4.0 < 7.0$ $\quad$ $|e_i| \sim 2.0 < 5.5 < 9.0$

Predictions

Partial Waves

χPT

HBχPT



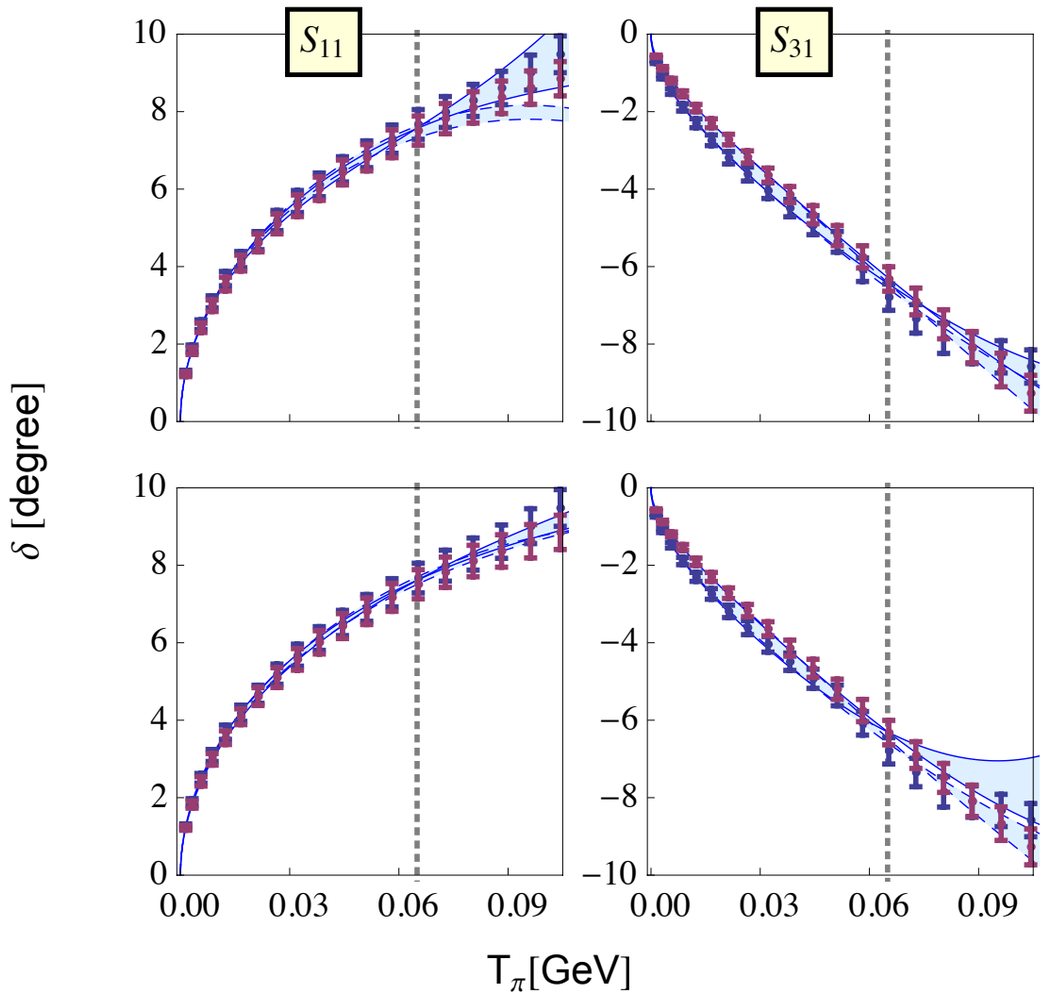
--- q^3

— q^4

Partial Waves

χPT

HBχPT



--- $q^3 + \delta^1$

— $q^4 + \delta^1$

P_{11} P_{31} P_{13} P_{33}

D_{13} D_{33} D_{15} D_{35}

δ [degree]

