EFFECTS OF THE SCALAR MESONS IN A SKYRME MODEL WITH HIDDEN LOCAL SYMMETRY

collaborator: Yong-Liang Ma
Masayasu Harada

Nstar @ Osaka University
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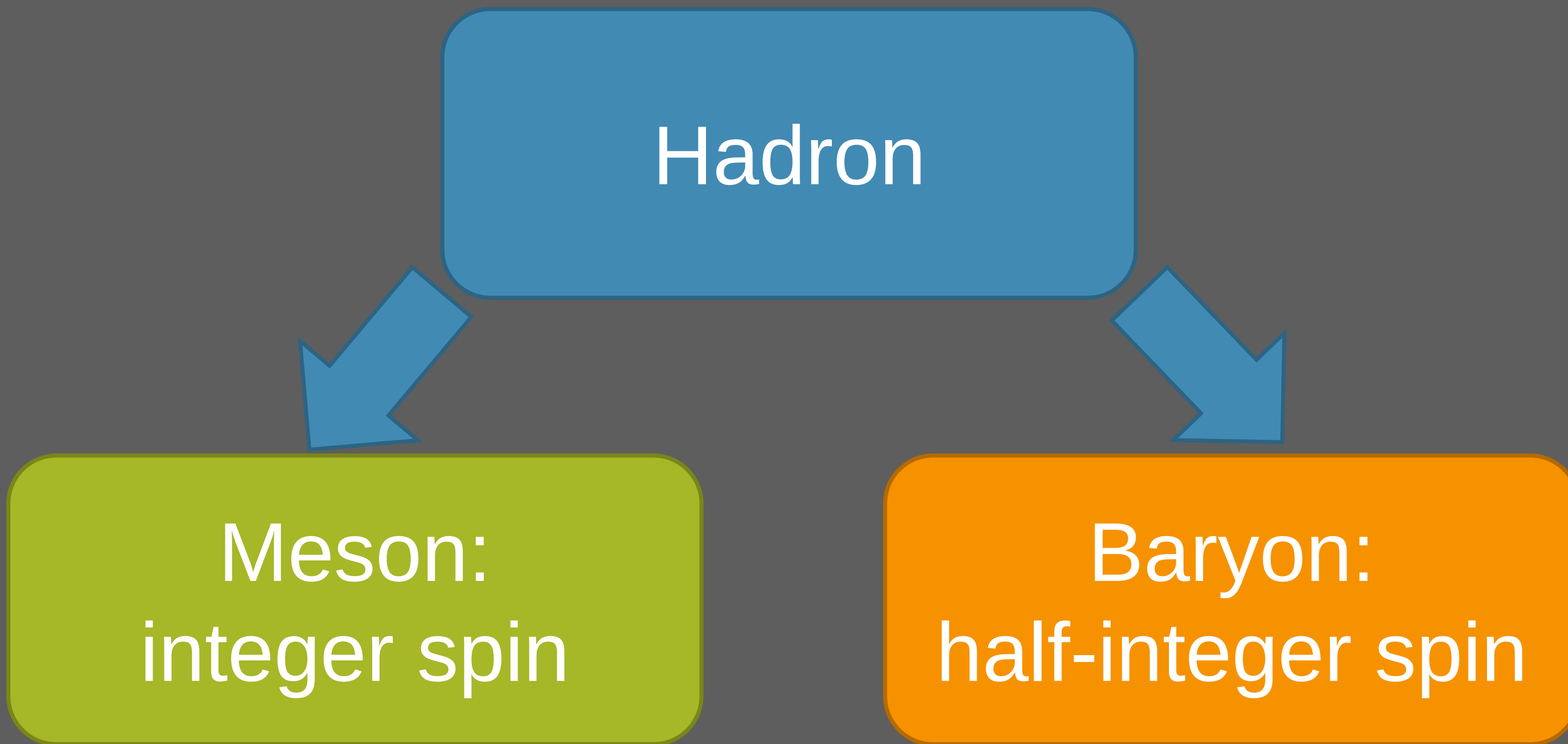
BingRan He
Nagoya University

Outline

- ***Introduction & Motivation***
- The Skyrmion model with π, ρ, ω
- Scalar meson mixing structure
- Numerical results
- Summary



A theory describe both meson sector and baryon sector in a consistent way?

