Baryon states with hidden charm in the extended local hidden gauge approach

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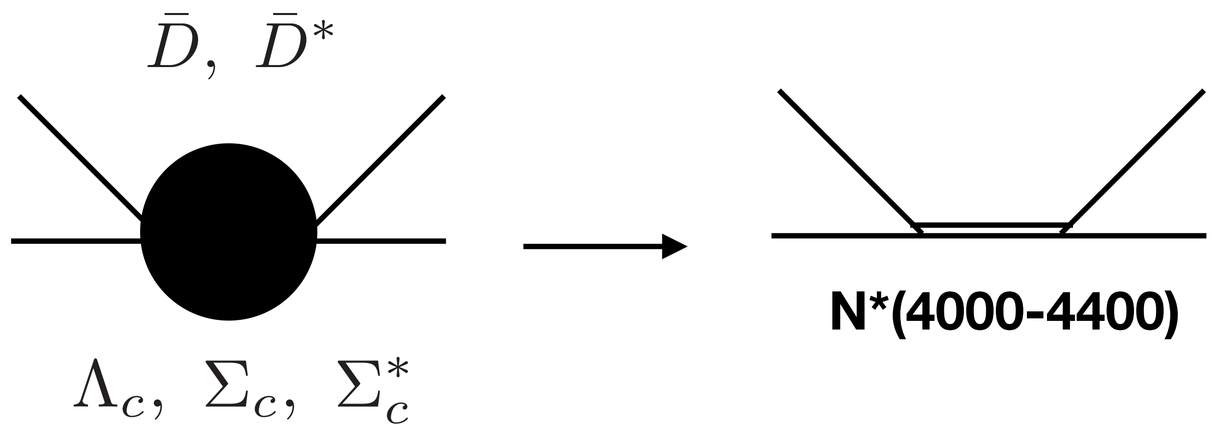
T. Uchino, W. H. Liang and E. Oset, arXiv:1504.05726 [hep-ph]

Hidden charm baryons

Nucleon resonances with negative parity are studied with the energy around 4000 - 4400 MeV as hidden charm molecules which are formed with one anti-charmed meson and one charmed baryon



Relevant channels: $\bar{D}\Lambda_c\ \ \bar{D}\Sigma_c\ \ \bar{D}^*\Lambda_c\ \ \bar{D}^*\Sigma_c\ \ \bar{D}\Sigma_c^*\ \ \bar{D}^*\Sigma_c^*$

Within a unitary coupled channels approach, s-wave meson-baryon is considered. By the use of the on-shell factorization, the amplitude is given by

$$T = V + VGT = \frac{V}{1 - VG}$$

with the G function regularized in the cutoff method

$$G_l(\sqrt{s}) = \int_{|\vec{q}| \leq q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_l + E_l}{2\omega_l E_l} \frac{2M_l}{P^{02} - (\omega_l + E_l)^2 + i\epsilon}$$

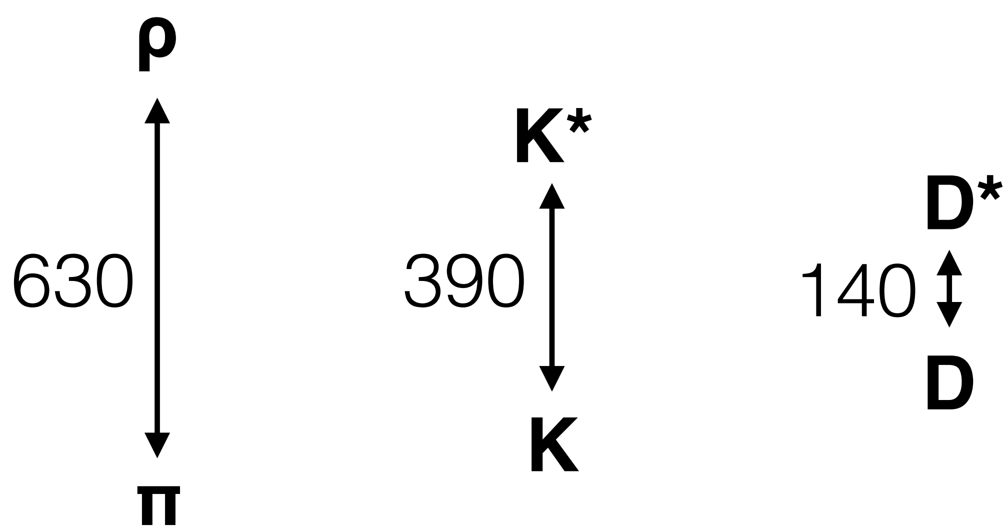
* the Dbar Λ_c channel is taken into account as an intermediate state

Hidden charm baryons

Interaction V: based on the extended local hidden gauge approach [1,2,3]

↓

PB-VB mixing: found to be important (even in the lighter sectors) [4,5]



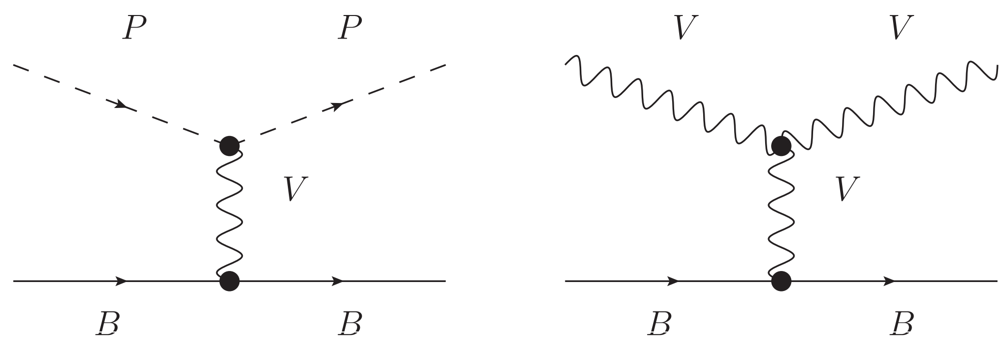
mass difference between P and V is small -> full coupled channels approach

[1] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. 54, 1215 (1985)
[2] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988)
[3] U. G. Meissner, Phys. Rept. 161, 213 (1988)
[4] K. P. Khemchandani, A. Martinez Torres, H. Kaneko, H. Nagahiro and A. Hosaka, Phys. Rev. D 84, 094018 (2011)
[5] E. J. Garzon and E. Oset, Phys. Rev. C 91, 025201 (2015)

