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# Parity doublet model with the $U(3)_L \times U(3)_R$ chiral symmetry

work in progress


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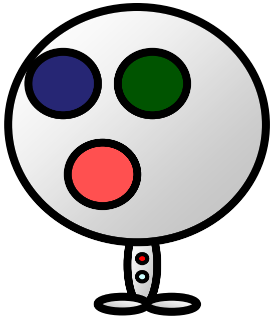


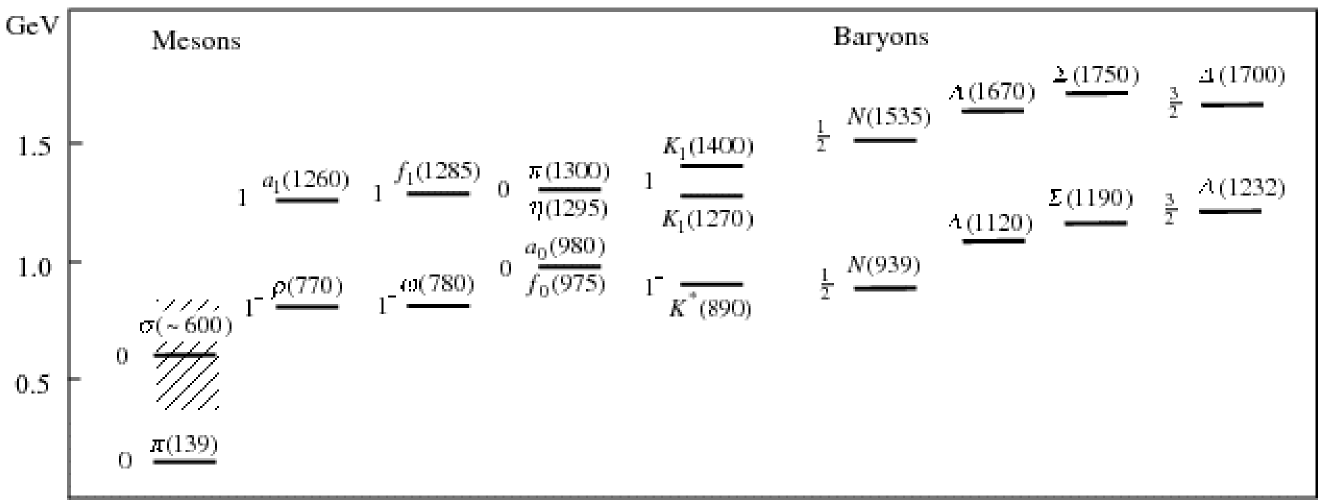
# Outline

1. Introduction
2. Three-flavor Parity doublet model
3. Extended Goldberger-Treiman relation
4. Summary

# Parity doubling structure

There are many hadrons.

D. Jido, M. Oka and A. Hosaka, Prog. Theor. Phys. **106**, 873 (2001)




## Parity doublet model

C. Dater & T. Kunihiro, PRD **39**, 2805 (1989)

Our purpose

Extend to three-flavor case

cf. H. X. Chen, V. Dmitrasinovic and A. Hosaka, Phys. Rev. D**83**, 014015 (2011)

# Outline

1. Introduction
2. **Three-flavor Parity doublet model**
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# Representations of the chiral sym.

Quarks belong to

$$q_L \sim \underline{(\mathbf{3},\mathbf{1})}_{-1} \qquad q_R \sim (\mathbf{1},\mathbf{3})_{\underline{+1}}$$

$\mathbf{SU(3)}_{L,R}$ $\qquad$ $\mathbf{U(1)}_A$ charge

Baryon

Rep. of 3q baryon

$$q_L \otimes q_R \otimes q_R \sim (\mathbf{3},\mathbf{1})_{-1} \otimes (\mathbf{1},\mathbf{3})_{+1} \otimes (\mathbf{1},\mathbf{3})_{+1}$$
$$\sim \underline{(\mathbf{3},\bar{\mathbf{3}})_{+1}} \oplus \underline{(\mathbf{3},\mathbf{6})_{+1}} \ ,$$

$$q_L \otimes q_L \otimes q_L \sim (\mathbf{3},\mathbf{1})_{-1} \otimes (\mathbf{3},\mathbf{1})_{-1} \otimes (\mathbf{3},\mathbf{1})_{-1}$$
$$\sim (\mathbf{1},\mathbf{1})_{-3} \oplus 2\underline{(\mathbf{8},\mathbf{1})_{-3}} \oplus (\mathbf{10},\mathbf{1})_{-3}$$

These rep. yield octet-baryons.