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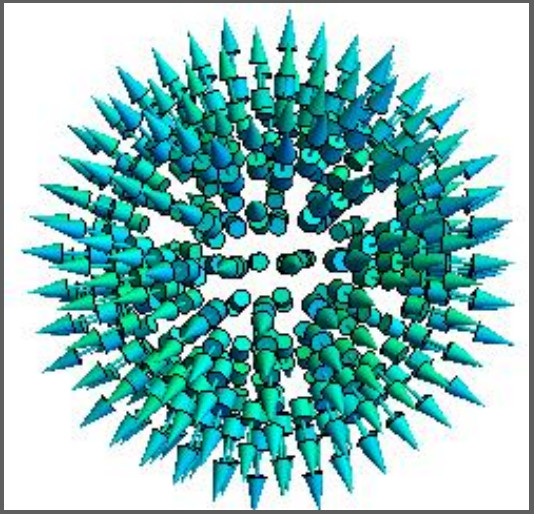
The original skyrmion mode



The nonlinear sigma model:

$$\begin{aligned} U &= \xi_L^\dagger \xi_R = e^{2i\frac{\pi(x)}{f_\pi}} \\ \xi_{L,R} &\rightarrow \xi_{L,R} \cdot g_{L,R}^\dagger \\ \xi_{L,R} &= e^{\mp i\frac{\pi(x)}{f_\pi}} \end{aligned}$$

The lagrangian:

Skyrmion term, repulsive force

$$\mathcal{L} = \frac{f_\pi^2}{16}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}([U^\dagger\partial_\mu U, U^\dagger \partial_\nu U]^2)$$

- Only pion is included. Due to the hedgehog ansatz, the isospin group $SU(2)$ mapping to space group $SO(3)$
- The skyrmion term, generating repulsive force to prevent the soliton shrink

T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962)

Incorporate the vector meson contribution

The hidden local symmetry:

$$
\begin{aligned}
U &= \xi_L^\dagger \xi_R = e^{2i\frac{\pi(x)}{f_\pi}} \\
\xi_{L,R} &\to h(x)\xi_{L,R} \cdot g_{L,R}^\dagger \\
\xi_{L,R} &= e^{i\frac{\sigma(x)}{f_\sigma}} e^{\mp i\frac{\pi(x)}{f_\pi}}
\end{aligned}
$$

$$h(x) \in H_{\mathrm{local}} \ , \quad g_{\mathrm{L,R}} \in G_{\mathrm{global}}$$

- The transformation for U do not changes, which seems that the freedom of vector meson is “hidden”

$$[SU(N_f)_L \times SU(N_f)_R]_{global} \times [SU(N_f)_V]_{local} \to [SU(N_f)_V]_{global}$$

The vector meson contribution

The lowest order HLS Lagrangian(π, ρ meson contribution)

$$\mathcal{L} = F_\pi^2 \,\mathrm{tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\right] + F_\sigma^2 \,\mathrm{tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu\right] - \frac{1}{2g^2}\,\mathrm{tr}\left[V_{\mu\nu}V^{\mu\nu}\right]$$

U. G. Meissner, N. Kaiser, A. Wirzba and W. Weise,
Phys. Rev. Lett. 57, 1676 (1986)

Repulsive force

$$\widehat{\alpha}_{\perp\mu} = \left(D_\mu\xi_\mathrm{R}\cdot\xi_\mathrm{R}^\dagger - D_\mu\xi_\mathrm{L}\cdot\xi_\mathrm{L}^\dagger\right)/(2i)\ ,$$

$$\widehat{\alpha}_{\parallel\mu} = \left(D_\mu\xi_\mathrm{R}\cdot\xi_\mathrm{R}^\dagger + D_\mu\xi_\mathrm{L}\cdot\xi_\mathrm{L}^\dagger\right)/(2i)\ .$$

$$D_\mu\xi_\mathrm{L} = \partial_\mu\xi_\mathrm{L} - iV_\mu\xi_\mathrm{L} + i\xi_\mathrm{L}\mathcal{L}_\mu\ ,$$

$$D_\mu\xi_\mathrm{R} = \partial_\mu\xi_\mathrm{R} - iV_\mu\xi_\mathrm{R} + i\xi_\mathrm{R}\mathcal{R}_\mu\ .$$

$$V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$$

$$V_\mu = \frac{g}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\left(\rho_\mu^0+\omega_\mu\right) & \rho_\mu^+ & K_\mu^{*,+} \\ \rho_\mu^- & -\frac{1}{\sqrt{2}}\left(\rho_\mu^0+\omega_\mu\right) & K_\mu^{*,0} \\ K_\mu^{*,-} & \bar{K}_\mu^{*,0} & \phi_\mu \end{pmatrix}$$

The Wess-Zumino term(ω meson contribution)

$$\int d^4x\mathcal{L}_\mathrm{anom} = \frac{N_c}{16\pi^2}\int_{M^4}\sum_{i=1}^{3} c_i\mathcal{L}_i$$

$$\mathcal{L}_1 = i\epsilon^{\mu\nu\sigma\rho}\,\mathrm{Tr}(\alpha_{L\mu}\alpha_{L\nu}\alpha_{L\sigma}\alpha_{R\rho} - \alpha_{R\mu}\alpha_{R\nu}\alpha_{R\sigma}\alpha_{L\rho})$$

$$\mathcal{L}_2 = i\epsilon^{\mu\nu\sigma\rho}\,\mathrm{Tr}(\alpha_{L\mu}\alpha_{R\nu}\alpha_{L\sigma}\alpha_{R\rho})$$

$$\mathcal{L}_3 = \epsilon^{\mu\nu\sigma\rho}\,\mathrm{Tr}[F_{V\mu\nu}(\alpha_{L\sigma}\alpha_{R\rho} - \alpha_{R\sigma}\alpha_{L\rho})]$$