Vector exchange driving force

Local hidden gauge approach extended to SU(4) provides relevant vertices [6]

$$\mathcal{L}_{PPV} = -ig\langle V^\mu [P, \partial_\mu P]\rangle$$



$$\mathcal{L}_{VVV} = ig\langle V^\mu [V^\nu, \partial_\mu V_\nu]\rangle$$

$$\mathcal{L}_{BBV} = g\left(\langle \bar{B}\gamma_\mu [V^\mu, B]\rangle + \langle \bar{B}\gamma_\mu B\rangle\langle V^\mu\rangle\right)$$

P, V, B: pseudoscalar, vector, baryon fields matrix in SU(4) $g = M_V/2f_\pi$

→**Weinberg-Tomozawa (WT) interaction**

$$V_{ij} = -\frac{C_{ij}}{4f^2}\left(2\sqrt{s} - M_{B_i} - M_{B_j}\right)\sqrt{\frac{M_{B_i}+E_i}{2M_{B_i}}}\sqrt{\frac{M_{B_j}+E_j}{2M_{B_j}}} \quad C_{ij} = \begin{cases} 1 & \bar{D}\Sigma_c(J=1/2),\ \bar{D}^*\Sigma_c(J=1/2,3/2) \\ & \bar{D}\Sigma_c^*(J=3/2),\ \bar{D}^*\Sigma_c^*(J=1/2,3/2,5/2) \\ -1 & \bar{D}\Lambda_c(J=1/2),\ \bar{D}^*\Lambda_c(J=1/2,3/2) \\ 0 & \mathrm{off-diagonal} \end{cases}$$

- **Heavy Quark Spin Symmetry (HQSS) is automatically fulfilled**
- **VB interaction has spin degeneracy [7]**
- **No off-diagonal potential**

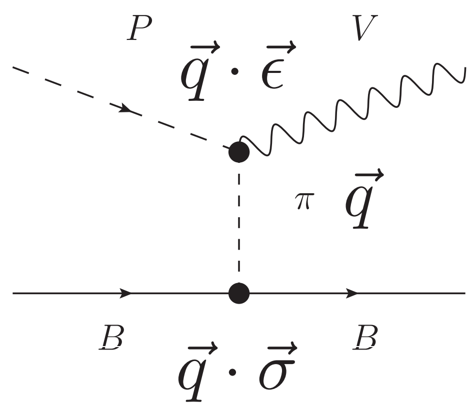
* In [6], the lighter states were found not to modify the energy and simply give ~30 MeV width to the generated states.

[6] J. -J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010)
[7] E. Oset and A. Ramos Eur. Phys. J. A 44, 445–454 (2010)

PB-VB mixing

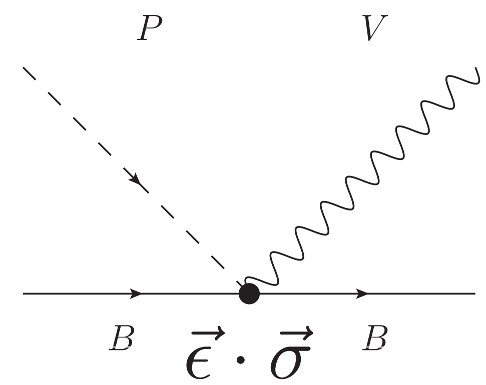
Following the local hidden gauge approach, we can have PB-VB mixing terms [8]

VPP vertices are given by the local hidden gauge: pion exchange term
Constraint of gauge invariance: contact term



pion exchange term

momentum dependent



Kroll-Ruderman contact term

momentum independent

Evaluation of BBπ vertices :

- $\pi\Sigma_c\Sigma_c^*$: negligible
- $\pi\Lambda_c\Sigma_c^*$: negligible
- $\pi\Lambda_c\Lambda_c$: prohibited from the isospin conservation

→

Transition including Σ_c Λ_c

$$\bar{D}\Sigma_c \leftrightarrow \bar{D}^*\Sigma_c$$
$$\bar{D}\Sigma_c \leftrightarrow \bar{D}^*\Lambda_c$$
$$\bar{D}\Lambda_c \leftrightarrow \bar{D}^*\Sigma_c$$

Transition including Σ_c^*

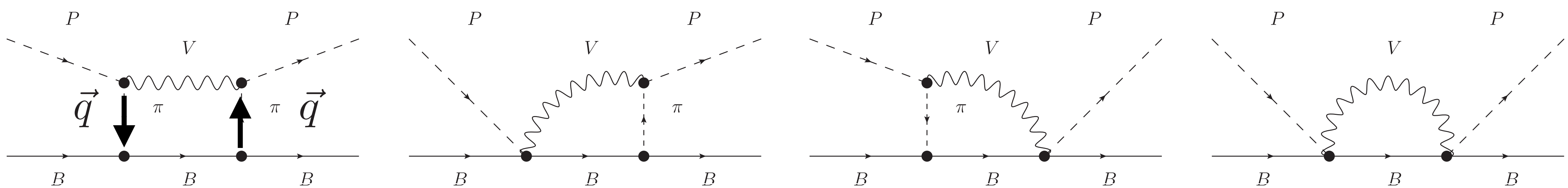
$$\bar{D}\Sigma_c^* \leftrightarrow \bar{D}^*\Sigma_c^*$$

PB-VB mixing intermediated by a pion depends on the momentum transfer
→ the box diagram

[8] E. J. Garzon and E. Oset, Eur. Phys. J. A 48, 5 (2012)

PB-VB-PB box potentials

Momentum transfer q is integrated out in the following box diagrams where one pion is emitted and later absorbed. These potentials are added to the PB driving force as momentum independent functions.



δV_{PP} δV_{CP} δV_{PC} δV_{CC}

s-wave VB intermediate states

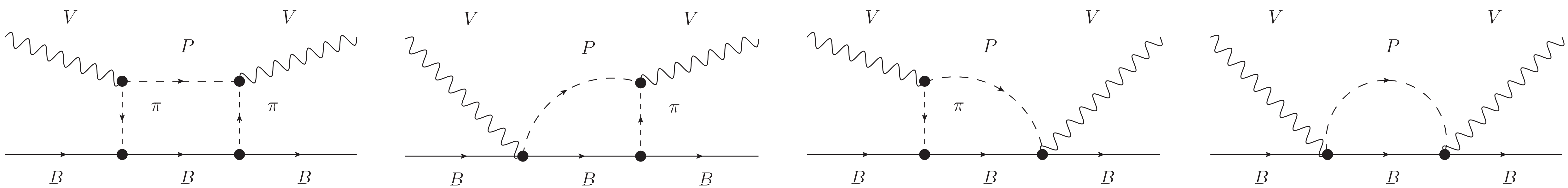
$$\begin{array}{ccccc} \bar{D}\Sigma_c & \to & \bar{D}^*\Sigma_c & \to & \bar{D}\Sigma_c \\ \bar{D}\Sigma_c & \to & \bar{D}^*\Lambda_c & \to & \bar{D}\Sigma_c \end{array} \quad \text{J=1/2} \quad \delta V = \delta V_{PP} + 2\delta V_{CP} + \delta V_{CC}$$

$$\bar{D}\Sigma_c^* \to \bar{D}^*\Sigma_c^* \to \bar{D}\Sigma_c^* \quad \text{J=3/2} \quad \delta V = \delta V_{PP} + 2\delta V_{CP} + \delta V_{CC}$$