# Introduce fields

6 baryon fields and one meson field are introduced as

Chirality

Baryon

$$\begin{cases} \Psi_{1l} \sim (\mathbf{3}, \bar{\mathbf{3}})_{+1} \\ \Psi_{1r} \sim (\bar{\mathbf{3}}, \mathbf{3})_{-1} \end{cases} \quad \begin{cases} \Psi_{2l} \sim (\bar{\mathbf{3}}, \mathbf{3})_{-1} \\ \Psi_{2r} \sim (\mathbf{3}, \bar{\mathbf{3}})_{+1} \end{cases}$$

$$\begin{cases} \eta_{1l} \sim (\mathbf{3}, \mathbf{6})_{+1} \\ \eta_{1r} \sim (\mathbf{6}, \mathbf{3})_{-1} \end{cases} \quad \begin{cases} \eta_{2l} \sim (\mathbf{6}, \mathbf{3})_{-1} \\ \eta_{2r} \sim (\mathbf{3}, \mathbf{6})_{+1} \end{cases}$$

$$\begin{cases} \chi_{1l} \sim (\mathbf{8}, \mathbf{1})_{-3} \\ \chi_{1r} \sim (\mathbf{1}, \mathbf{8})_{+3} \end{cases} \quad \begin{cases} \chi_{2l} \sim (\mathbf{1}, \mathbf{8})_{+3} \\ \chi_{2r} \sim (\mathbf{8}, \mathbf{1})_{-3} \end{cases}$$

Meson $\quad M \sim (\mathbf{3}\ ,\ \bar{\mathbf{3}})_{-2}$

In the following we focus the study on the spin 1/2 baryon.

# Allowed operators

Contract the indices corresponding to the chiral sym.

e.g.

$$\epsilon_{abc}\left(\bar{\Psi}_{1r}\right)^{a}_{\alpha}\left(M\right)^{b}_{\beta}\left(\eta_{1l}\right)^{(c,\alpha\beta)}$$

$a,\ b,\ c$ : $\mathbf{SU(3)}_L$
$\alpha,\ \beta$ : $\mathbf{SU(3)}_R$

$\mathbf{U(1)}_A$: $+1$ $-2$ $+1$ $=0$

Requiring
$\mathbf{U(3)}_L\times\mathbf{U(3)}_R$ , $P$, and $C$.

Lagrangian

Kinetic term

Yukawa interaction

Chiral inv. mass

$$\mathcal{L}=\sum_{k=1}^{6}\mathcal{K}^{(k)}+\sum_{k=1}^{4}g_k\mathcal{W}^{(k)}+\sum_{k=1}^{6}y_k\left(\mathcal{O}^{(k)}+\mathcal{O}^{(k)\dagger}\right)+\sum_{k=1}^{3}m_0^{(k)}\left(\mathcal{Q}^{(k)}+\mathcal{Q}^{(k)\dagger}\right)+\mathcal{L}_{\text{meson}}$$

**19 operators**

# Decomposition to irreducible rep.

$\langle M \rangle = \sigma 1 \neq 0$ : Chiral sym. $\xrightarrow{\text{SSB}}$ Flavor sym.

In the chiral braking phase, the chiral rep. are decomposed to irreducible rep. of the flavor symmetry.