Wess-Zumino term $\quad C_1 = -C_2 = \frac{2}{3}, C_3 = 0 \quad \omega_\mu B^\mu$

U. G. Meissner, N. Kaiser and W. Weise,
Nucl. Phys. A 466, 685 (1987)

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Scalar meson plays attractive effects

Model	Soliton mass [MeV]	$\sqrt{\langle r^2 \rangle_W}$ [fm]
π, ρ	1054.6	0.27
π, ρ, ω	1469.0	0.49
π, ρ, ω, χ	1408.3	0.51

- The incorporate of dilaton χ drops skyrmion mass
- The incorporate of dilaton χ changes charge radius of skyrmion

Quark model

When meson made by $\bar{q}q$ bound states

$$P = (-1)^{l+1},\ C = (-1)^{l+s}$$

$l = 0$

Pseudo-Scalar meson	$J^{PC} = 0^{-+}$	π
Vector meson	$J^{PC} = 1^{--}$	ω,ρ

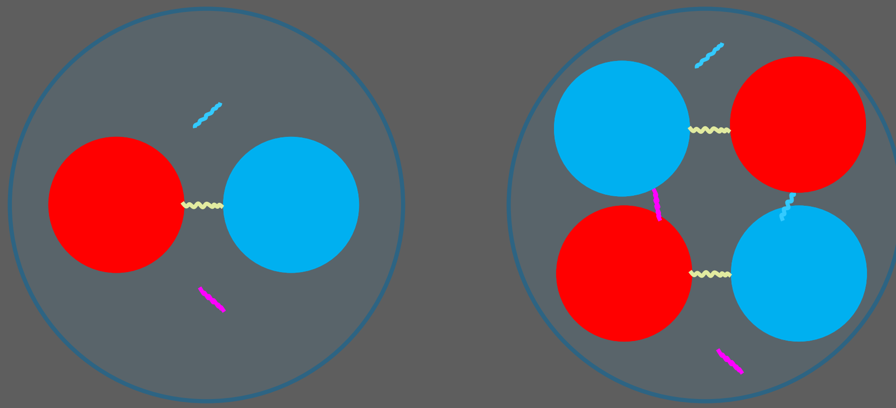
$l = 1$

Scalar meson	$J^{PC} = 0^{++}$	f_0
Axial-Vector meson	$J^{PC} = 1^{++,+-}$	a_1
Tenser	$J^{PC} = 2^{++}$	$f_2,\ a_2$

When meson made by $\overline{qq}qq$ bound states

Scalar meson can be made as $l = 0,\ J^{PC} = 0^{++}$

Scalar Meson Mixing mechanism



R. L. Jaffe, Phys. Rev. D15, 267 (1977)

Scalar meson below 1 GeV

$$I=0: m[f_0(600)] \approx 500 \text{ MeV}$$
$$I=1/2: m[\kappa] \approx 800 \text{ MeV}$$
$$I=0: m[f_0(980)] \approx 980 \text{ MeV}$$
$$I=1: m[a_0(980)] \approx 980 \text{ MeV}$$

State	$\bar{q}q\%$	$\bar{q}\bar{q}qq\%$	m (GeV)
a	24	76	0.984
a'	76	24	1.474
κ	8	92	1.067
κ'	92	8	1.624
f_1	40	60	0.742 $f_0(500)$
f_2	5	95	1.085
f_3	63	37	1.493 $f_0(1370)$
f_4	93	7	1.783

With q=u,d,s

H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D79, 074014 (2009)

The scalar mixing structure(q=u,d)

The 2 quark state and the 4 quark state for scalar meson

$$M_{(2)} = \frac{1}{2}\xi_L^\dagger \sigma \xi_R\,,\ \ M_{(4)} = \frac{1}{2}\phi$$