T_π[GeV]

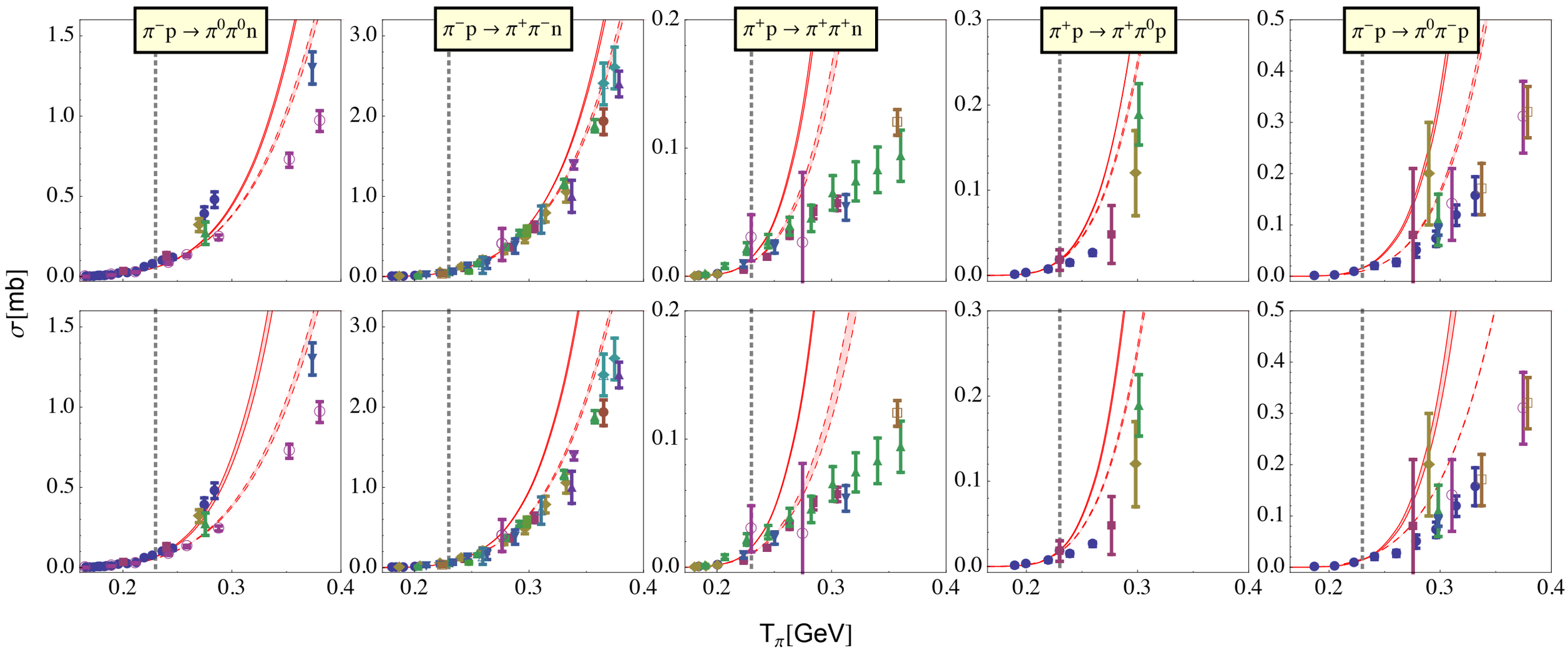
P_{11} P_{31} P_{13} P_{33}

D_{13} D_{33} D_{15} D_{35}

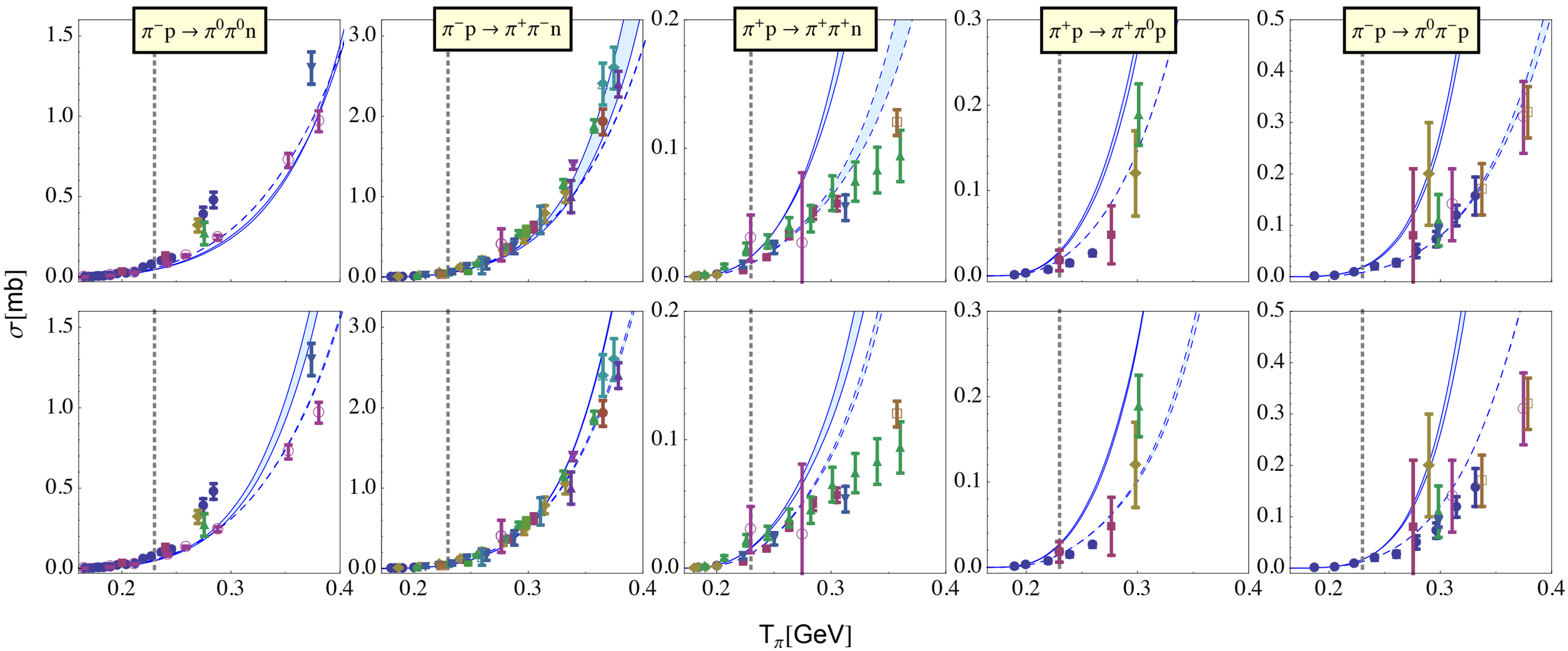
δ [degree]

T_π[GeV]

Cross Sections



Cross Sections



Summary

Good description of the phase shifts in $\pi N \to \pi N$

- Fits in q^3 & q^4 comparable $\Longrightarrow$ convergency
- χPT ~ HBχPT $\Longrightarrow$ $1/m_N$ contributions not that important
- higher energy predictions for P_{11} (R) and P_{33} (Δ) problematic

Fair description of the cross sections in $\pi N \to \pi\pi N$

- $q^3 > q^4 \Longrightarrow$ bad convergency (too large LECs from $\pi N \to \pi N$)
- χPT $\gtrsim$ HBχPT $\Longrightarrow$ $1/m_N$ contributions important
- role of Δ and R underestimated?

Future extensions of the combined fit

- deltaless: q^3 & q^4 + ΔNLO + RNLO
- deltafull: ε^3 + RNLO

Thanks !!