$$\Psi_i = B_i^{(1)} + \frac{1}{\sqrt{3}} \Lambda_i^{(4)} \mathbf{1} \ , \quad \longrightarrow B^{(1)}$$

$$\left\{ \begin{array}{l} \eta_{1l,2r}^{(a,\alpha\beta)} = \Delta_{1l,2r}^{a\alpha\beta} + \frac{1}{\sqrt{6}} \left( \epsilon^{\alpha ac} \delta_k^{\beta} + \epsilon^{\beta ac} \delta_k^{\alpha} \right) \left( B_{1l,2r}^{(2)} \right)_c^k \\ \eta_{1r,2l}^{(ab,\alpha)} = \Delta_{1r,2l}^{ab\alpha} + \frac{1}{\sqrt{6}} \left( \epsilon^{a\alpha c} \delta_k^{b} + \epsilon^{b\alpha c} \delta_k^{a} \right) \left( B_{1r,2l}^{(2)} \right)_c^k \end{array} \right. \quad \longrightarrow B^{(2)}$$

$$\chi_i = B_i^{(3)} \ . \quad \longrightarrow B^{(3)}$$

They are assigned as

Octet-baryon

$$B_b^a = \begin{pmatrix} B_1^1 & B_2^1 & B_3^1 \\ B_1^2 & B_2^2 & B_3^2 \\ B_1^3 & B_2^3 & B_3^3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & \frac{-1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & \frac{-2}{\sqrt{6}}\Lambda \end{pmatrix}$$

# F- and D-term

They are obtained from the kinetic terms as

Anti-commutation Commutation

$$\mathrm{Tr}\left[\bar{B}\gamma_5\gamma^\mu\,(D)_{6\times6}\,\{A_\mu, B\}\right] + \mathrm{Tr}\left[\bar{B}\gamma_5\gamma^\mu\,(F)_{6\times6}\,[A_\mu, B]\right]$$

Trace works in flavor space

Axial vector of the external gauge field

where

$$(F)_{6\times6} = \mathrm{diag}\left(0,\ \frac{2}{3},\ -1,\ 0,\ -\frac{2}{3},\ 1\right)$$

$$(D)_{6\times6} = \mathrm{diag}\,(-1,\ 1,\ 0,\ 1,\ -1,\ 0)\ ,$$

$$B^T \equiv \left(B_1^{(1)}\ \ B_1^{(2)}\ \ B_1^{(3)}\ \ B_2^{(1)}\ \ B_2^{(2)}\ \ B_2^{(3)}\right)$$

# Mass matrix

Mass term for the nucleons is obtained as $-\bar{N}' M'_N N'$

where

$$N'^T \equiv \begin{pmatrix} N'^{(1)}_1 & N'^{(2)}_1 & N'^{(3)}_1 & N'^{(1)}_2 & N'^{(2)}_2 & N'^{(3)}_2 \end{pmatrix}$$

$$M'_N = \begin{pmatrix} \frac{g_1}{2}f_\pi & -\frac{3y_1}{2\sqrt{6}}f_\pi & -\frac{y_2}{2}f_\pi & m_0^{(1)} & 0 & 0 \\ -\frac{3y_1}{2\sqrt{6}}f_\pi & \frac{g_3}{4}f_\pi & -\frac{3y_5}{2\sqrt{6}}f_\pi & 0 & m_0^{(2)} & 0 \\ -\frac{y_2}{2}f_\pi & -\frac{3y_5}{2\sqrt{6}}f_\pi & 0 & 0 & 0 & m_0^{(3)} \\ m_0^{(1)} & 0 & 0 & -\frac{g_2}{2}f_\pi & \frac{3y_3}{2\sqrt{6}}f_\pi & \frac{y_4}{2}f_\pi \\ 0 & m_0^{(2)} & 0 & \frac{3y_3}{2\sqrt{6}}f_\pi & -\frac{g_4}{4}f_\pi & \frac{3y_6}{2\sqrt{6}}f_\pi \\ 0 & 0 & m_0^{(3)} & \frac{y_4}{2}f_\pi & \frac{3y_6}{2\sqrt{6}}f_\pi & 0 \end{pmatrix}$$

The physical states are defined as

$$N^T_{\rm phys} = \begin{pmatrix} N(939) & N(1440) & N(1710) & \\ & N(1535) & N(1650) & \underline{N^{\rm 6th}} \end{pmatrix}_{\rm phys}$$

A candidate is $N(1895)$

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# Extended Goldberger-Treiman relation