2 quark

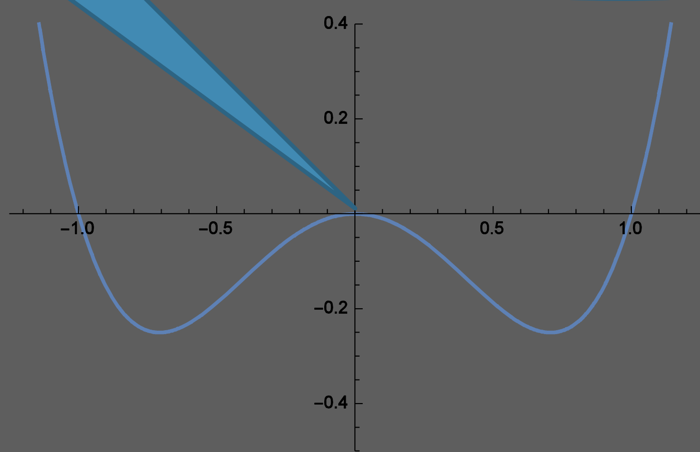
4 quark

The potential

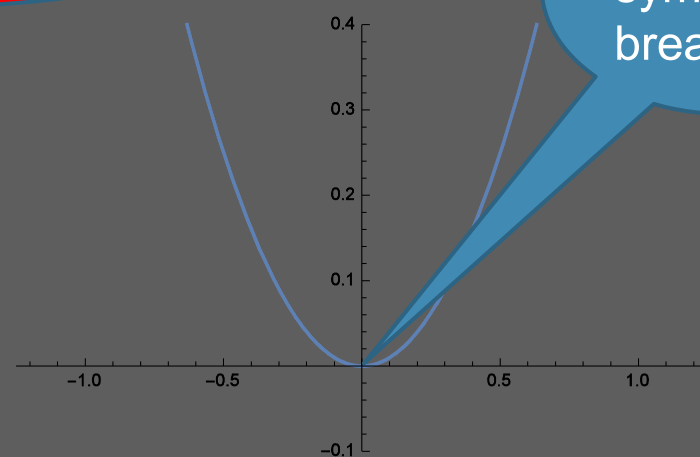
$$V_0 = \frac{1}{8}\lambda\sigma^4 - \frac{1}{2}m_2^2\sigma^2 + \frac{1}{2}m_4^2\phi^2 + \frac{1}{\sqrt{2}}A\sigma^2\phi$$

$$V_{\rm SB} = -\frac{1}{4}M_\pi^2 f_\pi \sigma\, \mathrm{Tr}(U + U^\dagger)$$

σ, before symmetry breaking



ϕ, before symmetry breaking



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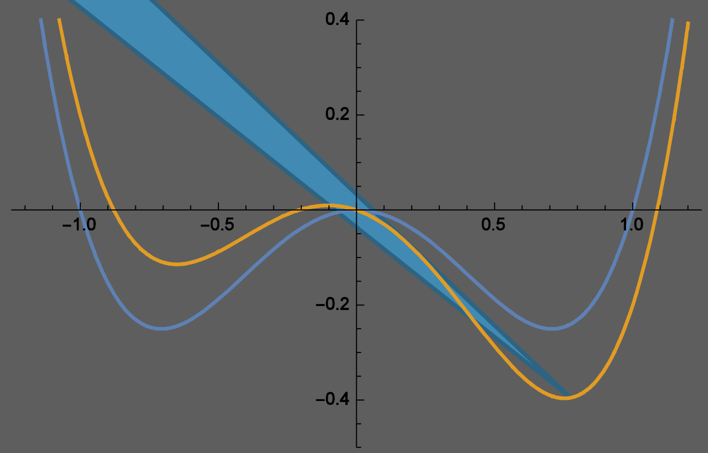
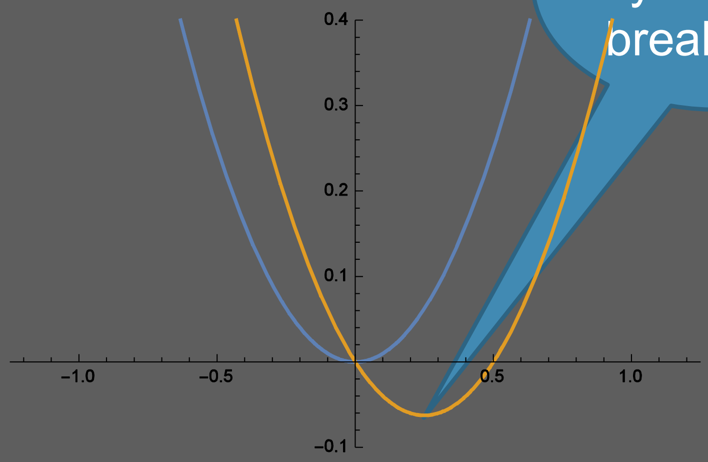
σ, after symmetry breaking



ϕ, after symmetry breaking



The scalar mixing structure

The scalar mesons

$$\sigma(x) = \bar{\sigma} + \tilde{\sigma}(x) = f_\pi + \tilde{\sigma}(x)$$

$$\phi(x) = \bar{\phi} + \tilde{\phi}(x)$$

- Scalar mesons have vacuum expectation value

The scalar mixing structure

$$\begin{pmatrix} f_{500} \\ f_{1370} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{\sigma} \\ \tilde{\phi} \end{pmatrix}$$

- The physical scalar meson are made by mixing 2-quark state and 4-quark state

The Lagrangian(q=u,d)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \sigma^2 \mathrm{Tr}(\alpha_{\perp\mu}\alpha_\perp^\mu) + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - (V_0 - \bar{V}_0) - (V_{\mathrm{SB}} - \bar{V}_{\mathrm{SB}})$$
$$+ a_{\mathrm{hls}}(s_0\sigma^2 + (1-s_0)f_\pi^2)\,\mathrm{Tr}(\alpha_{\parallel\mu}\alpha_\parallel^\mu) - \frac{1}{2g^2}\mathrm{Tr}(V_{\mu\nu}V^{\mu\nu}) + \mathcal{L}_{\mathrm{anom}}$$

