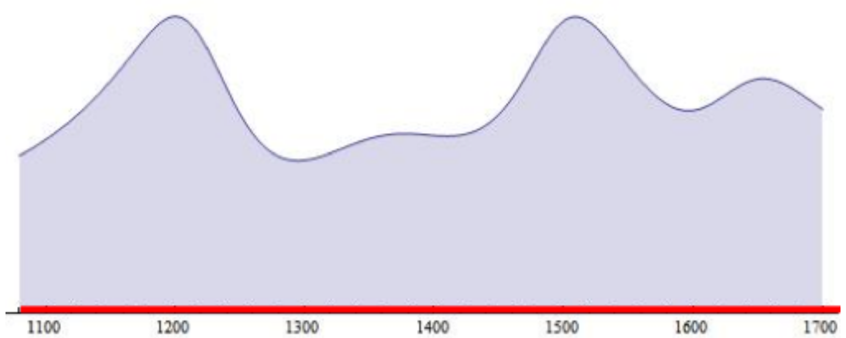
Hadron Spectroscopy and Interactions from Lattice QCD

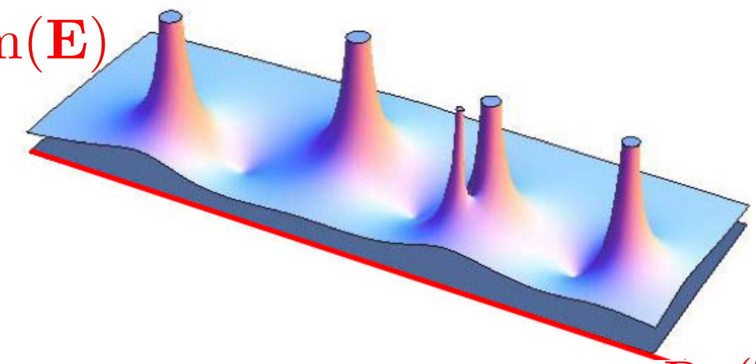
Robert Edwards

Jefferson Lab

Resonances

- Most hadrons are resonances
  - E.g., πN πN





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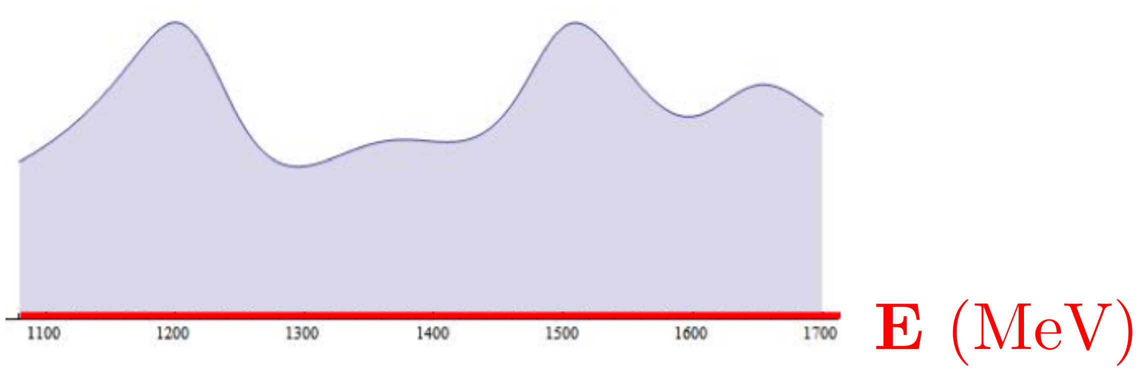
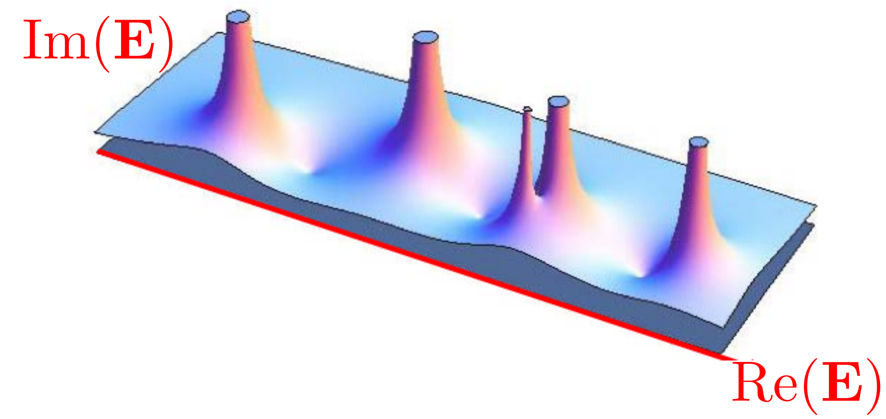
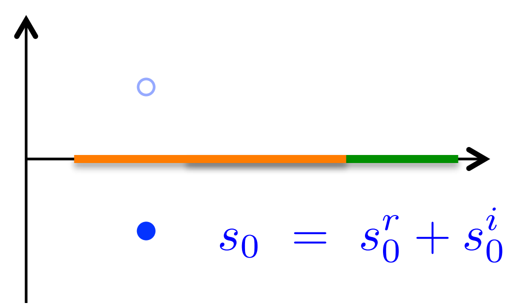




  - Formally defined as a pole in a partial-wave scattering amplitude

$$t_l(s) \sim \frac{R}{s_0 - s} + \ldots$$



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  - Pole structure gives decay information

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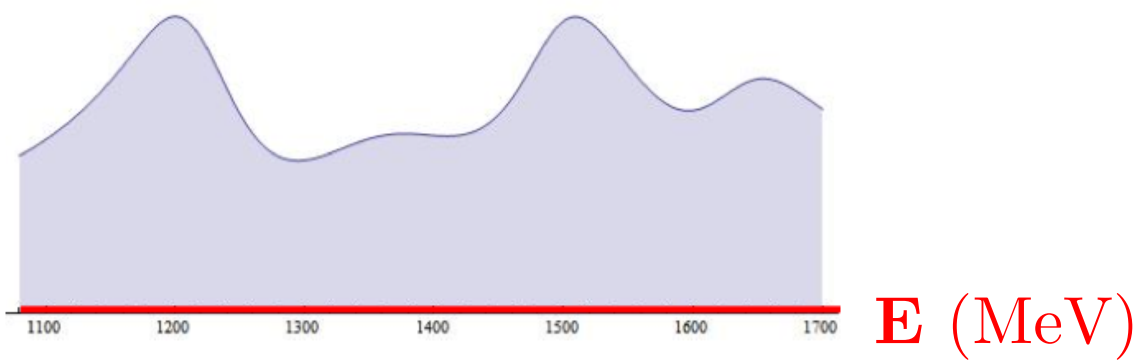

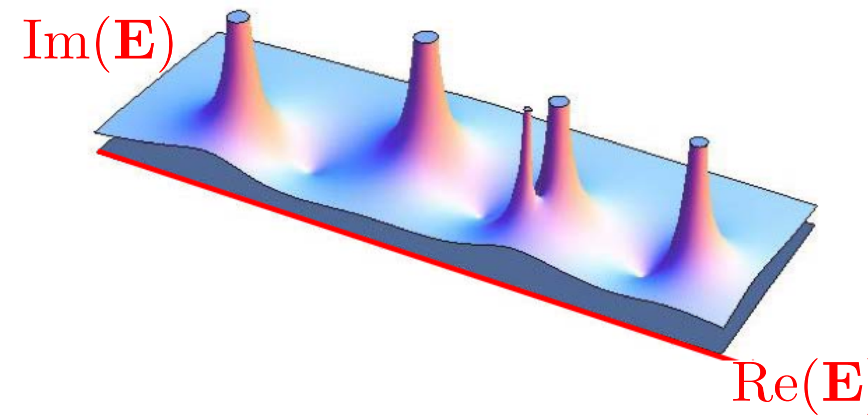

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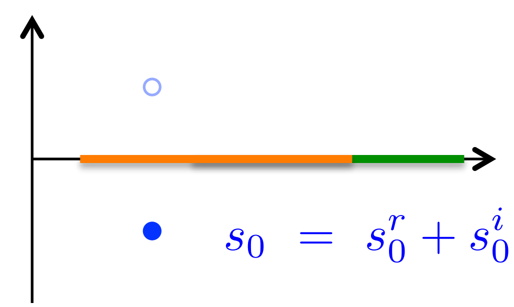

