

On the structure of $Zc(3900)$ from lattice QCD

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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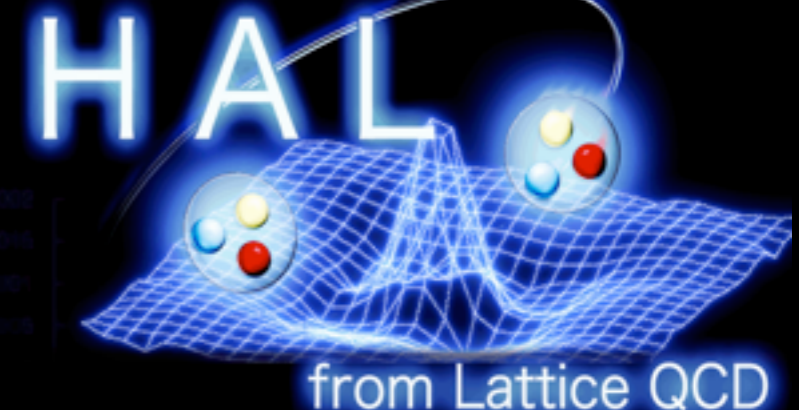
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Noriyoshi Ishii, Keiko Murano (RCNP, Osaka Univ.)

Hidekatsu Nemura, Kenji Sasaki (Univ. Tsukuba)

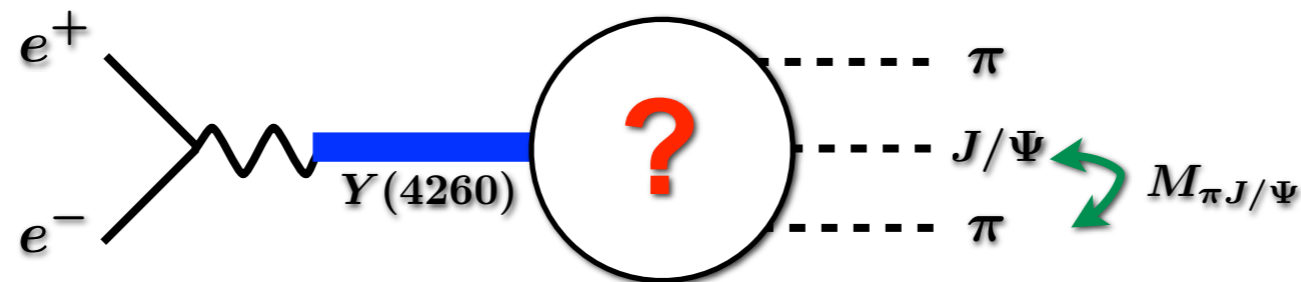
(Students) Daisuke Kawai, Takaya Miyamoto (YITP)

Hadrons to Atomic nuclei

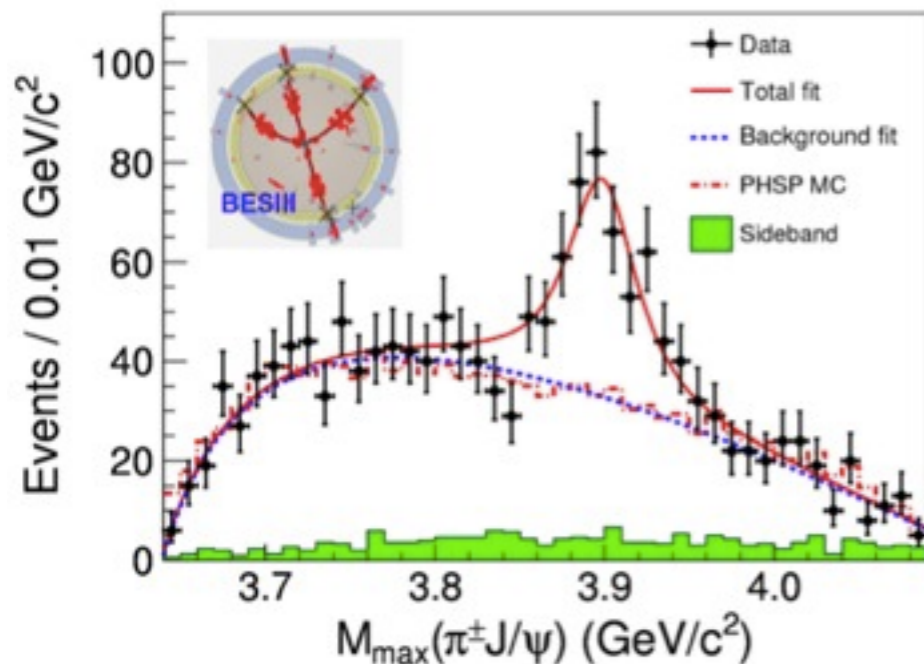


The 10th International Workshop on Physics of Excited Nucleon (NSTAR2015)
@Osaka Univ., May 25--28, 2015.

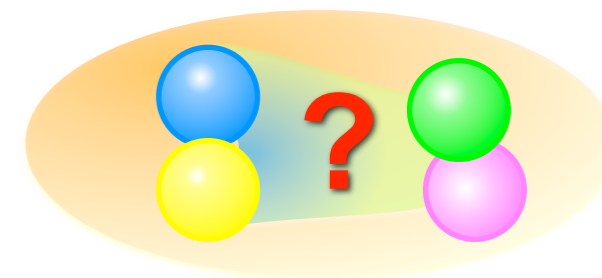
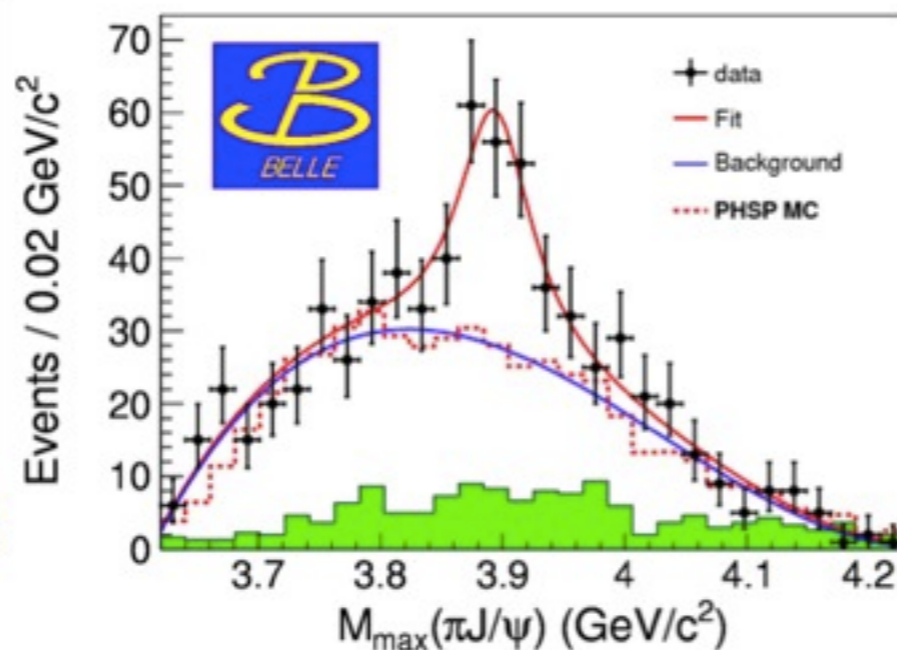
What is $Z_c(3900)$?



BESIII Coll., PRL110, 252001, (2013).

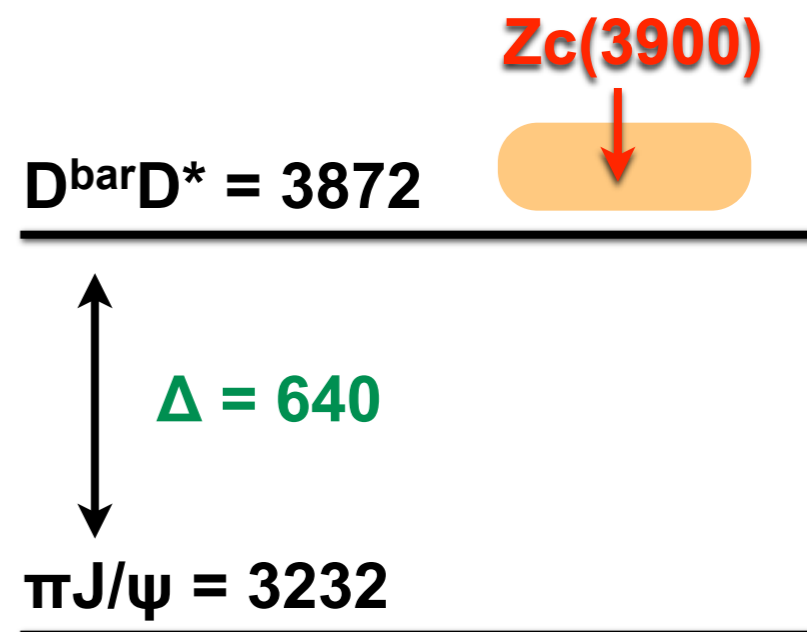


Belle Coll., PRL110, 252002, (2013).

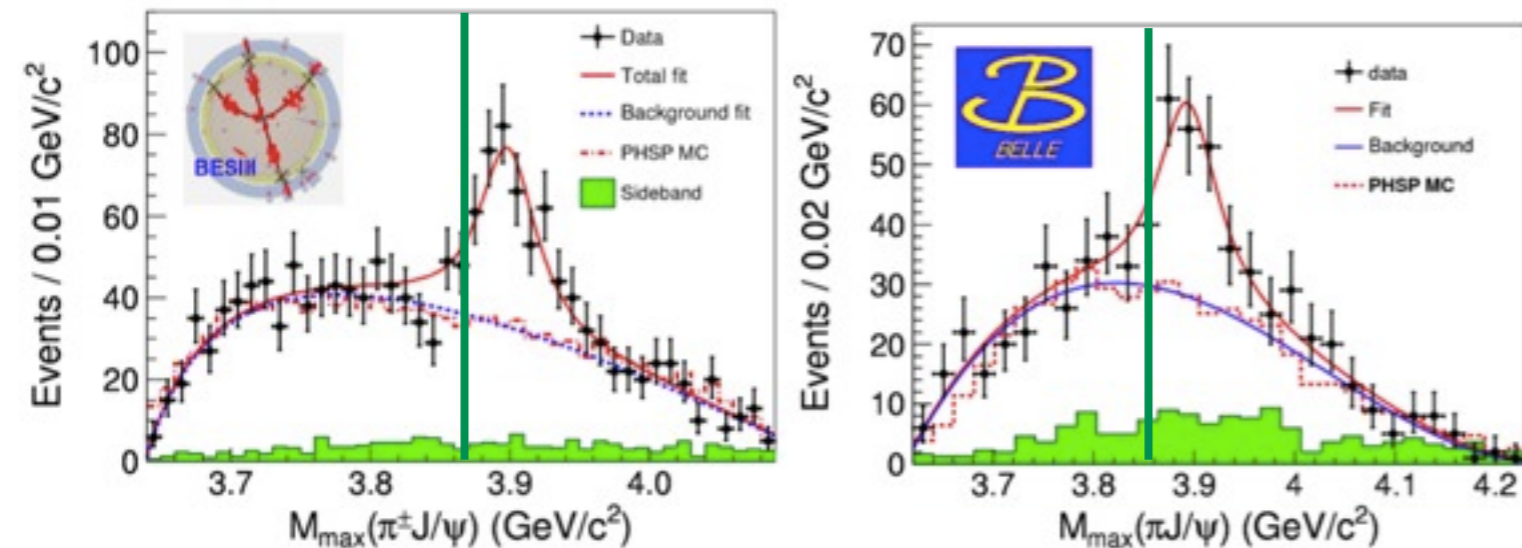


- $Z_c(3900)$ is observed in $\pi^\pm J/\psi$ invariant mass
- Charged tetraquarks? (4 quarks necessary)
- Isospin: $I^G=1^+$
- $M \sim 3900$, $\Gamma \sim 60$ MeV from BW line shape
- Peak confirmed by CLEO Coll.

Xiao et al., PLB727 (2013).



Status of $Z_c(3900)$



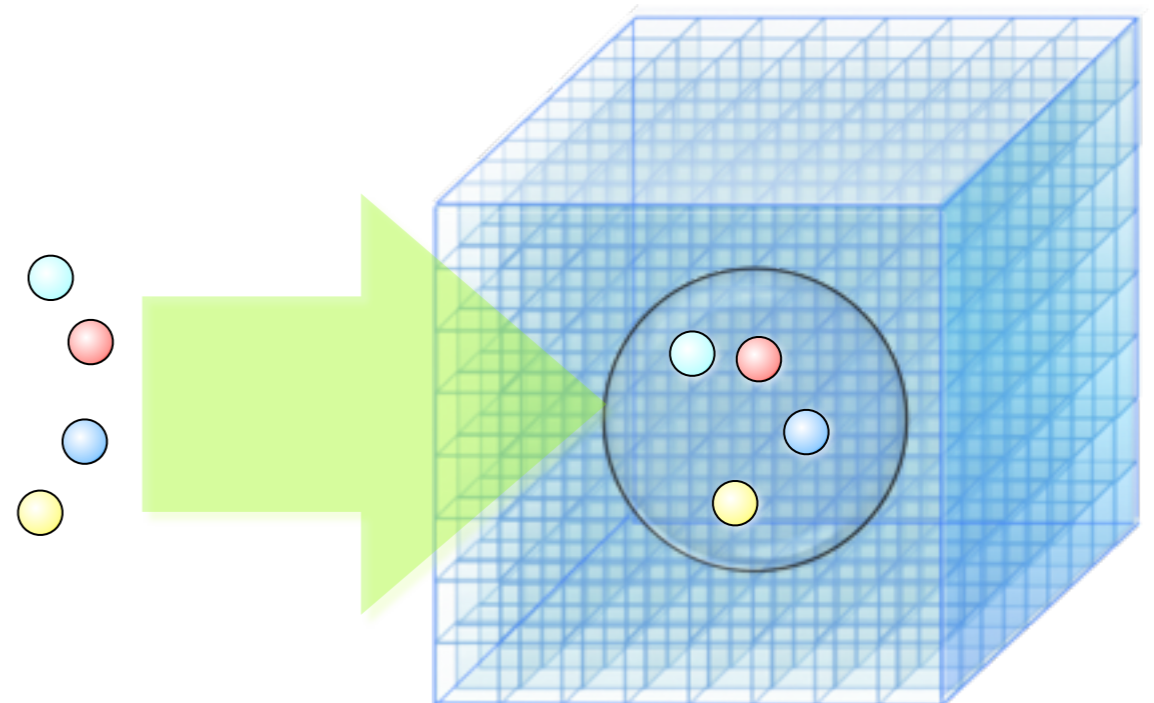
- observed in $\pi^\pm J/\psi$ & $D^{\text{bar}} D^*$ invariant mass spectra
- $J^P = 1^+$ seems most probable