Symmetry of the model

$SU(2)_L \times SU(2)_R \times U(1)_V$ → $SU(2)_V \times U(1)_V$

The model

Field	Operator	Physical fields
Pseudoscalar meson	$F(r)$	π
Vector meson	$W(r), G(r)$	ω, ρ
2-quark scalar meson	$\sigma(r) = f_\pi + \tilde{\sigma}$	$f_{500} = \cos(\theta)\,\tilde{\sigma} - \sin(\theta)\tilde{\phi}$
4-quark scalar meson	$\phi(r) = \phi_{vac} + \tilde{\phi}$	$f_{1370} = \sin(\theta)\,\tilde{\sigma} + \cos(\theta)\tilde{\phi}$

The properties of skrymion

The baryon number current

$$B_0 = -\frac{2}{3gr^2}\{f_\pi^2 g^2 r^2 a_{\rm hls} W\left[s_0\bar\sigma^2 + 2s_0\bar\sigma + 1\right] + F'\left[\alpha_2 - 2G\left(-\alpha_2+\alpha_3+\alpha_2\cos F\right) + \alpha_2\cos^2 F - 2\alpha_2\cos F + \left(\alpha_2-\alpha_3\right)G^2\right] - 2\alpha_3 \sin F G' + \alpha_1 \sin^2 F F F'\} - \frac{\sin^2 F}{2\pi^2 r^2} F'$$

- Baryon number current, by functional derivative the external field of Wess-Zumino action
- The integral of baryon number current over all space gives $\int dV\, B_0 = 1$

The charge radius(root-mean-square (rms) radius)

- Charge radius of the baryon-number current

$$\langle r^2\rangle_W^{1/2} = \sqrt{\int_0^\infty d^3r r^2 B(r)}$$

- Charge radius of the energy(soliton mass)

$$\langle r^2\rangle_E^{1/2} = \sqrt{\frac{1}{M_{\rm sol}}\int_0^\infty d^3r r^2 M_{\rm sol}(r)}$$

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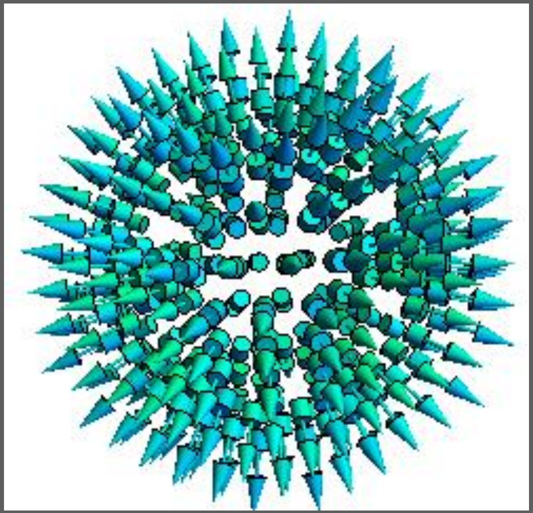
The ansatz

$$
\begin{aligned}
U &= \cos F(r) + i\vec{\tau}\cdot\hat{x}\sin F(r) \\
V_\mu &= \frac{1}{2}(g_\omega \omega_\mu + g_\rho \rho_\mu) \\
\rho^a_{\mu=i} &= \epsilon^{ika}\hat{r}^k \frac{G(r)}{g_\rho r} \\
\omega_\mu &= \delta_{\mu 0} W(r) \\
\sigma &= f_\pi(1+\bar{\sigma}(r)) \\
\phi &= \phi_{\text{vac}}(1+\bar{\phi}(r))
\end{aligned}
$$

Well known (V_μ, $\rho^a_{\mu=i}$, ω_μ)

Dilaton type (σ, ϕ)