  - Different channels should have same pole location
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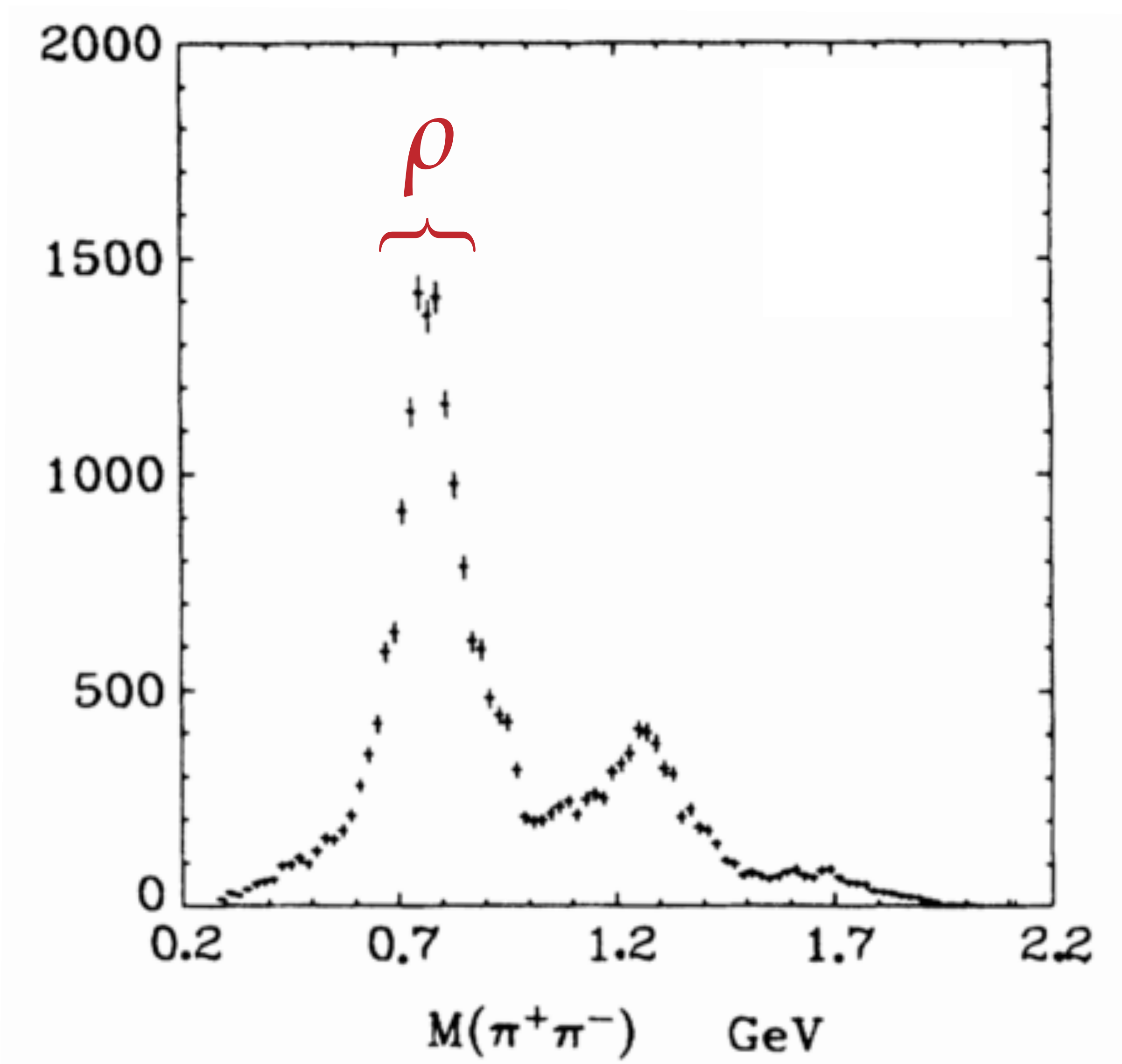
- Can we predict hadron properties from first principles?