[BESIII Coll., PRL112 \(2014\).](#)

★ structure of $Z_c(3900)$ from models → poor information on interactions

- Tetraquark? [Maiani et al. \(2013\).](#)
- $D^{\text{bar}} D^*$ molecule? [Nieves et al. \(2011\)](#)
+ many others
- $cc^{\text{bar}} + \pi$ cloud? [Voloshin \(2008\).](#)
- pole + $D^{\text{bar}} D^*$ cloud? [Guo et al. \(2015\)](#)
- cusp? [Chen et al. \(2013\), Swanson \(2015\).](#)

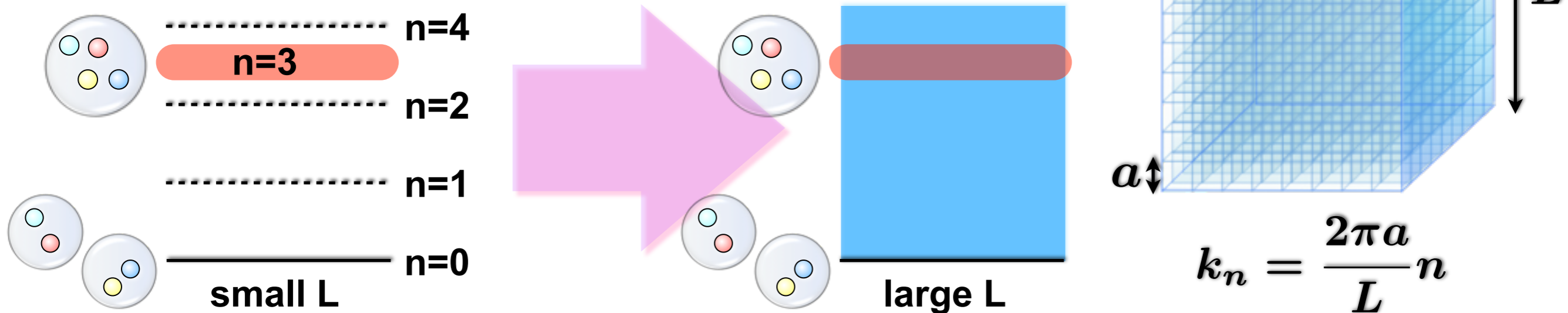
★ LQCD simulations for $Z_c(3900)$



How to find $Z_c(3900)$ on the lattice?

◆ Conventional approach: LQCD spectrum

⇒ identify all relevant $W_n(L)$ ($n=0,1,2,3,\dots$)



⇒ LQCD spectrum consistent with MM scattering up to $n=13$ ($L=2\text{fm}$)

[S. Prelovsek et al., PRD91, 014504 \(2015\).](#)

★ $Z_c(3900)$ can be resonance & coupled-channel system

• difficulties in finding broad resonances w/ large L

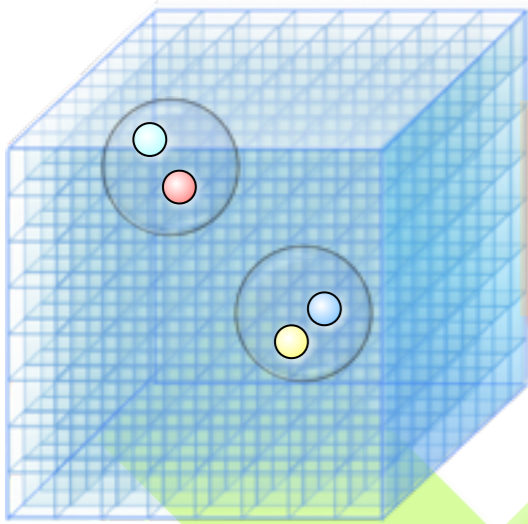
⇒ scatterings on the lattice w/ channel coupling to extract S-matrix

Lüscher's finite size formula

⇒ identify $W_{n=0,1,2,\dots}(L)$ & find degenerate $W_n(L)$ w/ different L

⇒ in practice, it is difficult to find degenerate $W_n(L)$ (QCD eigenenergy)

HAL QCD approach



- Lüscher's finite size formula
energy $W_n(L)$ --> phase shift, inelasticity

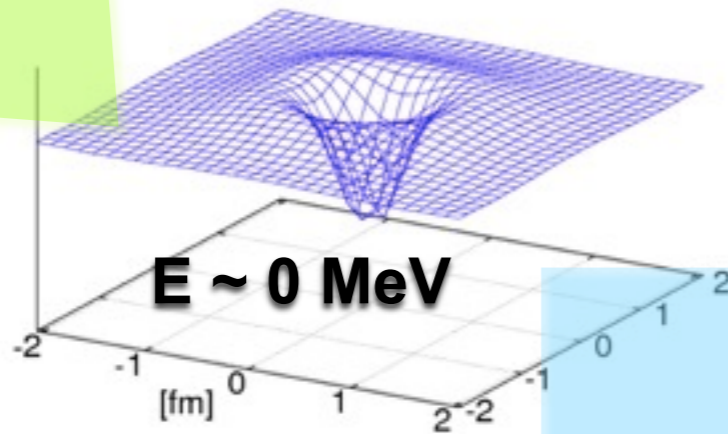
Lüscher, NPB354, 531 (1991).

- S-matrix

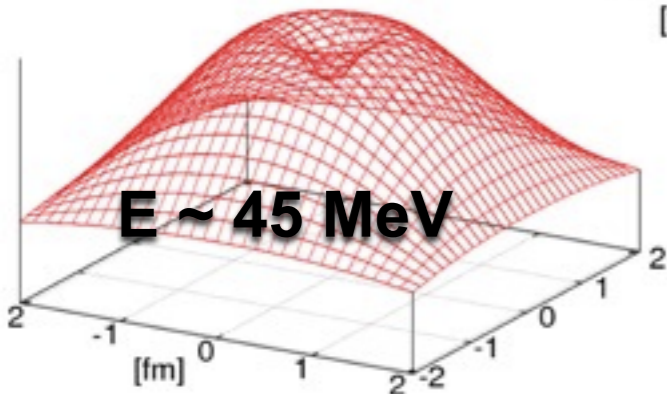
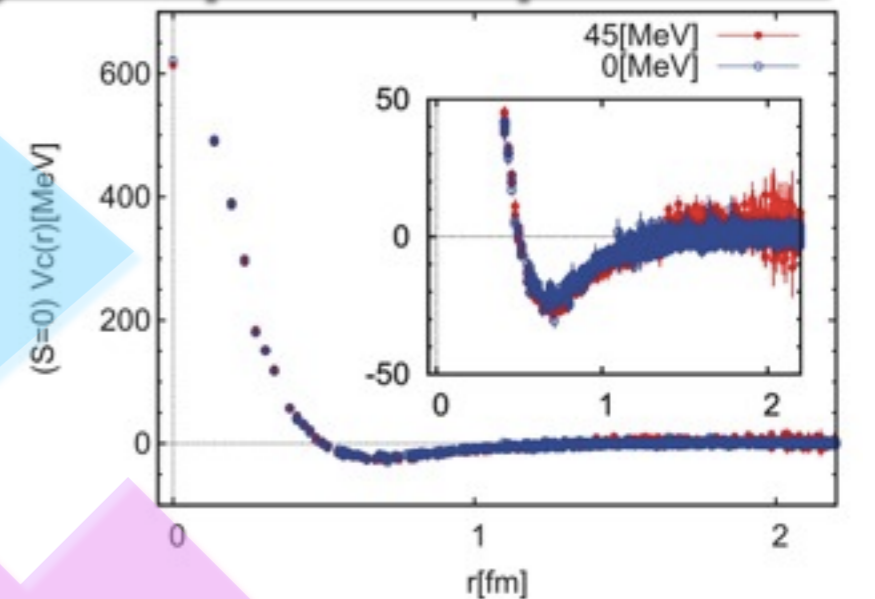
$$S(k) : \eta, \delta(k)$$

- **Guaranteed to be the same**
Kurth et al., JHEP 1312 (2013) 015.

- NBS wave function



- Energy-independent potential



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Aoki, Hatsuda, Ishii, PPTP123, 89 (2010).

✓ Advantages of HAL QCD method:

coupled-channel problem, resonance (pole position)

Contents

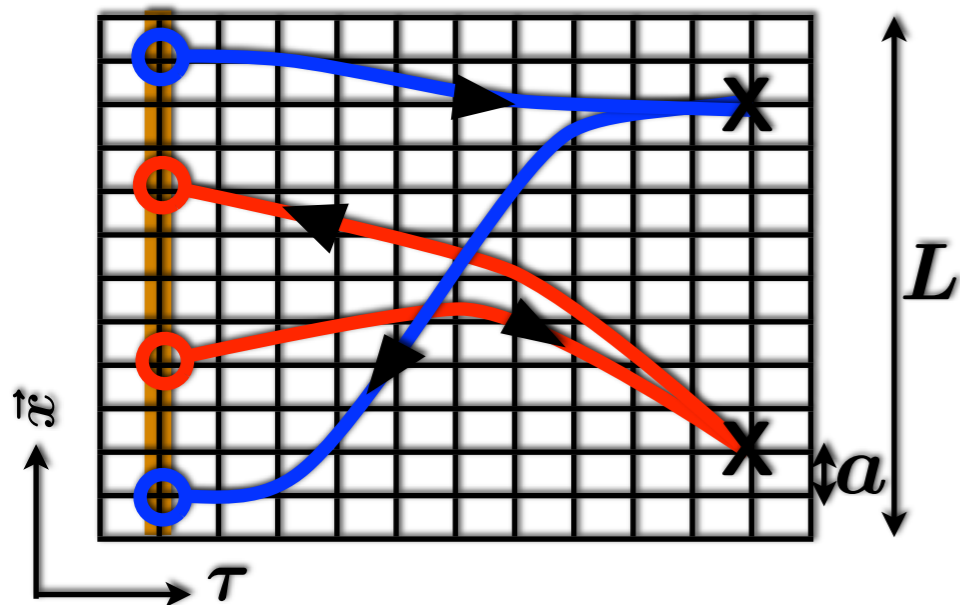
- Introduction
- **Brief introduction to HAL QCD method to define “potential”**
 - ➔ talk by K. Sasaki (Wed.)
- **Coupled-channel potential matrix for $Z_c(3900)$ in $I(J^P)=1(1^+)$**
- **Application of LQCD potential -- production reactions of $Z_c(3900)$ --**
- **Summary**

“Potentials” in LQCD

Hadron 4pt function & Nambu-Bethe-Salpeter (NBS) wave function

$$R^{ab}(\vec{r}, \tau) \equiv \sum_{\vec{x}} \langle 0 | \phi_1^a(\vec{x} + \vec{r}, \tau) \phi_2^a(\vec{x}, \tau) \mathcal{J}^{b\dagger}(\tau = 0) | 0 \rangle \frac{e^{(m_1^a + m_2^a)\tau}}{\sqrt{Z_1^a} \sqrt{Z_2^a}}$$

$$= \sum_n A_n^b \exp[-\Delta W_n \tau] \psi_n^a(\vec{r})$$



- Helmholtz eq. of NBS wave func.

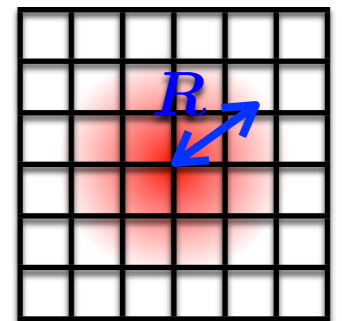
$$(\nabla^2 + (\vec{k}_n^a)^2) \psi_n^a(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

$$\psi_{W(\vec{k})}^{(l)}(r) \sim \frac{e^{i\delta_l(k)}}{kr} \sin(kr + \delta_l(k) - l\pi/2)$$

- NBS wave func. in QFT ~ wave func. in Q.M.

- Coupled-channel potential matrix (faithful to S-matrix)

$$\left(\nabla^2 + (\vec{k}_n^a)^2 \right) \psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r}, \vec{r}') \psi_n^b(\vec{r}')$$



Aoki et al. [HAL QCD Coll.], Proc. Jpn. Acad., Ser. B, 87 (2011); PTEP 2012, 01A105 (2012).

- Coupled-channel potentials are energy-independent (non-local in general)

HAL QCD method

- ✓ Define **energy-independent coupled-channel potentials** :

$$\psi_n^a(\vec{r}) = \langle 0 | \phi_1^a(\vec{r} + \vec{x}) \phi_2^a(\vec{x}) | W_n; I, J^P \rangle$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

$$\left(\nabla^2 + (\vec{k}_n^a)^2 \right) \psi_n^a(\vec{r}) = 2\mu \sum_c \int d\vec{r}' U^{ac}(\vec{r}, \vec{r}') \psi_n^c(\vec{r}')$$

- ✿ Since **energy-independent potential** can produce all scattering states, we do NOT have to identify energy eigenstate $|W_n, I, J^P\rangle$

- ✓ Extract **energy-independent potential** from time-dependent Schrödinger-type eq.