Interaction term is written as $\bar{N}'C'_{\pi NN}i\gamma_5\pi N'$

Coupling constant in the matrix form

$$C'_{\pi NN} = \begin{pmatrix} -\frac{g_1}{2} & -\frac{y_1}{2\sqrt{6}} & \frac{y_2}{2} & 0 & 0 & 0 \\ -\frac{y_1}{2\sqrt{6}} & \frac{5g_3}{12} & \frac{y_5}{2\sqrt{6}} & 0 & 0 & 0 \\ \frac{y_2}{2} & \frac{y_5}{2\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{g_2}{2} & \frac{y_3}{2\sqrt{6}} & -\frac{y_4}{2} \\ 0 & 0 & 0 & \frac{y_3}{2\sqrt{6}} & -\frac{5g_4}{12} & -\frac{y_6}{2\sqrt{6}} \\ 0 & 0 & 0 & -\frac{y_4}{2} & -\frac{y_6}{2\sqrt{6}} & 0 \end{pmatrix}$$

$$C'_{\pi NN} = \frac{1}{2f_\pi}\left\{(F+D)_{6\times 6}\,,M'_N\right\}$$

Extended Goldberger-Treiman relation

cf.

GT-relation

$$g_{\pi NN} = \frac{m_N}{f_\pi}g_A$$

# Upper bound - Derivation

The GT relation gives us an upper bound of the decay width

$$C'_{\pi NN} = \frac{1}{2f_\pi}\{(F+D)_{6\times6}\,,M'_N\}$$

Eigenvector of the mass matrix $M'_N$

sandwich in $u$

Eigenvalue

$$g'_{\pi N_i N_j} \equiv u_i^T \cdot C'_{\pi NN} \cdot u_j = \frac{\lambda_{N_i}+\lambda_{N_j}}{2f_\pi}\left[u_i^T \cdot (F+D)_{6\times6} \cdot u_j\right]$$

Using completeness: $\sum_j u_j^T \cdot u_j = 1$

$$\sum_j \frac{f_\pi^2}{(\lambda_{N_i}+\lambda_{N_j})^2}\left(g'_{\pi N_i N_j}\right)^2 = \frac{1}{4}\left[u_i^T \cdot (F+D)^2_{6\times6} \cdot u_i\right] \le \frac{25}{36}$$

$\propto$ Decay width

for any $i$

# Upper bound - Estimate

The upper bound is rewritten as

$$\sum_j \frac{f_\pi^2}{\left(\lambda_{N_i}+\lambda_{N_j}\right)^2}\left(g'_{\pi N_i N_j}\right)^2 \le \frac{25}{36}$$

$$\frac{1}{4}\left(g_A\right)^2+\sum_{N^*}\tilde{\Gamma}\left(N^*\right)\le\frac{25}{36}$$

where $\tilde{\Gamma}\left(N^*\right)\equiv\frac{16\pi}{3}\frac{m_{N^*}}{|\vec{p}|}\left[\sqrt{|\vec{p}|^2+m_N^2}\mp m_N\right]^{-1}\frac{f_\pi^2}{\left(m_{N^*}\pm m_N\right)^2}$