Isospin=1 ππ P-wave



expand angular dependence
in *partial waves*

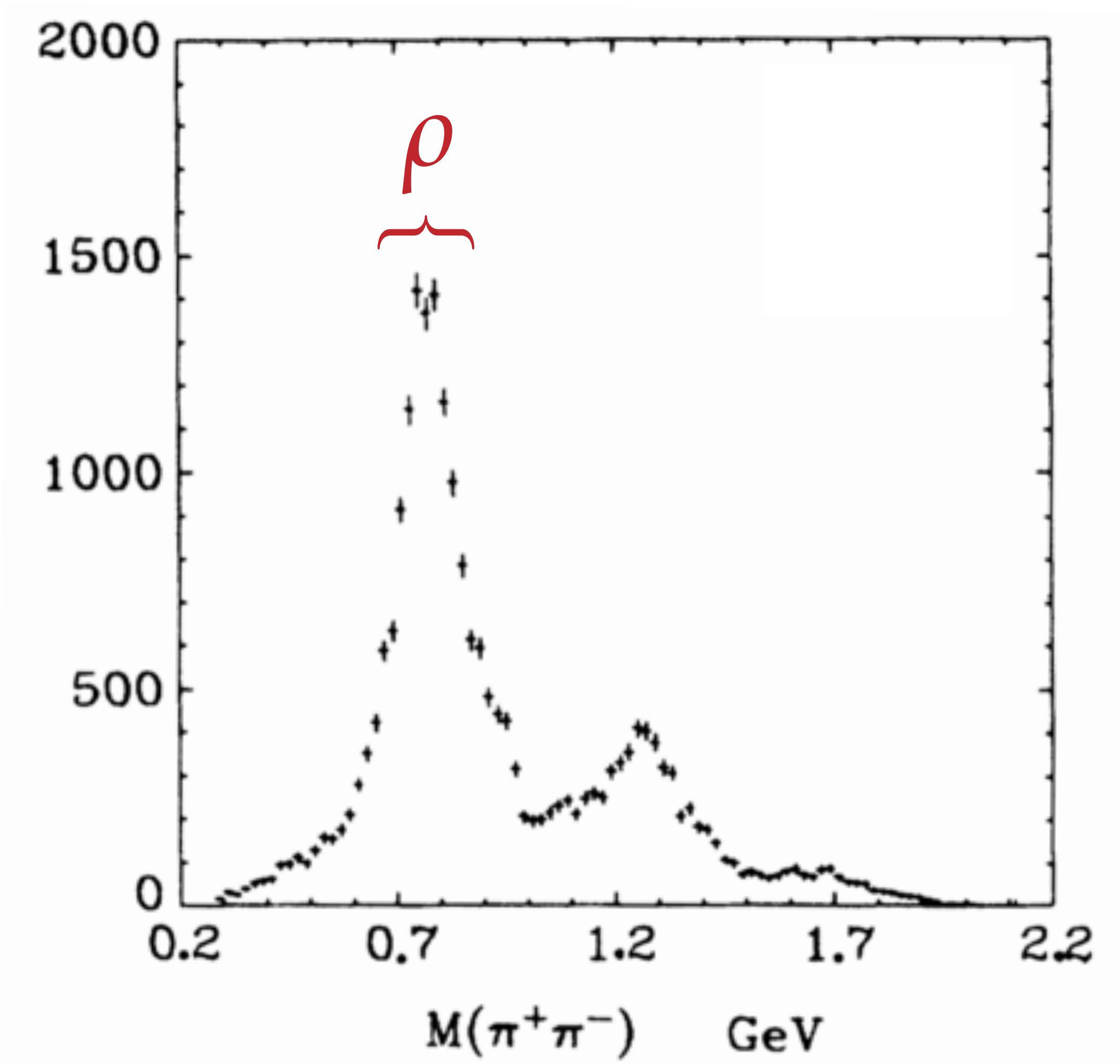
PARTIAL WAVE AMPLITUDE

$$f_\ell = \frac{1}{2i}\left(\eta_\ell\, e^{2i\delta_\ell} - 1\right)$$

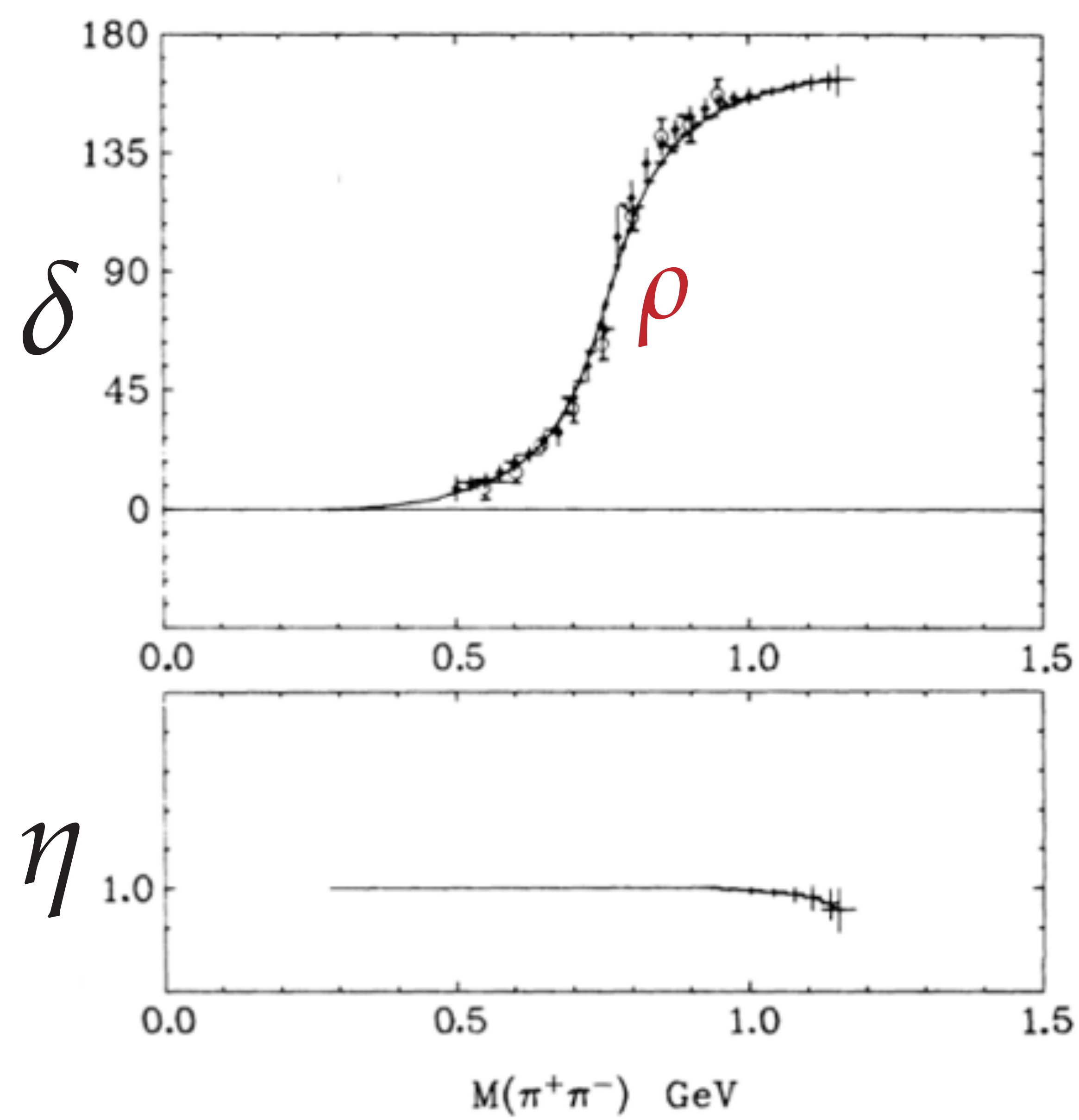
$\eta = 1$ elastic

$\eta \leq 1$ inelastic

Isospin=1 ππ P-wave



RESONANT PHASE SHIFT



Finite-volume

- Where's the meson-meson continuum ?
  - **there isn't one !**

Finite-volume

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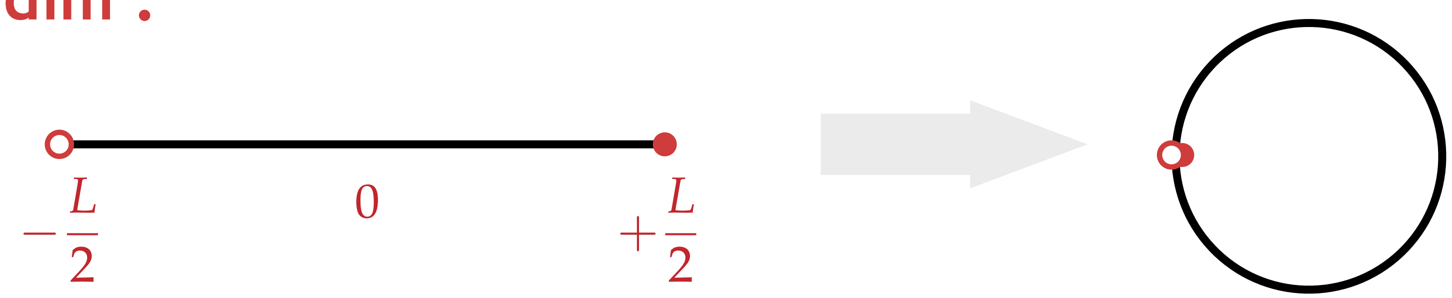
  - in a finite-volume the spectrum is discrete

one-dim :



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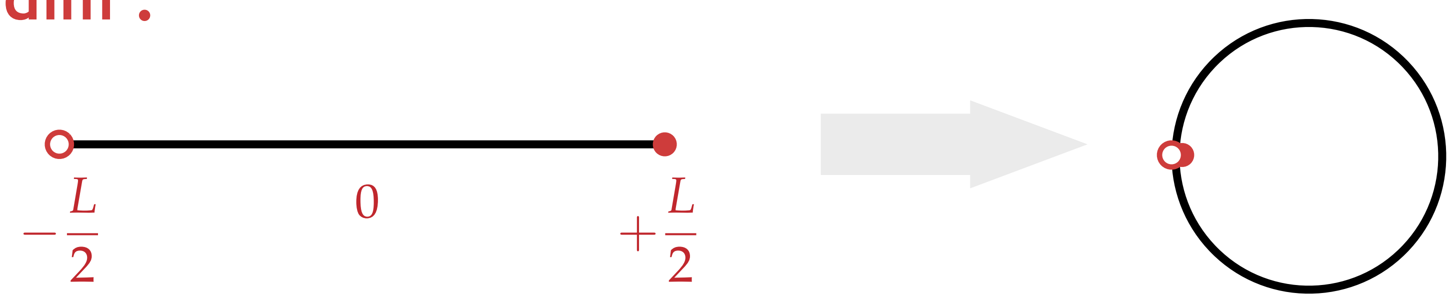

e.g. a free particle

$$\psi(x) \sim e^{ipx}$$

» periodic boundary condition

$$\psi(x) = \psi(x+L)$$

$$e^{ipx} = e^{ip(x+L)}$$

$$e^{ipL} = 1$$

$$p = \frac{2\pi}{L} n$$

discrete energy spectrum

Interacting particles in a finite-volume

- Two identical bosons **interacting** through a finite-range potential

$$\psi(z) \sim \cos\left[\, p|z| + \delta(p)\right]$$ outside the range of the potential, $|z| > R$

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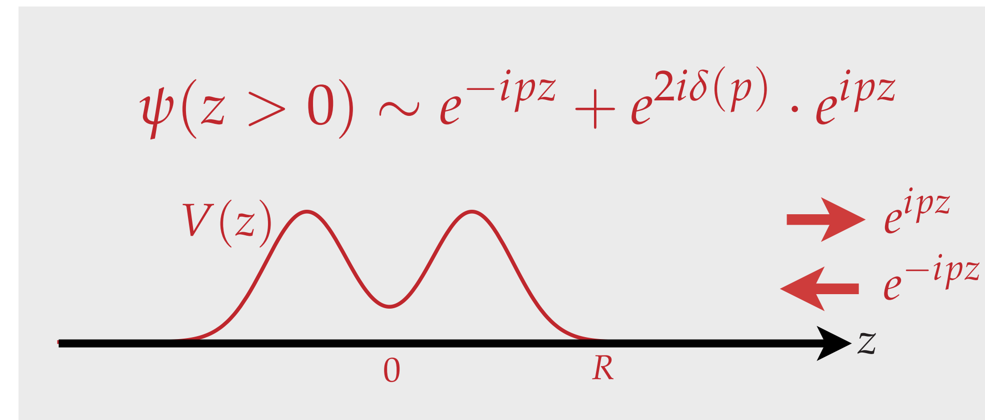

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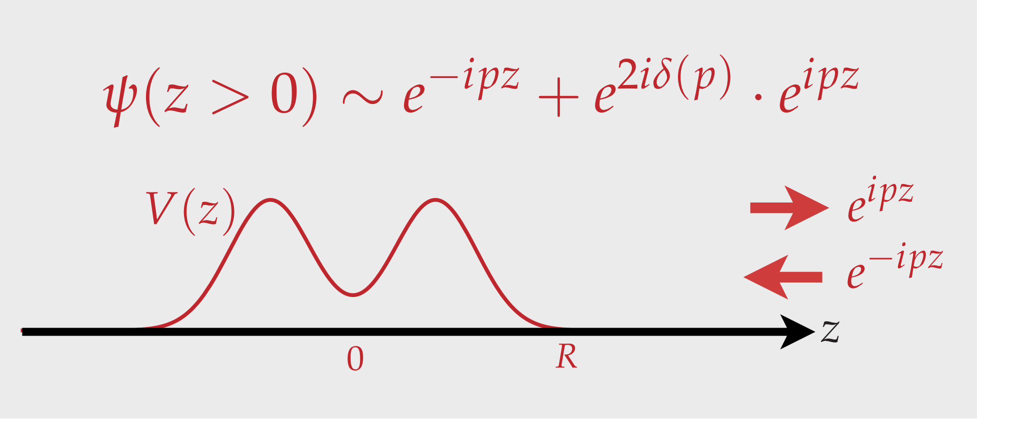

- periodic boundary conditions

$$\left.\begin{aligned} \psi\left(-\tfrac{L}{2}\right) &= \psi\left(\tfrac{L}{2}\right) \\ \frac{d\psi}{dz}\left(-\tfrac{L}{2}\right) &= \frac{d\psi}{dz}\left(\tfrac{L}{2}\right) \end{aligned}\right\} \implies 0 = \sin\left[\frac{pL}{2} + \delta(p)\right]$$

$$\frac{pL}{2} + \delta(p) = n\pi$$

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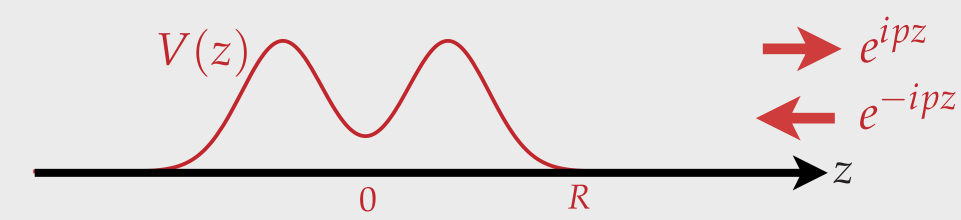
$$\psi(z>0) \sim e^{-ipz} + e^{2i\delta(p)} \cdot e^{ipz}$$