Ishii et al.(HAL QCD), PLB712, 437(2012).

$$\left[-\partial_\tau + \nabla^2 / 2\mu^a + \partial_\tau^2 / 8\mu^a + \mathcal{O}(\delta^2) \right] R^{ab}(\vec{r}, \tau) = \sum_c \int d\vec{r}' \Delta^{ac} U^{ac}(\vec{r}, \vec{r}') R^{cb}(\vec{r}', \tau)$$

$$\delta = \frac{m_1^a - m_2^a}{m_1^a + m_2^a} \quad \Delta^{ac} = \frac{e^{(m_1^a + m_2^a)\tau}}{e^{(m_1^c + m_2^c)\tau}}$$

- ✓ Velocity expansion:

$$U(\vec{r}, \vec{r}') = V(\vec{r}, \nabla) \delta(\vec{r} - \vec{r}')$$

(LO)

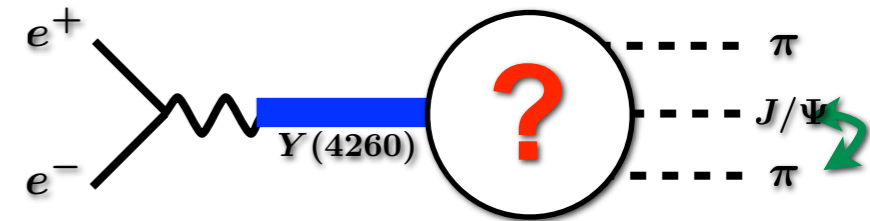
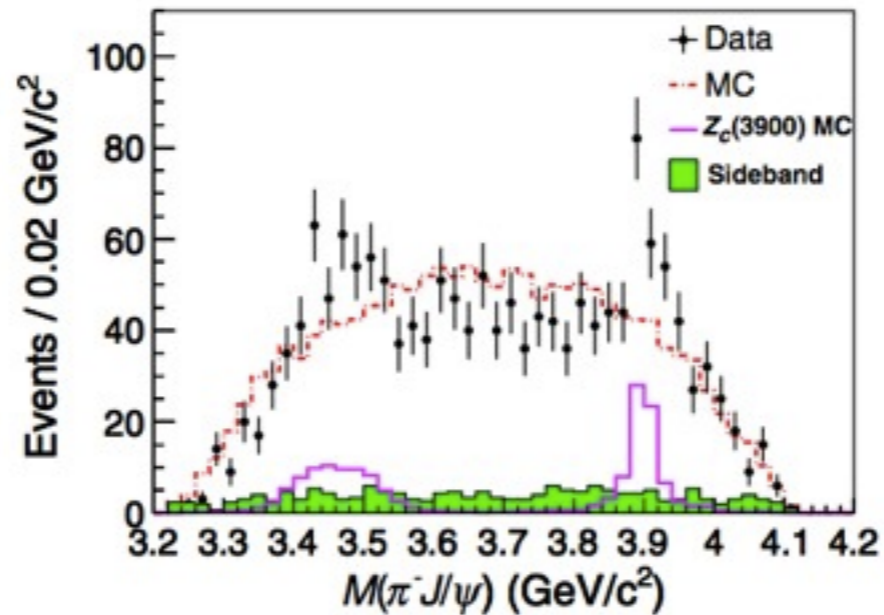
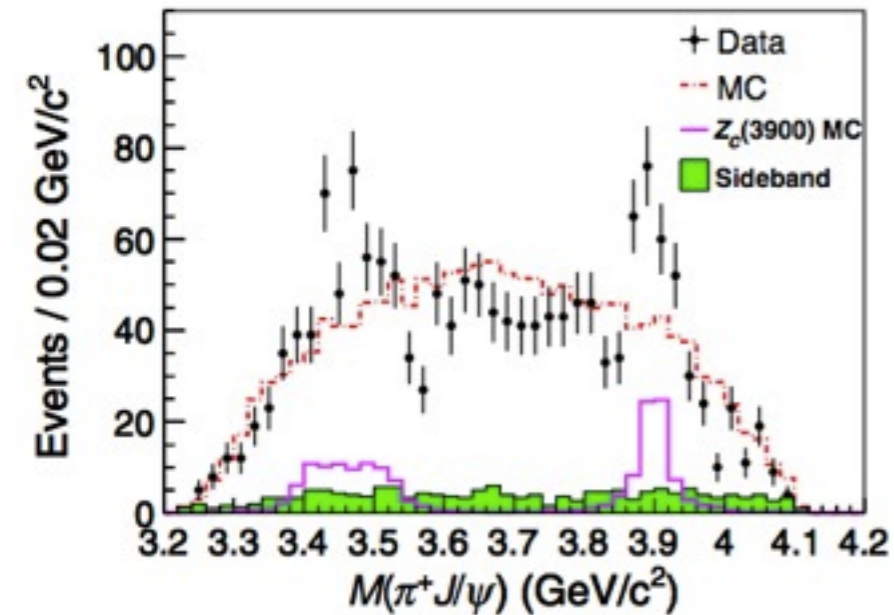
(NLO)

$$\longrightarrow V(\vec{r}, \nabla) = V_C(\vec{r}) + \vec{L} \cdot \vec{S} V_{LS}(\vec{r}) + \mathcal{O}(\nabla^2)$$

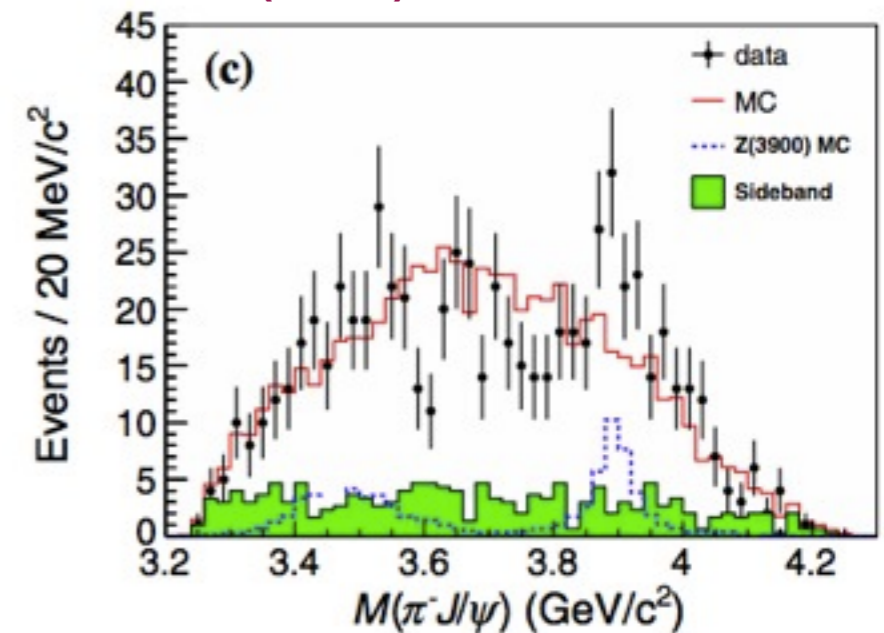
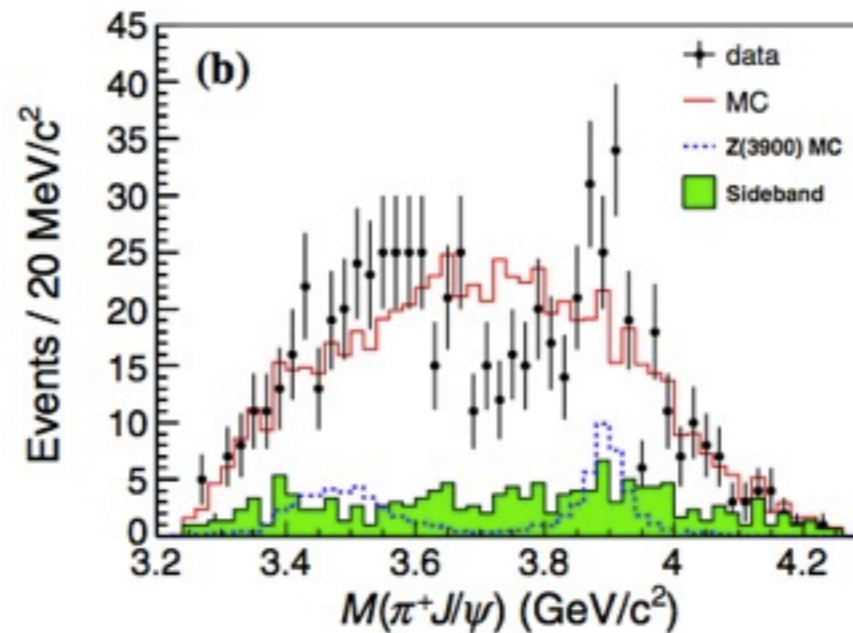
- ✓ Calculate observable: phase shift, binding energy, pole position, ...

Lattice QCD simulations for $Z_c(3900)$ in $I^G(J^P)=1^+(1^+)$

BESIII Coll., PRL110, 252001, (2013).



Belle Coll., PRL110, 252002, (2013).



Lattice QCD setup

$N_f=2+1$ full QCD configurations generated by PACS-CS Coll.

[PACS-CS Coll., S. Aoki et al., PRD79, 034503, \(2009\).](#)

- Iwasaki gauge & $O(a)$ -improved Wilson quark actions
- $a=0.0907(13)$ fm $\rightarrow L\sim 2.9$ fm ($32^3 \times 64$)

Light meson mass [conf.1, conf.2, conf.3] (MeV)

$M_\pi=701(1), 572(1), 411(1)$ [PDG:135 (π^0)]

$M_\rho=1097(4), 1000(5), 896(8)$ [PDG:770 (ρ^0)]

Tsukuba-type Relativistic Heavy Quark (RHQ) action for charm quark

[S. Aoki et al., PTP109, 383 \(2003\).](#)

➔ remove leading cutoff errors $O(m_c a)$, $O(\Lambda_{\text{QCD}} a)$, ...

- We are left with $O((a\Lambda_{\text{QCD}})^2)$ error (\sim a few %)
- We employ RHQ parameters tuned by Namekawa et al.

[Y. Namekawa et al., PRD84, 074505 \(2011\).](#)

Charmed meson mass [conf.1, conf.2, conf.3] (MeV)

$M_{\eta_c}=3024(1), 3005(1), 2988(1)$ [PDG:2981]

$M_{J/\psi}=3143(1), 3118(1), 3097(1)$ [PDG:3097]

$M_D=2000(1), 1947(1), 1903(1)$ [PDG:1865 (D^0)]

$M_{D^*}=2159(2), 2101(2), 2056(3)$ [PDG:2007 (D^{*0})]

Lattice QCD setup : thresholds

✦ Thresholds in $I^G J^P = 1^+ 1^+$ channel

Physical thresholds

$$\underline{D^{\text{bar}}D^* = 3872}$$

$$\underline{\pi\psi' = 3821}$$

$$\underline{\pi\pi\eta_c = 3256}$$

$$\underline{\pi J/\psi = 3232}$$

LQCD simulation

$$\underline{D^{\text{bar}}D^* = 4159, 4048, 3959}$$

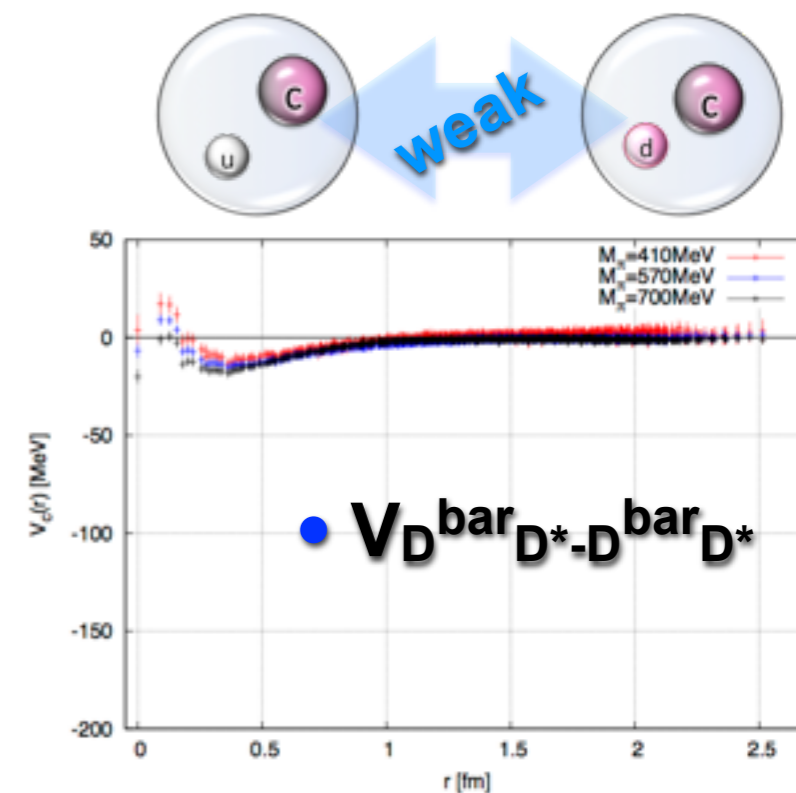
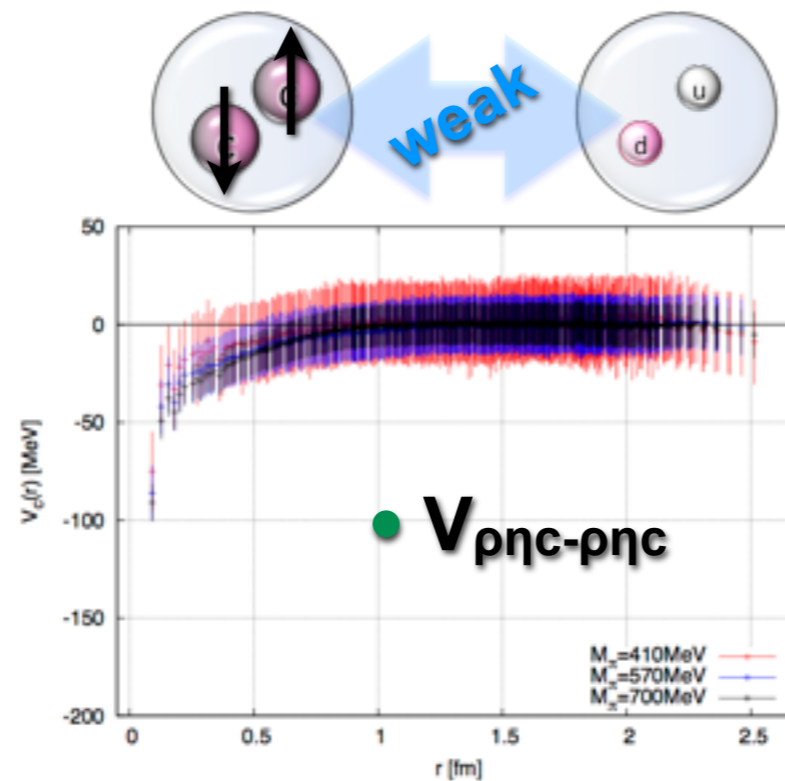
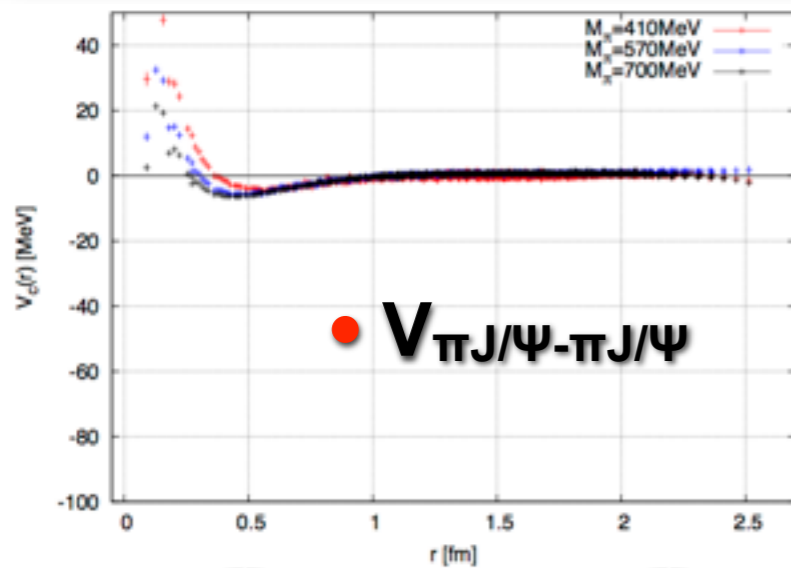
$$\underline{\rho\eta_c = 4121, 4005, 3884}$$

$$\underline{\pi J/\psi = 3844, 3688, 3508}$$

- $M_{\pi\psi'} > M_{D^{\text{bar}}D^*}$ due to heavy pion mass
- $\rho \rightarrow \pi\pi$ decay not allowed w/ $L \sim 3\text{fm}$