PP box: VB intermediate states are not necessarily only in the s-wave
CP, PC, PP boxes: VB intermediate states are in the s-wave

VB-PB-VB box potentials (breaking the degeneracy)

Corrections to the VB driving force with PB intermediate states



$\delta V'_{PP}$ $\delta V'_{CP}$ $\delta V'_{PC}$ $\delta V'_{CC}$

s-wave PB intermediate states

$$\begin{array}{ccccc} \bar{D}^*\Sigma_c & \to & \bar{D}\Sigma_c & \to & \bar{D}^*\Sigma_c \\ \bar{D}^*\Sigma_c & \to & \bar{D}\Lambda_c & \to & \bar{D}^*\Sigma_c \\ \bar{D}^*\Lambda_c & \to & \bar{D}\Sigma_c & \to & \bar{D}^*\Sigma_c \end{array}$$

J=1/2 $\delta V' = \delta V'_{PP} + 2\delta V'_{CP} + \delta V'_{CC}$

J=3/2 $\delta V' = \delta V'_{PP}$

$$\bar{D}^*\Sigma_c^* \to \bar{D}\Sigma_c^* \to \bar{D}^*\Sigma_c^*$$

J=1/2, 5/2 $\delta V' = \delta V'_{PP}$

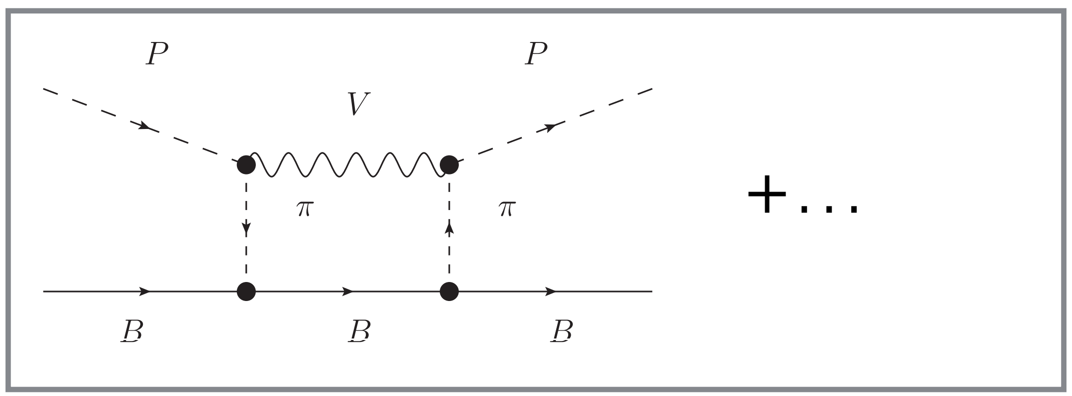
J=3/2 $\delta V' = \delta V'_{PP} + 2\delta V'_{CP} + \delta V'_{CC}$

PP box: VB intermediate states are not necessarily only in the s-wave
CP, PC, PP boxes: VB intermediate states are in the s-wave

→These VB box potentials resolve the spin degeneracy of the VB sectors

Effective transition potential

In order to fully take into account the PB-VB mixing, one step further, we implement the full coupled channels calculation by constructing **the effective PB-VB transition potential** [9]



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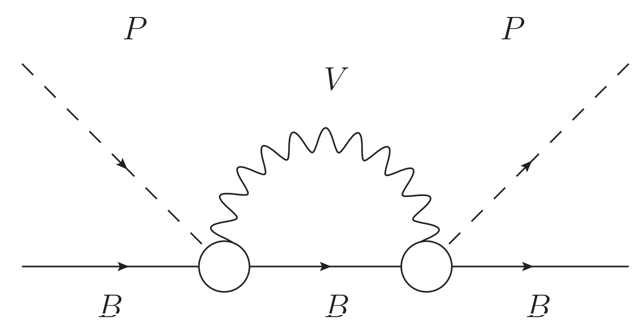
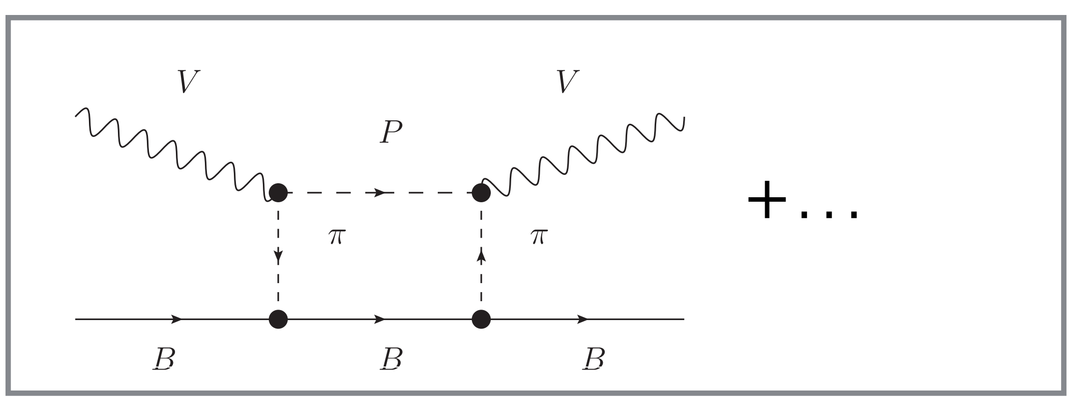


$$\delta V_s(PB \to VB \to PB) \qquad \tilde{V}_{\text{eff}} G_{VB} \tilde{V}_{\text{eff}}$$



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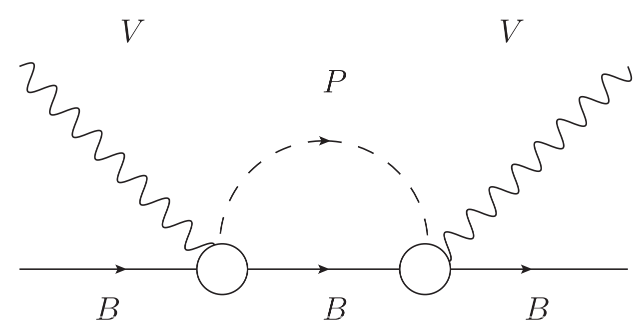


$$\delta V_s^{'}(VB \to PB \to VB) \qquad \tilde{V}_{\text{eff}}^{'} G_{PB} \tilde{V}_{\text{eff}}^{'}$$

$$\longrightarrow \; V_{\text{eff}} = \frac{1}{2}\left(\tilde{V}_{\text{eff}} + \tilde{V}_{\text{eff}}^{'}\right)$$