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discrete energy spectrum

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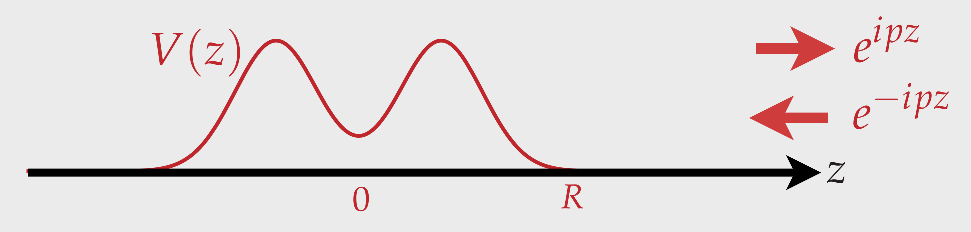

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discrete energy spectrum

discrete energy spectrum is determined by the scattering amplitude

(or vice-versa)

Scattering in a finite cubic volume

- Expect a discrete spectrum in a finite periodic volume $\psi(x+L) = \psi(x)$

e.g. free particle $e^{ip(x+L)} = e^{ipx}$

quantized momentum $p = \frac{2\pi}{L} n$

- For an interacting theory $$\cot \delta_\ell(E) = \mathcal{M}_\ell(E, L)$$ LÜSCHER ...

elastic scattering phase-shift

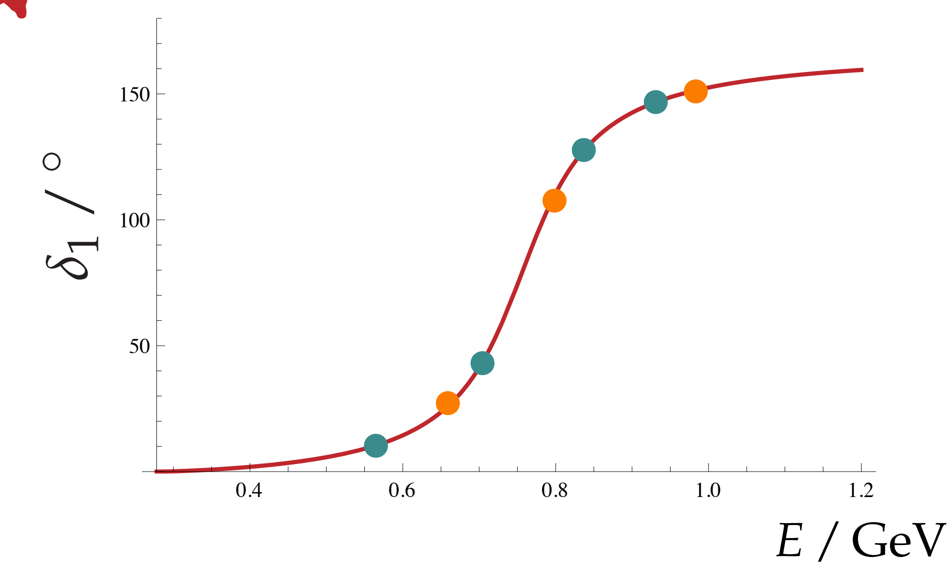
known function

Discrete energies in a finite-volume ⟷ Discrete values of the phase-shift

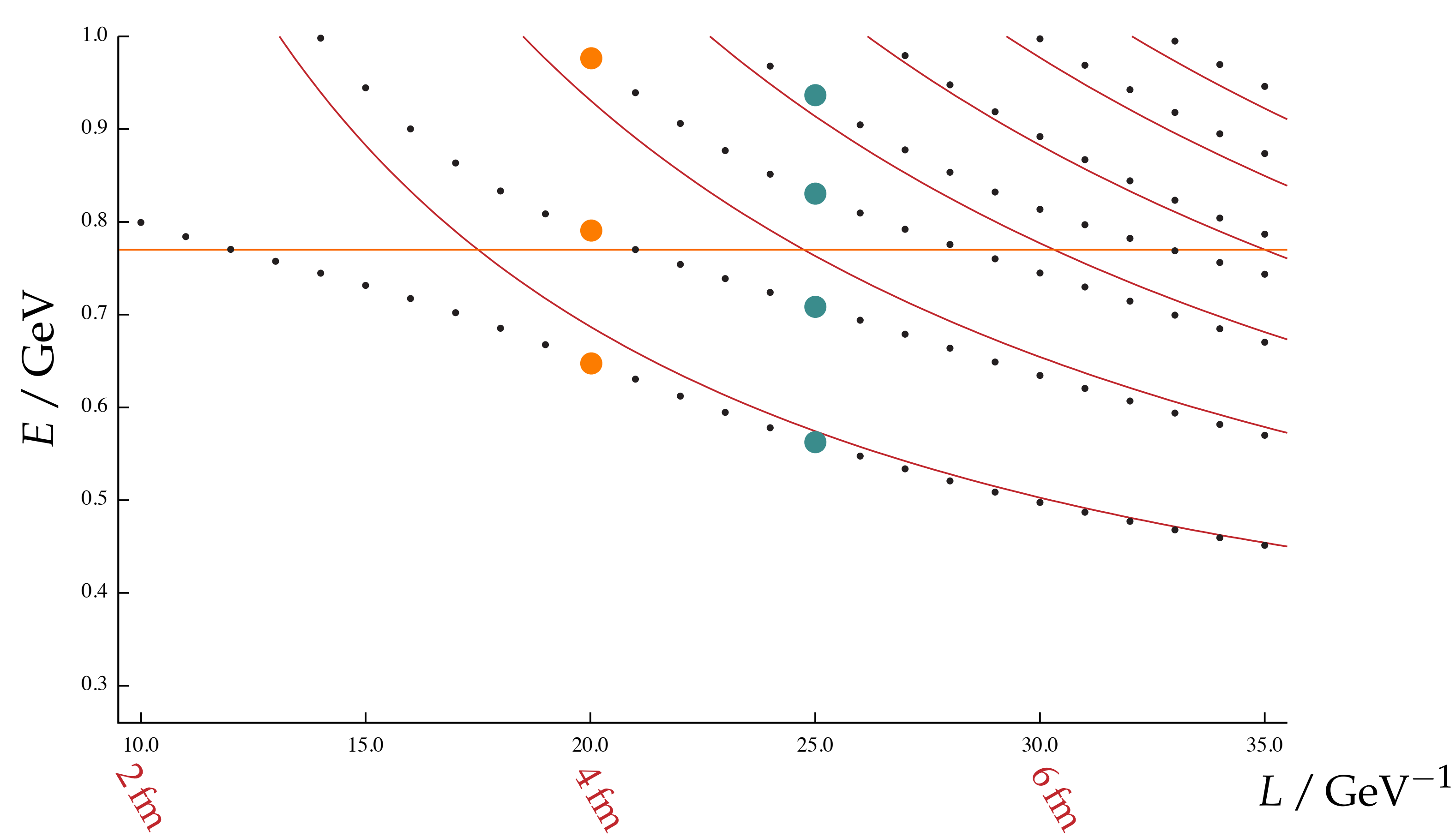
Scattering in a finite cubic volume

- Experimental $\pi\pi$ I=1 P-wave scattering amplitude

P-WAVE PHASE SHIFT



CUBIC BOX SPECTRUM



Coupled-channel scattering

HE, JHEP 0507 011
HANSEN, PRD86 016007
BRICENO, PRD88 094507
GUO, PRD88 014051

- Finite-volume formalism recently derived (multiple methods)

$$\det\left[\left(\left[t^{(\ell)}(E)\right]^{-1}_{ij} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell\big(p_i(E)L\big)\right] = 0$$

scattering matrix

phase space

known functions

matrices in partial-wave space ...

- However, this is **one equation** for **multiple unknowns** (per energy level) $\frac{1}{2}N(N+1)$ for N channels
  - parameterize the energy dependence of *t*
  - try to describe a spectrum globally

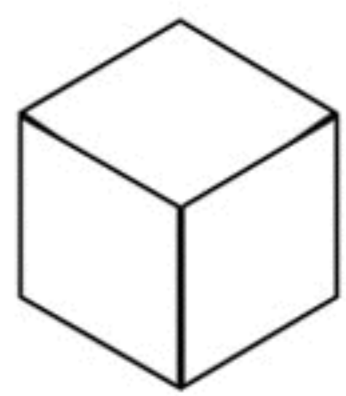
"Energy-dependent" analysis

Finite volume QCD & the hadron spectrum

- Compute **correlation functions** as an average over field configurations

e.g. $\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu \; \bar{\psi}\Gamma\psi(t)\,\bar{\psi}\Gamma\psi(0)\; e^{-\int d^4x\, \mathcal{L}_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$

'sum' 'field correlation' 'probability weight'



Field integration within a finite, but continuous, hypercube

Need some kind of ultraviolet regulator....

