Nucleon Form Factors and Polarizabilities at Very Low Q

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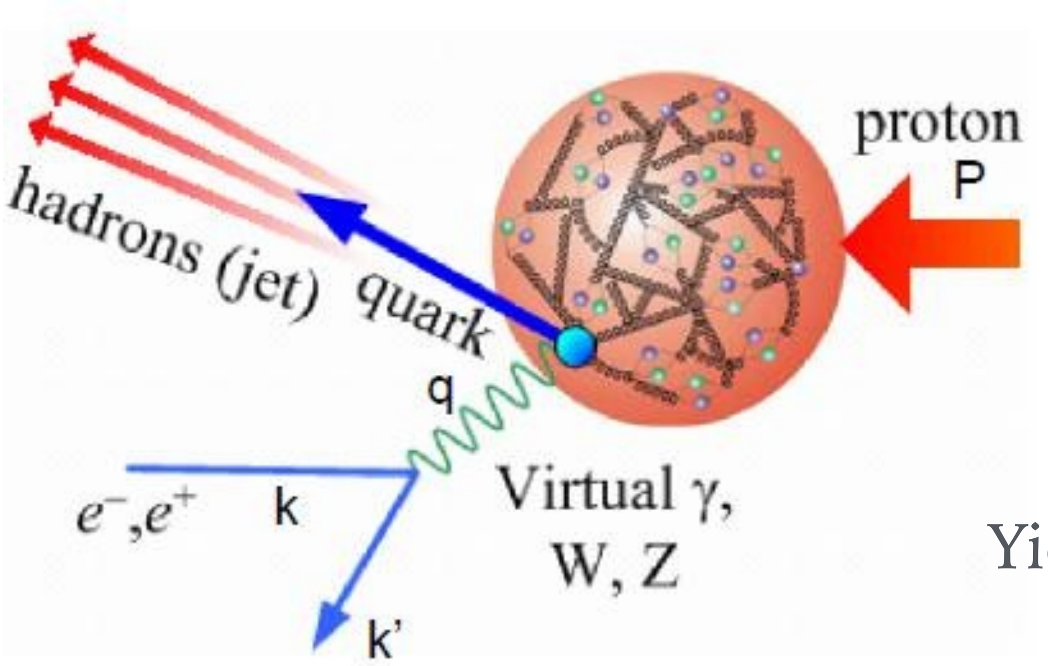
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Traditional tool — Electron Scattering



$$Q^2 = -(k' - k)^2$$
$$x = Q^2/(2M_p \nu)$$

Yields 4 Structure Functions

1) elastic part given by **form factors**

$$\begin{aligned} f_1^{\rm el}(\nu, Q^2) &= \tfrac{1}{2} G_M^2(Q^2)\, \delta(1-x), \\ f_2^{\rm el}(\nu, Q^2) &= \frac{1}{1+\tau}\left[G_E^2(Q^2) + \tau G_M^2(Q^2)\right] \delta(1-x), \\ g_1^{\rm el}(\nu, Q^2) &= \tfrac{1}{2} F_1(Q^2) G_M(Q^2)\, \delta(1-x), \\ g_2^{\rm el}(\nu, Q^2) &= -\tfrac{1}{2}\tau F_2(Q^2) G_M(Q^2)\, \delta(1-x), \end{aligned}$$

where $\tau = Q^2/4M^2$ and $G_E(Q^2)$, $G_M(Q^2)$ are the Sachs FFs,

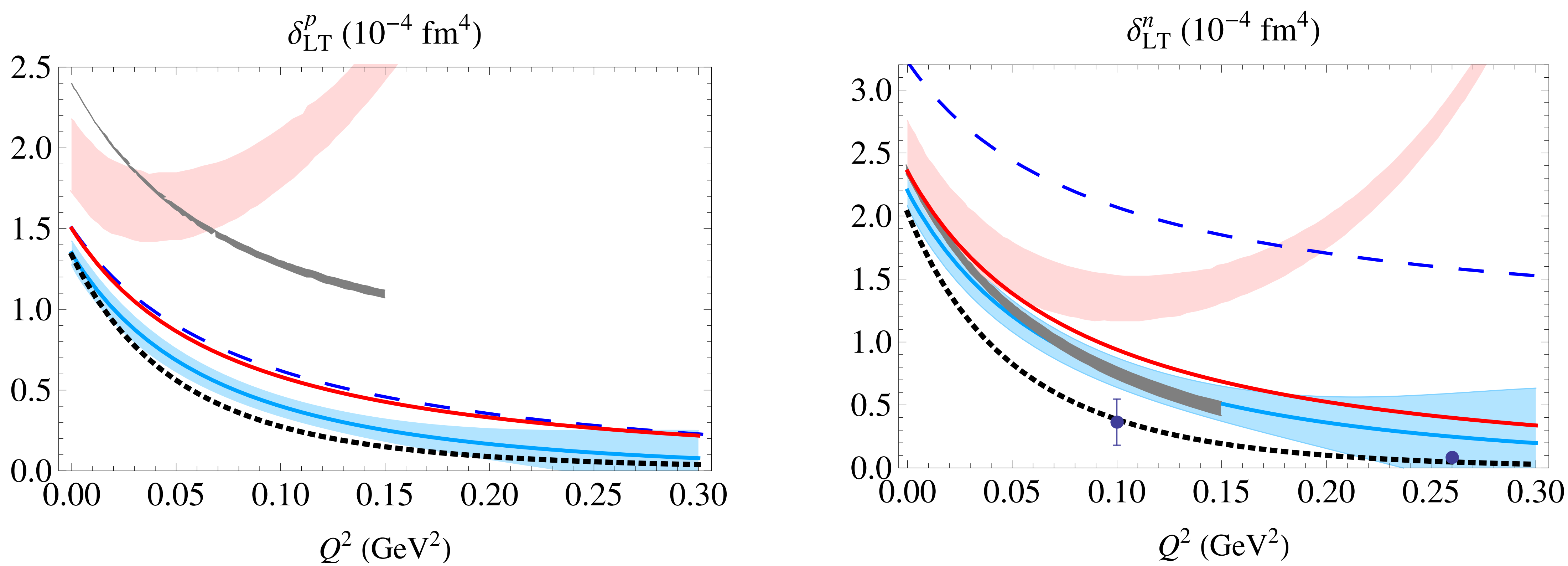
2) moments of the inelastic structure functions related to **polarizabilities**

For example

$$\alpha_{E1}(Q^2) + \beta_{M1}(Q^2) = \frac{8\alpha M_N}{Q^4} \int_0^{x_0} \mathrm{d}x \, x F_1(x, Q^2),$$

$$\gamma_0(Q^2) = \frac{16\alpha M_N^2}{Q^6} \int_0^{x_0} \mathrm{d}x \, x^2 g_{TT}(x, Q^2), \qquad g_{TT} \; = \; g_1 - (4M_N^2 x^2/Q^2) g_2$$

$$\delta_{LT}(Q^2) = \frac{16\alpha M_N^2}{Q^6} \int_0^{x_0} \mathrm{d}x \, x^2 \Big[g_1(x, Q^2) + g_2(x, Q^2) \Big]$$



DeltaLT puzzle — none of chiral PT calculation describe neutron deltaLT.

Proton Form Factors and RMS Radii

FF interpretation: Fourier transforms of charge and magnetization distributions

$$\rho(r) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} G(\boldsymbol{q}^2) e^{-i\boldsymbol{q}\boldsymbol{r}}$$

$$G_E(Q^2) = 1 - 1/6\, R_E^2\, Q^2 + \cdots$$

root-mean-square (rms) charge radius:

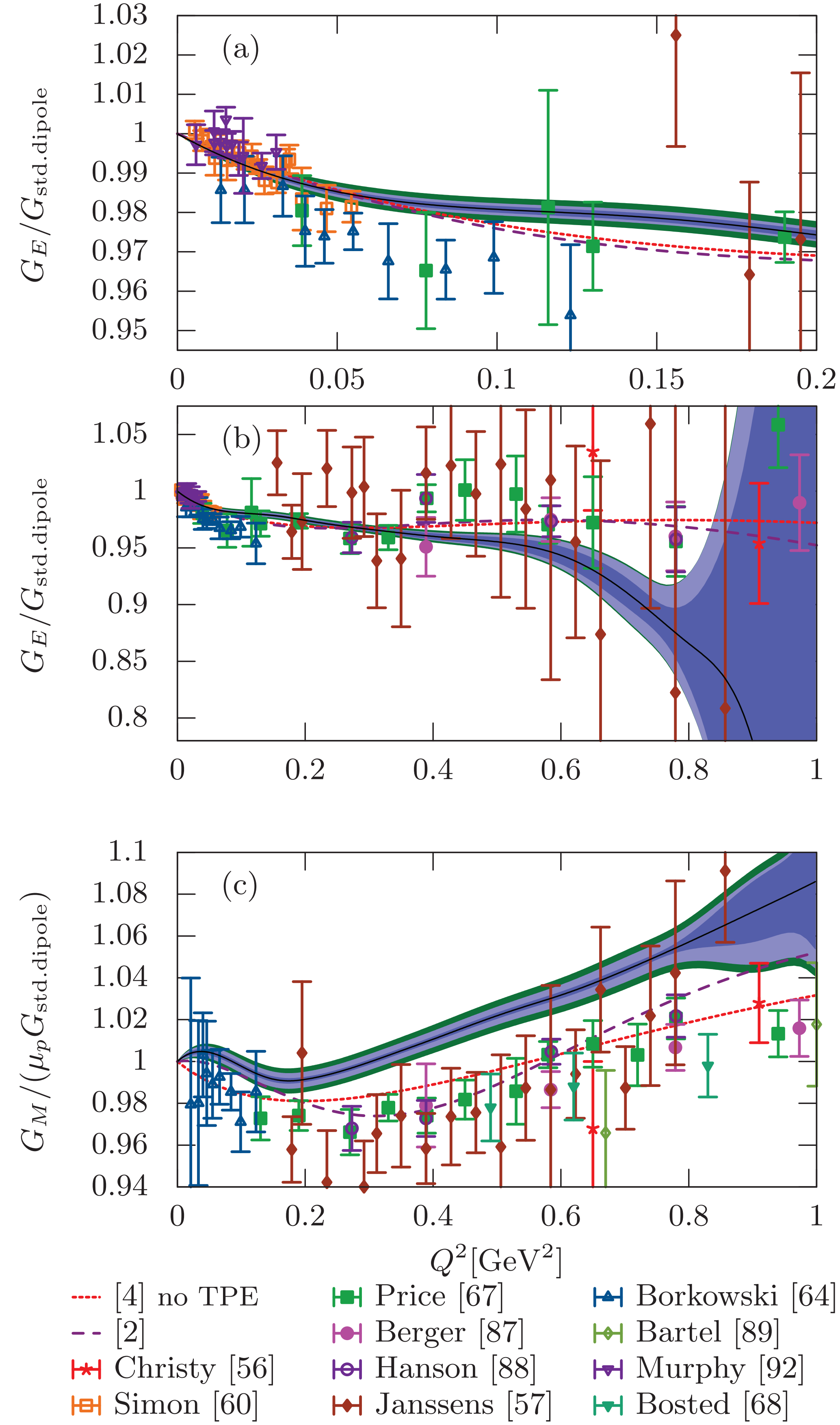
$$R_E = \sqrt{\langle r^2 \rangle_E}$$

$$\langle r^2 \rangle_E \equiv \int \mathrm{d}\boldsymbol{r}\, r^2\, \rho_E(\boldsymbol{r}) = -6 \frac{\mathrm{d}}{\mathrm{d}Q^2} G_E(Q^2)\Big|_{Q^2=0}$$

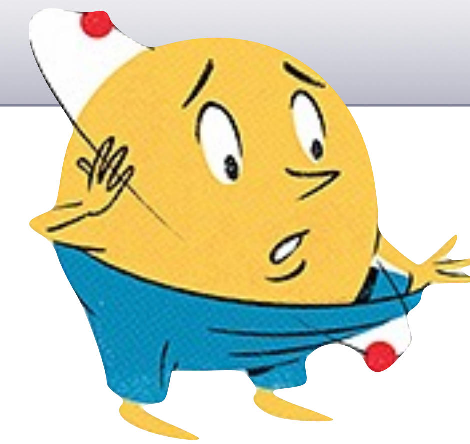
$$R_E = 0.879(5)_{\mathrm{stat}}(4)_{\mathrm{syst}}(2)_{\mathrm{model}}(4)_{\mathrm{group}}\ \mathrm{fm},$$

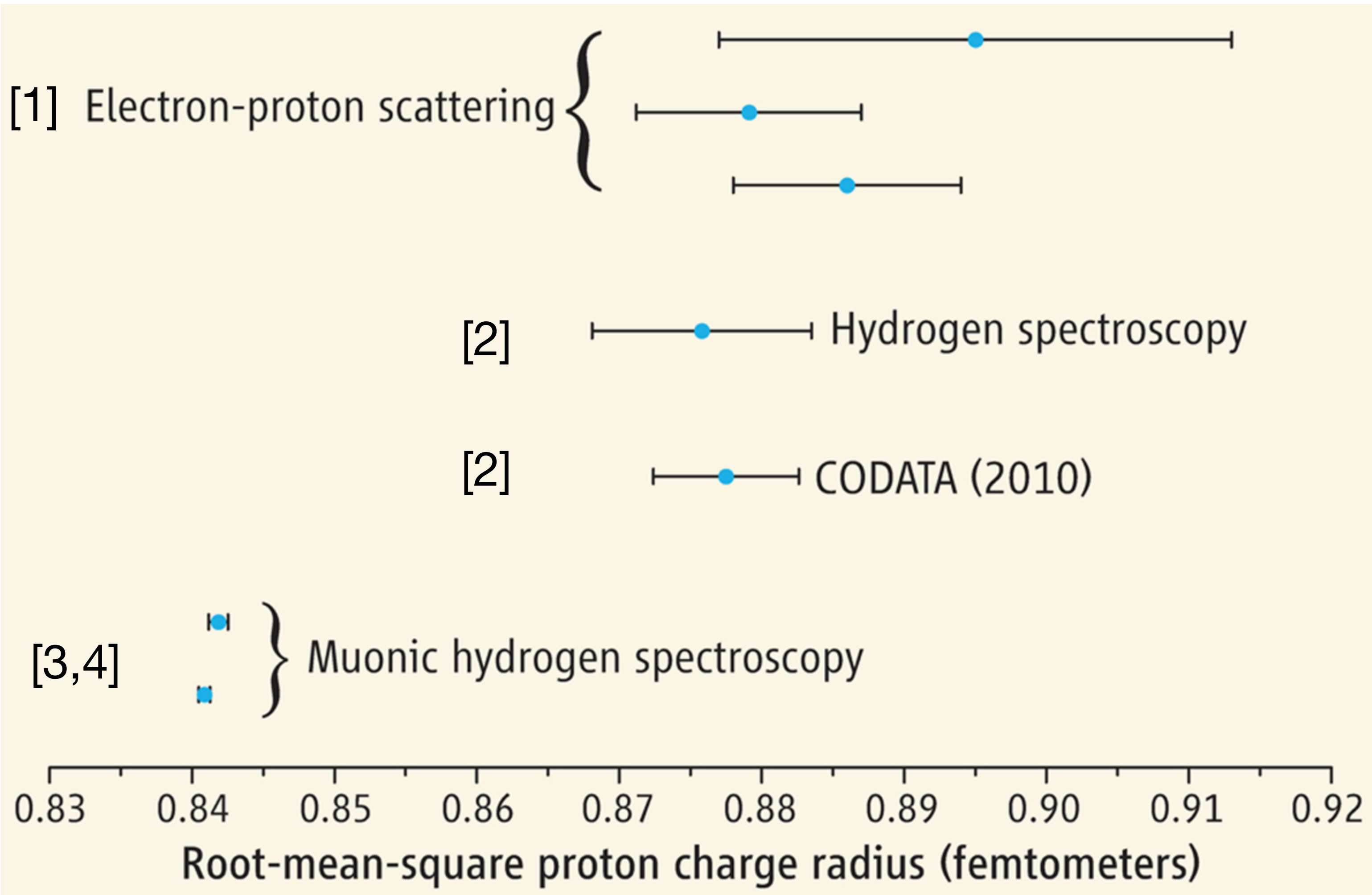
$$R_M = 0.777(13)_{\mathrm{stat}}(9)_{\mathrm{syst}}(5)_{\mathrm{model}}(2)_{\mathrm{group}}\ \mathrm{fm}.$$

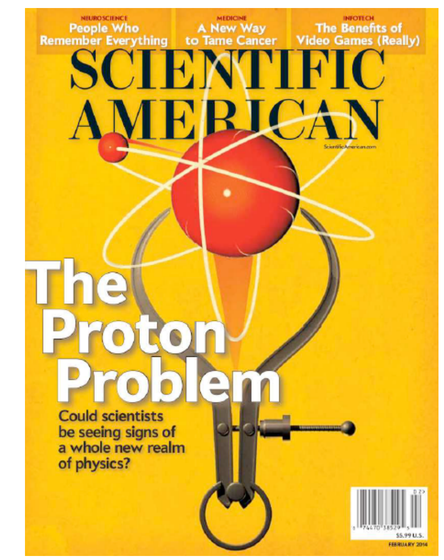
J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).