✦ **S-wave $\pi J/\psi - \rho\eta_c - D^{\text{bar}}D^*$ coupled-channel analysis**

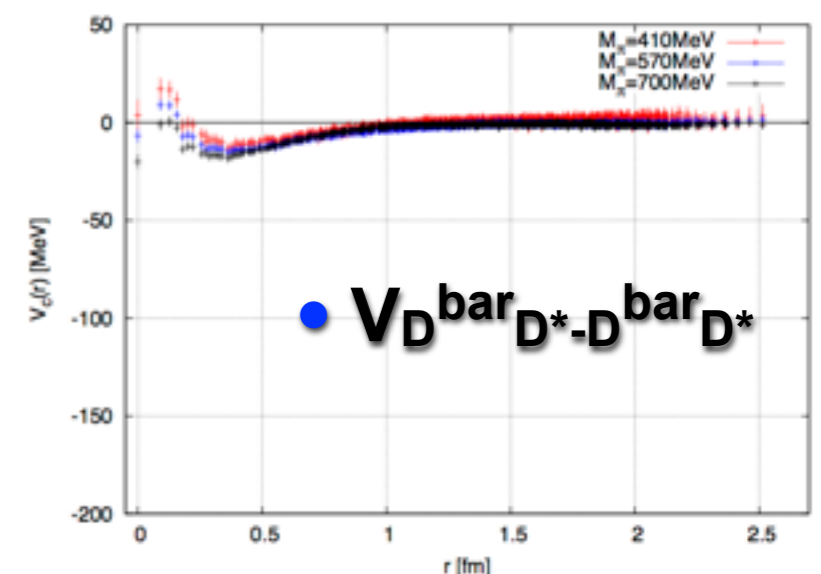
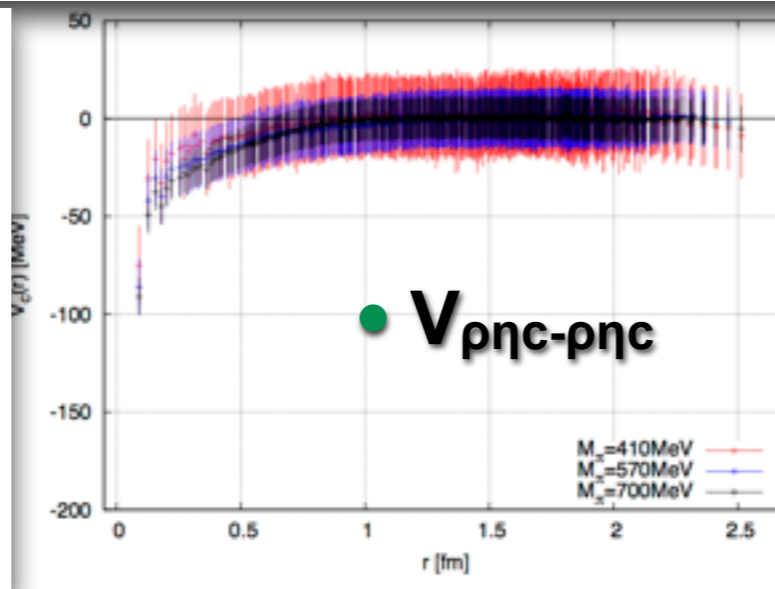
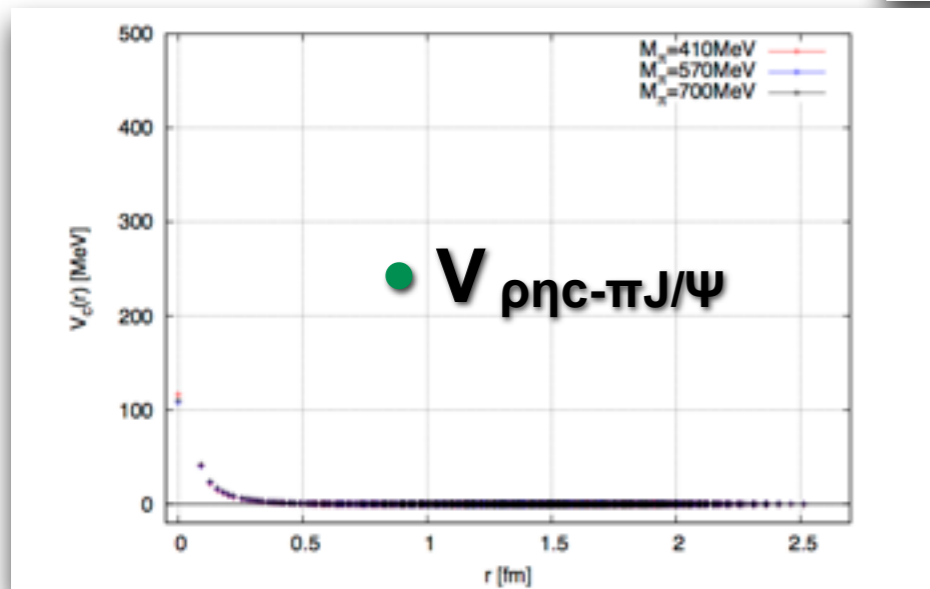
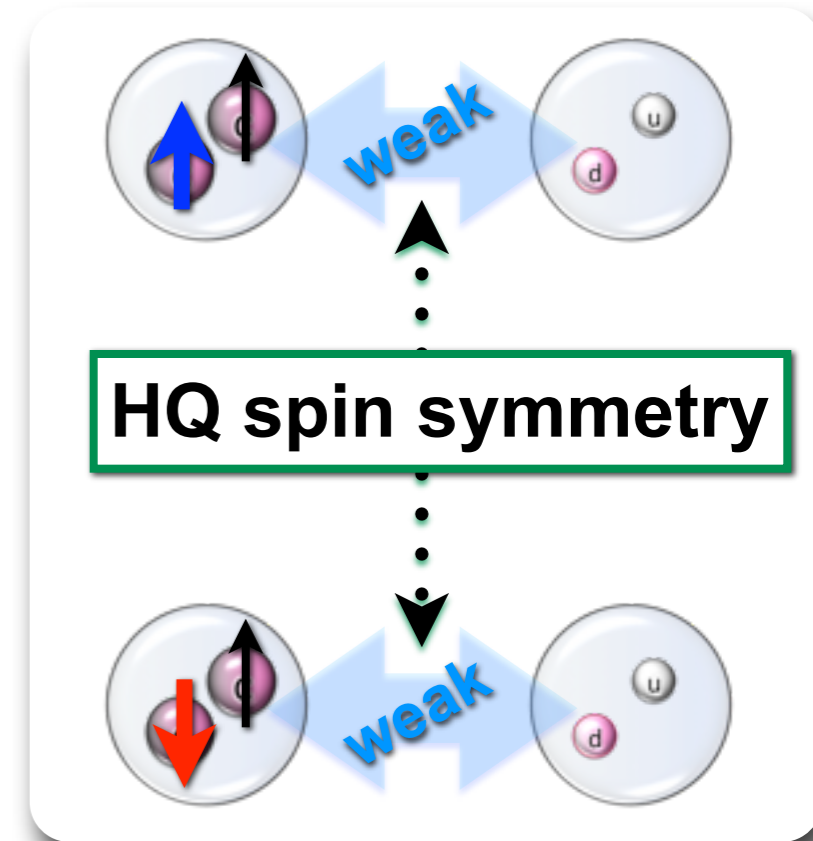
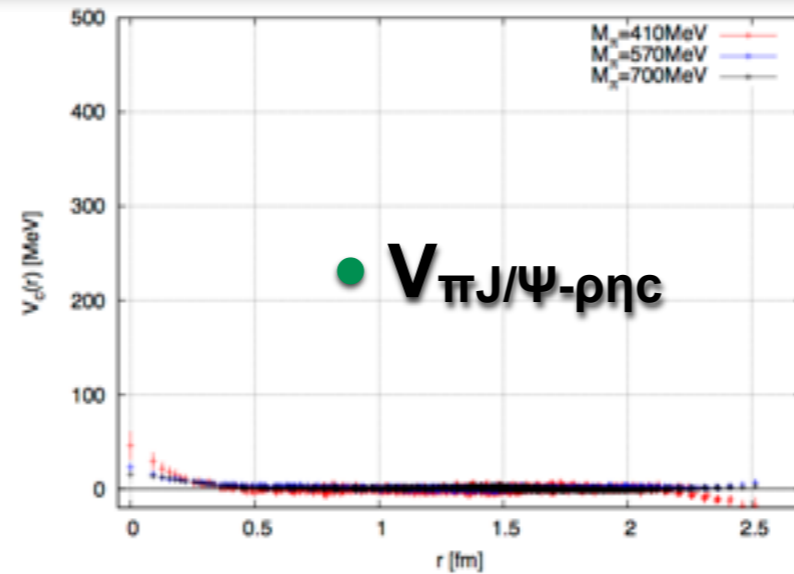
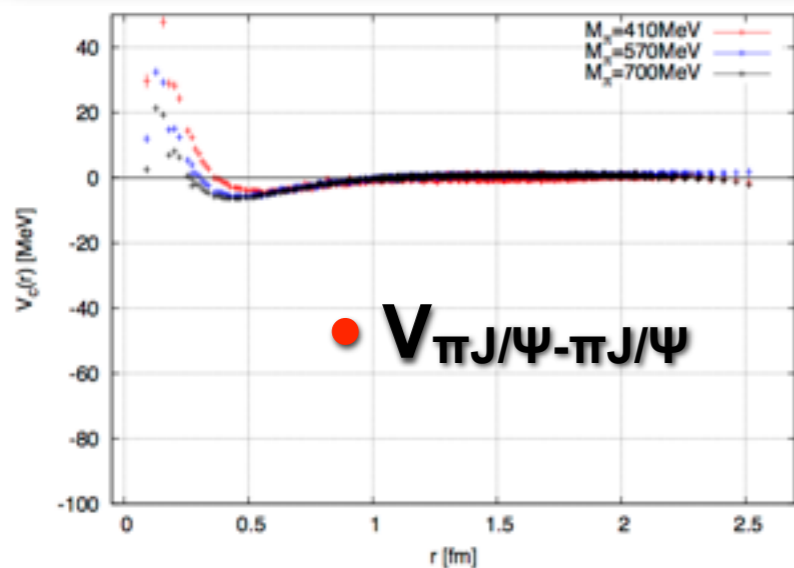
Potential matrix ($\pi J/\psi$ - $\rho \eta_c$ - $D^{\text{bar}} D^*$)



- All diagonal potentials are weak

➔ no bound $D^{\text{bar}} D^*$

Potential matrix ($\pi J/\psi$ - $\rho \eta_c$ - $D^{\text{bar}} D^*$)