- Spectrum from two-point correlation functions

$$\begin{aligned} C(t) &= \langle 0|\mathcal{O}(t)\,\mathcal{O}^\dagger(0)|0\rangle \\ &= \sum_{\mathfrak{n}} e^{-E(\mathfrak{n})t}\,\langle 0|\mathcal{O}(0)|\mathfrak{n}\rangle\,\langle \mathfrak{n}|\mathcal{O}^\dagger(0)|0\rangle \end{aligned}$$

Lattice QCD & the hadron spectrum

- Compute **correlation functions** as a Monte Carlo average over field configurations

$$\text{e.g.} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \; \bar{\psi}\Gamma\psi(t)\; \bar{\psi}\Gamma\psi(0) \;\; e^{-\int d^4x\, \mathcal{L}_{\rm QCD}(\psi,\bar{\psi},A_\mu)}$$

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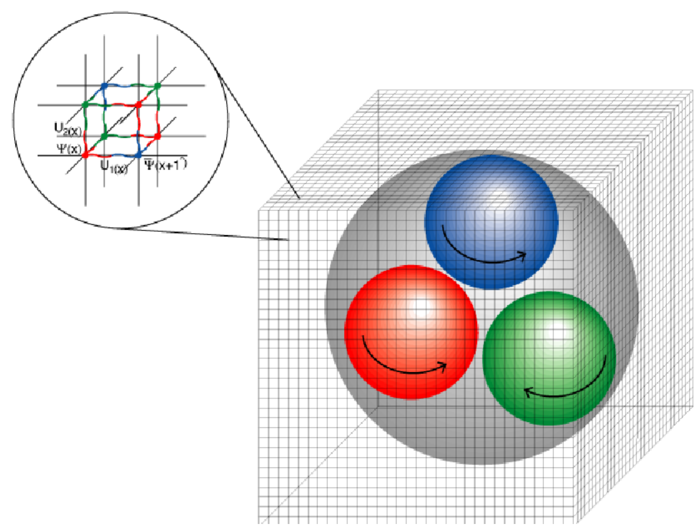

Discretize the action over sites

Serves as an ultraviolet regulator

- Spectrum from two-point correlation functions

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Excited states from correlators

- how to get at excited QCD eigenstates ?
  - optimal operator for state $|\mathfrak{n}\rangle$: $\Omega_{\mathfrak{n}}^\dagger \sim \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$

    for a basis of meson operators $\{\mathcal{O}_i\}$

  - can be obtained (in a variational sense) from the matrix of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) | 0 \rangle$$

  - by solving a generalized eigenvalue problem

'diagonalize the correlation matrix'

$$C(t) v^{(\mathfrak{n})} = C(t_0) v^{(\mathfrak{n})} \lambda_{\mathfrak{n}}(t)$$

eigenvalues

$$\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$$

  - a large basis can be constructed using covariant derivatives :

$$\mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$$

Including multi-meson operators

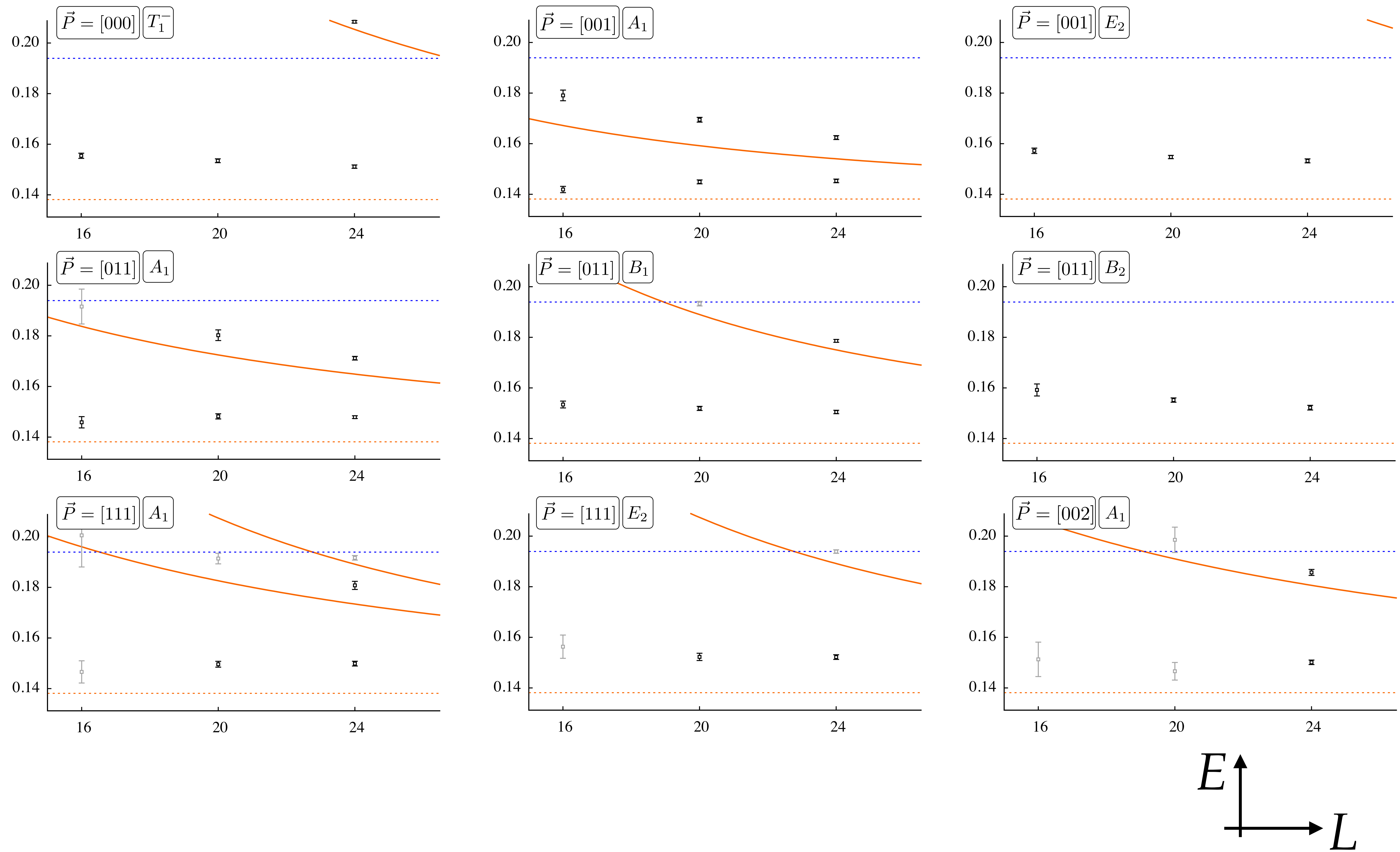
- form correlator matrix with both $\bar{\psi}\boldsymbol{\Gamma}\psi$ and $\pi\pi$-like

- include operators which resemble a pair of pions $$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\,\pi^\dagger(\vec{k}_1)\,\pi^\dagger(\vec{k}_2)$$

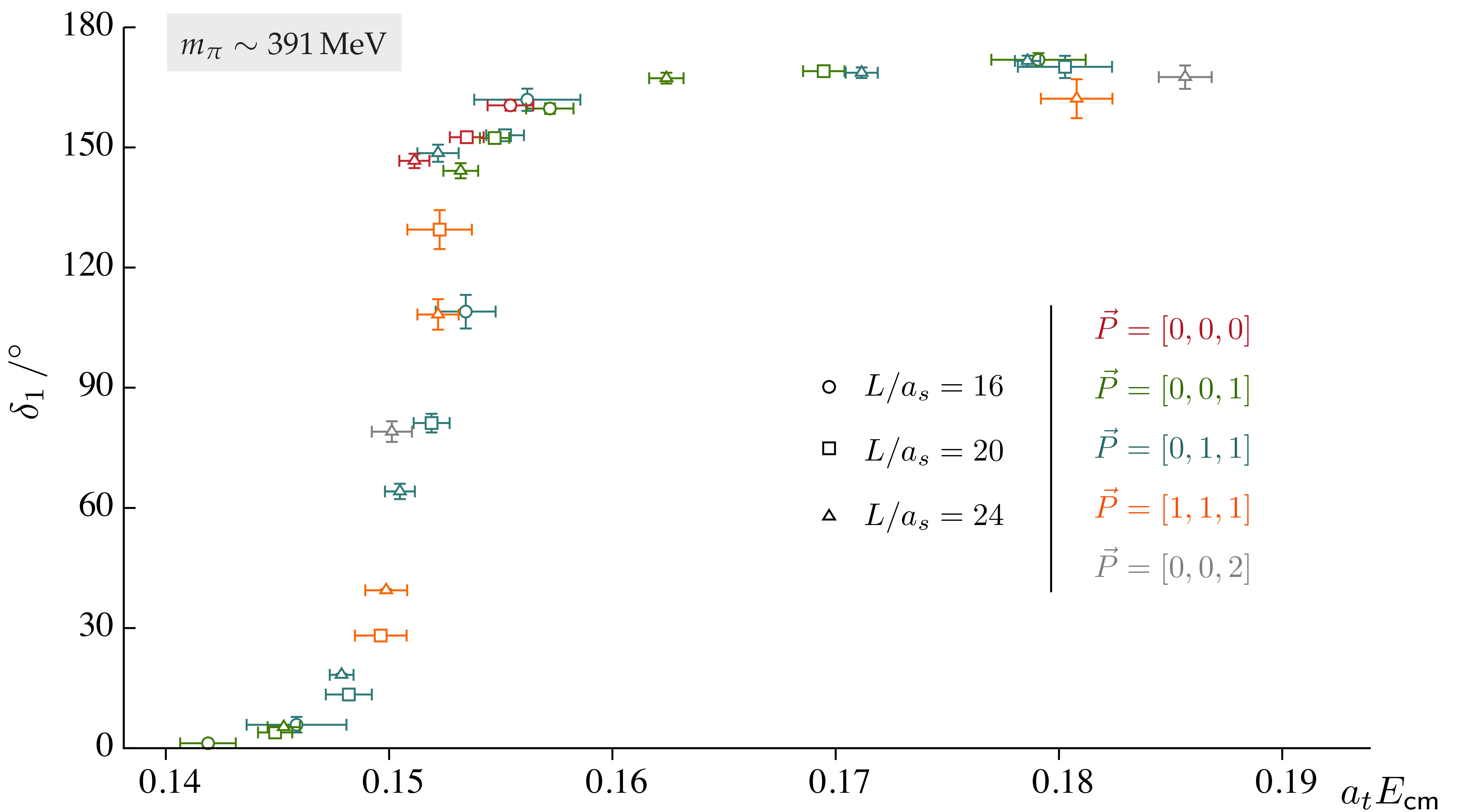
$$\pi^\dagger \sim \bar{\psi}\boldsymbol{\Gamma}\psi$$