Proton radius puzzle





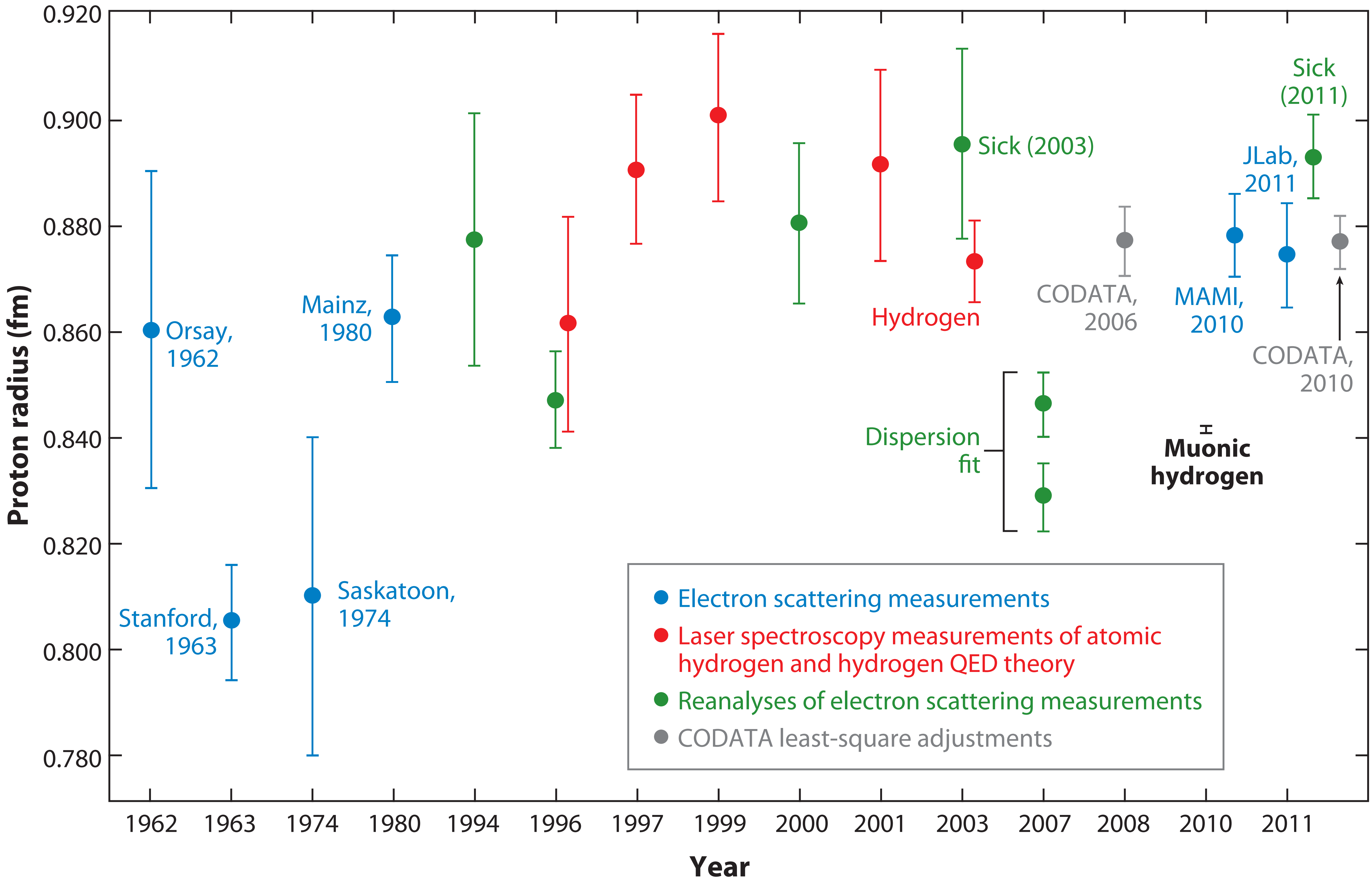


[1] J. C. Bernauer *et al.*, Phys. Rev. Lett. **105**, 242001 (2010).
[2] P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012).
[3] R. Pohl, A. Antognini *et al.*, Nature **466**, 213 (2010).
[4] A. Antognini *et al.*, Science **339**, 417 (2013).

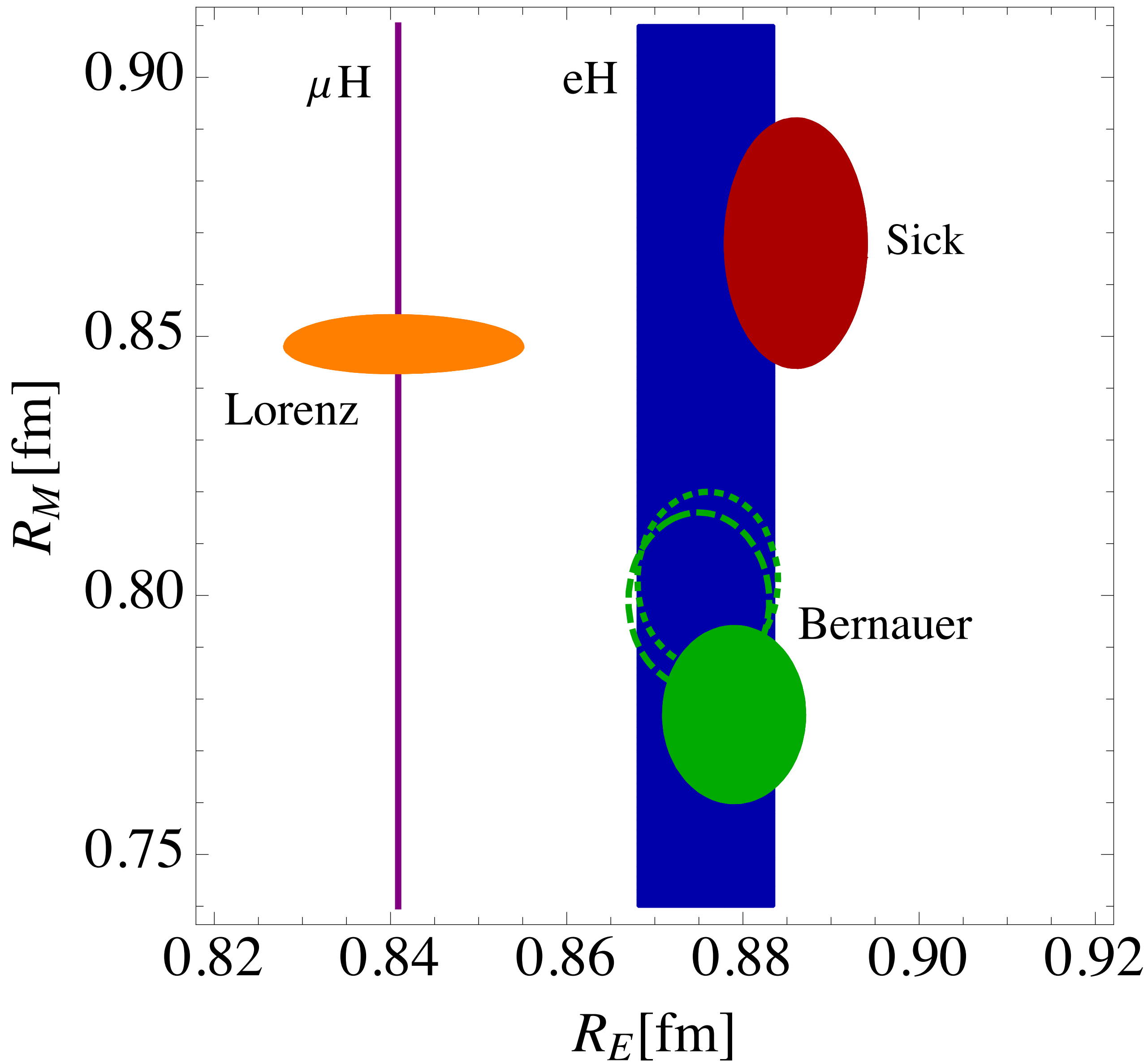
7σ discrepancy

$$[R_E^{\mu\mathrm{H}} = 0.84087(39)\ \mathrm{fm}] \longleftrightarrow [R_E^{\mathrm{CODATA\ 2010}} = 0.8775(51)\ \mathrm{fm}]$$

Proton Radius — Historical Perspective



Present Status



- H: $R_E = 0.8758(77)$ fm;
- μH: $R_E = 0.84087(39)$ fm;
- Sick: $R_E = 0.886(8)$ fm, $R_M = 0.868(24)$ fm;
- Lorenz et al.: $R_E = 0.840\,[0.828\ldots0.855]$ fm, $R_M = 0.848\,[0.843\ldots0.854]$ fm;
- Bernauer et al.:

$$
\begin{aligned}
R_E &= 0.879(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(4)_{\rm group}\ \text{fm},\\
R_E^{\text{TPE-a}} &= 0.876(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(5)_{\rm group}\ \text{fm},\\
R_E^{\text{TPE-b}} &= 0.875(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(5)_{\rm group}\ \text{fm},\\
R_M &= 0.777(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(2)_{\rm group}\ \text{fm},\\
R_M^{\text{TPE-a}} &= 0.803(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(3)_{\rm group}\ \text{fm},\\
R_M^{\text{TPE-b}} &= 0.799(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(3)_{\rm group}\ \text{fm}.
\end{aligned}
$$

[1] P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012).

[2] A. Antognini, et al., Science **339** (2013) 417– 420.

[3] I. Sick, Prog. Part. Nucl. Phys. **67** (2012) 473–478.

[4] I. Lorenz, et al., Phys. Rev. **D**91 (2015) 014023.

[5] J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).

Isovector Dirac and Pauli radii from Lattice QCD
Phys.Lett. B734 (2014) [arXiv:1209.1687]

Nucleon Structure from Lattice QCD Using a Nearly Physical Pion Mass

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[2] *Physics Department, New Mexico State University, Las Cruces, New Mexico 88003, USA*
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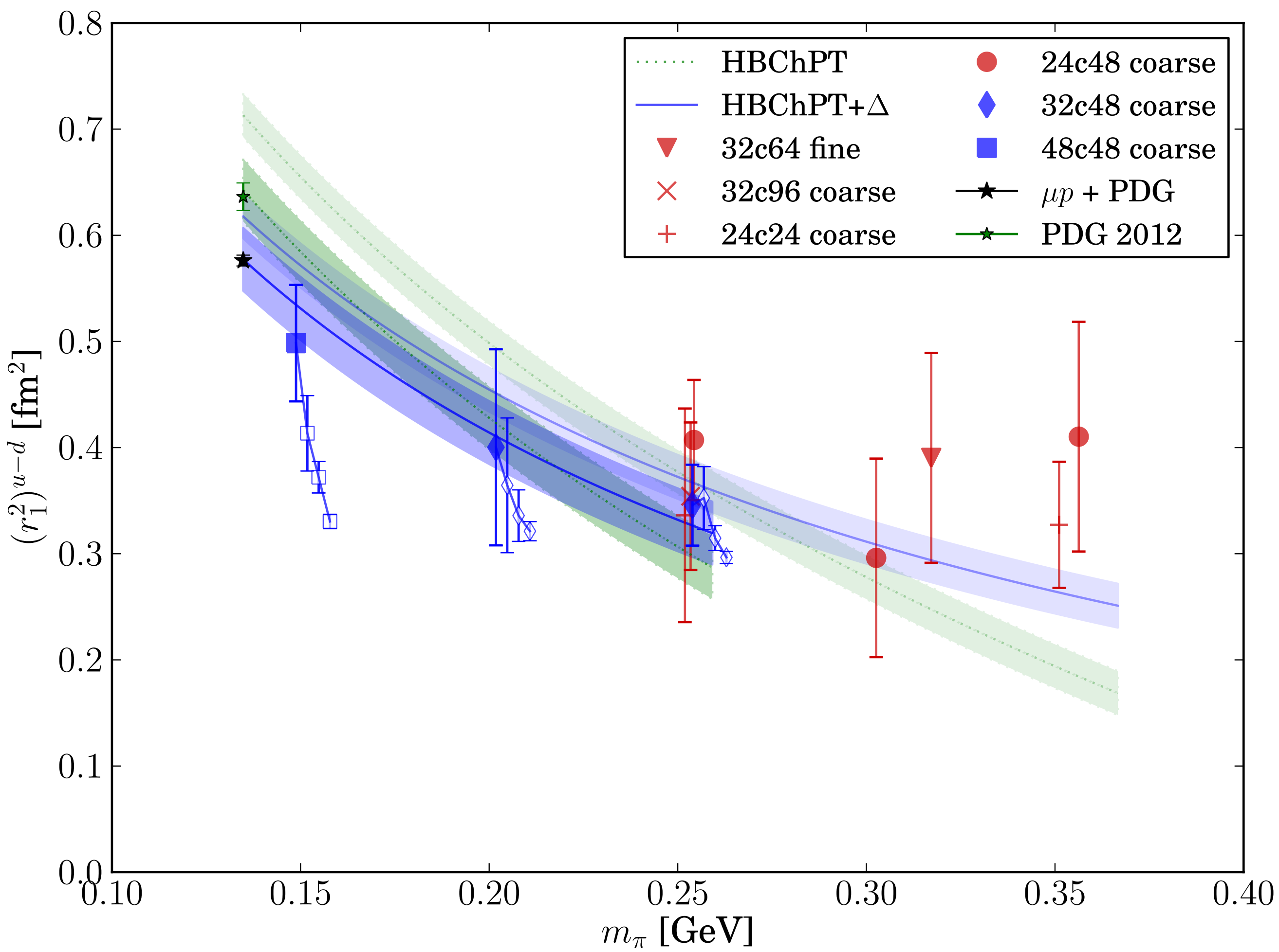
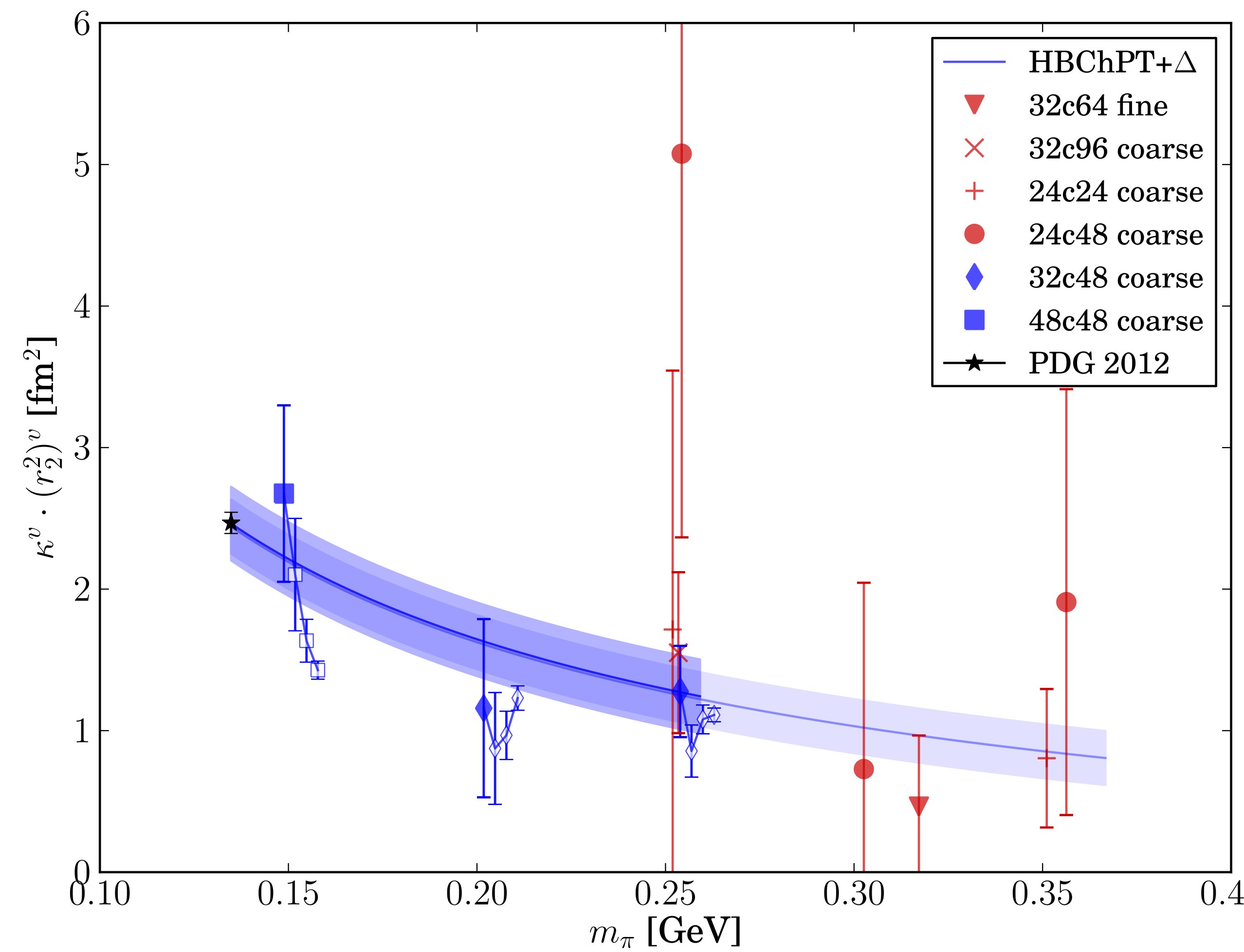
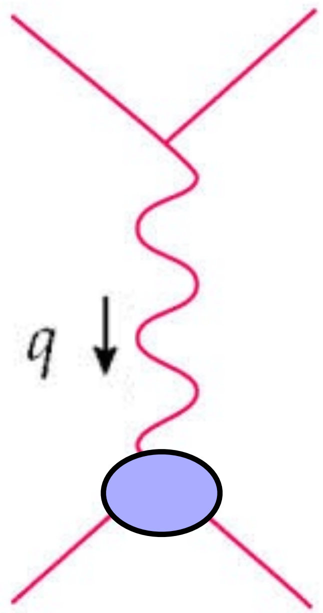
(Dated: September 11, 2012)