- s-wave components: used to construct the effective potentials
- d-wave (and other) components: kept in the box potentials yet

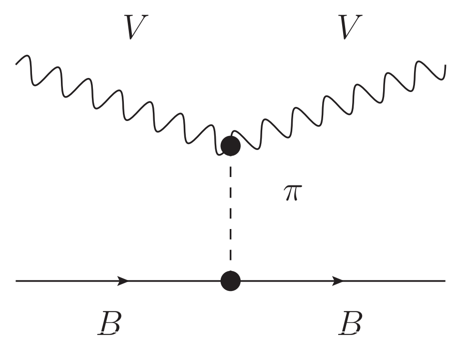
[9] W. H. Liang, T. Uchino, C. W. Xiao and E. Oset, Eur. Phys. J. A 51 (2015) 2, 16

Box diagram potential - Anomalous VVP term

Anomalous VVP vertex also provides the VB-VB potential via pion exchange [10,11]

$$\mathcal{L}_{VVP} = \frac{G}{\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\langle\partial_\mu V_\nu \partial_\alpha V_\beta\rangle$$

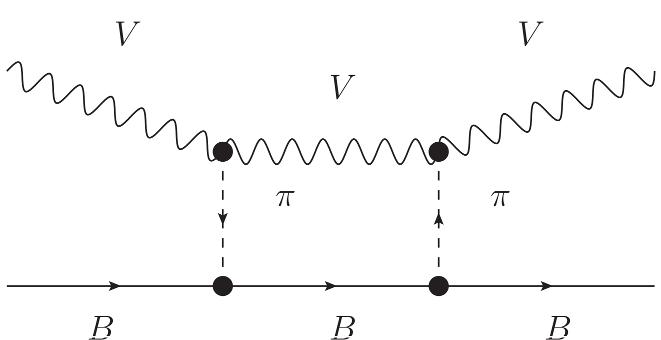
$$G = 3M_V^2/16\pi^2 f_\pi^2 \sim 14\mathrm{GeV}^{-1}$$



This VB - VB potential does not interfere with the WT interaction at tree level

→ These contributions are taken into account as box diagrams

$\delta V_{\mathrm{an}} =$



Box diagram including Σ_c Λ_c

$$\bar{D}^*\Sigma_c \to \bar{D}^*\Sigma_c \to \bar{D}^*\Sigma_c$$
$$\bar{D}^*\Sigma_c \to \bar{D}^*\Lambda_c \to \bar{D}^*\Sigma_c$$
$$\bar{D}^*\Lambda_c \to \bar{D}^*\Sigma_c \to \bar{D}^*\Sigma_c$$

Box diagram including Σ_c^*

$$\bar{D}^*\Sigma_c^* \to \bar{D}^*\Sigma_c^* \to \bar{D}^*\Sigma_c^*$$

<u>These box potentials just give extra contributions (attractions) only to VB potentials</u>

*An anomalous process, like the V V P interaction, is one that does not conserve “natural” parity. The “natural” parity of a particle is defined as follows: it is +1 if the particle transforms as a true tensor of that rank, and −1 if it transforms as a pseudotensor, e.g. π, γ, ρ and a_1 have “natural” parity −1, +1, +1 and −1, respectively.

[10] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B 344, 240 (1995)
[11] E. Oset, J. R. Pelaez and L. Roca, Phys. Rev. D 67, 073013 (2003)

The generated states: parameters, analysis

Cutoff parameters

Three cutoff parameters, two for the G functions of PB and of VB intermediate states and one for all the box diagrams, are used.

	set I	set II	set III
$q^B_{\rm max}$	600	800	1000
$q^V_{\rm max}$	771	737	715
$q^P_{\rm max}$	527	500	483

These values were chosen to reproduce two Lambda_c resonances in the previous work [9]

Analysis of the generated states

All the generated states are measured on the real axis, not in the complex energy plane

bound states

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_B} \longrightarrow \lim_{\sqrt{s}\to M_B} (\sqrt{s} - M_B) T_{ij}(\sqrt{s}) = g_i g_j \longrightarrow g_i G_i(M_B)$$

bound pole

resonances

coupling constants

wave function at the origin*

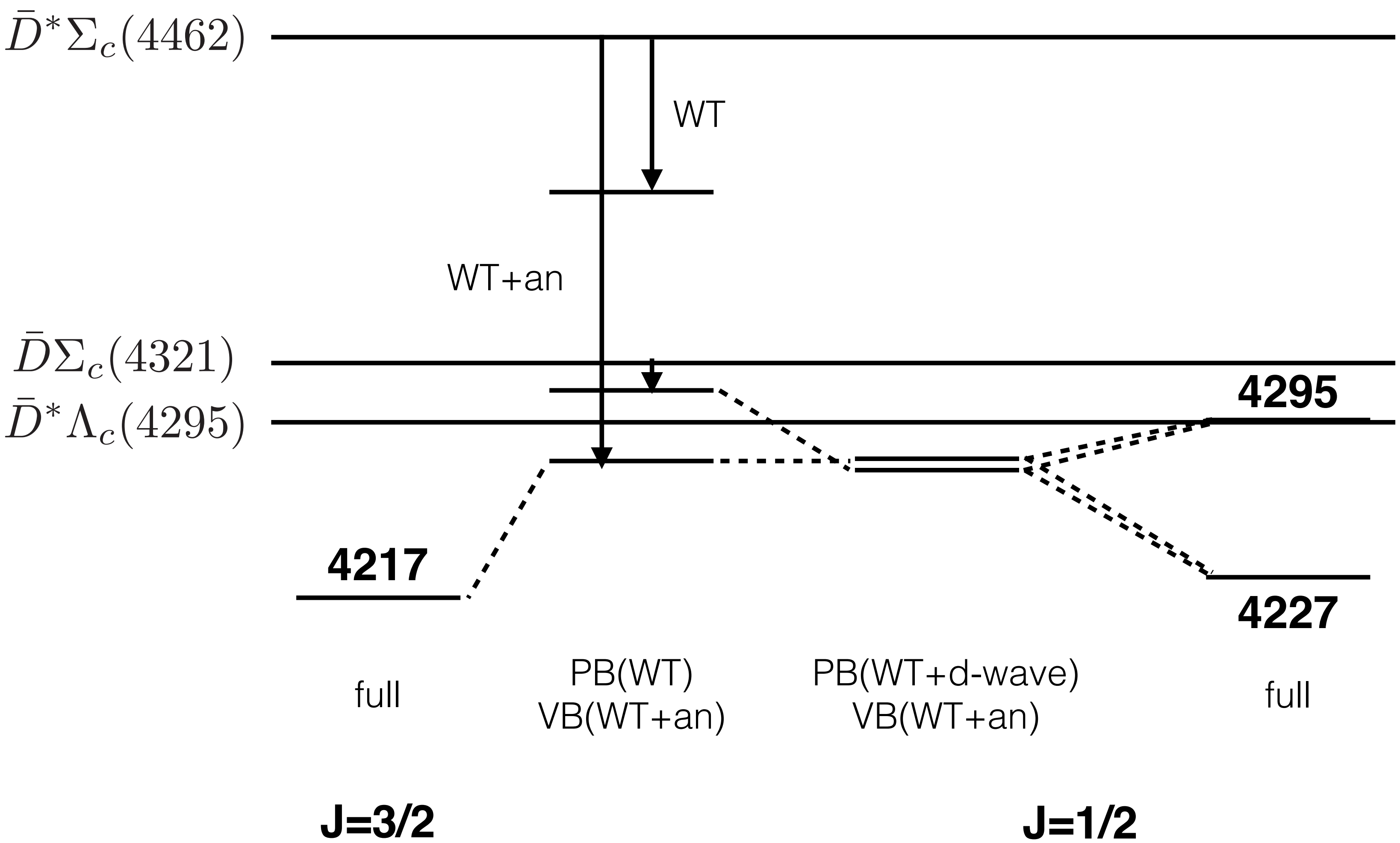
$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2} \longrightarrow {\rm Im} T_{ij}(M_R) \sim -\frac{2}{\Gamma_R} g_i g_j \longrightarrow g_i G_i(M_R)$$