$$\times\,\Gamma(N^*\to N+\pi)$$

Input experimental data

$$\frac{1}{4}g_A^2=0.4026\pm 0.0004$$

$$\tilde{\Gamma}^{\rm exp}\left(N\left(1535\right)\right)=0.047\ \pm\ 0.013$$

$$\tilde{\Gamma}^{\rm exp}\left(N\left(1650\right)\right)=0.042\ \pm\ 0.015$$

$$\tilde{\Gamma}^{\rm exp}\left(N\left(1710\right)\right)=0.0046\ \pm\ 0.0071$$

We set

$$\tilde{\Gamma}\left(N^{6\rm th}\right)=0$$

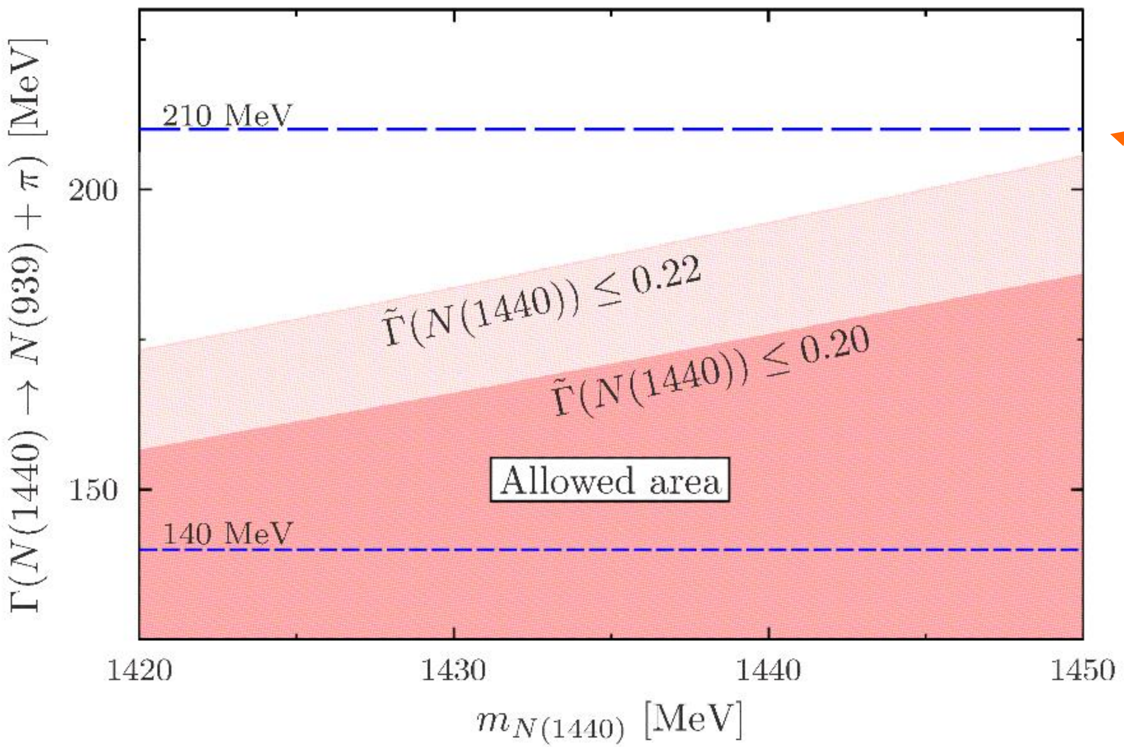
$$\Rightarrow\quad\tilde{\Gamma}\left(N(1440)\right)\le 0.20\pm 0.02$$

PDG + T. Yamanishi, PRD **76**, 014006 (2007)

# Allowed area

The upper bound is obtain from the extended GT relation

$$\tilde{\Gamma}\,(N(1440)) \le 0.20 \pm 0.02$$




$N(1440) \to N\pi$

Exp. (PDG)
210 $\pm$ 70 MeV

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# Summary

- We constructed a parity doublet model
  with the $U(3) \times U(3)$ chiral symmetry

- We found the extended Goldberger-Treiman relation

- This gives us an upper bound of the decay width

<u>Future work</u>

- Hyperons sector
- Finite density & temperature



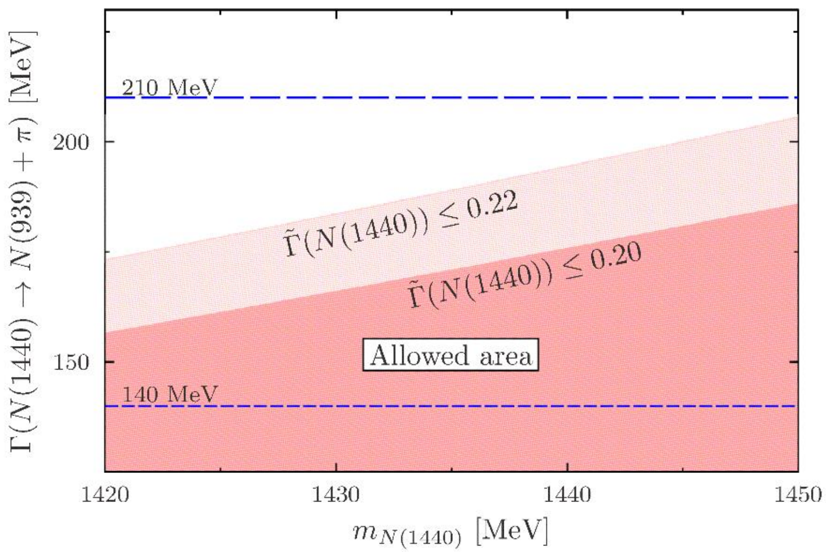
*Thank you for your attentions.*

Back up

# Test of the model

When the experimental data are improved

$$\tilde{\Gamma}\left(N(1440)\right) \leq 0.20 \pm 0.02$$




If it appear out of the allowed area

We have to take account of a molecular state.