hedgehog



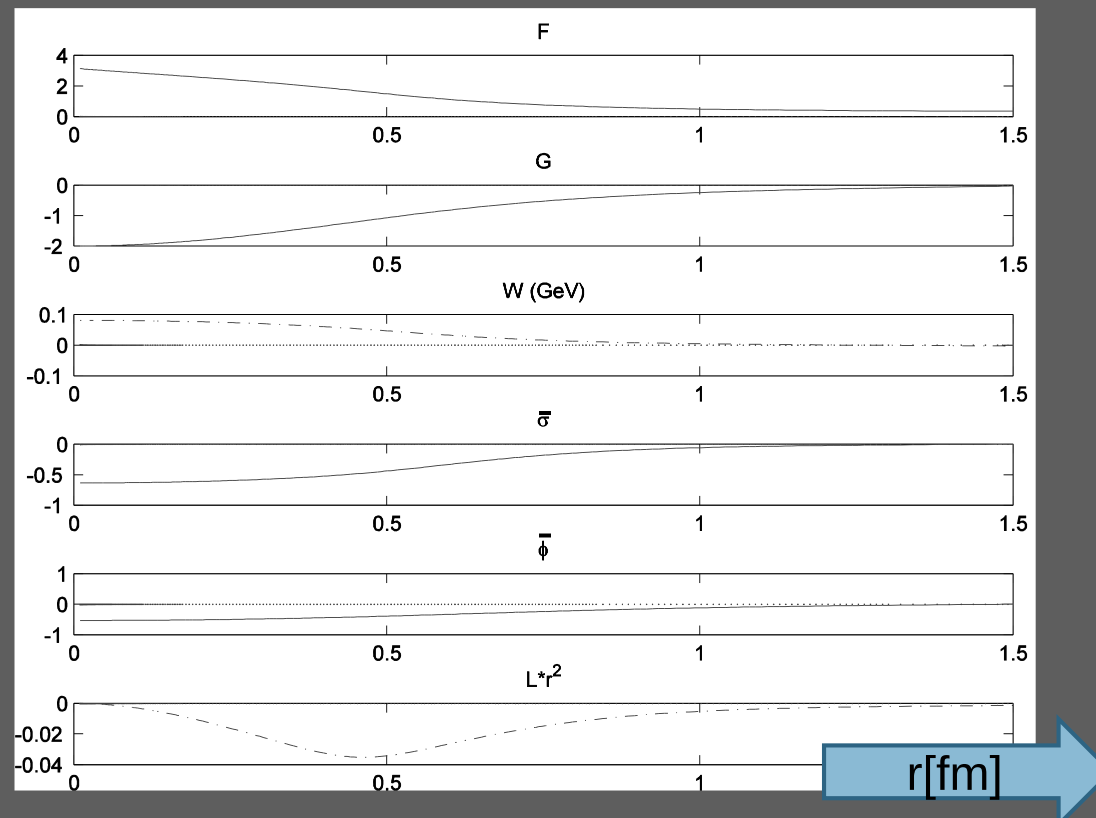
The ansatz for π, ρ, ω are consistent with literatures. Considering $U(2)_V$ symmetry, we take $g_\omega = g_\rho = g$.

The ansatz for σ, ϕ do not contains either space index nor isospin index, which are same with dilaton type scalar meson.

The boundary condition and profiles(B=1)

Boundary Condition

	$r \to 0$	$r \to \infty$
$F(r)$	π	0
$F'(r)$	-	-
$G(r)$	-2	0
$G'(\mathrm{r})$	-	-
$W(r)$	-	0
$W'(r)$	0	-
$\bar{\sigma}(r)$	-	0
$\bar{\sigma}'(r)$	0	-
$\bar{\phi}(r)$	-	0
$\bar{\phi}'(r)$	0	-



The boundary condition for π, ρ, ω are consistent with literatures, while the boundary condition for $\bar{\sigma}, \bar{\phi}$ are taken for the asymptotic solution when taking $r \to 0$, similar as dilaton case.

When scalar meson is made by pure 2 quark state

When taking $A = 0$

$$V_0 = \frac{1}{8}\lambda\sigma^4 - \frac{1}{2}m_2^2\sigma^2 + \frac{1}{2}m_4^2\phi^2 + \frac{1}{4}A\sigma^2$$

ϕ only have a trivial solution

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \sigma^2\,\mathrm{Tr}(\alpha_{\perp\mu}\alpha_\perp^\mu) - (V_\sigma - \bar{V}_\sigma) - (V_{\mathrm{SB}} - \bar{V}_{\mathrm{SB}}) \\ &+ a_{\mathrm{hls}}(s_0\sigma^2 + (1 - s_0)f_\pi^2)\,\mathrm{Tr}(\alpha_{\parallel\mu}\alpha_\parallel^\mu) - \frac{1}{2g^2}\,\mathrm{Tr}(V_{\mu\nu}V^{\mu\nu}) + \mathcal{L}_{\mathrm{anom}}\end{aligned}$$

$$V_\sigma = \frac{1}{8}\lambda\sigma^4 - \frac{1}{2}m_2^2\sigma^2$$

$$\lambda = (m_\sigma^2 - m_\pi^2)/f_\pi^2\,, \quad m_2^2 = \frac{1}{2}\left(m_\sigma^2 - 3m_\pi^2\right)\,.$$