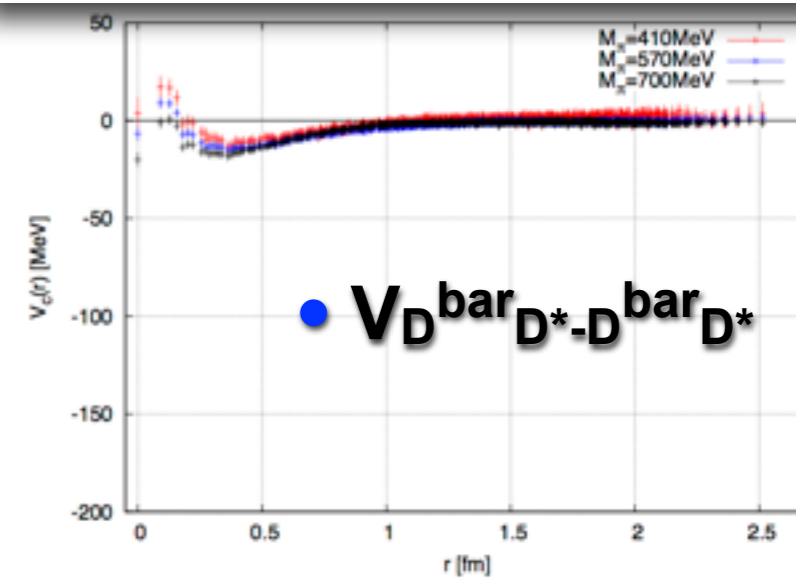
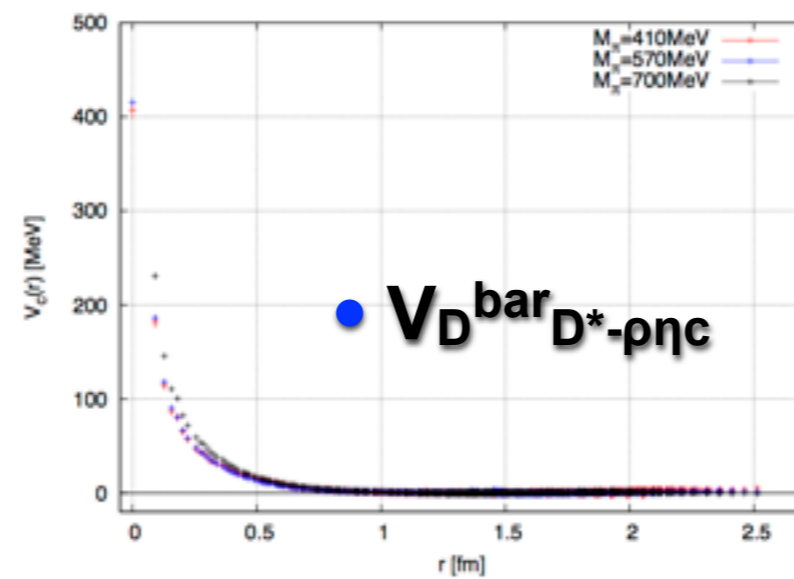
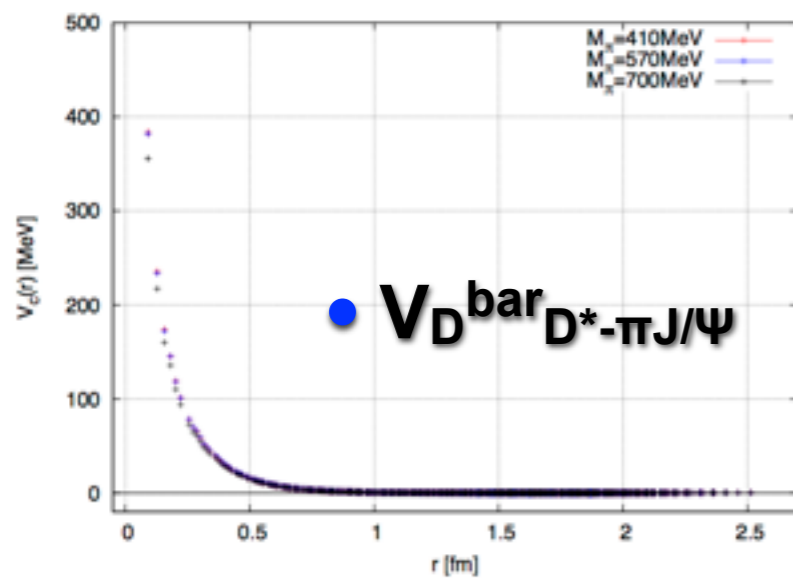
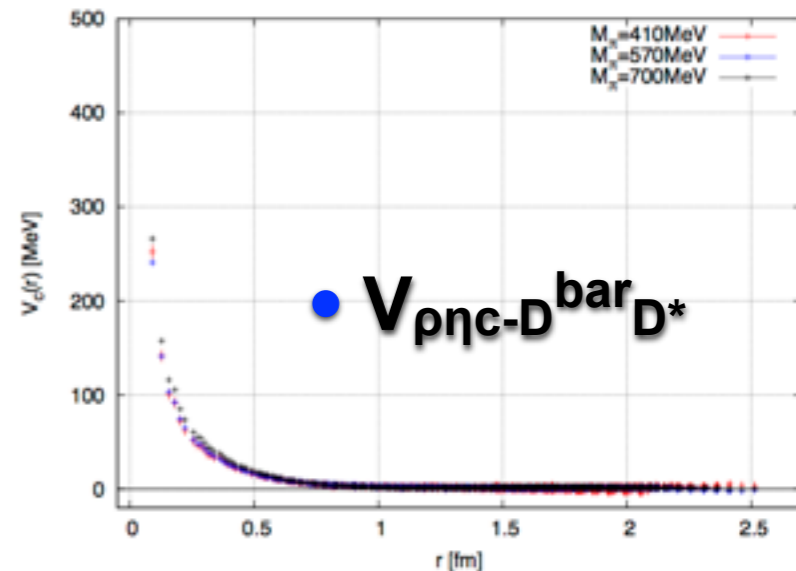
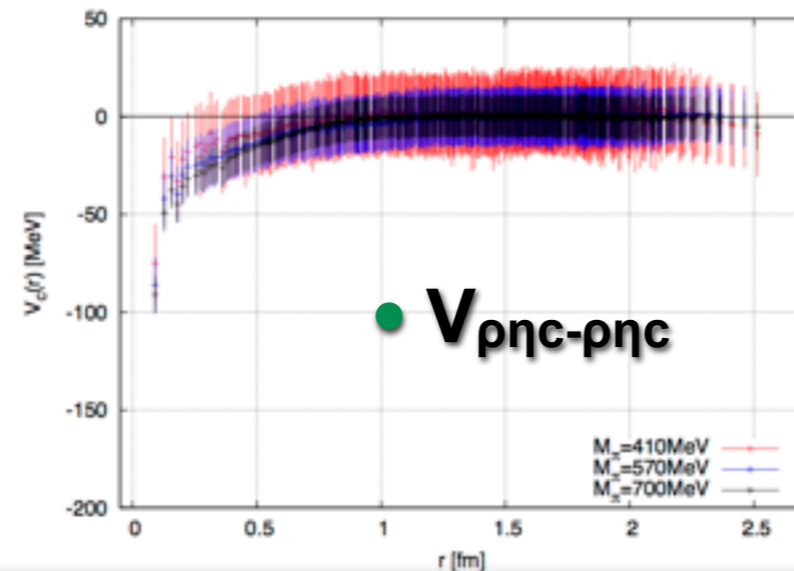
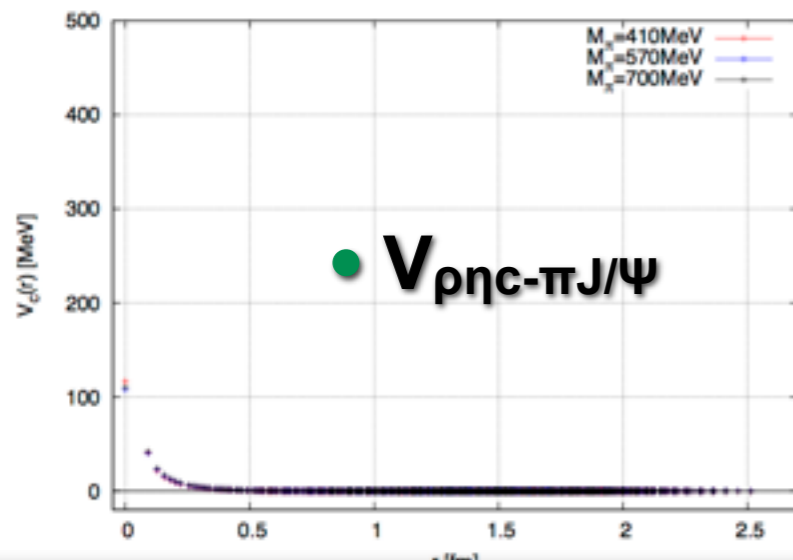
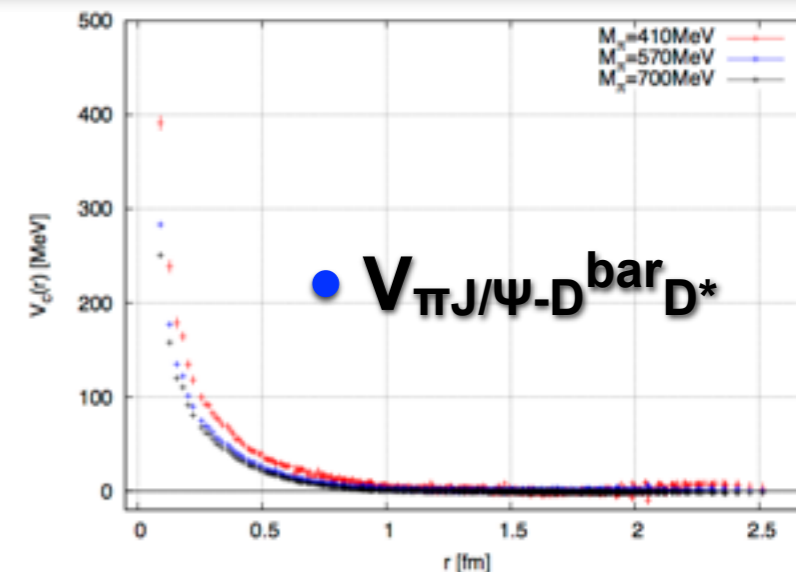
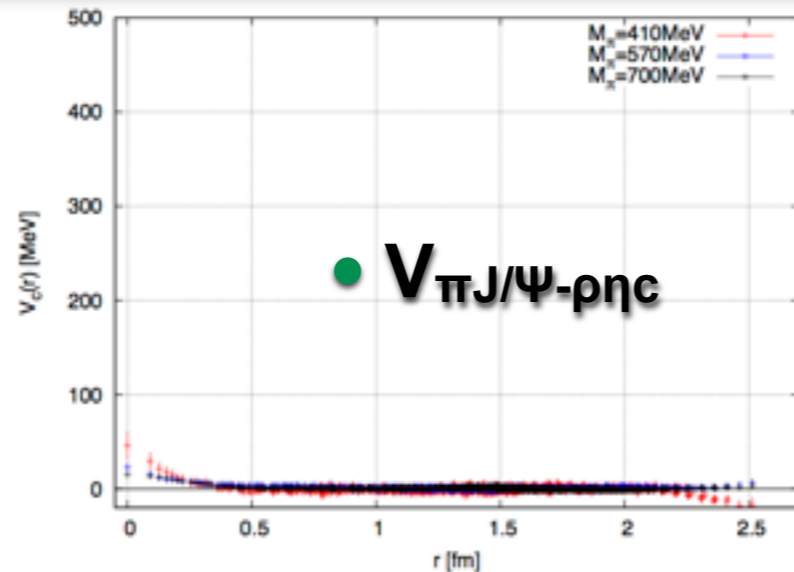
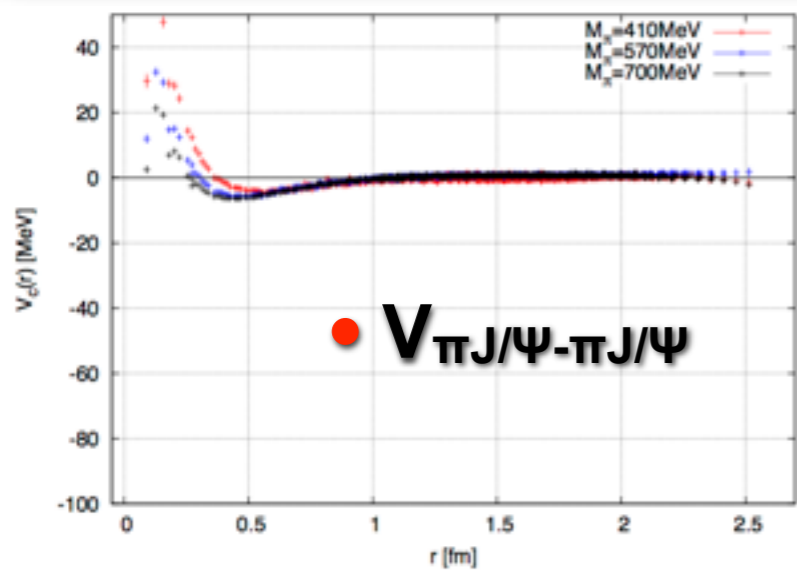