This could be a test whether $N(1440)$ is a molecular state or not.

# Redefinition

For convenience, we redefine the baryon field as

$$\psi'_1 = \psi_1 \ , \quad \psi'_2 = \gamma_5 \psi_2$$

⇓

After this, we can regard these as positive parity fields.

Corresponding to this.

$M$ ⇒ $M'$

$C$ ⇒ $C'$

One with a prime does not include Dirac matrices $\gamma_5$

# Explicit form

$$\mathcal{K} \equiv \mathcal{K}_L + \mathcal{K}_R \ , \quad \mathcal{W} \equiv \mathcal{W}_L + \mathcal{W}_R$$
$$\mathcal{O} \equiv \mathcal{O}_L + \mathcal{O}_R \ , \quad \mathcal{Q} \equiv \mathcal{Q}_L + \mathcal{Q}_R \ .$$

Lists of the allowed operators

| $k$ | $\mathcal{O}_R^{(k)}$ | $\mathcal{O}_L^{(k)}$ |
|---|---|---|
| 1 | $\epsilon_{\alpha\beta\sigma}\left(\bar{\Psi}_{1l}\right)_a^{\alpha}\left(M^{\dagger}\right)_b^{\beta}\left(\eta_{1r}\right)^{(ab,\sigma)}$ | $\epsilon_{abc}\left(\bar{\Psi}_{1r}\right)_{\alpha}^{a}\left(M\right)_{\beta}^{b}\left(\eta_{1l}\right)^{(c,\alpha\beta)}$ |
| 2 | $\mathrm{Tr}\left[\bar{\Psi}_{1l}M\chi_{1r}\right]$ | $\mathrm{Tr}\left[\bar{\Psi}_{1r}M^{\dagger}\chi_{1l}\right]$ |
| 3 | $\epsilon_{abc}\left(\bar{\Psi}_{2l}\right)_{\alpha}^{a}\left(M\right)_{\beta}^{b}\left(\eta_{2r}\right)^{(c,\alpha\beta)}$ | $\epsilon_{\alpha\beta\sigma}\left(\bar{\Psi}_{2r}\right)_a^{\alpha}\left(M^{\dagger}\right)_b^{\beta}\left(\eta_{2l}\right)^{(ab,\sigma)}$ |
| 4 | $\mathrm{Tr}\left[\bar{\Psi}_{2l}M^{\dagger}\chi_{2r}\right]$ | $\mathrm{Tr}\left[\bar{\Psi}_{2r}M\chi_{2l}\right]$ |
| 5 | $\epsilon^{\alpha\sigma\rho}\left(\bar{\eta}_{1l}\right)_{(a,\alpha\beta)}\left(M\right)_{\sigma}^{a}\left(\chi_{1r}\right)_{\rho}^{\beta}$ | $\epsilon^{acd}\left(\bar{\eta}_{1r}\right)_{(ab,\alpha)}\left(M^{\dagger}\right)_c^{\alpha}\left(\chi_{1l}\right)_d^{b}$ |
| 6 | $\epsilon^{acd}\left(\bar{\eta}_{2l}\right)_{(ab,\alpha)}\left(M^{\dagger}\right)_c^{\alpha}\left(\chi_{2r}\right)_d^{b}$ | $\epsilon^{\alpha\sigma\rho}\left(\bar{\eta}_{2r}\right)_{(a,\alpha\beta)}\left(M\right)_{\sigma}^{a}\left(\chi_{2l}\right)_{\rho}^{\beta}$ |