We report the first lattice QCD calculation using the almost physical pion mass $m_\pi = 149$ MeV that agrees with experiment for four fundamental isovector observables characterizing the gross structure of the nucleon: the Dirac and Pauli radii, the magnetic moment, and the quark momentum fraction. The key to this success is excluding the contributions of excited states. An analogous calculation of the nucleon axial charge governing beta decay fails to agree with experiment, and we discuss possible sources of error.







- one-photon exchange

$$= \left(2E_k\, 2E_{k'}\, 2E_p\, 2E_{p'}\right)^{-1/2} \bar{u}(\boldsymbol{k}')(-e\gamma^\mu)u(\boldsymbol{k})\, \Delta_{\mu\nu}(q)\, \bar{N}(\boldsymbol{p}')e\Gamma^\nu N(\boldsymbol{p})$$

photon propagator:

$$-\frac{1}{q^2}\left[g_{\mu\nu} - \frac{1}{\boldsymbol{q}^2 + t}\left(q^\mu q^\nu - \chi^\mu q^\nu - \chi^\nu q^\mu\right)\right]$$

in Coulomb gauge $\chi = (0, \boldsymbol{q})$

electromagnetic vertex:

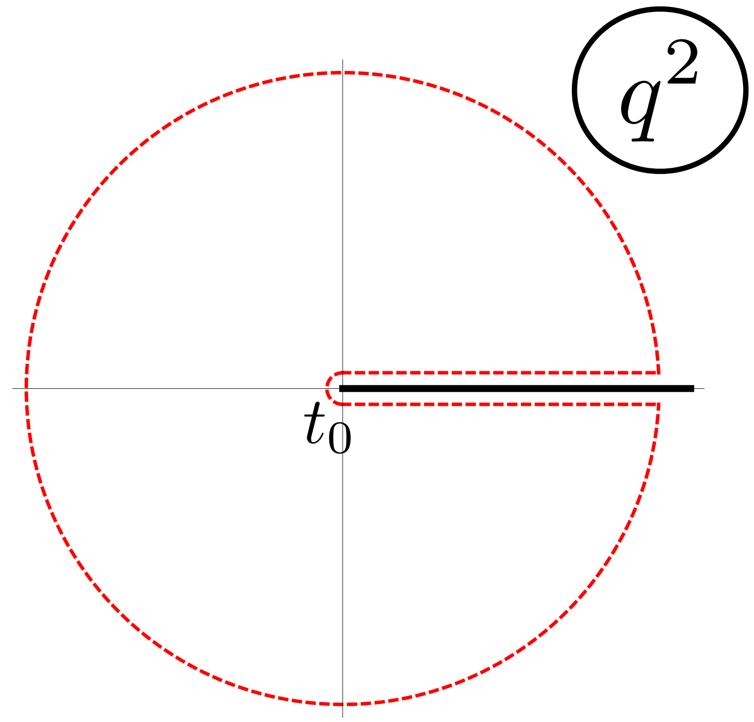
$$\Gamma^\mu = Z\gamma^\mu F_1(Q^2) - \frac{1}{2M}\gamma^{\mu\nu} q_\nu F_2(Q^2)$$

where $Q^2 = -q^2$

Dispersion relation

- Dirac & Pauli FFs:

$$F_i(q^2) = \frac{1}{\pi}\int_{t_0}^{\infty} dt\, \frac{\mathrm{Im}\, F_i(q^2)}{t - q^2 - i\epsilon}$$



t_0 is the lowest particle production threshold

(timelike photon is unstable)

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **91**, 040502(R) (2015)

Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution

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(Received 13 February 2015; published 20 April 2015)

Yukawa-type potential: $V_Y(r) = \frac{Z\alpha}{r}\frac{1}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\, e^{-r\sqrt{t}}\,\mathrm{Im}\, G_E(t)$

electric FF correction to the Coulomb potential $-Z\alpha/r$

- contribution of $V_Y(r)$ to classic Lamb shift at 1st-order perturbation theory (PT):

$$\Delta E^{\mathrm{FF}(1)}_{2P-2S} = \langle 2P_{1/2}\,|V_Y|\,2P_{1/2}\rangle - \langle 2S_{1/2}\,|V_Y|\,2S_{1/2}\rangle$$

$$= -\frac{(Z\alpha)^4 m_r^3}{2\pi}\int_{t_0}^{\infty}\mathrm{d}t\,\frac{\mathrm{Im}\, G_E(t)}{(\sqrt{t}+Z\alpha m_r)^4}$$

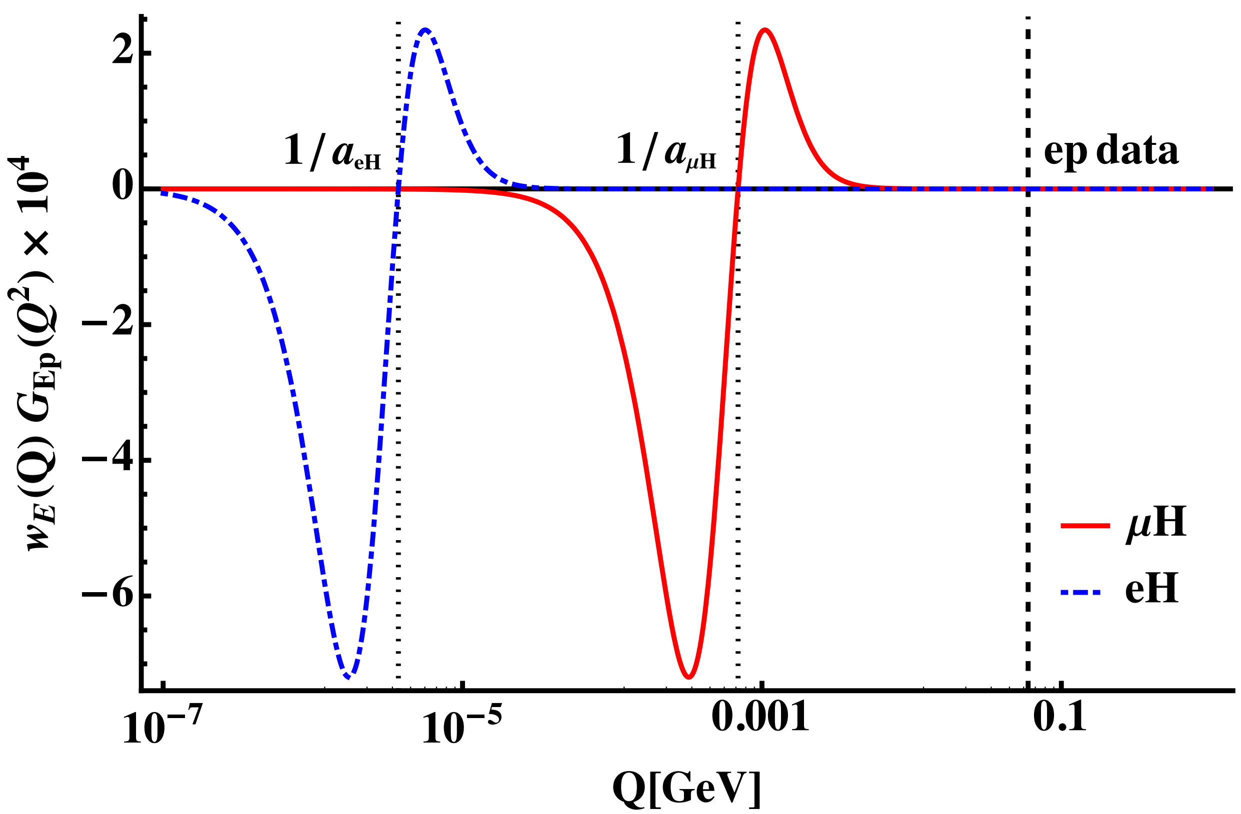
$$= -\frac{(Z\alpha)^4 m_r^3}{12}\sum_{k=0}^{\infty}\frac{(-Z\alpha m_r)^k}{k!}\langle r^{k+2}\rangle_E$$

$$= -\frac{(Z\alpha)^4 m_r^3}{12}\left[\langle r^2\rangle_E - Z\alpha m_r\langle r^3\rangle_E\right] + O(\alpha^6)$$

convergence radius of the expansion is limited by t_0

FF effect on the Lamb shift

$$E^{\mathrm{FF}(1)}_{2P-2S} = \int_0^\infty \mathrm{d}Q\, w(Q)\, G_E(Q^2)\,, \quad \text{with} \quad w(Q) = -\frac{4}{\pi}(Z\alpha)^5 m_r^4\, Q^2 \frac{(Z\alpha m_r)^2 - Q^2}{[(Z\alpha m_r)^2 + Q^2]^4}$$



$$w(Q) = -\frac{4}{\pi}(Z\alpha)^5 m_r^4\, Q^2 \frac{(Z\alpha m_r)^2 - Q^2}{[(Z\alpha m_r)^2 + Q^2]^4}$$

Dipole FF:
$G_{Ep} = (1 + Q^2/0.71\,\mathrm{GeV}^2)^{-2}$

alternatively:

$$E^{\mathrm{FF}(1)}_{2P-2S} = -\frac{1}{3}\pi(Z\alpha)^4 m_r^3 \int_0^\infty \mathrm{d}r\, r^4 e^{-r/a} \rho_E(r) \quad \text{with} \quad \rho_E(r) = \frac{1}{(2\pi)^2 r}\int_{t_0}^\infty \mathrm{d}t\, \mathrm{Im}\, G_E(t)\, e^{-r\sqrt{t}}$$

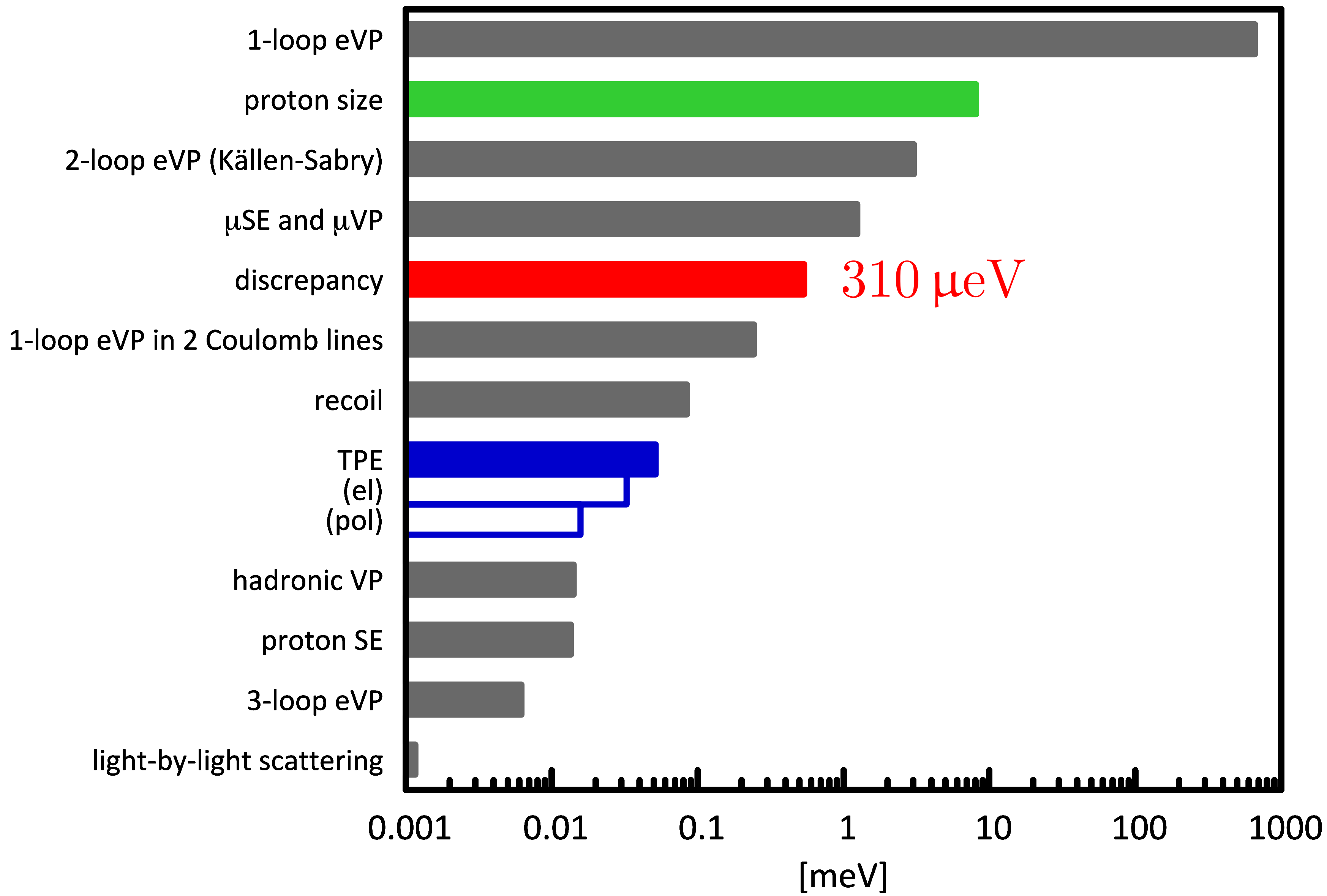
$a = 1/(Z\alpha m_r)$ Bohr radius

Muonic Hydrogen Lamb shift

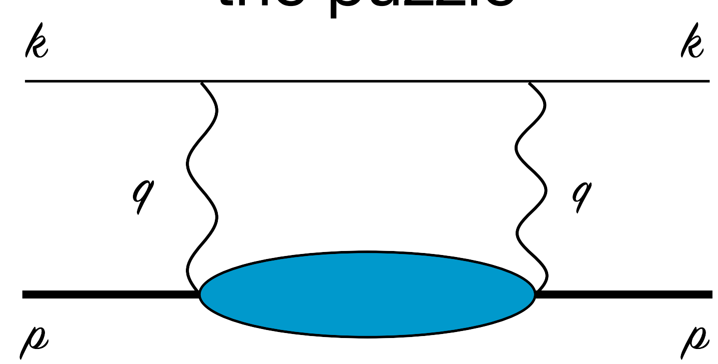
$$\Delta E^{\rm th}_{\rm LS} = 206.0668(25) - 5.2275(10)\,(R_E/{\rm fm})^2$$

numerical values reviewed in: A. Antognini *et al.*, Annals Phys. **331**, 127-145 (2013).