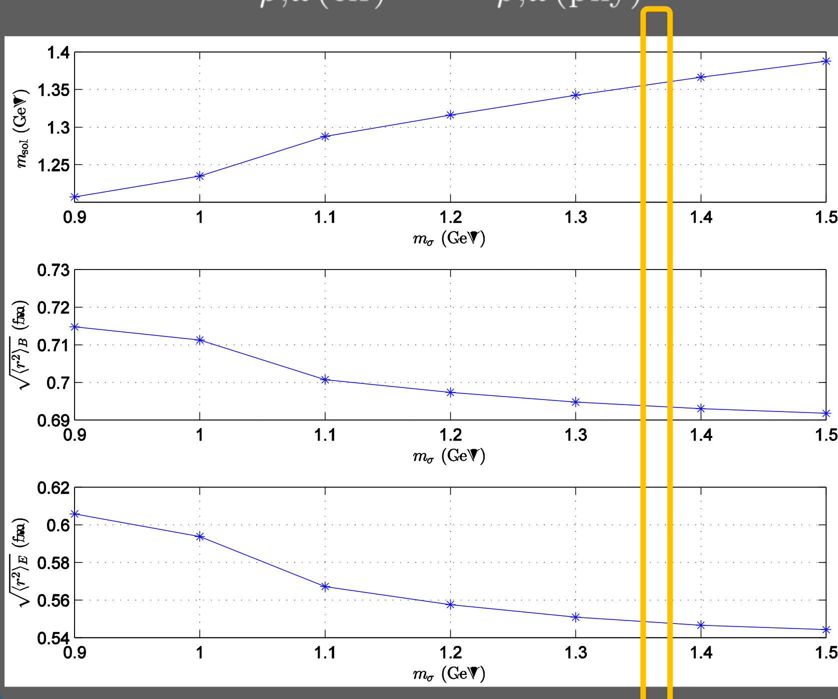
$$m^2_{\rho,\omega(\mathrm{eff})} = a g^2_{\rho,\omega} f_\pi^2 (1 - s_0)$$

m_σ modify potential part for scalar meson, s_0 modify effective vector meson mass inside skyrmion

Scalar meson is made by pure 2 quark state

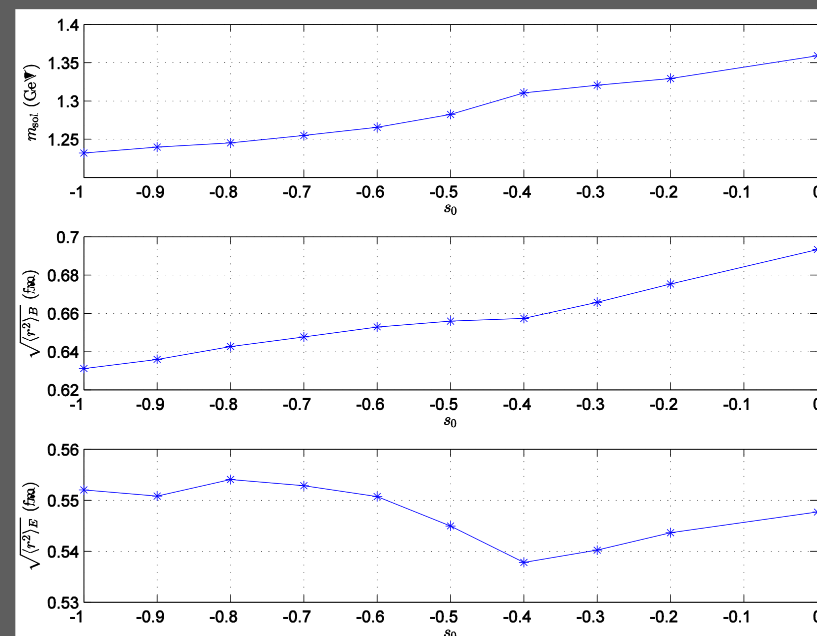
When taking $S_0 = 0$

$$m^2_{\rho,\omega(\text{eff})} = m^2_{\rho,\omega(\text{phy})}$$



When taking $m_\sigma = 1.37$ GeV

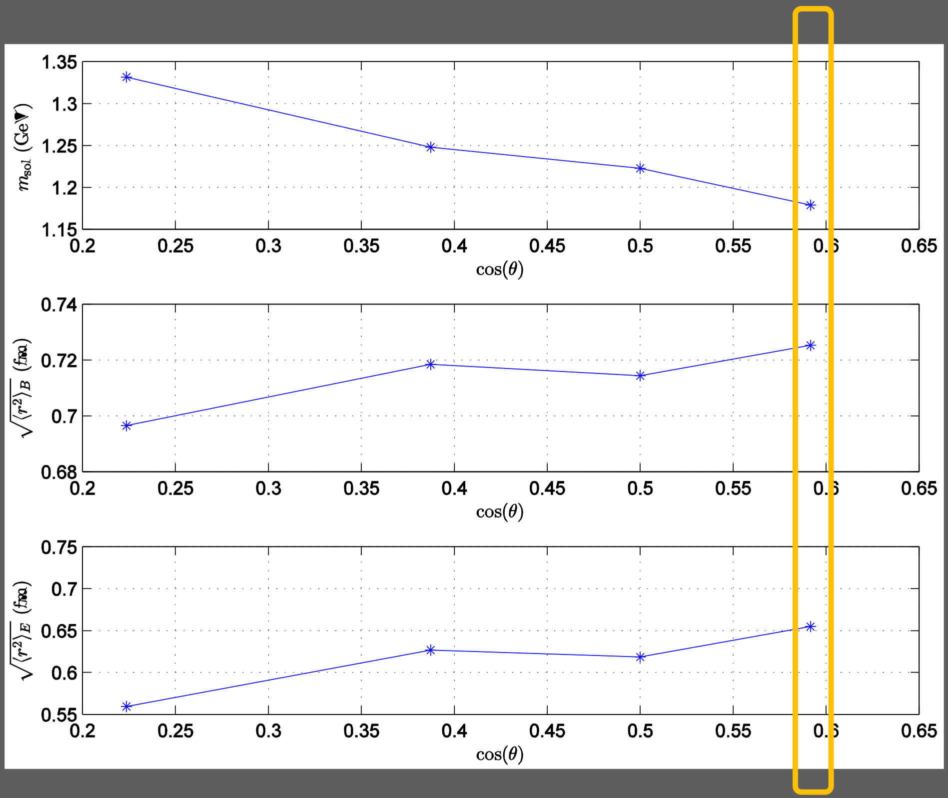
$$m^2_{\rho,\omega(\text{eff})} = a g^2_{\rho,\omega} f^2_\pi (1 - s_0)$$