resonance peak

*The value gG at the energy of generated states provides the wave function at the origin [12]
-> evaluate the component of the generated states in the coupled channel

[12] D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D 81, 014029 (2010)

Results I: states generated by $\bar{D}\Sigma_c$ $\bar{D}^*\Lambda_c$ $\bar{D}^*\Sigma_c$



coupling constants and wave functions at the origin

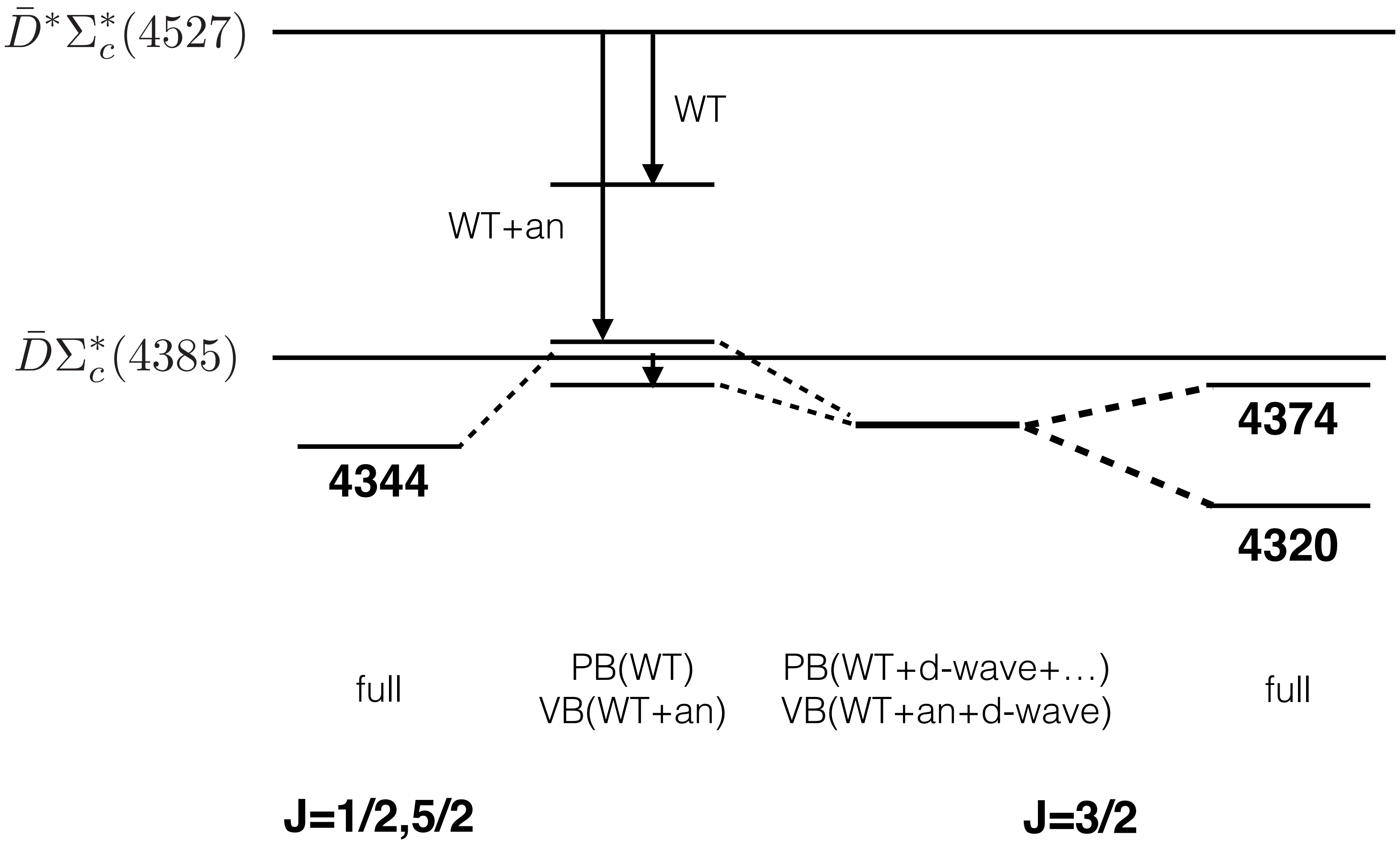
(4227.6, 21.1)	$\bar{D}\Sigma_c$	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Lambda_c$
g_i	4.40	5.39	0.39
$g_i G_i$	−15.66	−24.17	−3.31
(4295.1, 10.6)	$\bar{D}\Sigma_c$	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Lambda_c$
g_i	−1.27	2.28	−0.11
$g_i G_i$	8.46	−12.60	$2.09 + i0.04$

Two orthogonal states emerge

$$\sim \frac{1}{\sqrt{2}}\left(|\bar{D}^*\Sigma_c\rangle \pm |\bar{D}\Sigma_c\rangle\right)$$