Finite-volume spectrum - moving frames

$m_\pi \sim 391\,\text{MeV}$

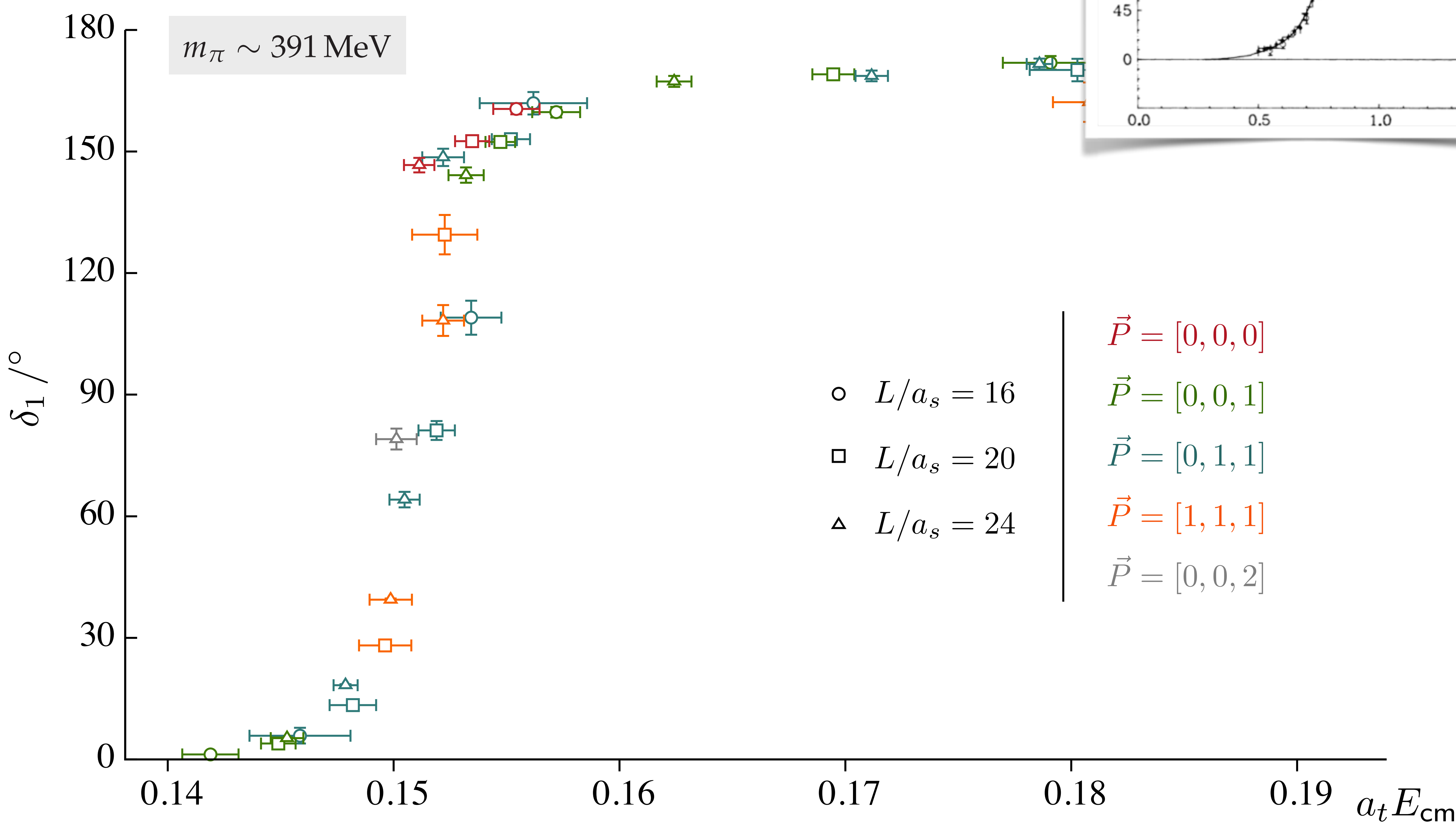


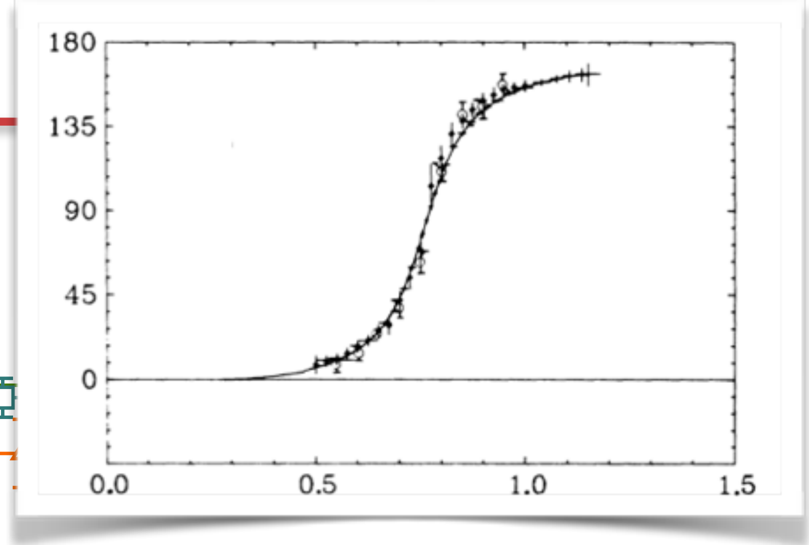
ππ *P*-wave phase-shift



PRD87 034505 (2013)

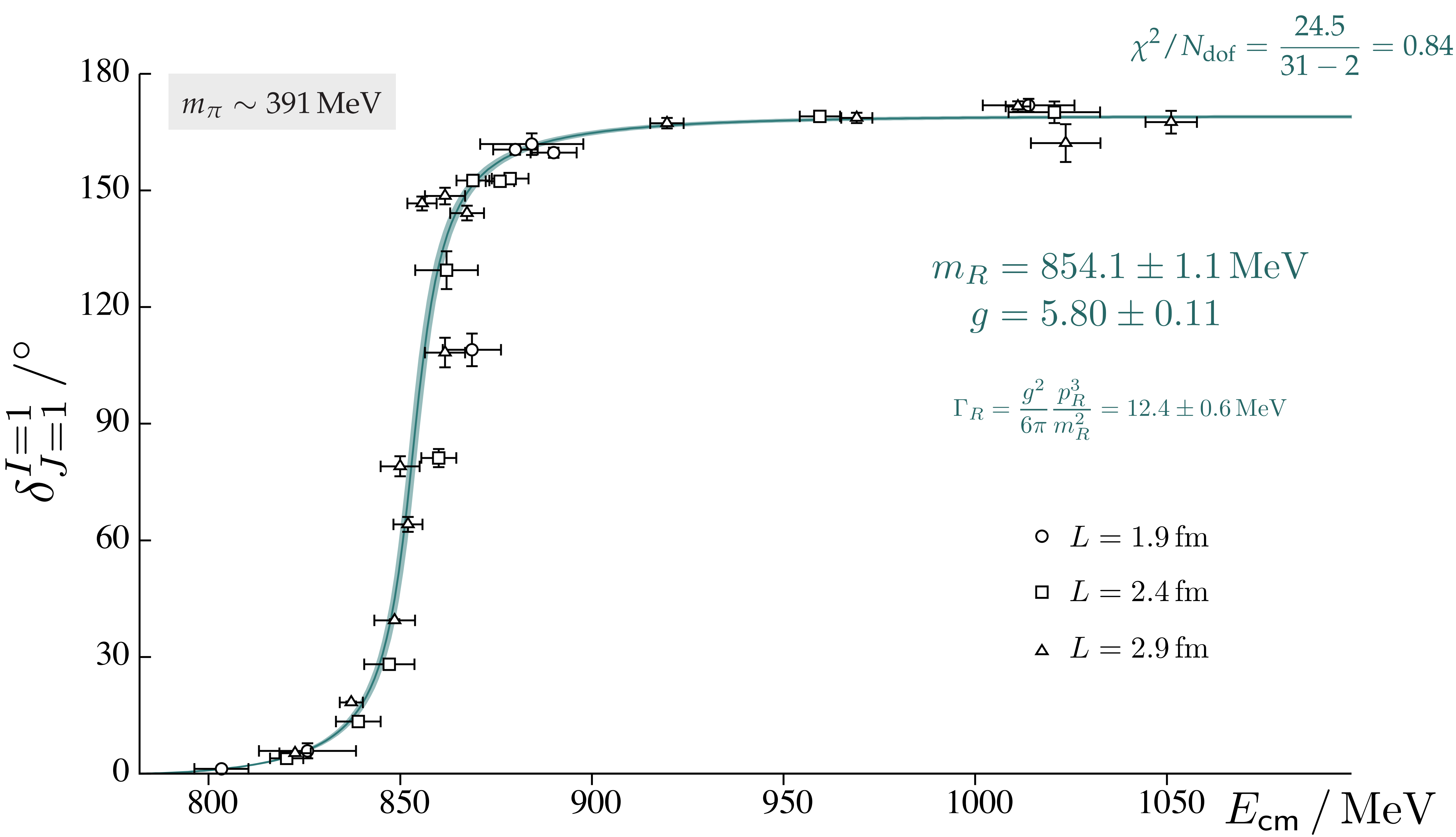
$\pi\pi$ *P*-wave phase-shift





PRD87 034505 (2013)