theory uncertainty: $2.5\ \mu eV$



subleading effects of proton structure proposed to resolve the puzzle

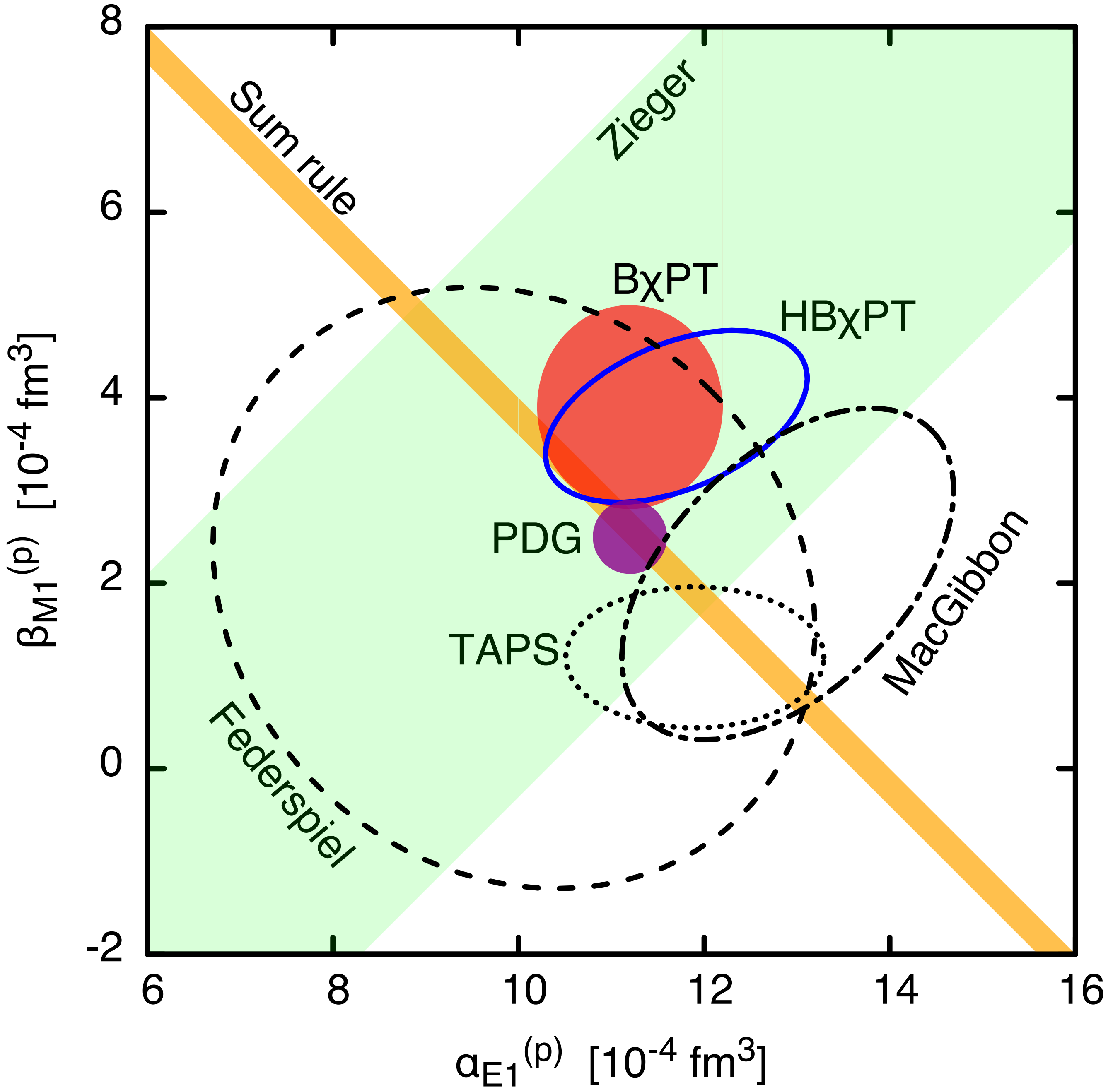


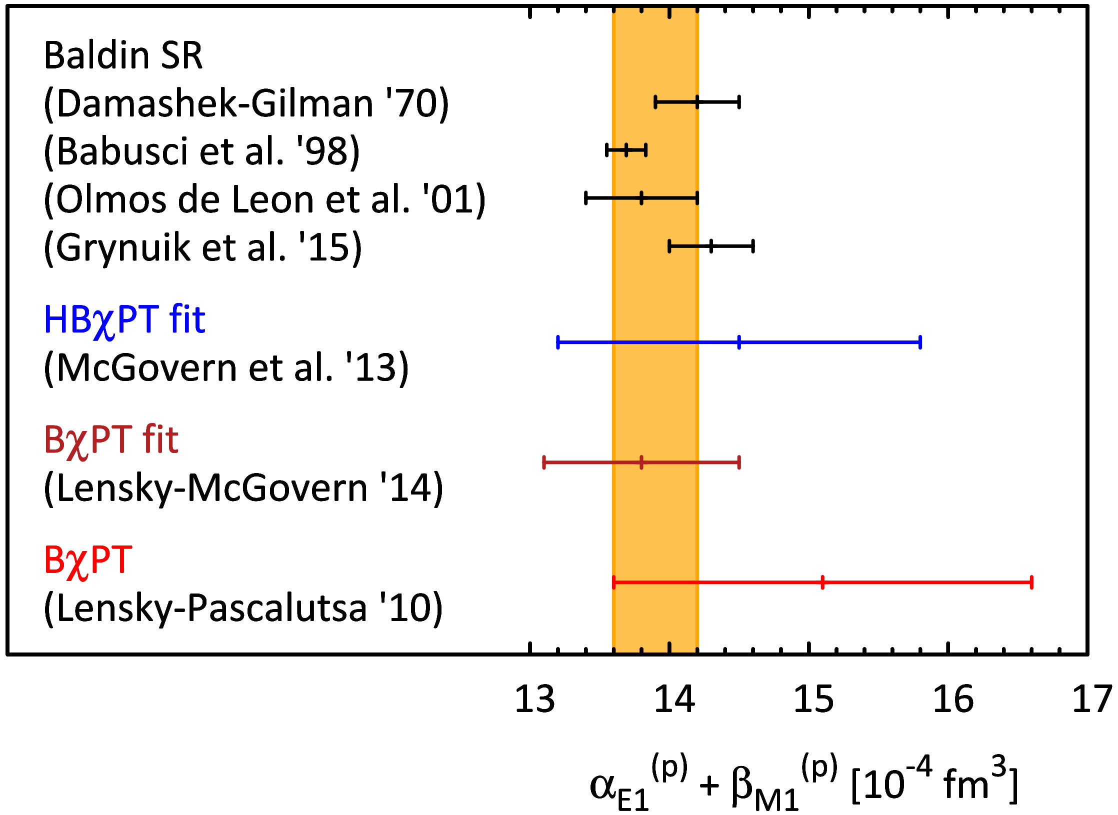
$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\rm elastic} + \delta V^{(2\gamma)}_{\rm polariz.}$$

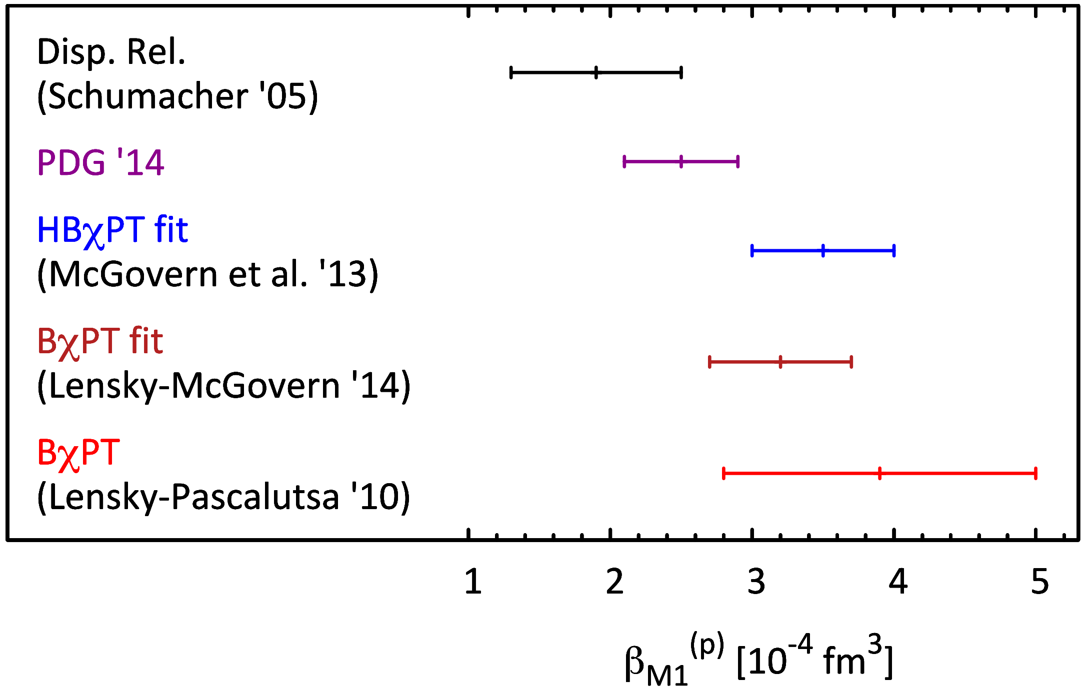
A. De Rujula, Phys. Lett. B693 (2010)

G. A. Miller, Phys. Lett. B718 (2013)

Proton Dipole Polarizabilities — Status



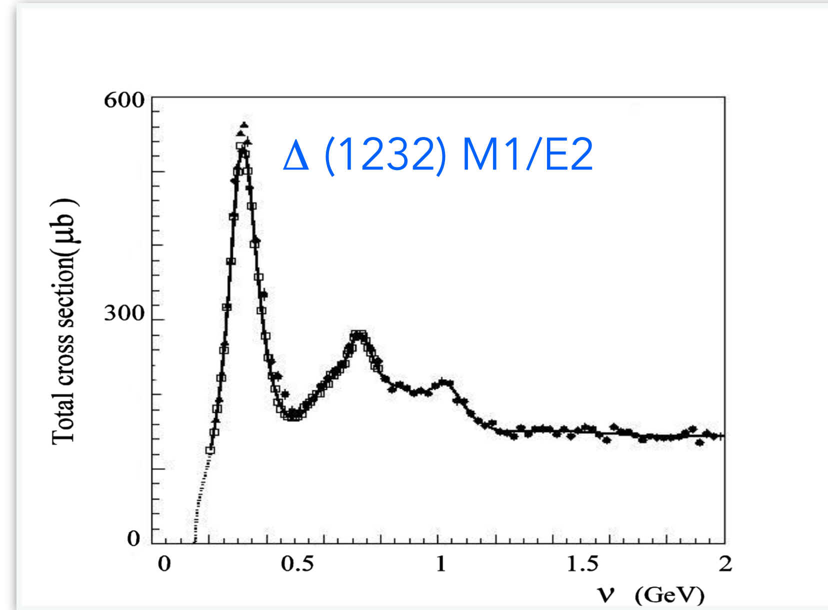


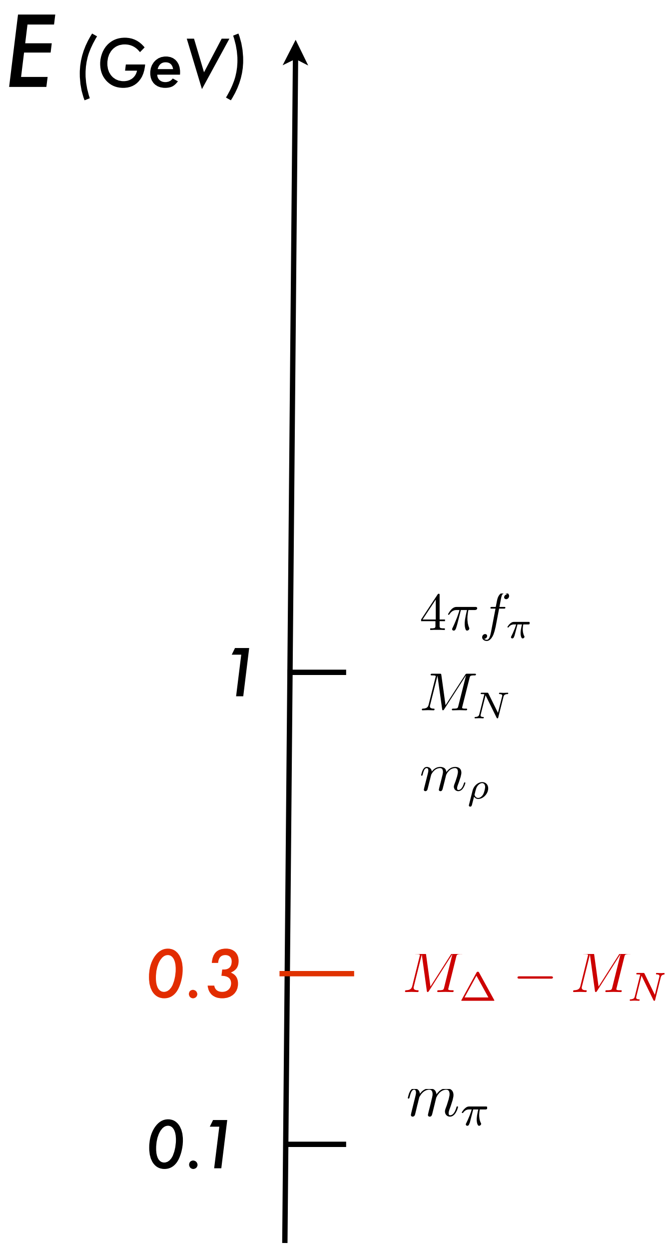


Baryon ChPT

Not just the pion cloud: Delta(1232) excitation

Jenkins & Manohar, PLB (1991)
Hemmert, Holstein, Kambor, JPhysG (1998)
V.P. & Phillips, PRC (2003)