- One VB bound state and two PB-VB admixture states appear
- In both admixture states, VB component is stronger than PB component
- Two admixture states are orthogonal: the relative sign of gG
- In the coupled channel, without the transition potential, energy of the two states are very close
- The mass difference between two orthogonal states is ~ 60 MeV

Results II: states generated by $\bar{D}\Sigma_c^*$ $\bar{D}^*\Sigma_c^*$



coupling constants and wave functions at the origin

4324.85	$\bar{D}\Sigma_c^*$	$\bar{D}^*\Sigma_c^*$
g_i	3.61	4.89
$g_i\, G_i^{II}$	−16.54	−24.22
4378.84	$\bar{D}\Sigma_c^*$	$\bar{D}^*\Sigma_c^*$
g_i	−1.25	3.00
$g_i\, G_i^{II}$	12.30	−17.57

Two orthogonal states emerge

$$\sim \frac{1}{\sqrt{2}}\left(|\bar{D}^*\Sigma_c^*\rangle \pm |\bar{D}\Sigma_c^*\rangle\right)$$

- One VB bound state and two PB-VB admixture states appear
- In both admixture states, VB component is stronger than PB component
- Two admixture states are orthogonal: the relative sign of gG
- In the coupled channel, without the transition potential, energy of the two states are very close
- The mass difference between two orthogonal states is ~ 60 MeV

Summary

- Negative parity nucleon resonances with energy 4000-4400 are studied at the viewpoint of the hidden charm hadron molecule.
- Based on the local hidden gauge approach extended to SU(4), we have the WT term and the PB-VB mixing term. In addition, we also consider the extra VB potential from the anomalous VVP interaction.
- The full coupled channels calculation is implemented with the effective transition potential from the relevant parts of the PB-VB mixing term.
- Very similar results are obtained in the two sectors, one contains the spin 1/2 baryon and the other contains the 3/2 baryons → HQSS as to Σ_c and Σ_c*

    - One VB bound states and two PB-VB admixture states
    - From the evaluation of the states with the wave function at the origin, this two admixture states are found to be roughly orthogonal

Summary of the numerical results

main channel	J	$(E,\,\Gamma)$ [MeV]	main decay channels
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c + \bar{D}\Sigma_c)$	$1/2$	$4228,\ 21(51)$	$\bar{D}\Lambda_c$
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c - \bar{D}\Sigma_c)$	$1/2$	$4295,\ 11(41)$	$\bar{D}\Lambda_c$
$\bar{D}^*\Sigma_c$	$3/2$	$4218,\ 103$	$\bar{D}\Lambda_c$
$\bar{D}^*\Sigma_c^*$	$1/2,\ 5/2$	$4344,\ 0$	$-$
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c^* + \bar{D}\Sigma_c^*)$	$3/2$	$4325,\ 0$	$-$
$\frac{1}{\sqrt{2}}(\bar{D}^*\Sigma_c^* - \bar{D}\Sigma_c^*)$	$3/2$	$4378,\ 0$	$-$

The width in brackets is the effect of the couplings to the lighter sector

Thank you!

Backup slides

Full coupled channels I: $\bar{D}\Sigma_c \;\; \bar{D}^*\Lambda_c \;\; \bar{D}^*\Sigma_c$

J=1/2: coupled channels

$$
V = \left(\begin{array}{c|ccc}
 & \bar{D}\Sigma_c & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline
\bar{D}\Sigma_c & V_{WT} & 0 & 0 \\
\bar{D}^*\Sigma_c & & V_{WT} & 0 \\
\bar{D}^*\Lambda_c & & & V_{WT}
\end{array}\right)
+
\left(\begin{array}{c|ccc}
 & \bar{D}\Sigma_c & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline
\bar{D}\Sigma_c & \delta V_d(\bar{D}^*\Sigma_c) + \delta V_d(\bar{D}^*\Lambda_c) & V_{\rm eff} & V_{\rm eff} \\
\bar{D}^*\Sigma_c & & 0 & 0 \\
\bar{D}^*\Lambda_c & & & 0
\end{array}\right)
$$

$$
+ \left(\begin{array}{c|ccc}
 & \bar{D}\Sigma_c & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline
\bar{D}\Sigma_c & 0 & 0 & 0 \\
\bar{D}^*\Sigma_c & & \delta V_{\rm an}(\bar{D}^*\Sigma_c) + \delta V_{\rm an}(\bar{D}^*\Lambda_c) & 0 \\
\bar{D}^*\Lambda_c & & & \delta V_{\rm an}(\bar{D}^*\Sigma_c)
\end{array}\right)
$$

d-wave box potentials are added only to PB sector

J=3/2: two single channels

$$
V = \left(\begin{array}{c|cc}
 & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline
\bar{D}^*\Sigma_c & V_{WT} & 0 \\
\bar{D}^*\Lambda_c & & V_{WT}
\end{array}\right)
+
\left(\begin{array}{c|cc}
 & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline
\bar{D}^*\Sigma_c & \delta V(\bar{D}\Sigma_c) + \delta V(\bar{D}\Lambda_c) & 0 \\
\bar{D}^*\Lambda_c & & \delta V(\bar{D}\Sigma_c)
\end{array}\right)
$$

$$
+ \left(\begin{array}{c|cc}
 & \bar{D}^*\Sigma_c & \bar{D}^*\Lambda_c \\ \hline
\bar{D}^*\Sigma_c & \delta V_{\rm an}(\bar{D}^*\Sigma_c) + \delta V_{\rm an}(\bar{D}^*\Lambda_c) & 0 \\
\bar{D}^*\Lambda_c & & \delta V_{\rm an}(\bar{D}^*\Sigma_c)
\end{array}\right)
$$

There is no transition between two channels -> two single channels

Full coupled channels II: $\bar{D}\Sigma_c^* \quad \bar{D}^*\Sigma_c^*$

J=1/2, 5/2: single channel

$$
V = \left(\begin{array}{c|c} & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}^*\Sigma_c^* & V_{WT} \end{array}\right) + \left(\begin{array}{c|c} & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}^*\Sigma_c^* & \delta V(\bar{D}\Sigma_c^*) \end{array}\right) + \left(\begin{array}{c|c} & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}^*\Sigma_c^* & \delta V_{\rm an}(\bar{D}^*\Sigma_c^*) \end{array}\right)
$$