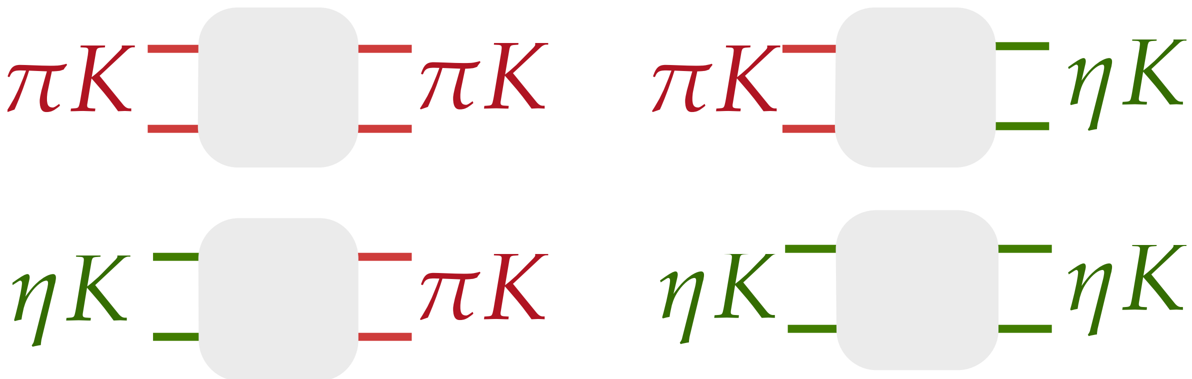
ρ resonance



$g_{phys.} = 5.95(2)$ *PDG*

$\pi K/\eta K$ scattering & kaon resonances

- Example of coupled-channel scattering



- Compute finite-volume spectrum

$$\bar{u}\mathbf{\Gamma}s$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\ \pi^\dagger(\vec{k}_1)\ K^\dagger(\vec{k}_2)$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\ \eta^\dagger(\vec{k}_1)\ K^\dagger(\vec{k}_2)$$

PRL 113 182001
PRD 91 054008

$\pi K/\eta K$ scattering & kaon resonances

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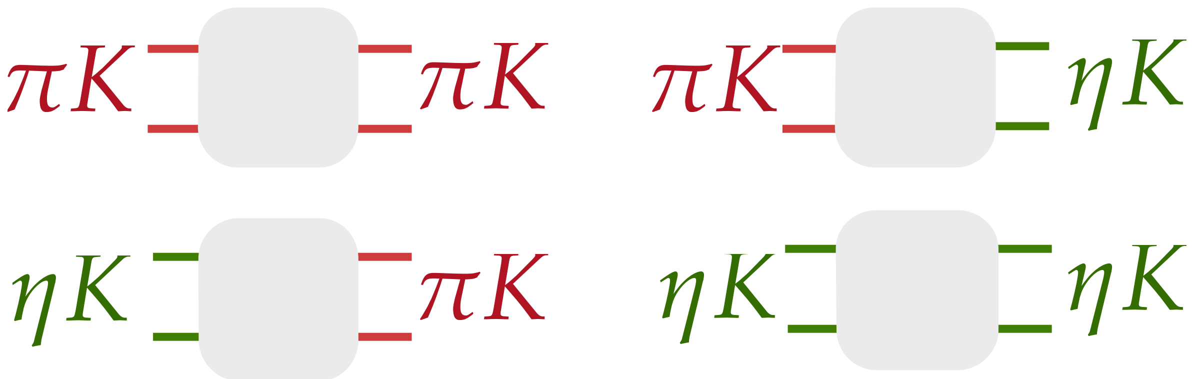

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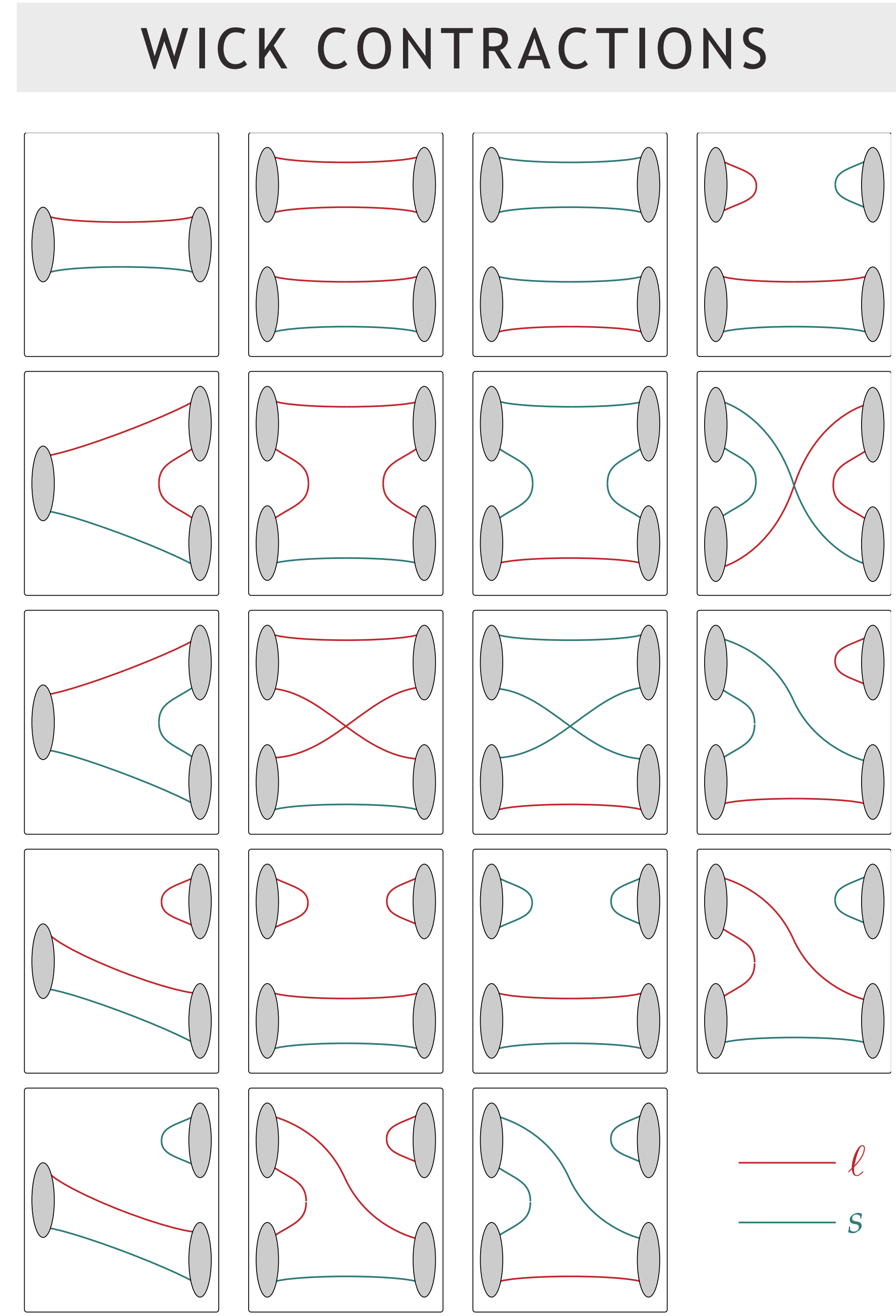
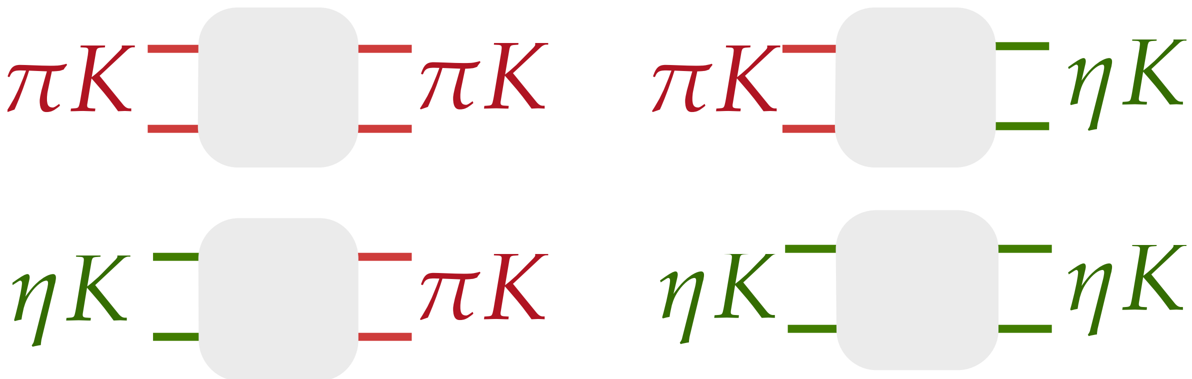