| $k$ | $\mathcal{K}_L^{(k)}$ | $\mathcal{K}_R^{(k)}$ |
|---|---|---|
| 1 | $\mathrm{Tr}\left[\bar{\Psi}_{1l}i\not{D}\Psi_{1l}\right]$ | $\mathrm{Tr}\left[\bar{\Psi}_{1r}i\not{D}\Psi_{1r}\right]$ |
| 2 | $\mathrm{Tr}\left[\bar{\Psi}_{2l}i\not{D}\Psi_{2l}\right]$ | $\mathrm{Tr}\left[\bar{\Psi}_{2r}i\not{D}\Psi_{2r}\right]$ |
| 4 | $\left(\bar{\eta}_{1l}\right)_{(a,\alpha\beta)}i\not{D}\left(\eta_{1l}\right)^{(a,\alpha\beta)}$ | $\left(\bar{\eta}_{1r}\right)_{(ab,\alpha)}i\not{D}\left(\eta_{1r}\right)^{(ab,\alpha)}$ |
| 3 | $\left(\bar{\eta}_{2l}\right)_{(ab,\alpha)}i\not{D}\left(\eta_{2l}\right)^{(ab,\alpha)}$ | $\left(\bar{\eta}_{2r}\right)_{(a,\alpha\beta)}i\not{D}\left(\eta_{2r}\right)^{(a,\alpha\beta)}$ |
| 5 | $\mathrm{Tr}\left[\bar{\chi}_{1l}i\not{D}\chi_{1l}\right]$ | $\mathrm{Tr}\left[\bar{\chi}_{1r}i\not{D}\chi_{1r}\right]$ |
| 6 | $\mathrm{Tr}\left[\bar{\chi}_{2l}i\not{D}\chi_{2l}\right]$ | $\mathrm{Tr}\left[\bar{\chi}_{2r}i\not{D}\chi_{2r}\right]$ |

| $k$ | $\mathcal{W}_R^{(k)}$ | $\mathcal{W}_L^{(k)}$ |
|---|---|---|
| 1 | $\epsilon^{abc}\epsilon_{\alpha\beta\sigma}\left(\bar{\Psi}_{1l}\right)_a^{\alpha}\left(M^{\dagger}\right)_b^{\beta}\left(\Psi_{1r}\right)_c^{\sigma}$ | $\epsilon_{abc}\epsilon^{\alpha\beta\sigma}\left(\bar{\Psi}_{1r}\right)_{\alpha}^{a}\left(M\right)_{\beta}^{b}\left(\Psi_{1l}\right)_{\sigma}^{c}$ |
| 2 | $\epsilon_{abc}\epsilon^{\alpha\beta\sigma}\left(\bar{\Psi}_{2l}\right)_{\alpha}^{a}\left(M\right)_{\beta}^{b}\left(\Psi_{2r}\right)_{\sigma}^{c}$ | $\epsilon^{abc}\epsilon_{\alpha\beta\sigma}\left(\bar{\Psi}_{2r}\right)_a^{\alpha}\left(M^{\dagger}\right)_b^{\beta}\left(\Psi_{2l}\right)_c^{\sigma}$ |
| 3 | $\left(\bar{\eta}_{1l}\right)_{(a,\alpha\beta)}\left(M^{\dagger}\right)_b^{\alpha}\left(\eta_{1r}\right)^{(ab,\beta)}$ | $\left(\bar{\eta}_{1r}\right)_{(ab,\alpha)}\left(M\right)_{\beta}^{a}\left(\eta_{1l}\right)^{(b,\alpha\beta)}$ |
| 4 | $\left(\bar{\eta}_{2l}\right)_{(ab,\alpha)}\left(M\right)_{\beta}^{a}\left(\eta_{2r}\right)^{(b,\alpha\beta)}$ | $\left(\bar{\eta}_{2r}\right)_{(a,\alpha\beta)}\left(M^{\dagger}\right)_b^{\alpha}\left(\eta_{2l}\right)^{(ab,\beta)}$ |

| $k$ | $\mathcal{Q}_R^{(k)}$ | $\mathcal{Q}_L^{(k)}$ |
|---|---|---|
| 1 | $\mathrm{Tr}\left[\bar{\Psi}_{1l}\Psi_{2r}\right]$ | $-\mathrm{Tr}\left[\bar{\Psi}_{1r}\Psi_{2l}\right]$ |
| 2 | $\left(\bar{\eta}_{1l}\right)_{(a,\alpha\beta)}\left(\eta_{2r}\right)^{(a,\alpha\beta)}$ | $-\left(\bar{\eta}_{1r}\right)_{(ab,\alpha)}\left(\eta_{2l}\right)^{(ab,\alpha)}$ |
| 3 | $\mathrm{Tr}\left[\bar{\chi}_{1l}\chi_{2r}\right]$ | $-\mathrm{Tr}\left[\bar{\chi}_{1r}\chi_{2l}\right]$ |