• Weak charm spin-flip potential

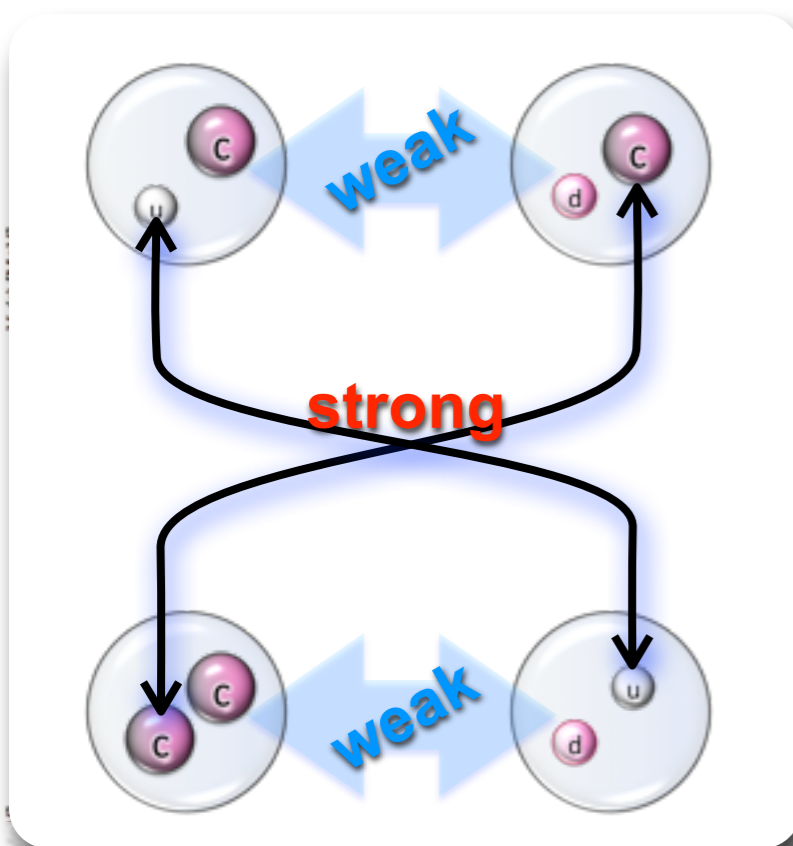
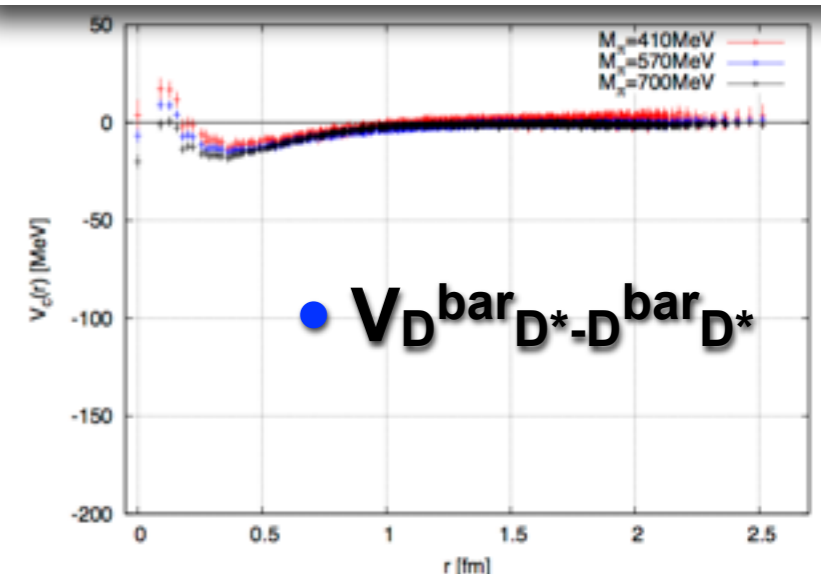
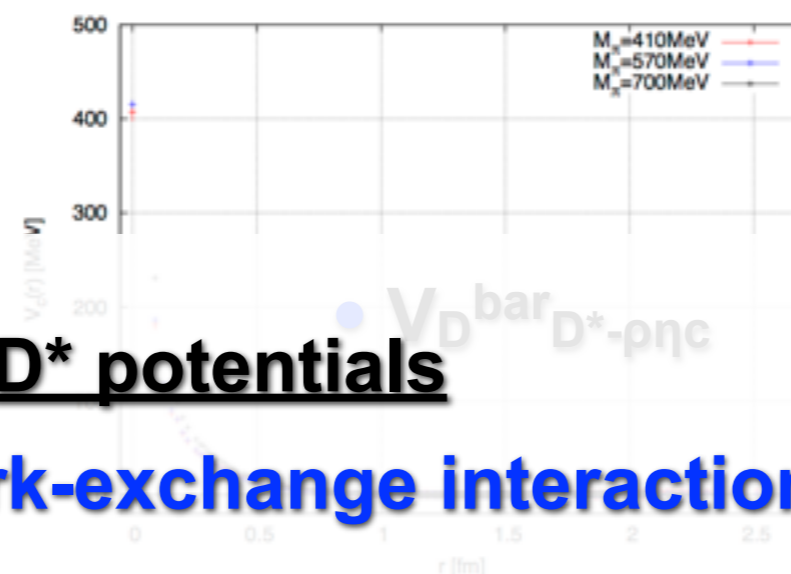
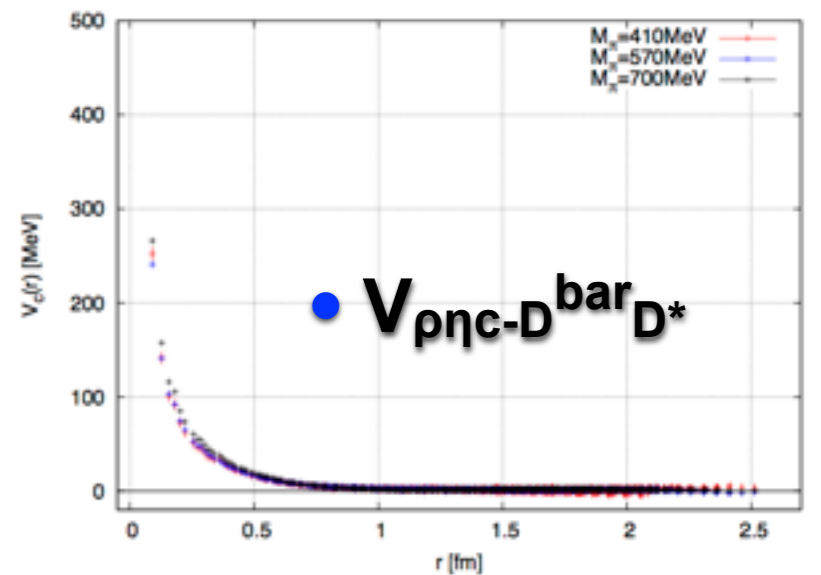
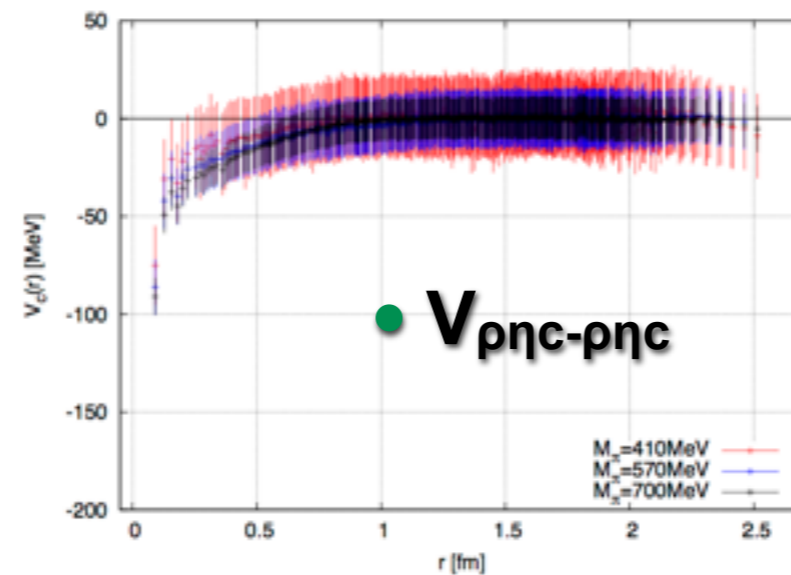
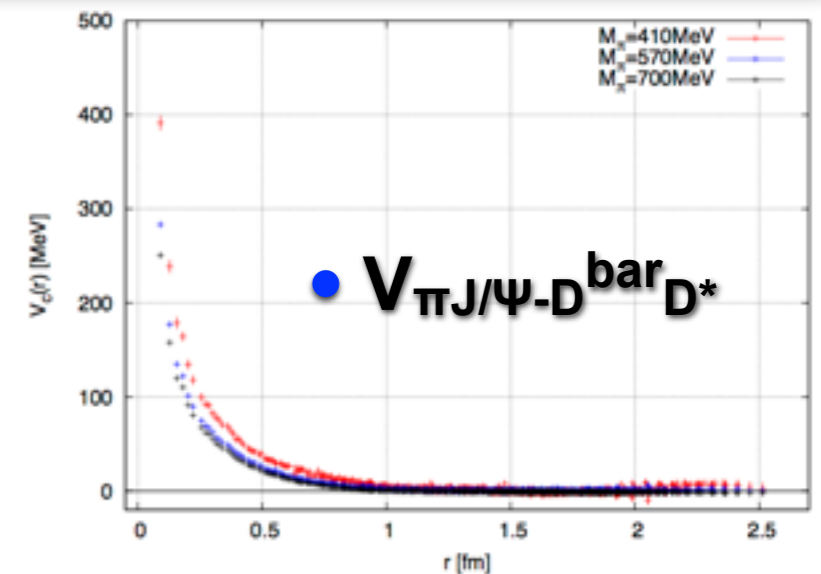
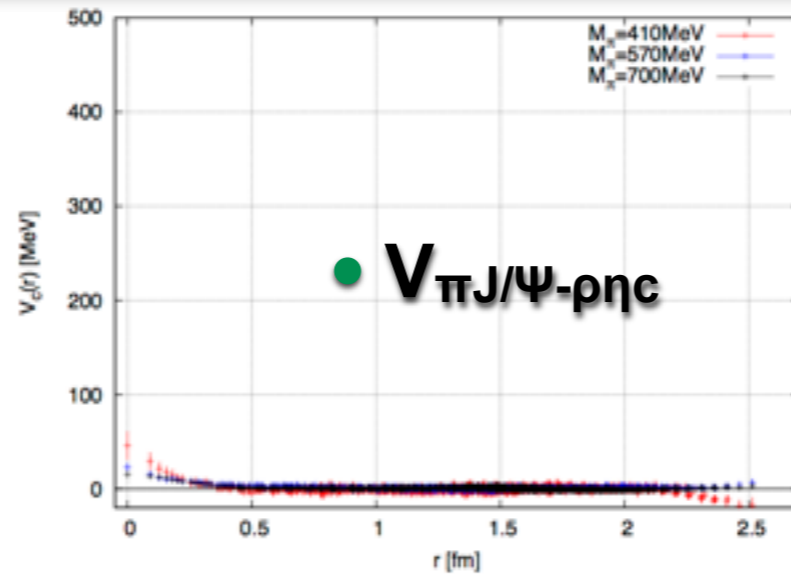
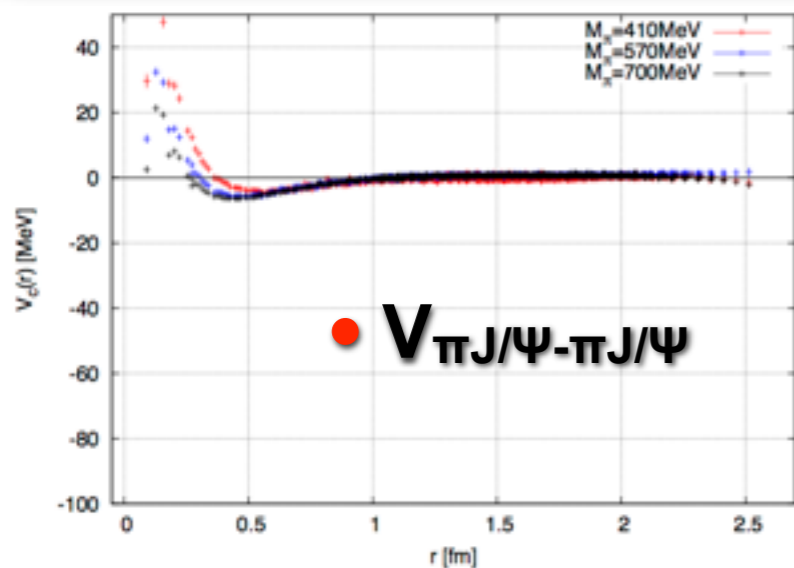
➔ heavy quark spin symmetry

(charm quark spin-flip amplitude is suppressed)

Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



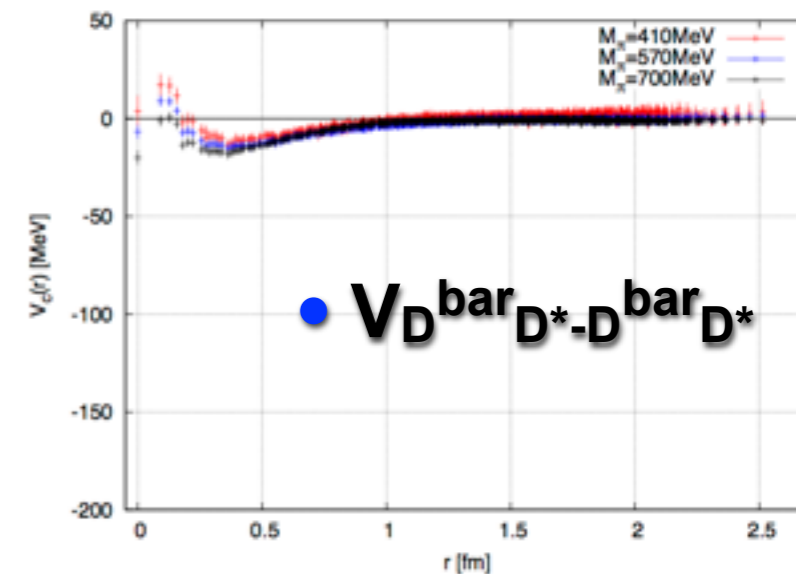
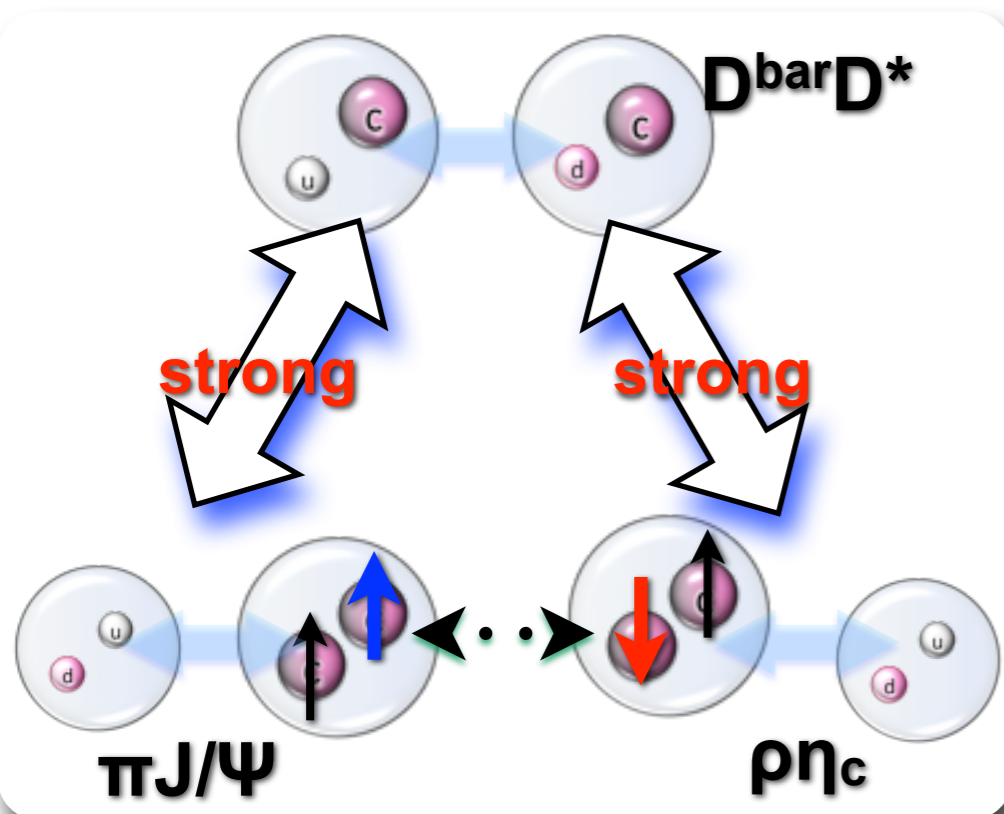
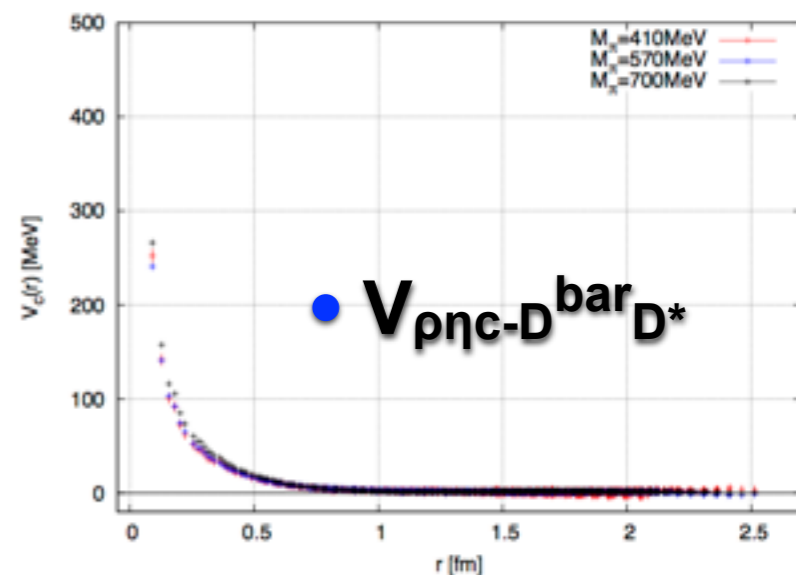
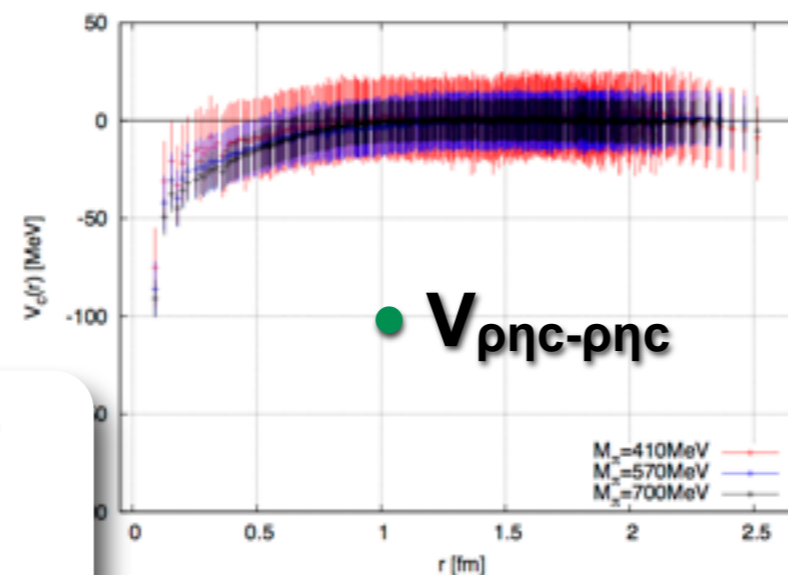
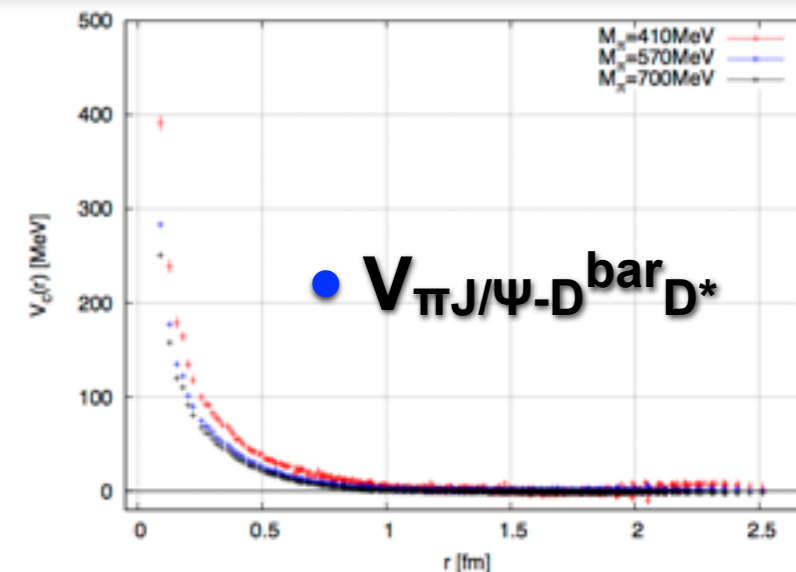
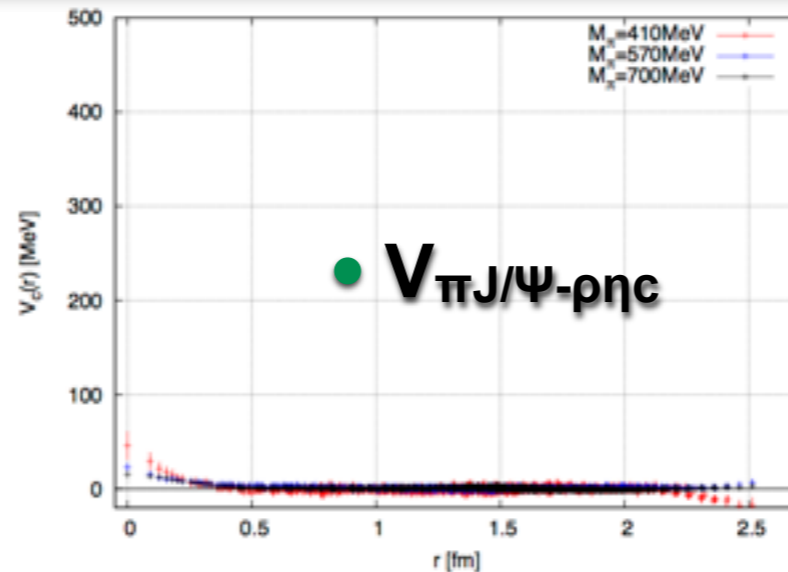
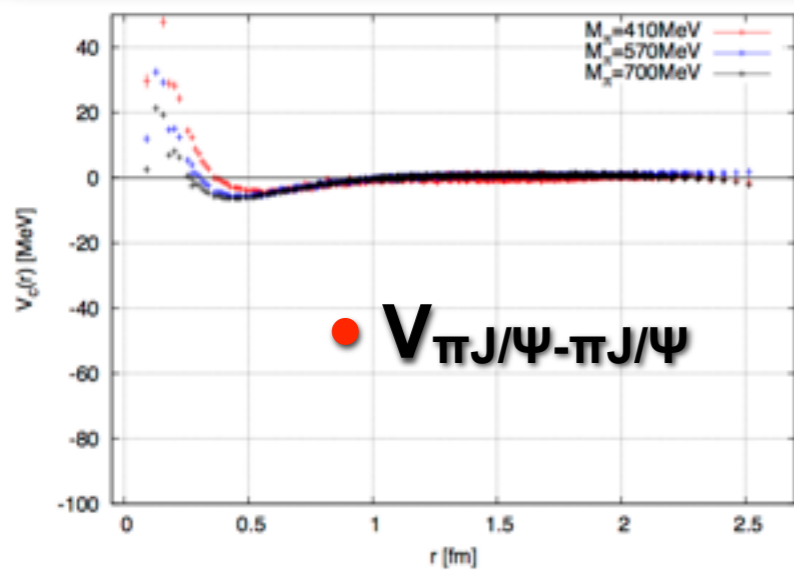
Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