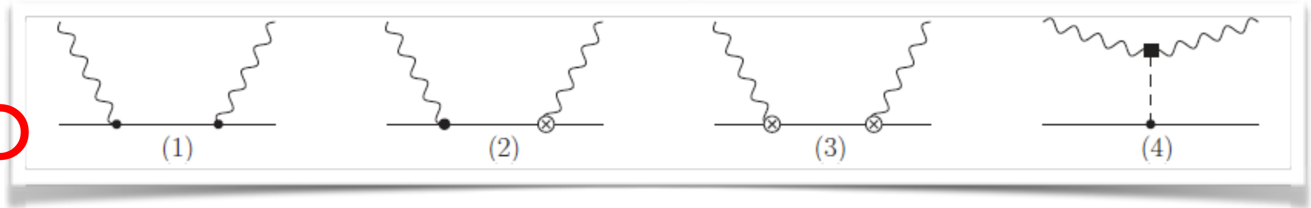


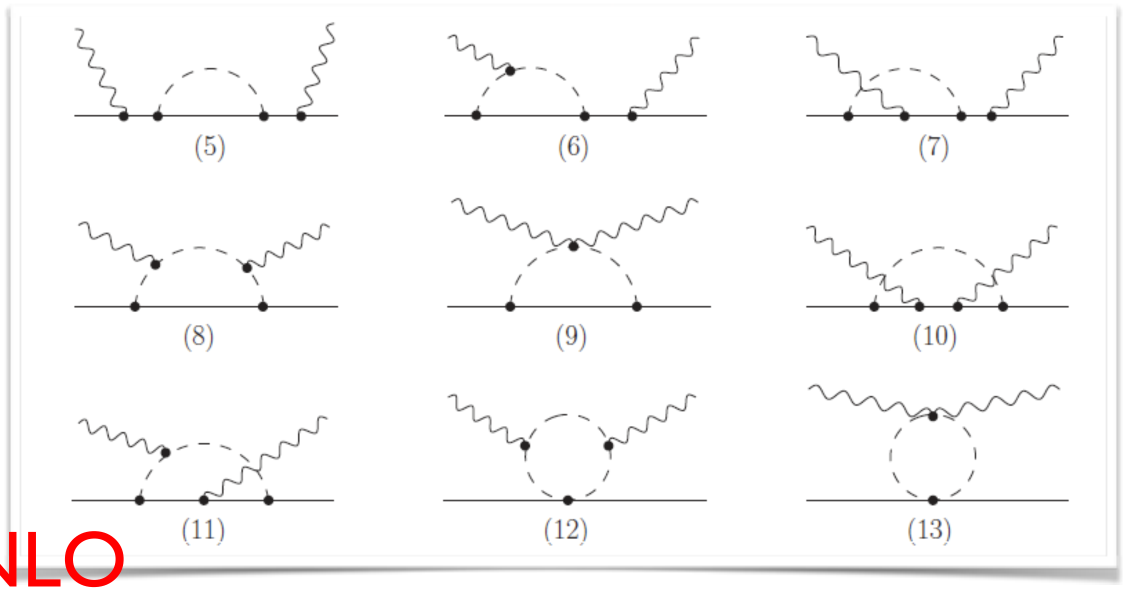


- The 1st nucleon excitation — Delta(1232) is within reach of chiral perturbation theory (293 MeV excitation energy is a light scale)
- Include into the chiral effective Lagrangian as explicit dof
- Power-counting for Delta contributions (SSE, ``delta-counting") depends on what chiral order is assigned to the excitation scale.

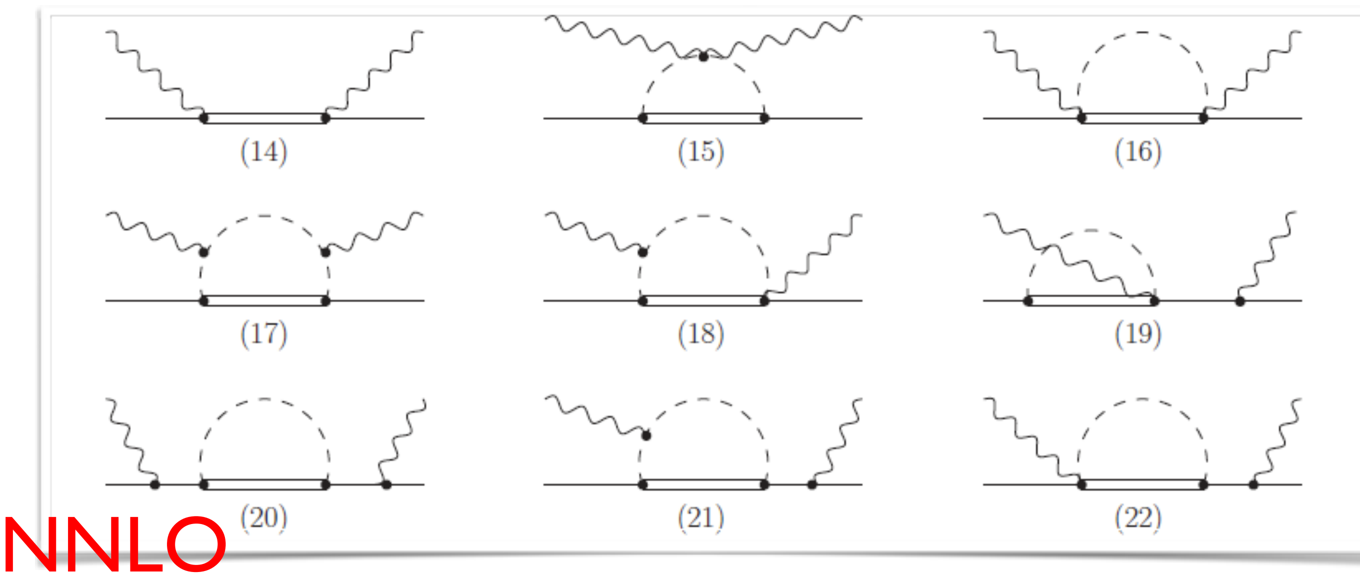
ChPT of Compton scattering (RCS) on proton

LO



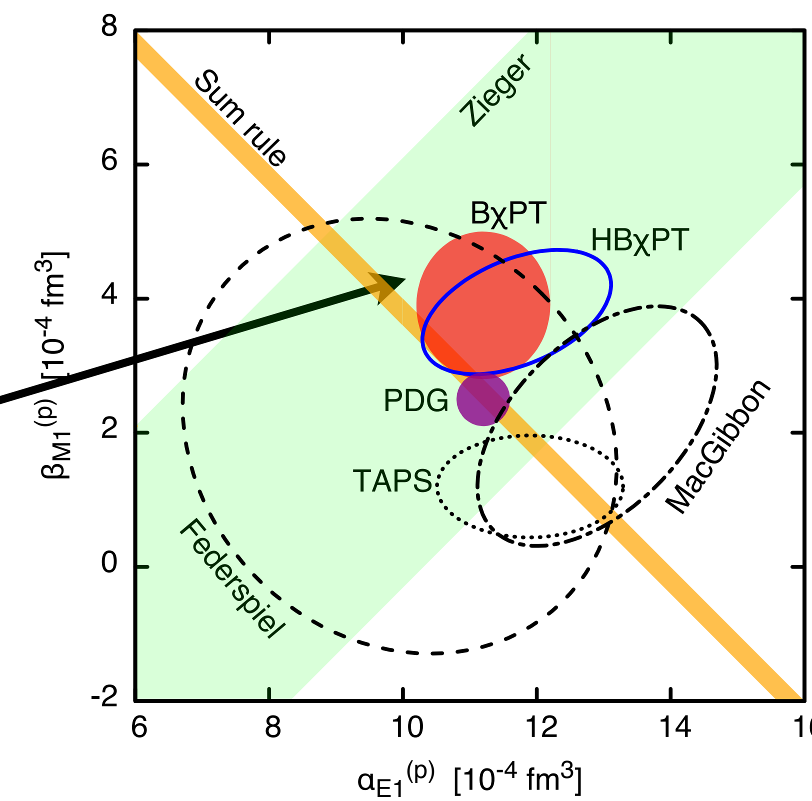


NLO



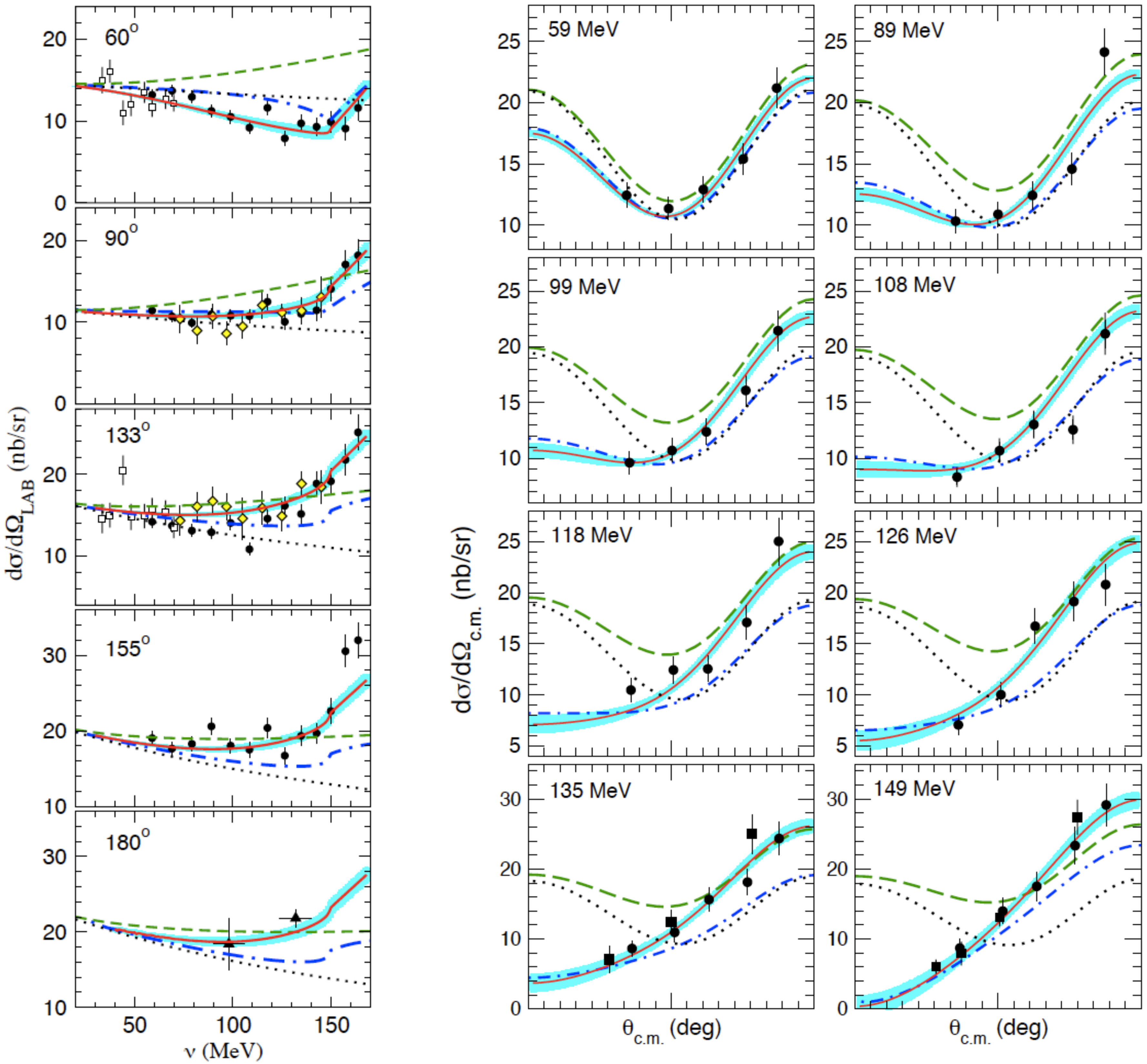
NNLO

$\mathcal{O}(p^2)$	$\frac{e^2}{4\pi} = \frac{1}{137}$, $M_N = 938.3$ MeV, $\hbar c = 197$ MeV$\cdot$fm
$\mathcal{O}(p^3)$	$g_A = 1.267$, $f_\pi = 92.4$ MeV, $m_\pi = 139$ MeV, $m_{\pi^0} = 136$ MeV, $\kappa_p = 1.79$
$\mathcal{O}(p^4/\Delta)$	$M_\Delta = 1232$ MeV, $h_A = 2.85$, $g_M = 2.97$, $g_E = -1.0$
$\mathcal{O}(p^4)$	$\alpha_0, \beta_0 = \pm\frac{e^2}{4\pi M_N^3}$ size of the red blob



Lensky & V.P., EPJC (2010)

Unpolarized cross sections for RCS



Data points:
MAMI/TAPS (2001)
SAL (1993)
Illinois (1991)

Curves:

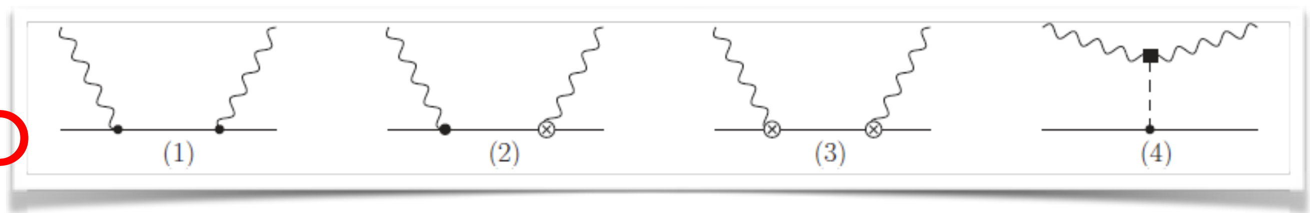
- Klein-Nishina
- Born + WZW
- + p-qube
- Total NNLO

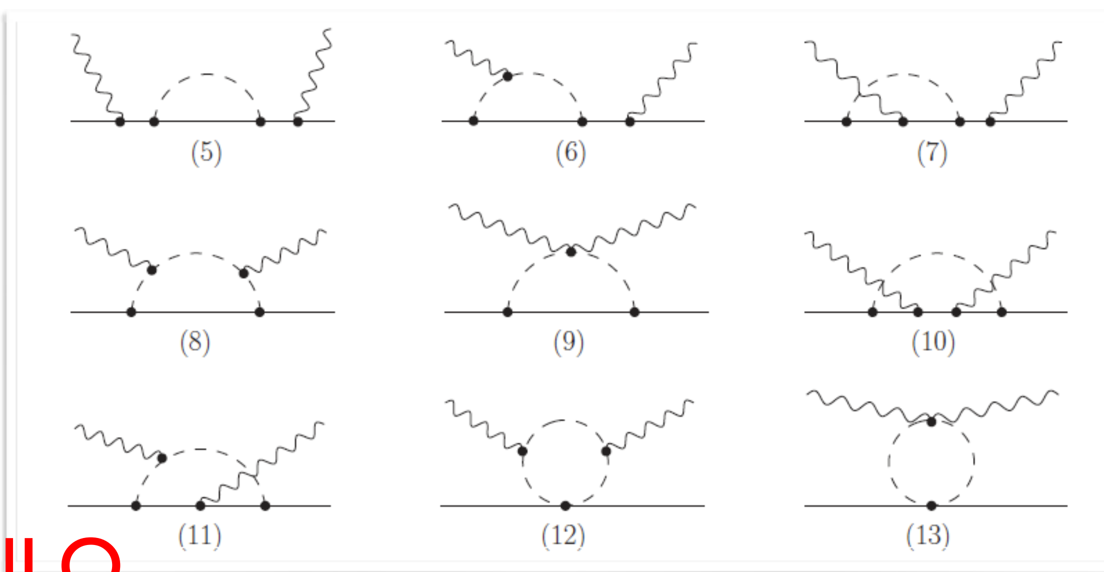
Lensky & V.P.,EPJC (2010)