# Properties for $P$ and $C$

Properties of fields

$$\psi_{1l,1r} \xrightarrow{\mathcal{P}} \gamma_0 \; \psi_{1r,1l} \;, \qquad \psi_{2l,2r} \xrightarrow{\mathcal{P}} -\gamma_0 \; \psi_{2r,2l} \;,$$

$$\psi_{1l,1r} \xrightarrow{\mathcal{C}} C \left(\bar{\psi}_{1r,1l}\right)^T \;, \quad \psi_{2l,2r} \xrightarrow{\mathcal{C}} -C \left(\bar{\psi}_{2r,2l}\right)^T$$

$$\mathcal{K}_L \overset{\mathcal{P}}{\leftrightarrows} \mathcal{K}_R \;, \quad \mathcal{W}_L \overset{\mathcal{P}}{\leftrightarrows} \mathcal{W}_R \;, \quad \mathcal{O}_L \overset{\mathcal{P}}{\leftrightarrows} \mathcal{O}_R \;, \quad \mathcal{Q}_L \overset{\mathcal{P}}{\leftrightarrows} \mathcal{Q}_R$$

$$\mathcal{K} \overset{C}{\leftrightarrows} \mathcal{K} \;, \quad \mathcal{W} \overset{C}{\leftrightarrows} \mathcal{W} \;, \quad \mathcal{O} \overset{C}{\leftrightarrows} \mathcal{O}^\dagger \;, \quad \mathcal{Q} \overset{C}{\leftrightarrows} \mathcal{Q}^\dagger$$

# Experimental data

PDG

| Baryon | $N(939)$ | $N(1440)$ | $N(1535)$ | $N(1650)$ | $N(1710)$ |
|---|---|---|---|---|---|
| Mass (MeV) | $939.0 \pm 1.3$ | $1430^{+20}_{-10}$ | $1535 \pm 10$ | $1655^{+15}_{-10}$ | $1710^{+40}_{-30}$ |
| Partial width (MeV) $\Gamma_{N^* \to N\pi}$ | | $210 \pm 70$ | $70 \pm 19$ | $100 \pm 35$ | $13^{+20}_{-13}$ |

$$g_A = 1.269 \pm 0.006$$

T. Yamanishi, Phys. Rev. D **76**, 014006 (2007)

| $J^P$ | Octets | | | |
|---|---|---|---|---|
| $1/2^+$ | $N(939)$ | $\Lambda(1116)$ | $\Sigma(1193)$ | $\Xi(1318)$ |
| $1/2^+$ | $N(1440)$ | $\Lambda(1600)$ | $\Sigma(1660)$ | $\Xi(1690)$ |
| $1/2^-$ | $N(1535)$ | $\Lambda(1670)$ | $\Sigma(1620)$ | $\Xi(?)$ |
| $1/2^-$ | $N(1650)$ | $\Lambda(1800)$ | $\Sigma(1750)$ | $\Xi(?)$ |
| $1/2^+$ | $N(1710)$ | $\Lambda(1810)$ | $\Sigma(1880)$ | $\Xi(?)$ |

$\tilde{\Gamma}^{\mathrm{exp}}(N(1440)) = 0.24 \pm 0.09$

# $\mathbf{U(1)}_A$ symmetry

This symmetry is broken by the anomaly and spontaneous symmetry breaking.

In the linear sigma model

$$G\left(\ln\det M - \ln\det M^\dagger\right)$$

this term reproduces the $\mathbf{U(1)}_A$ anomaly,
which is in the meson sector.

For the baryon sector we require the $\mathbf{U(1)}_A$ symmetry.