- Strong off-diagonal $D^{\text{bar}}D^*$ potentials

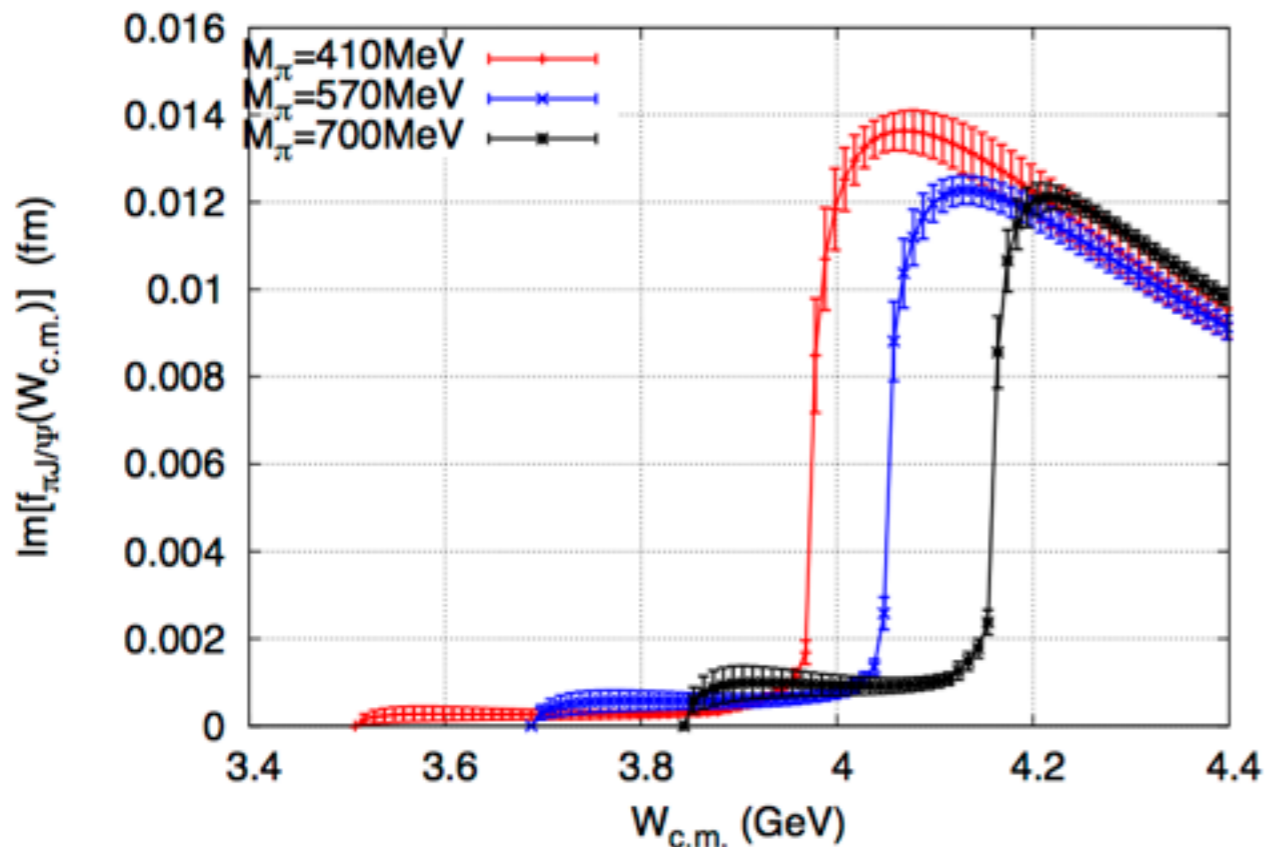
✓ strong charm-quark-exchange interactions

Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)

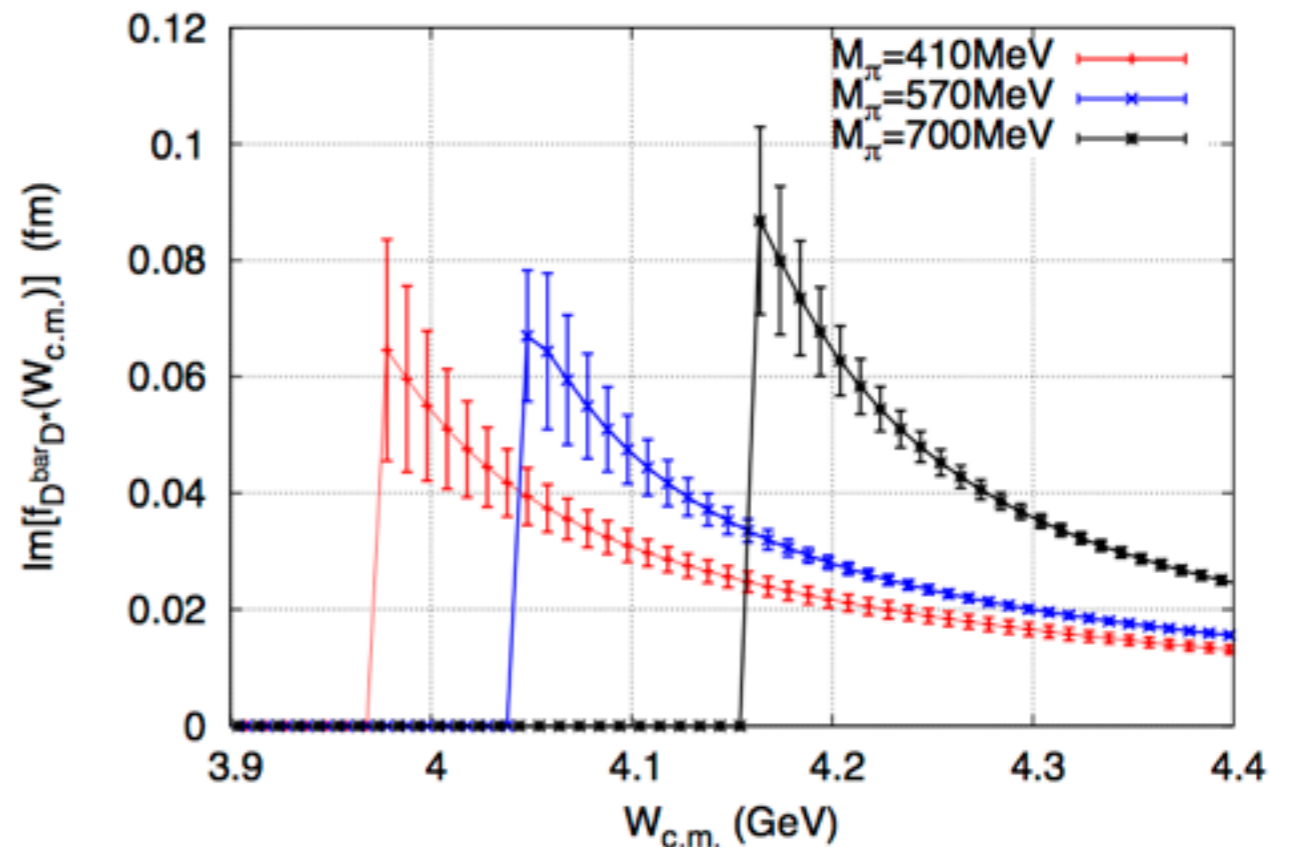


2-body scattering & inv. mass spectra

- $\pi J/\psi$ invariant mass



- $D^{\text{bar}}D^*$ invariant mass



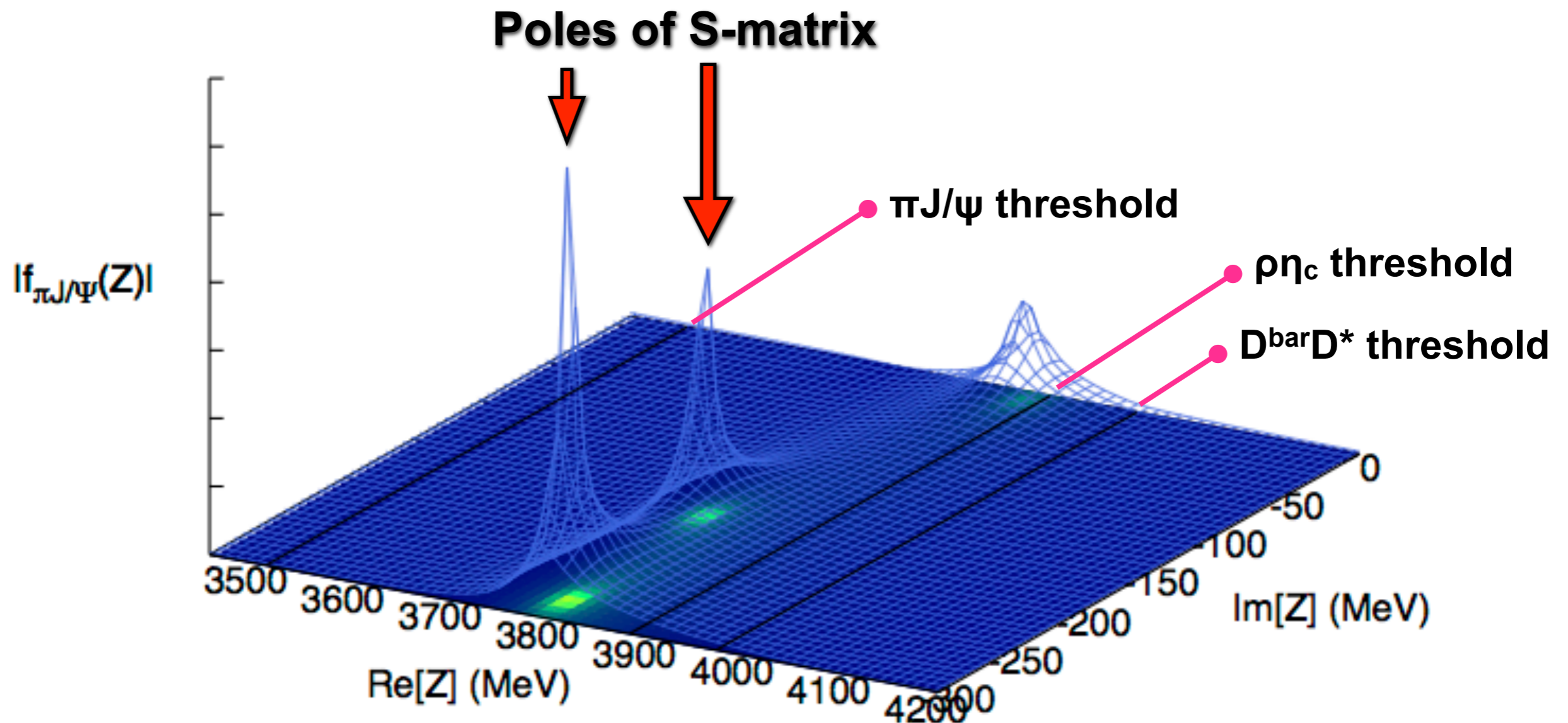
✓ enhancement near $D^{\text{bar}}D^*$ threshold due to large $\pi J/\psi$ - $D^{\text{bar}}D^*$ coupling

- peak in $\pi J/\psi$ invariant mass
- enhancement (**cusp?**) in $D^{\text{bar}}D^*$ invariant mass