ChPT of forward VVCS on the nucleon, extension to virtual photons

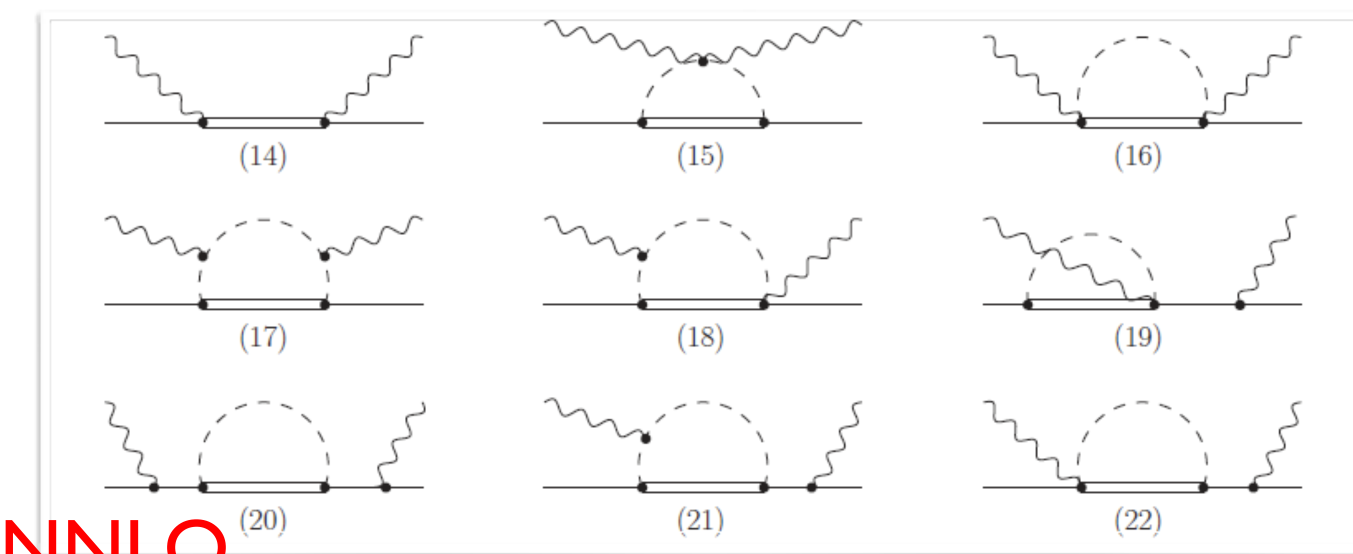
Alarcon, Lensky & VP, PRC90(2014)

LO





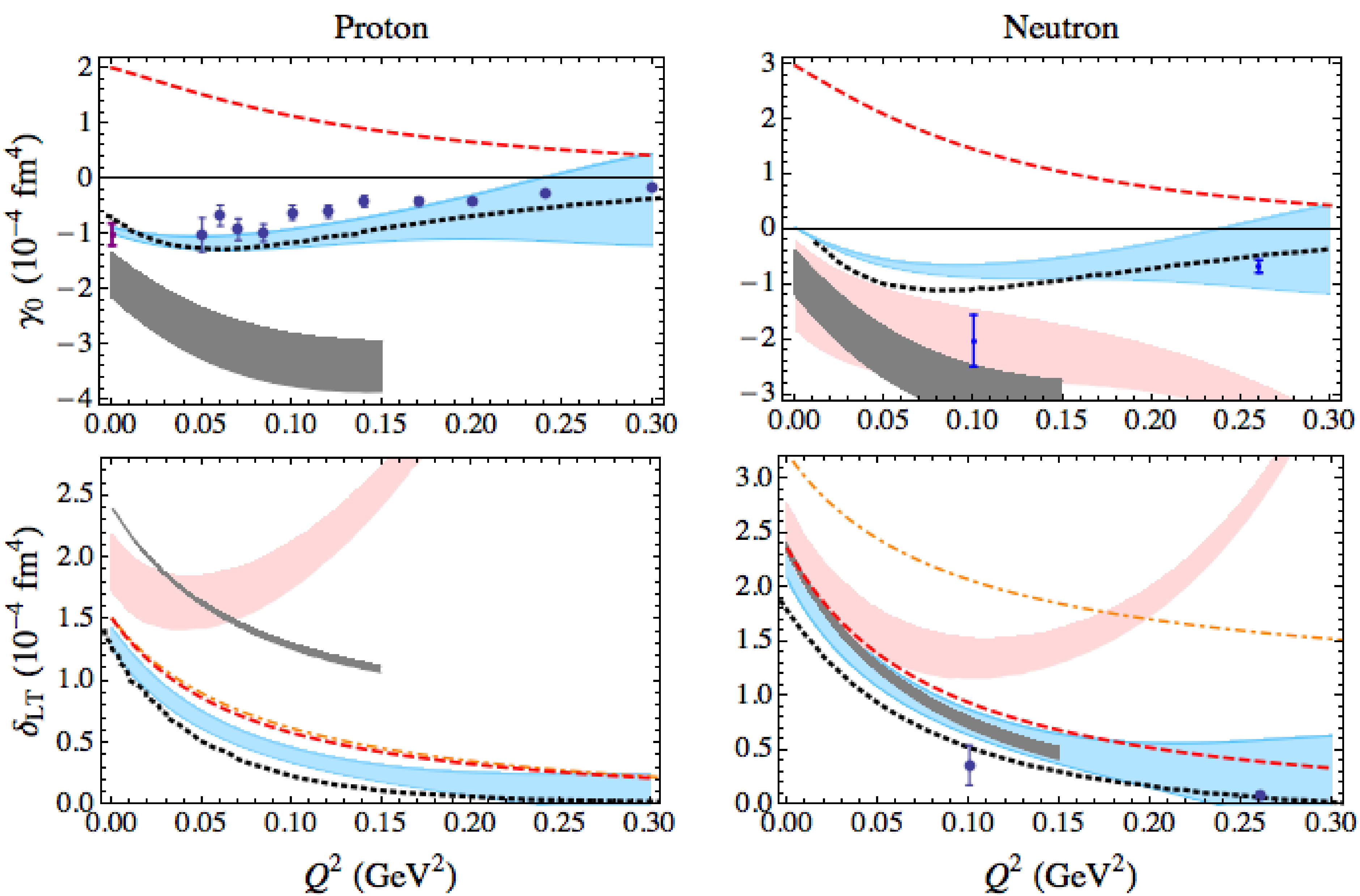
NLO



NNLO

BChPT for polarised VVCS (deltaLT puzzle)

Alarcon, Lensky & VP, PRC90(2014)



Curves:

MAID (empir.)

NLO-HBChPT

LO-BChPT

NLO-BChPT

NLO-BChPT [Bernard,Krebs, Epelbaum, Meissner (2013)]

Data points:
JLab spin program

Polarizability contribution in ChPT

Eur. Phys. J. C (2014) 74:2852
DOI 10.1140/epjc/s10052-014-2852-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution

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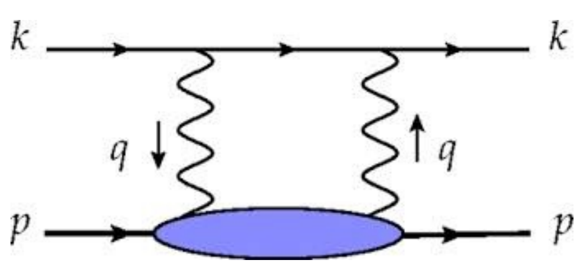

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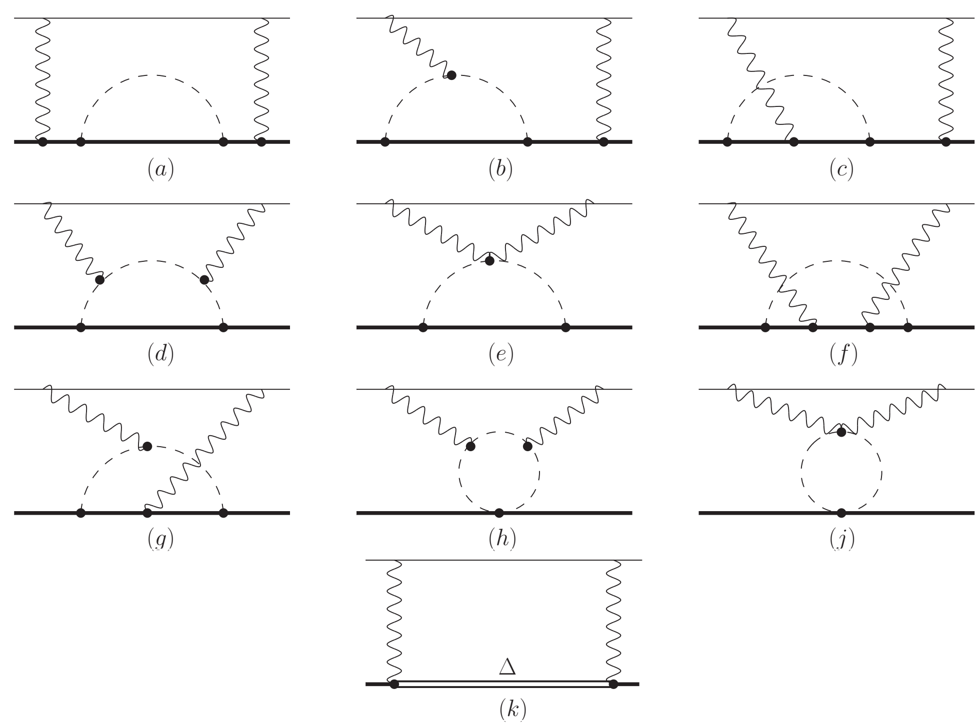


with corrections to elastic proton FFs subtracted, i.e. "polarizability" alone

Proton polarizability effect in mu-H

Heavy-Baryon (HB)ChPT

[Alarcon, Lensky & VP, EPJC (2014)]

(μeV)	Pachucki [9]	Martynenko [10]	Nevado and Pineda [11]	Carlson and Vanderhaeghen [12]	Birse and McGovern [13]	Gorchtein et al. [14]	LO-BχPT [this work]
$\Delta E_{2S}^{(\mathrm{subt})}$	1.8	2.3	–	5.3 (1.9)	4.2 (1.0)	−2.3 (4.6)[a]	−3.0
$\Delta E_{2S}^{(\mathrm{inel})}$	−13.9	−13.8	–	−12.7 (5)	−12.7 (5)[b]	−13.0 (6)	−5.2
$\Delta E_{2S}^{(\mathrm{pol})}$	−12 (2)	−11.5	−18.5	−7.4 (2.4)	−8.5 (1.1)	−15.3 (5.6)	$-8.2(^{+1.2}_{-2.5})$

[a] Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the 'elastic' and 'polarizability' contributions
[b] Taken from Ref. [12]

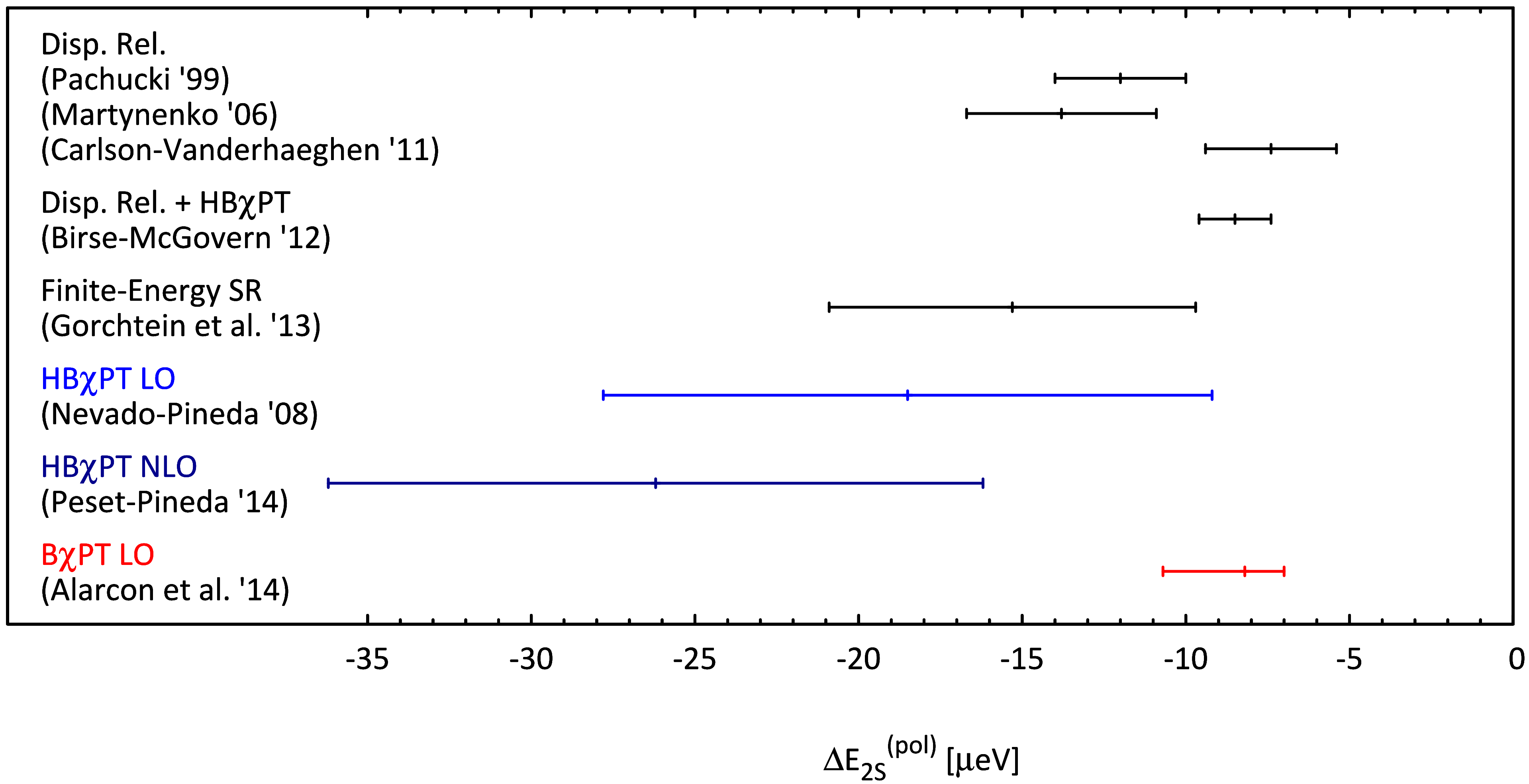
[9] K. Pachucki, Phys. Rev. A **60**, 3593 (1999).
[10] A. P. Martynenko, Phys. Atom. Nucl. **69**, 1309 (2006).
[11] D. Nevado and A. Pineda, Phys. Rev. C **77**, 035202 (2008).
[12] C. E. Carlson and M. Vanderhaeghen, Phys. Rev. A **84**, 020102 (2011).
[13] M. C. Birse and J. A. McGovern, Eur. Phys. J. A **48**, 120 (2012).
[14] M. Gorchtein, F. J. Llanes-Estrada and A. P. Szczepaniak, Phys. Rev. A 87, **052501** (2013).

$$\Delta E_{2S}^{(\mathrm{pol})}(\text{LO-HB}\chi\text{PT}) \approx \frac{\alpha_{\mathrm{em}}^5 m_r^3 g_A^2}{4(4\pi f_\pi)^2}\,\frac{m_\mu}{m_\pi}(1-10G+6\log 2) = -16.1\ \mu\mathrm{eV},$$

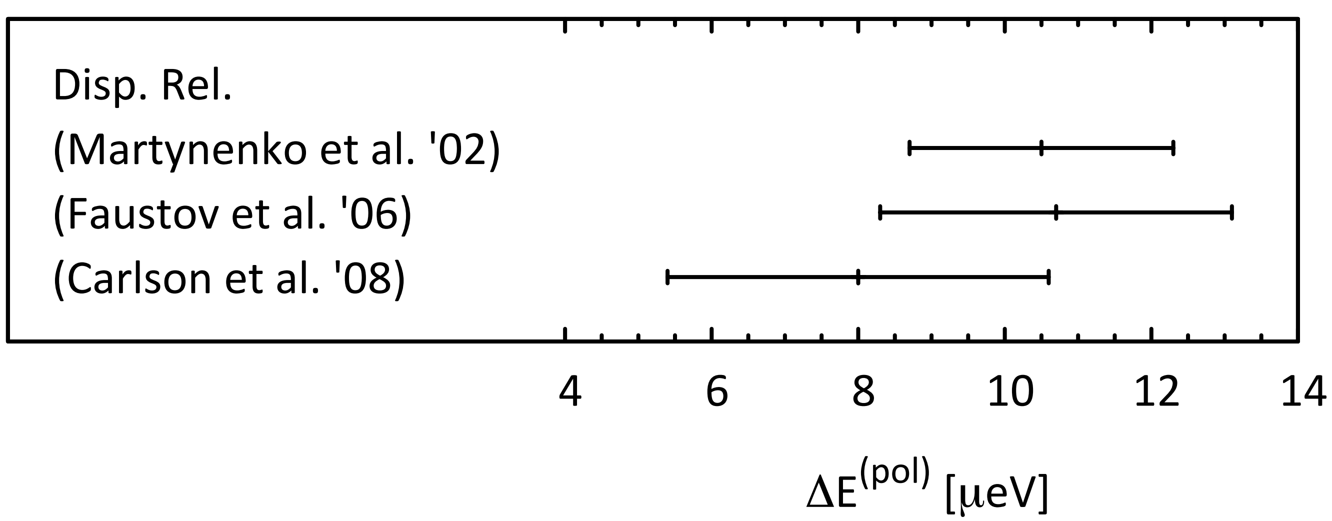
$G \simeq 0.9160$ is the Catalan constant.