J=3/2: coupled channels

$$
V = \left(\begin{array}{c|cc} & \bar{D}\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}\Sigma_c^* & V_{WT} & 0 \\ \bar{D}^*\Sigma_c^* & & V_{WT} \end{array}\right) + \left(\begin{array}{c|cc} & \bar{D}\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}\Sigma_c^* & \delta V_{\neq s}(\bar{D}^*\Sigma_c^*) & V_{\rm eff} \\ \bar{D}^*\Sigma_c^* & & \delta V_d(\bar{D}\Sigma_c^*) \end{array}\right)
$$

$$
+ \left(\begin{array}{c|cc} & \bar{D}\Sigma_c^* & \bar{D}^*\Sigma_c^* \\ \hline \bar{D}\Sigma_c^* & 0 & 0 \\ \bar{D}^*\Sigma_c^* & & \delta V_{\rm an}(\bar{D}^*\Sigma_c^*) \end{array}\right)
$$

It is not simple to extract the s-wave component of the VB intermediate state

$$
V_{\rm eff} = \tilde{V}_{\rm eff}' = \sqrt{\frac{\delta V_s'(\bar{D}\Sigma_c^*)}{G_{\bar{D}\Sigma_c^*}}} \longrightarrow \delta V_{\neq s}(\bar{D}^*\Sigma_c^*) = \delta V_{\rm total}(\bar{D}^*\Sigma_c^*) - V_{\rm eff} G_{\bar{D}^*\Sigma_c^*} V_{\rm eff}
$$

s-wave

Uncertainty from the cutoff

	set I	set II	set III
$q^B_{\max}$	600	800	1000
$q^V_{\max}$	771	737	715
$q^P_{\max}$	527	500	483

	set I	set II	set III
peak 1	4241.7	4227.6	4218.6
width 1	19.5	21.1	21.5
peak 2	4296.8	4295.1	4294.5
width 2	13.1	10.6	9.6

J=1/2 $\bar{D}\Sigma_c$ $\bar{D}^*\Sigma_c$ $\bar{D}^*\Lambda_c$

	set I	set II	set III
peak	4250.5	4217.7	4205.8
width	140.8	103.2	82.0

***J=3/2** $\bar{D}^*\Sigma_c$ $\bar{D}^*\Lambda_c$

	set I	set II	set III
Pole	4354.5	4344.1	4337.5

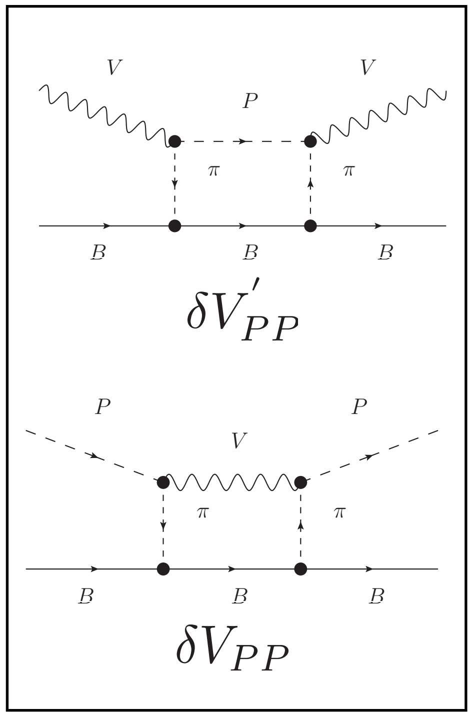
J=1/2, 5/2 $\bar{D}^*\Sigma_c^*$

	set I	set II	set III
Pole 1	4330.6	4324.9	4319.9
Pole 2	4384.1	4377.8	4374.4

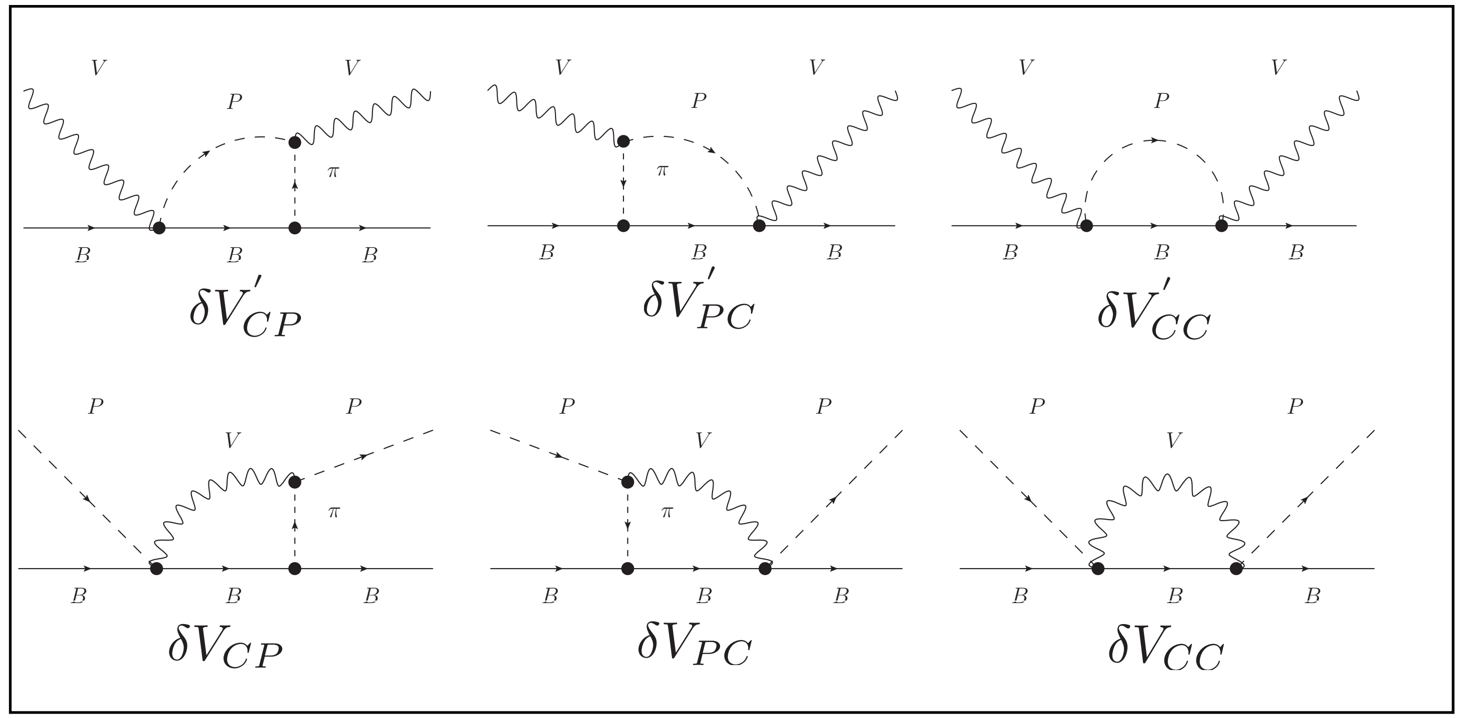
J=3/2 $\bar{D}\Sigma_c^*$ $\bar{D}^*\Sigma_c^*$