(No m_q dependence on qualitative behaviors of line shapes)

Pole search ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{\text{bar}}D^*$:2nd)

✿ input : LQCD potential matrix @ $m_\pi=410\text{MeV}$

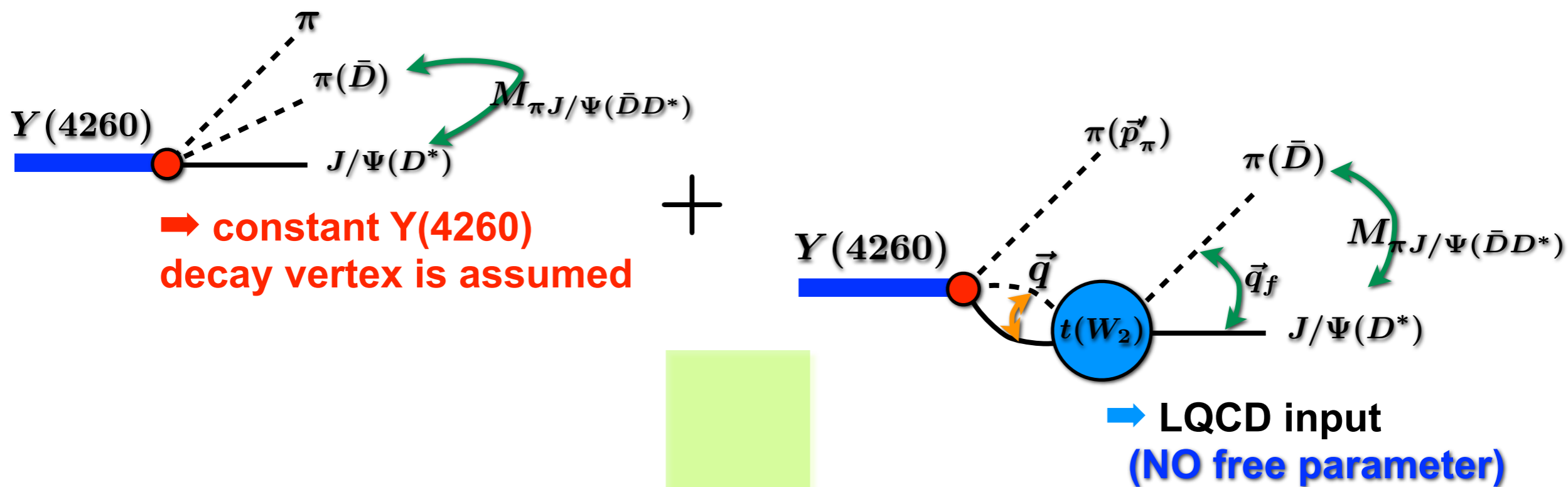


✓ **“Virtual (shadow)”** poles on the most adjacent complex energy plane for $Z_c(3900)$ energy region are found

✓ These poles contribute to threshold enhancement of amplitude

Three-body decay of $Y(4260)$

$$d\Gamma \propto (2\pi)^4 \delta^4(P - p'_\pi - p_{\pi(\bar{D})} - p_{J/\Psi(D^*)}) d^3 p'_\pi d^3 p_{\pi(\bar{D})} d^3 p_{J/\Psi(D^*)} \sum_f |T_{if}|^2$$



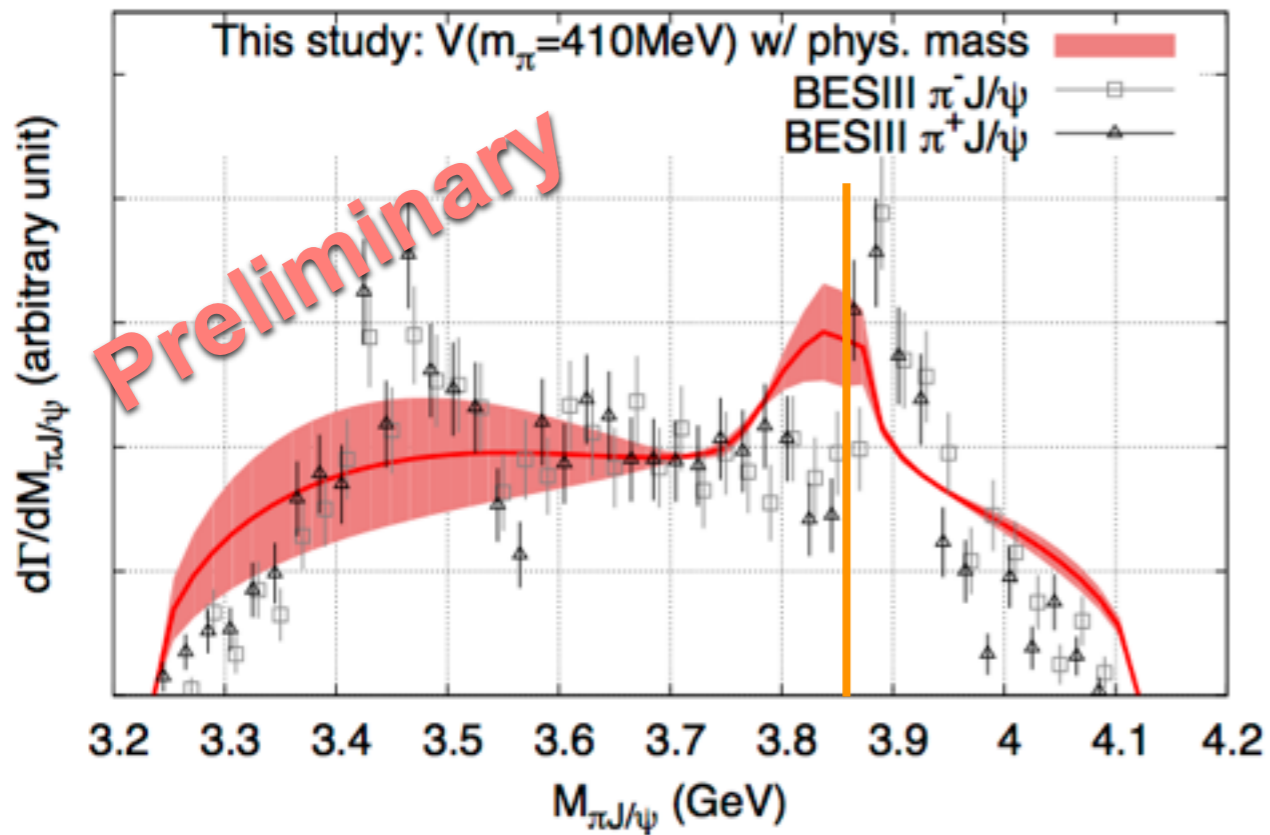
$$T_{if}(\vec{p}'_\pi, \vec{q}_f; W_3) = \sum_{n=\pi J/\Psi, \bar{D}D^*} C_n \left[\delta_{nf} + \int d^3 q \frac{1}{W_3 - E_\pi(\vec{p}'_\pi) - E_n(\vec{q}, \vec{p}'_\pi) + i\epsilon} t(\vec{q}, \vec{q}_f) \right]$$

physical hadron masses employed & S-wave calculation

✓ comparison with expt. data

✓ property of $Y(4260)$ decay

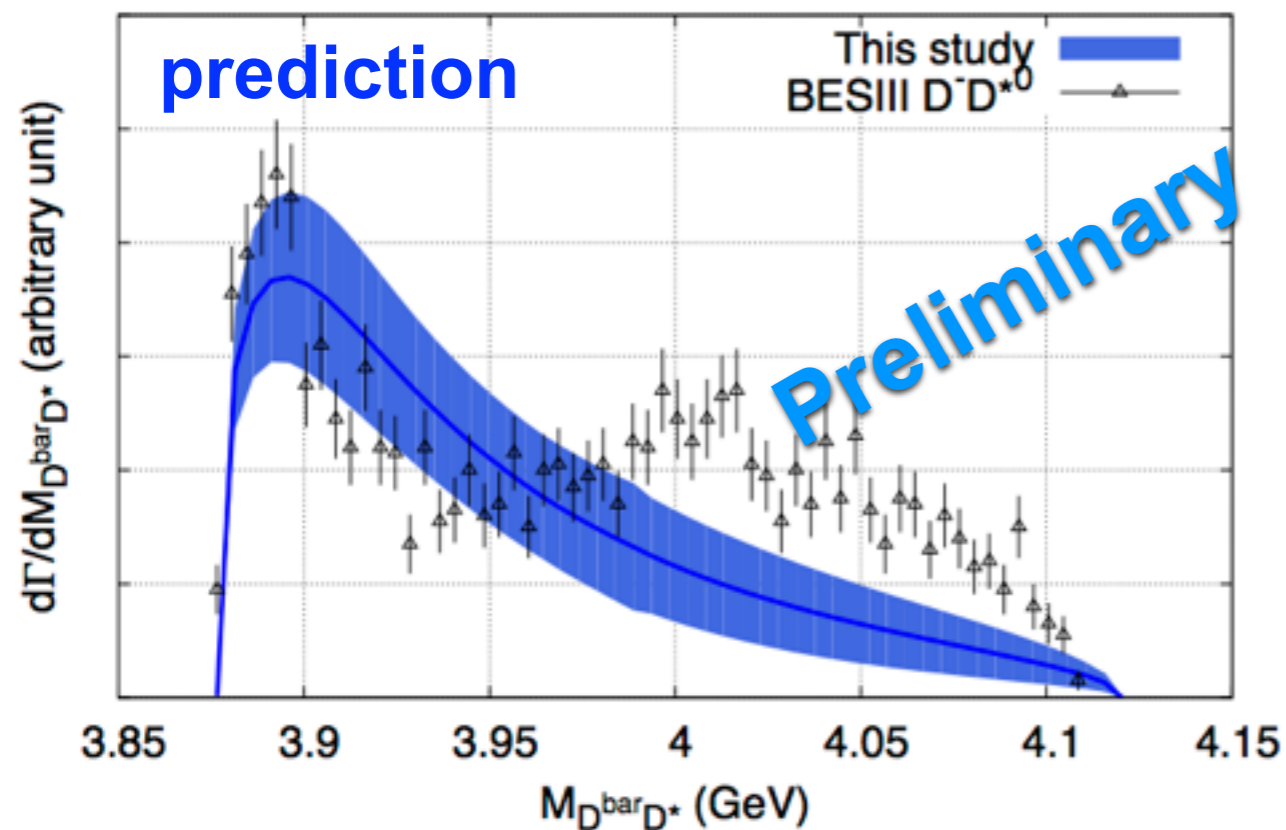
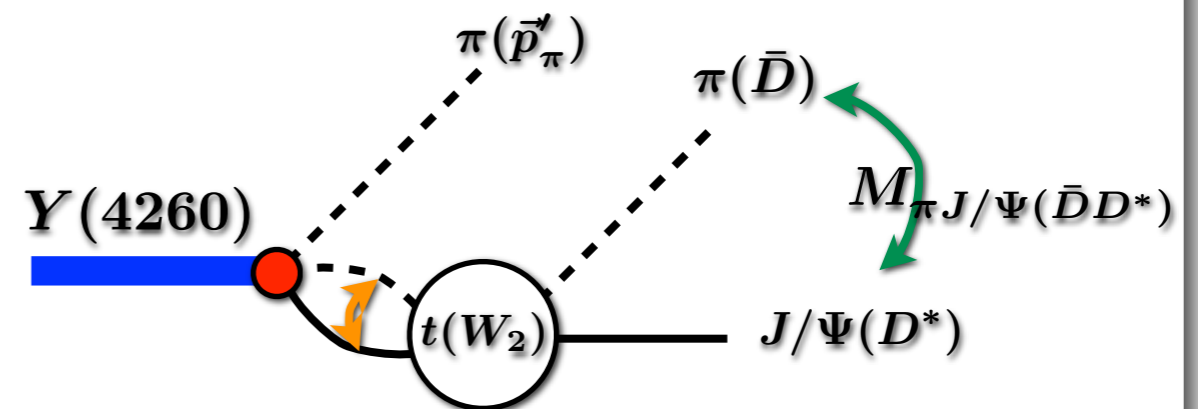
Invariant mass spectra ($\pi J/\psi$ & $D^{\text{bar}} D^*$)



parameters: $C_{D^{\text{bar}} D^*}/C_{\pi J/\psi} = R e^{i\theta}$

--> $R=0.95(18)$, $\theta=-58(44)$ deg.

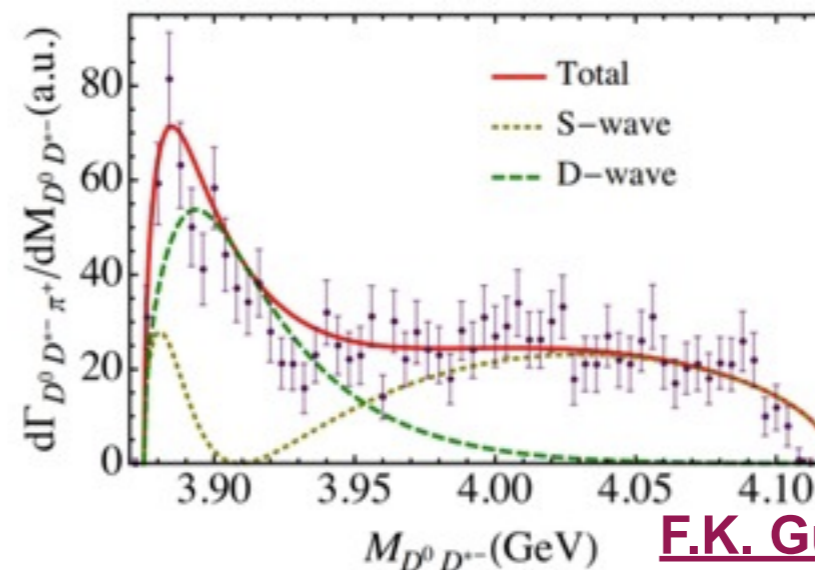
- Not only $D^{\text{bar}} D^*$ but also $\pi J/\psi$ loop is important for $Y(4260)$ decay



- Qualitatively good line shape

- Deviation from EXPT. data at high energies

--> higher partial wave??



Summary

❖ Applications of coupled-channel HAL QCD method

• Zc(3900) in $I^G(J^P)=1^+(1^+)$ channel on the lattice @ $m_\pi=410-700\text{MeV}$

➔ **Large channel coupling between $\pi J/\Psi$ and $D^{\text{bar}}D^*$ is a key**

➔ **Heavy quark spin symmetry is seen in c.c. potentials**

➔ **Similar line shape of inv. mass to EXPT. is observed**

▶ Zc(3900) is neither simple $D^{\text{bar}}D^*$ molecule nor $J/\Psi + \pi$ -cloud

▶ shadow **poles** on complex energy plane are found (w/ relatively large width)

❖ Physical point simulation is the next step

❖ Future targets

▶ other systems : Zc(4025), X(3872)

▶ extension to bottom systems