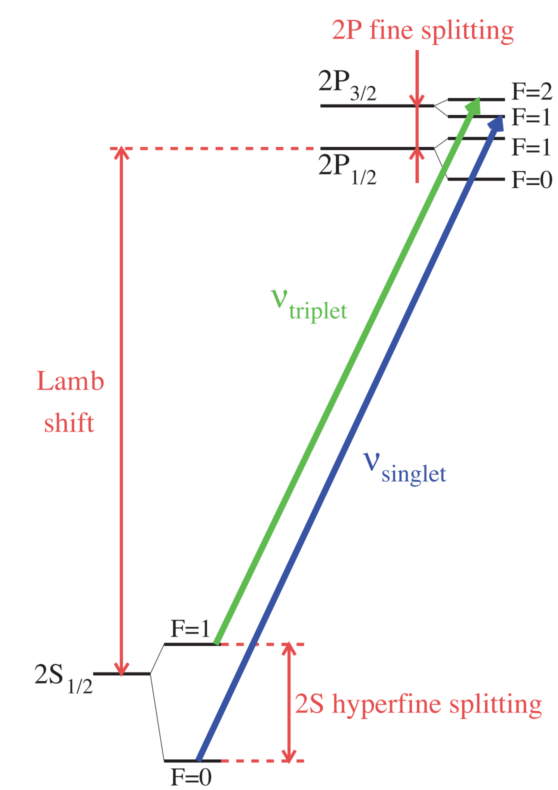
Summary of polarizability in muonic hydrogen

2S shift



2S HFS





Proton radius puzzle: possible explanations

$$\Delta E_{LS} = 209.9779\,(49) - 5.2262\ R_E^{\,2} + 0.00913\ R^3_{(2)}\ \text{meV}$$

different radii

Lamb shift difference of 310 ± 2 μeV

1

eH theory wrong ?

eH expt. wrong ? -> R_∞ wrong

+ ep scattering wrong:

- 2γ corrections
- low Q^2 extrapolation

2

μH expt. wrong ?

μH theory wrong ?

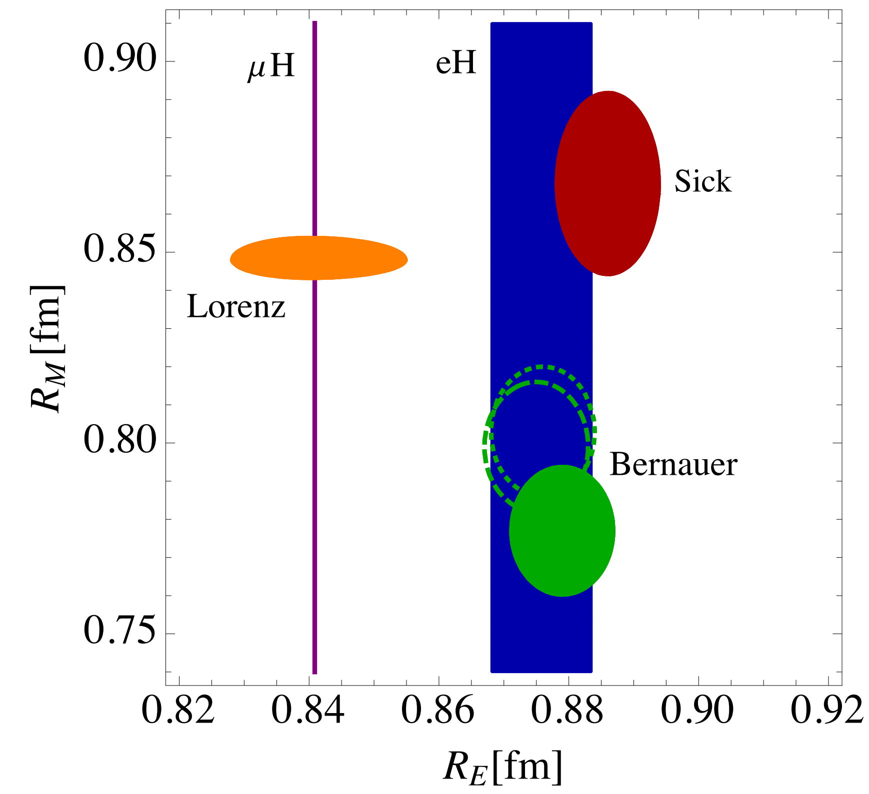
- missed QED or EW corrections
- soft hadronic corrections

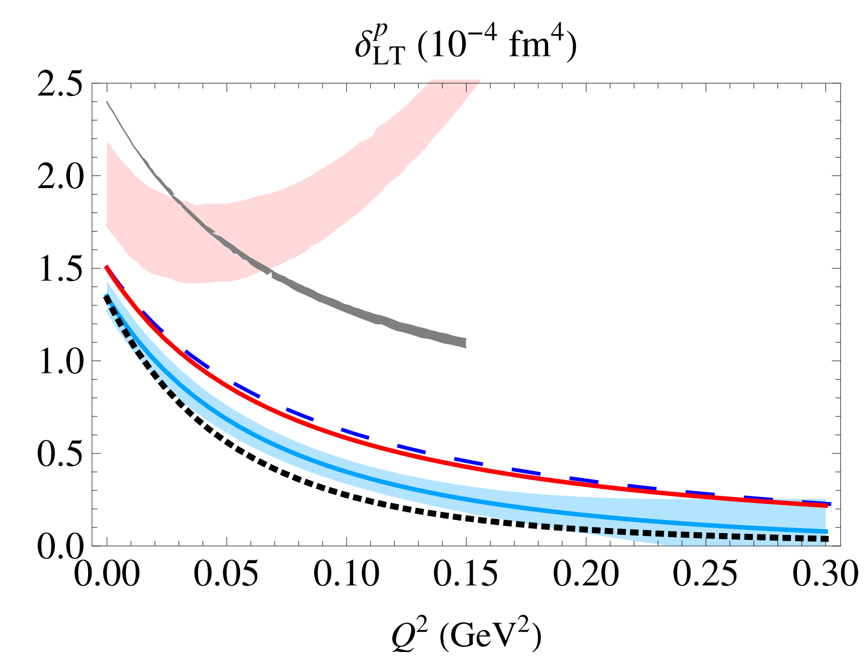
3

Beyond **Standard Model**

Conclusion

- 'proton radius puzzle' — 5 years later, many scenarios excluded.
  - expansion in radii, not applicable in some scenarios
  - 'soft' nucleon structure not ruled out by exp data, but concrete realisations needed
  - chiral PT contradicts the scenario of a huge polarizability contribution (to muH Lamb shift) resolving the puzzle. HFS in chiral PT to be done in near future.
- 'deltaLT' — seems to be resolved, pending publication of new JLab proton data
- 'magnetic polarizability of the proton' — awaits new MAMI data on beam asymmetry



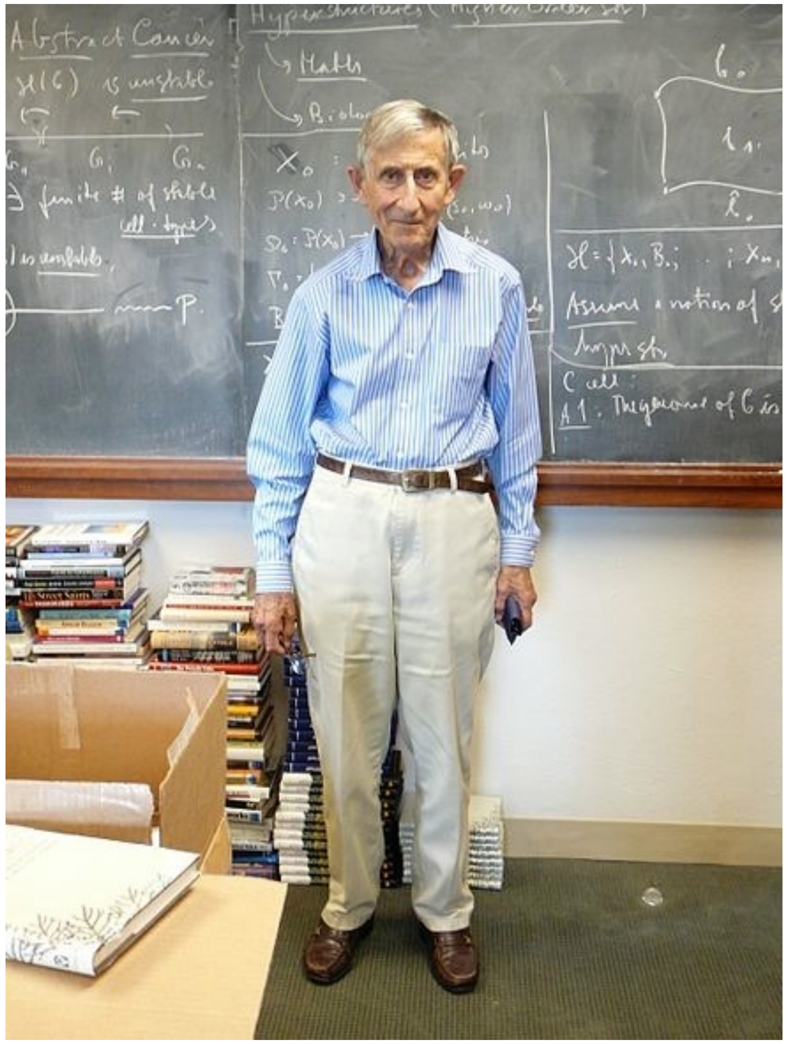


Breaking through frontiers

Freeman Dyson on 16 discoveries awarded the Nobel Prize between 1945 and 2008:

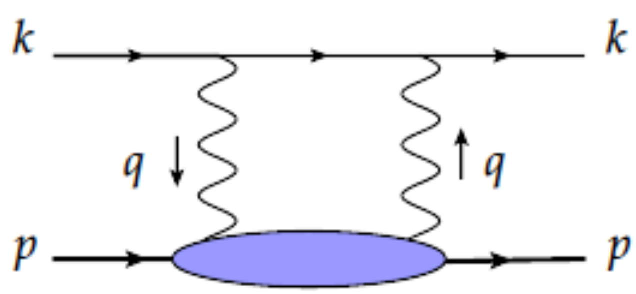
"four discoveries on the energy frontier, *four on the* rarity frontier, *eight on the* accuracy frontier. *Only a quarter of the discoveries were made on the energy frontier, while half of them were made on the accuracy frontier.* ***For making important discoveries, high accuracy was more useful than high energy.****"*

(Freeman Dyson, review of The Lightness of Being, F. Wilczek, The New York Review of Books, April 2009)



Backup slides

Lamb shift in terms of VVCS amplitudes



empirically known 'inelastic'

unknown 'subtraction'

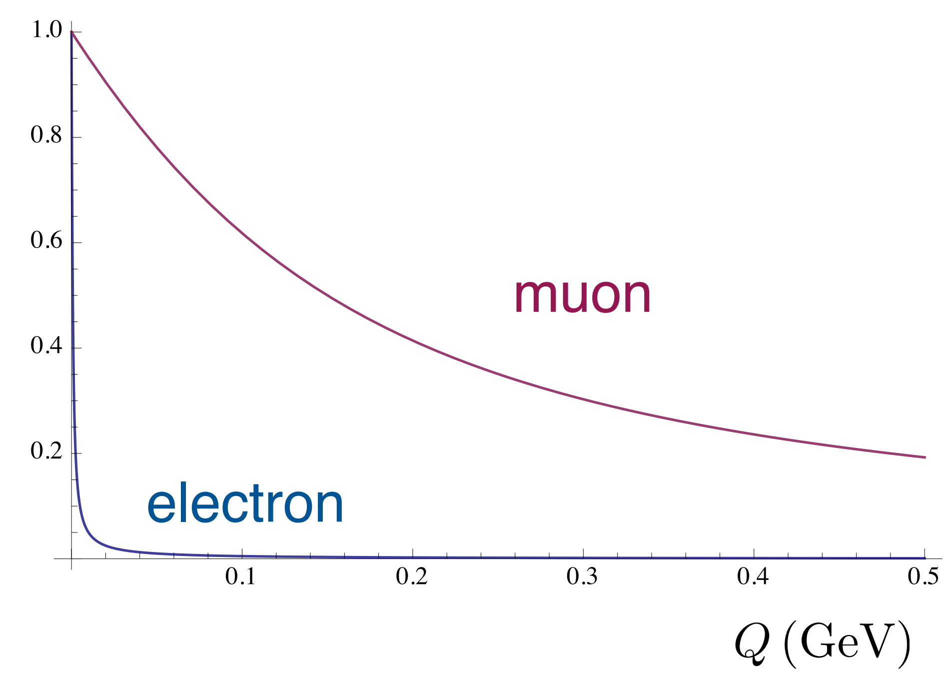
$$\Delta E^{(\mathrm{pol})}_{nS} = -4\alpha_{em}\phi_n^2 \int_0^\infty \frac{dQ}{Q^2}\, w(Q^2/4m_\ell^2)\, \left[T_2^{(\mathrm{NB})}(0,Q^2) - T_1^{(\mathrm{NB})}(0,Q^2)\right]$$

where unpolarized, **forward** Doubly-Virtual Compton scattering (VVCS) amplitude:

$$\phi_n^2(0) = m_r^3\alpha^3/(\pi n^3)$$

$$\begin{aligned} T^{\mu\nu}(p,q) &= \frac{i}{8\pi M}\int d^4x\, e^{iqx}\langle p|Tj^\mu(x)j^\nu(0)|p\rangle \\ &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) \\ &+ \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu,Q^2) \end{aligned}$$

$$w_\ell(Q) = \sqrt{1+\frac{Q^2}{4m_\ell^2}} - \frac{Q}{2m_\ell}$$



NB stands for non-Born, i.e. w/o elastic FFs

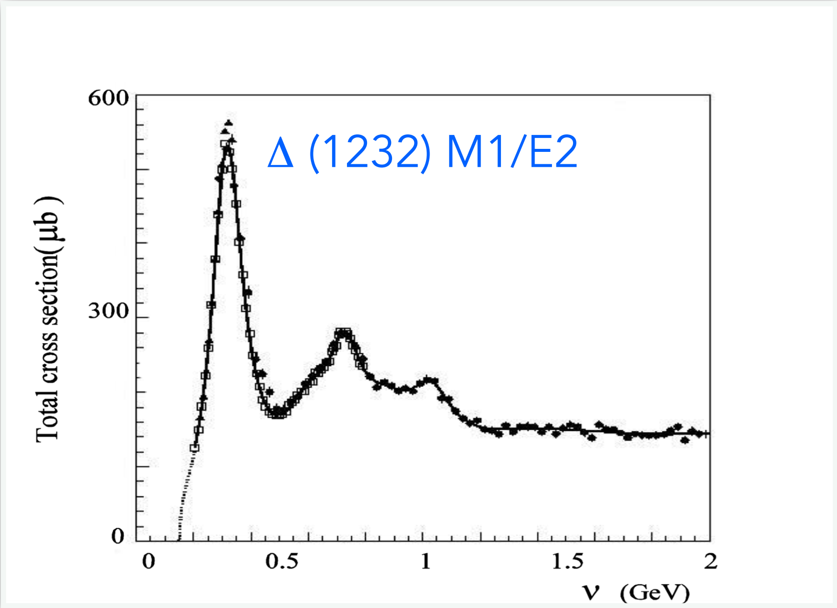
$$T_1^{(\mathrm{NB})}(0,Q^2) \simeq Q^2\beta_{M1}$$