The pure 2 quark scalar meson will drop about 100 MeV soliton mass, similar with dilaton scalar meson case.

The tendency of “charge radius - scalar meson mass” relation, depend on the way how scalar meson is incorporated.

Scalar meson is made by the mixing structure



State	$\bar{q}q\%$	$\bar{q}\bar{q}qq\%$	m (GeV)
$f_0(500)$	40	60	0.742
$f_0(1370)$	60	40	1.493

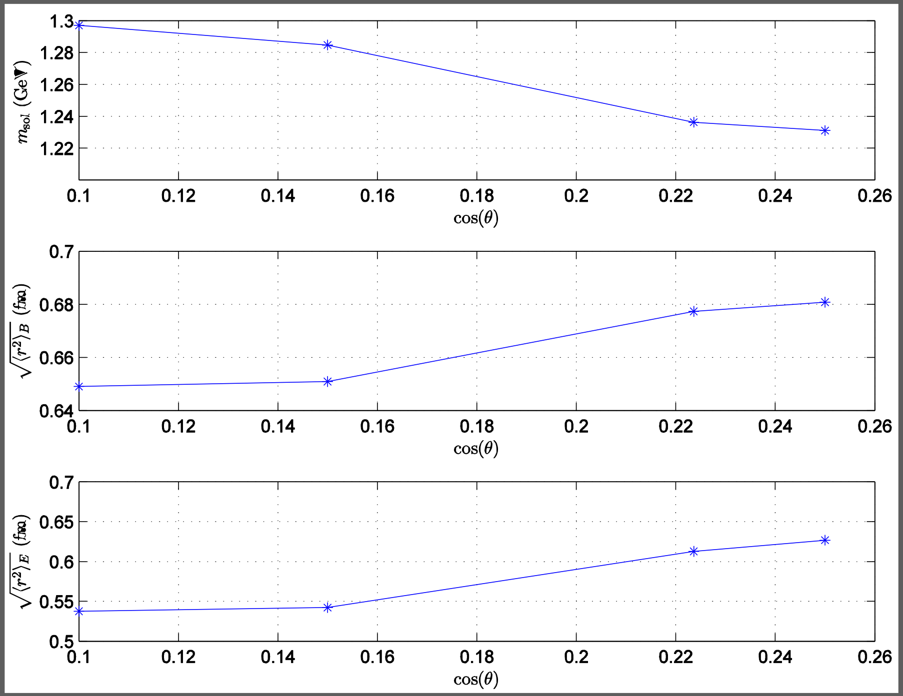
With q=u,d

$$\begin{pmatrix} \tilde{\sigma} \\ \tilde{\phi} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} f_{500} \\ f_{1370} \end{pmatrix}$$

The mixing structure of 2 quark state and 4 quark state will drop about 180 MeV soliton mass than pure 2 quark case, to get a more physical baryon mass.

Some alternative way to incorporate scalar meson

When taking $S_0 = -0.5$



The tendency are all same, but the allowed mixing strength is smaller than $S_0 = 0$ case, this is because $S_0 = -0.5$ corresponding to a heavier effective vector meson mass and weaker repulsive strength. So the allowed attractive force made by scalar meson becomes smaller.

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- The incorporate of scalar meson will drop the soliton mass.
- The result shows a lighter 2-quark state scalar meson reduce more soliton mass.
- When 2 quark and 4 quark mix with each other, the lighter “effective 2 quark scalar meson” mass is, the more lighter soliton mass becomes.
- The tendency of “charge radius - scalar meson mass” relation, depend on the way how scalar meson is incorporated.

Future works:

Modify the interaction term, Quantization the soliton, Dense media effects, EOS for neutron starts, …

Thank you for your attention!