Except the * sector, the uncertainty is ~ ± 10 MeV

Extract the s-wave component from the box



intermediate states are
not necessarily in the s-wave



intermediate states are
always in the s-wave

When an intermediate state of a PP box has s- wave and d-wave components…

$$q_i q_j \to \frac{1}{3} q^2 \delta_{ij} \qquad (\vec{\sigma} \cdot \vec{q})\,(\vec{\epsilon} \cdot \vec{q}) \to \epsilon_i q_i \sigma_j q_j = \epsilon_i \sigma_j \left\{ \frac{1}{3} q^2 \delta_{ij} + \left(q_i q_j - \frac{1}{3} q^2 \delta_{ij} \right) \right\}$$

s-wave

$$\delta V_{PP} \to \begin{cases} \delta V^s_{PP} = \frac{1}{3} \delta V_{PP} \\ \delta V^d_{PP} = \frac{2}{3} \delta V_{PP} \end{cases}$$

In the same manner, one can also decompose the the VB-PB-VB box potentials with the same factor, 1/3 and 3/2

Sign of the effective transition potential

From the definition, each ingredient of the effective potential is found to be a doubled-value function.

$$\tilde{V}_{\text{eff}} = \pm\sqrt{\frac{\delta V(PB \to VB \to PB)}{G_{VB}}} \qquad \tilde{V}'_{\text{eff}} = \pm\sqrt{\frac{\delta V'(VB \to PB \to VB)}{G_{PB}}}$$

Not to be cancelled out, the two signs should be taken as the same

$$V_{\text{eff}} = \pm\frac{1}{2}\left(\tilde{V}_{\text{eff}} + \tilde{V}'_{\text{eff}}\right)$$

In the case of the two coupled channels, the change of sign of the transition potentials

$$V = \begin{pmatrix} V_{11} & \pm V_{12} \\ & V_{22} \end{pmatrix} \xrightarrow{\text{T=V+VGT}} T = \begin{pmatrix} T_{11} & \pm T_{12} \\ & T_{22} \end{pmatrix} \qquad T_{12}(\sqrt{s}) \sim \pm\frac{g_1 g_2}{\sqrt{s} - M_R + i\Gamma_R}$$

close to the pole

Energies of generated states: not depend

Relative sign of coupling constants (and wave function at the origin): depend

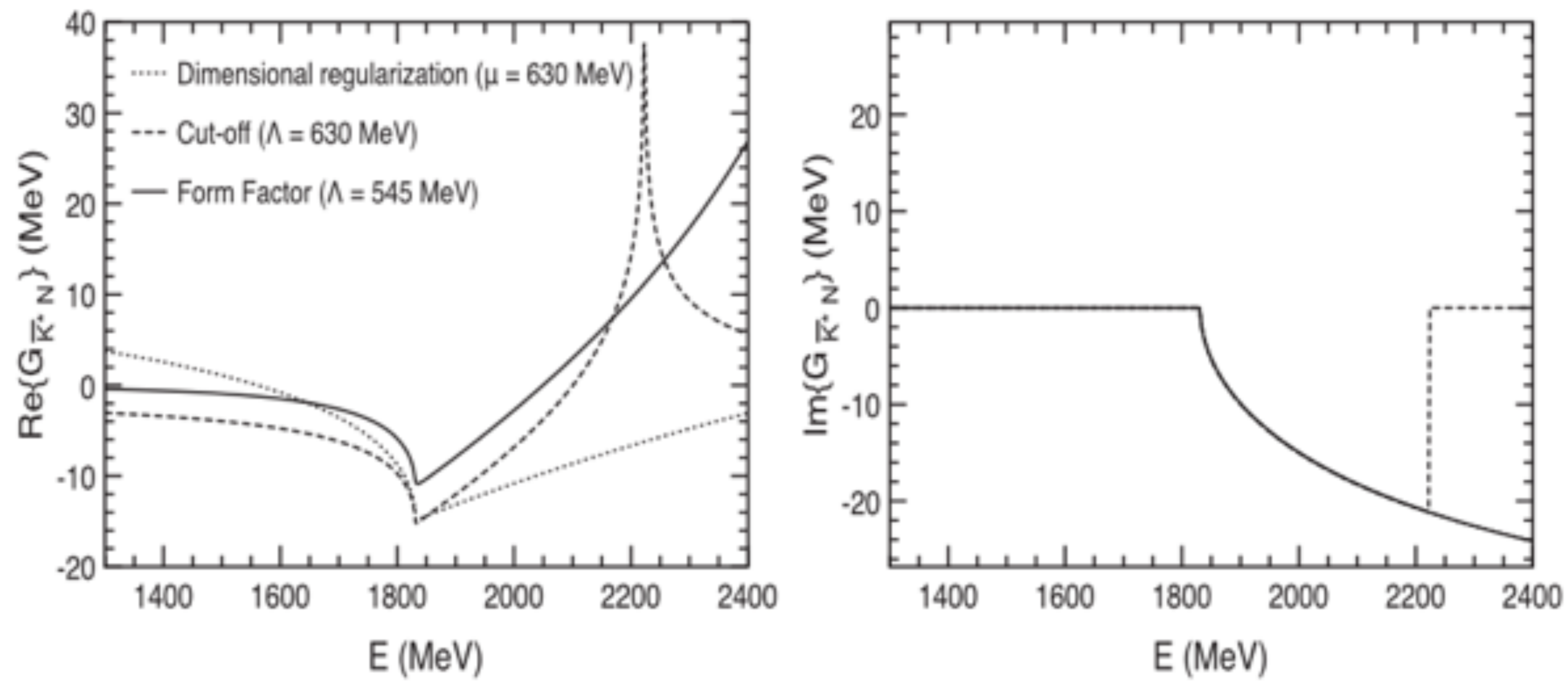
↓

In the present work, we utilize the effective potentials with negative real part

(as virtual pion is exchanged)

G function: dimensional regularisation vs cutoff

Real and imaginary part of the G function regularised in several ways



K. P. Khemchandani, A. Martinez Torres, H. Kaneko, H. Nagahiro and A. Hosaka, Phys. Rev. D 84, 094018 (2011)

Recalling the form of the scattering amplitude $T = V + VGT = \dfrac{V}{1 - VG}$

Even with repulsive interactions (ReV > 0), the denominator of the amplitude can be 0

Amplitude can have unphysical poles
below the threshold: dimensional regularisation
above the threshold: cut off

$\bar{D}\Lambda_c$ has the repulsive interaction and its threshold is below the generated states