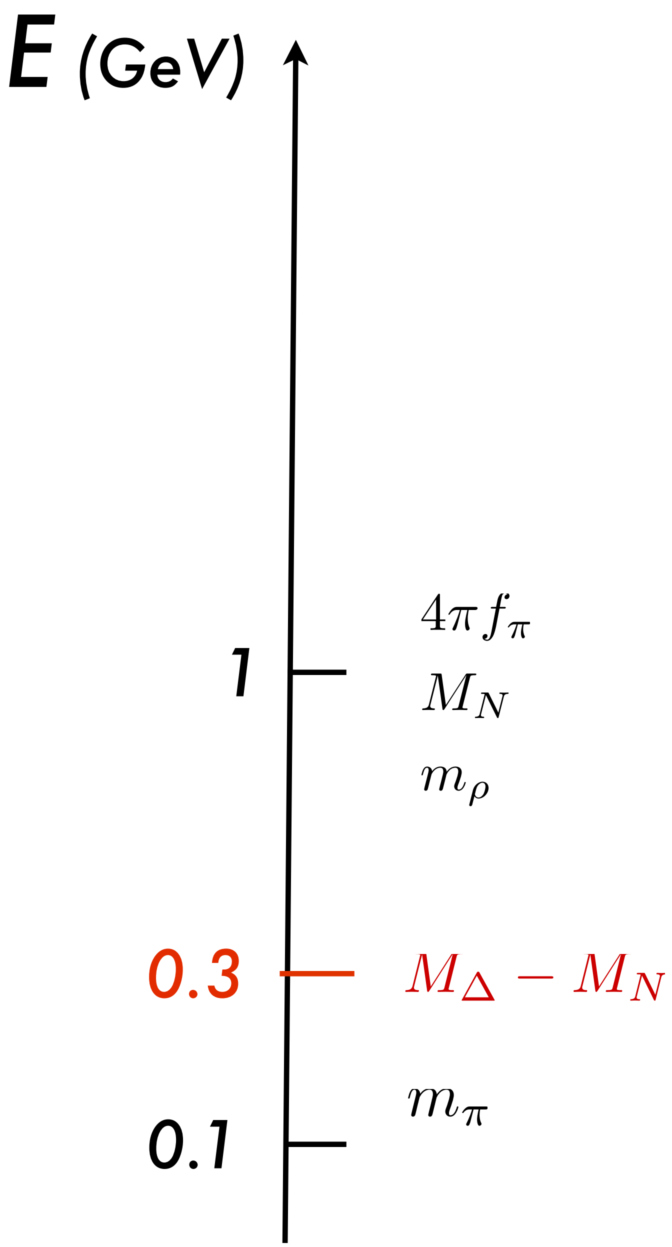
$$T_2^{(\mathrm{NB})}(0,Q^2) \simeq Q^2(\alpha_{E1}+\beta_{M1}), \quad \text{for low } Q$$

Baryon ChPT

pion cloud + Delta(1232) excitation

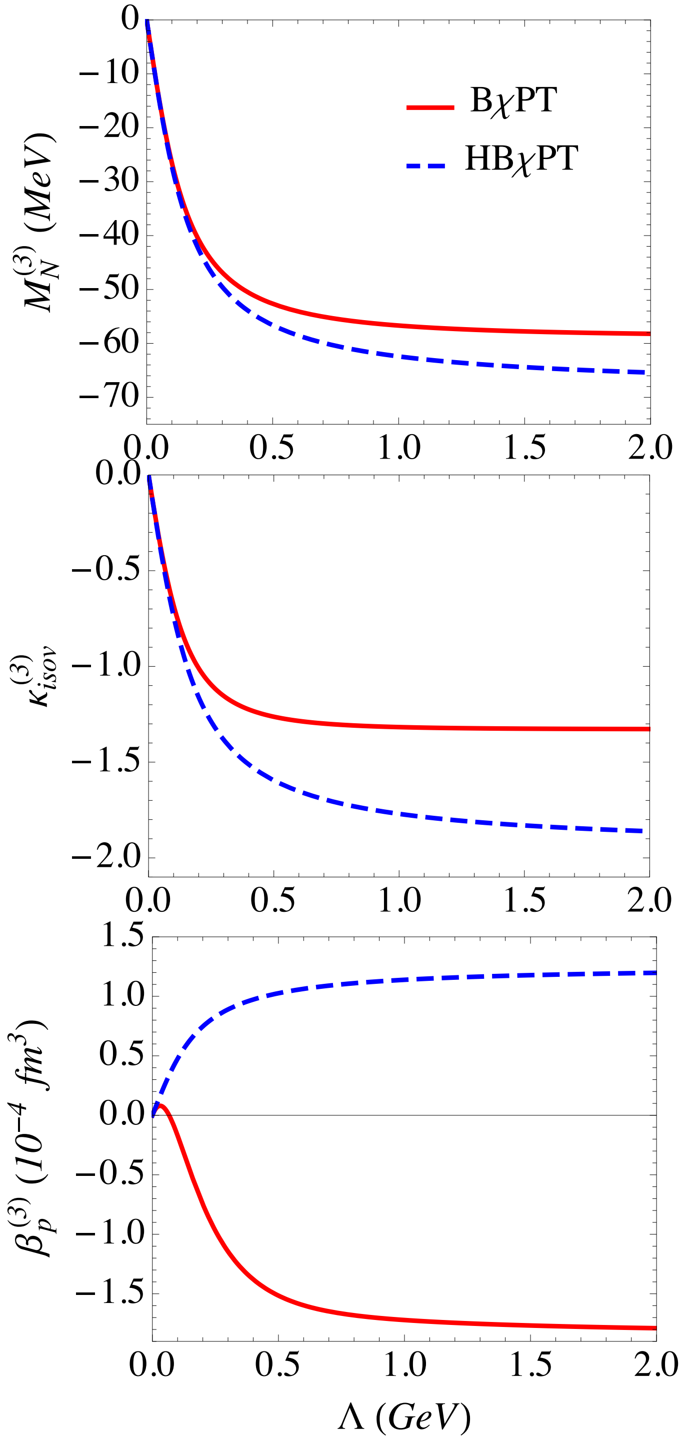
Jenkins & Manohar, PLB (1991)
Hemmert, Holstein, Kambor, JPhysG (1998)
V.P. & Phillips, PRC (2003)





- The 1st nucleon excitation — Delta(1232) is within reach of chiral perturbation theory (293 MeV excitation energy is a light scale)
- Include into the chiral effective Lagrangian as explicit dof
- Power-counting for Delta contributions (SSE, ``delta-counting") depends on what chiral order is assigned to the excitation scale.

UV dependence in HB- vs B-ChPT



$$M_N \sim m_\pi^3$$

$$\kappa \sim m_\pi$$

$$\beta_M \sim \frac{1}{m_\pi}$$

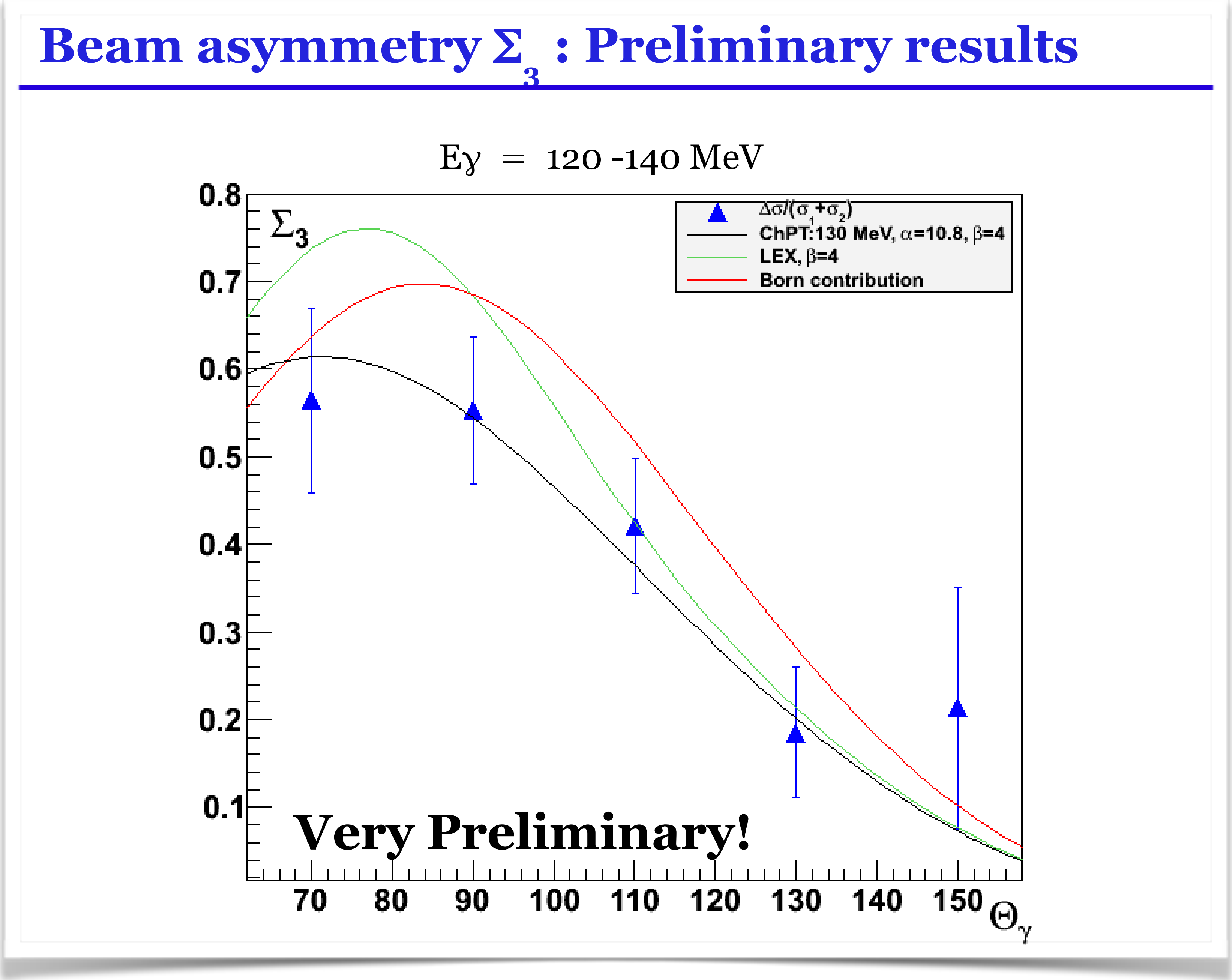
Heavy-Baryon expansion fails for quantities where the leading chiral-loop effects scales with a negative power of pion mass

E.g.: the effective range parameters of the NN force are such quantities -- hope for “perturbative pions” (KSW) in BChPT

New Mainz data for Compton beam asymmetry

Data taken: 28.05. – 17.06.2013, 327 h

V. Sokhoyan, E. Downie et al. [A2 Coll.]



first data on this observable below pion production threshold!

better precision needed!!

Predictions of HBChPT vs BChPT

HBChPT@LO

Bernard, Keiser, Meissner
Int J Mod Phys(1995)

$$\alpha_p = \alpha_n = \frac{5\pi\alpha}{6m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 12.2\times 10^{-4}\ \ \mathrm{fm}^3,$$

$$\beta_p = \beta_n = \frac{\pi\alpha}{12m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 1.2\times 10^{-4}\ \ \mathrm{fm}^3,$$

BChPT@NLO

Lensky & V.P.

$$\alpha = \underbrace{6.8}_{\mathcal{O}(p^3)} + (\ldots$$

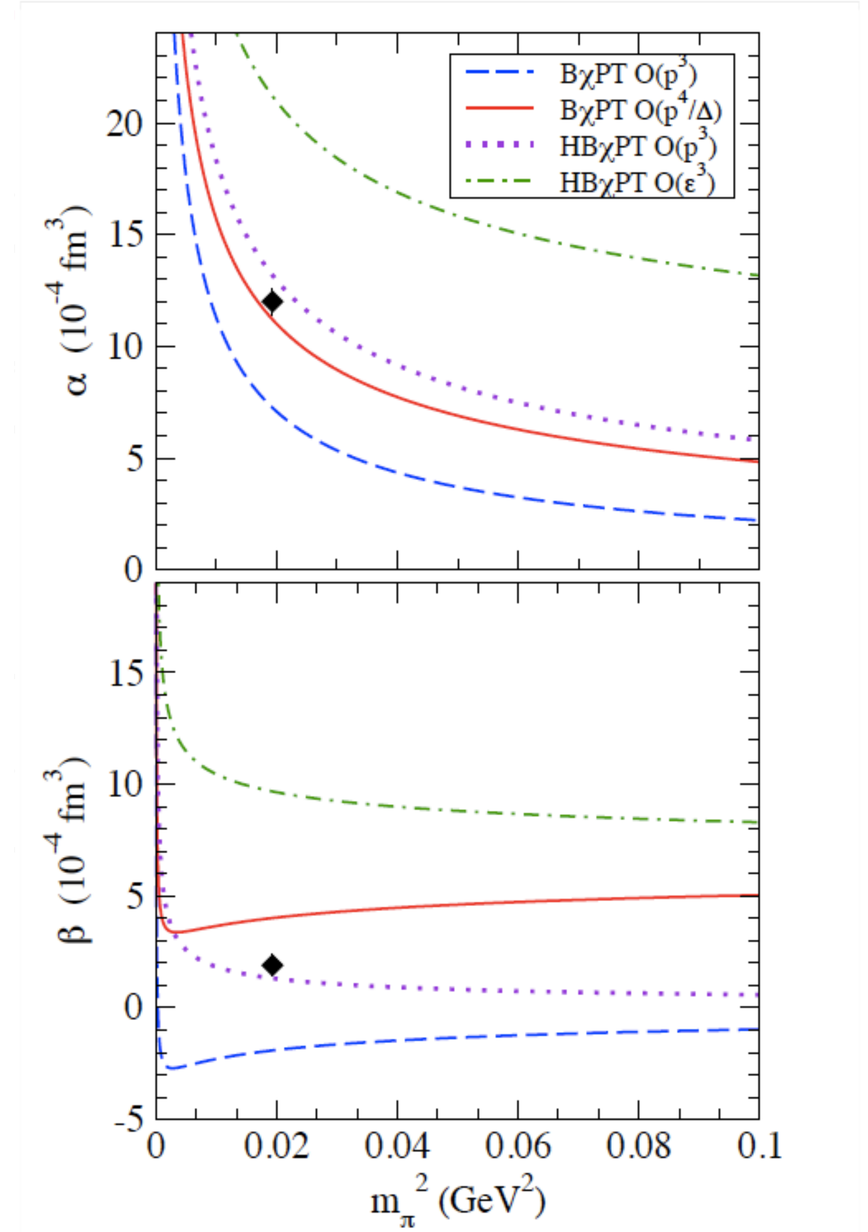
$$\beta = \underbrace{-1.8}_{\mathcal{O}(p^3)} + \ldots$$

paramagnetic

diamagnetic

$$\mu = m_\pi / M_N$$

$$\beta = \frac{e^2 g_A^2}{192\pi^3 F^2 M_N}\left[\frac{\pi}{4\mu} + 18\log\mu + \frac{63}{2} - \frac{981\pi\mu}{32} - (100\log\mu \ldots\right.$$



HBChPT@NLO:
Griesshammer & Hemmert (2004)
Griesshammer, McGovern, Phillips (2012)

The Delta contribution is accompanied by "promoted" LECs, hence not predictive

Lattice QCD data expected soon