$\pi K/\eta K$ scattering & kaon resonances

- Parameterize the t-matrix in a unitarity conserving way



$$t^{-1}_{ij}(E) = K^{-1}_{ij}(E) + \delta_{ij}\, I_i(E)$$

$$K_{ij}(E) = \frac{g_i\, g_j}{m^2 - E^2} + \gamma_{ij}$$

- Vary the parameters, solving

$$\det\left[\left(\left[t^{(\ell)}(E)\right]^{-1}_{ij} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell(E,L)\right] = 0$$

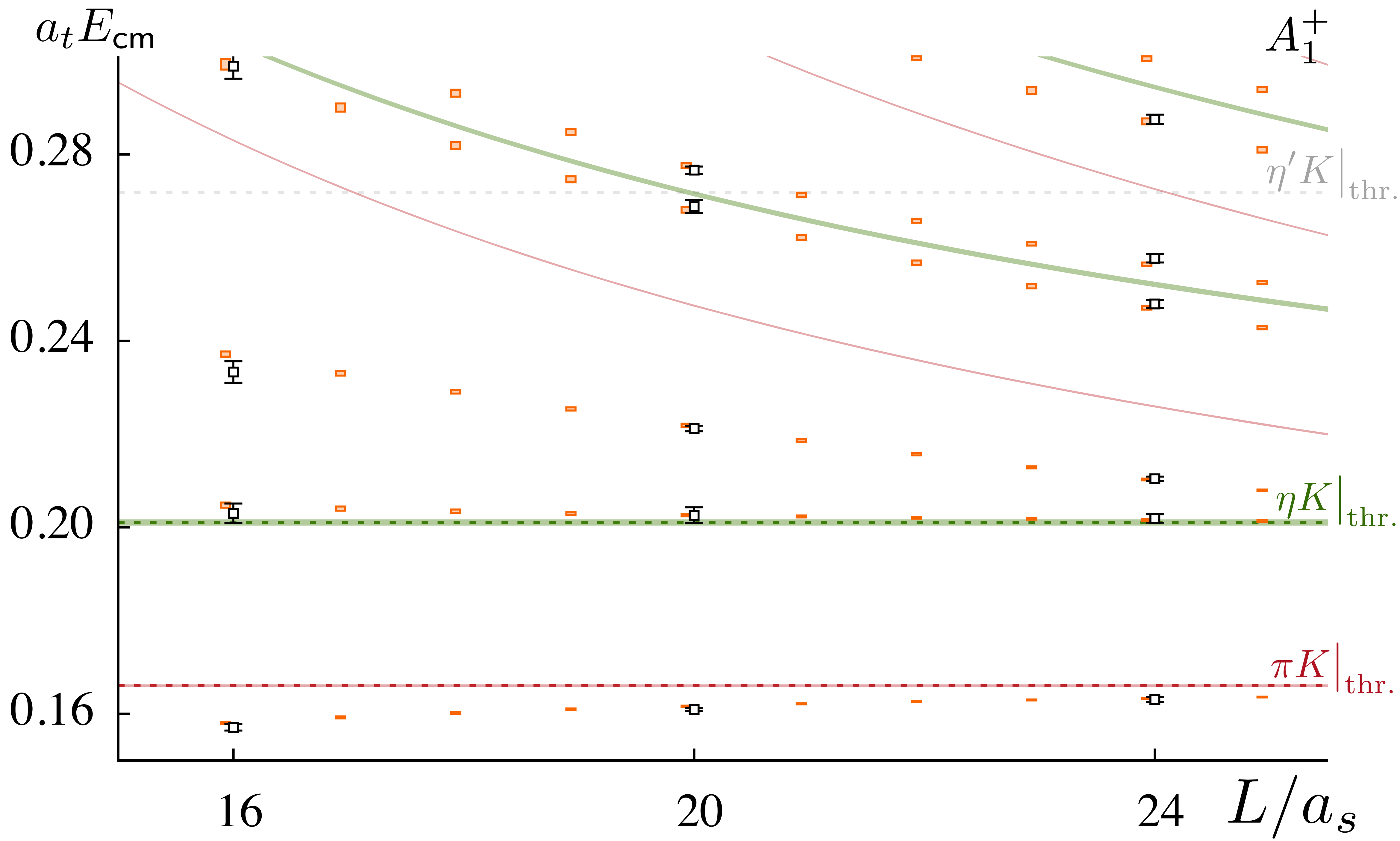
for the spectrum in each irreducible representation & momentum

Want pole mass and couplings of t-matrix

$\pi K/\eta K$ scattering

Model description $\chi^2/N_{\mathrm{dof}} = \frac{6.40}{15-6} = 0.71$

$m_\pi \sim 391\,\mathrm{MeV}$



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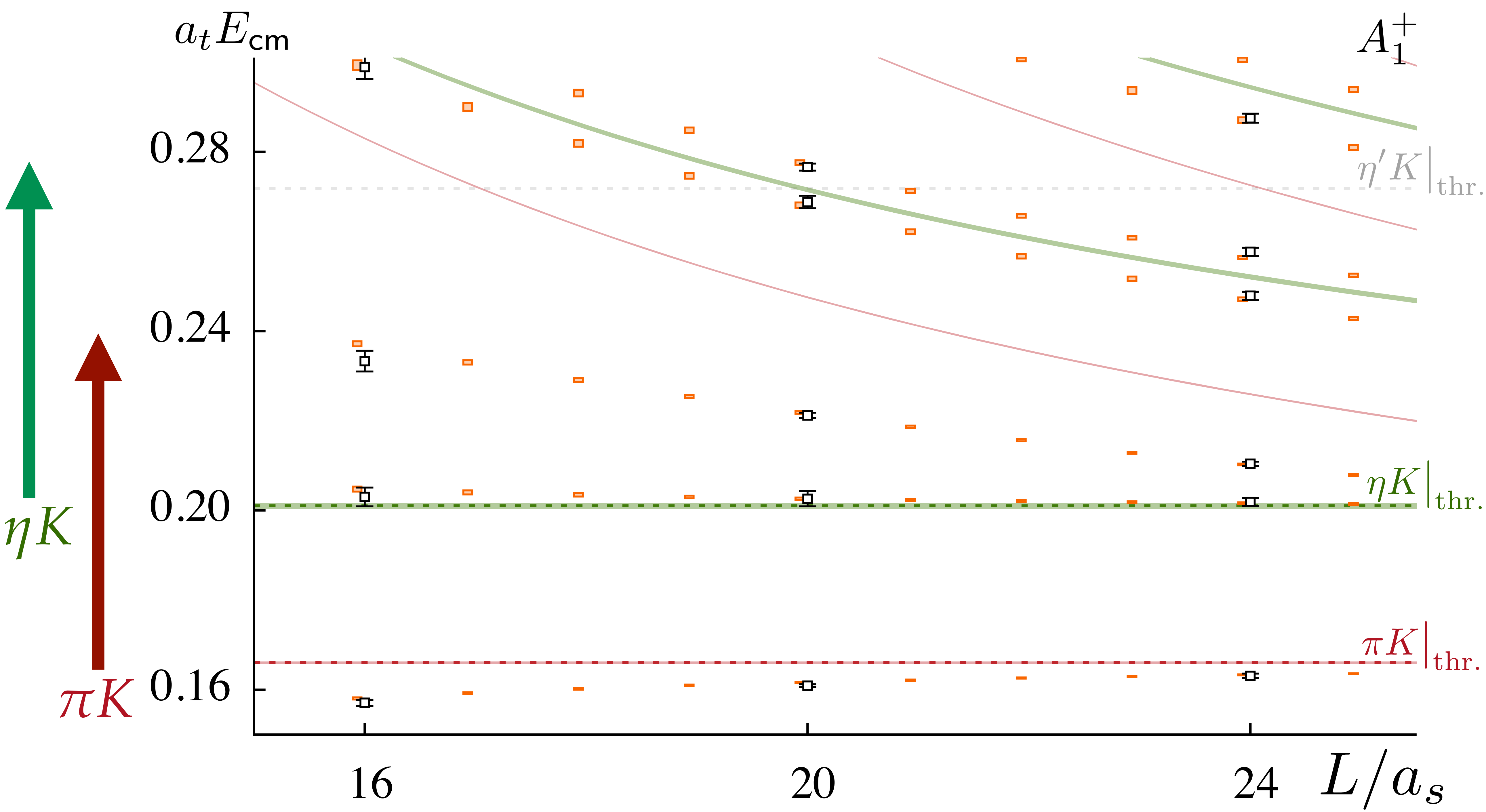

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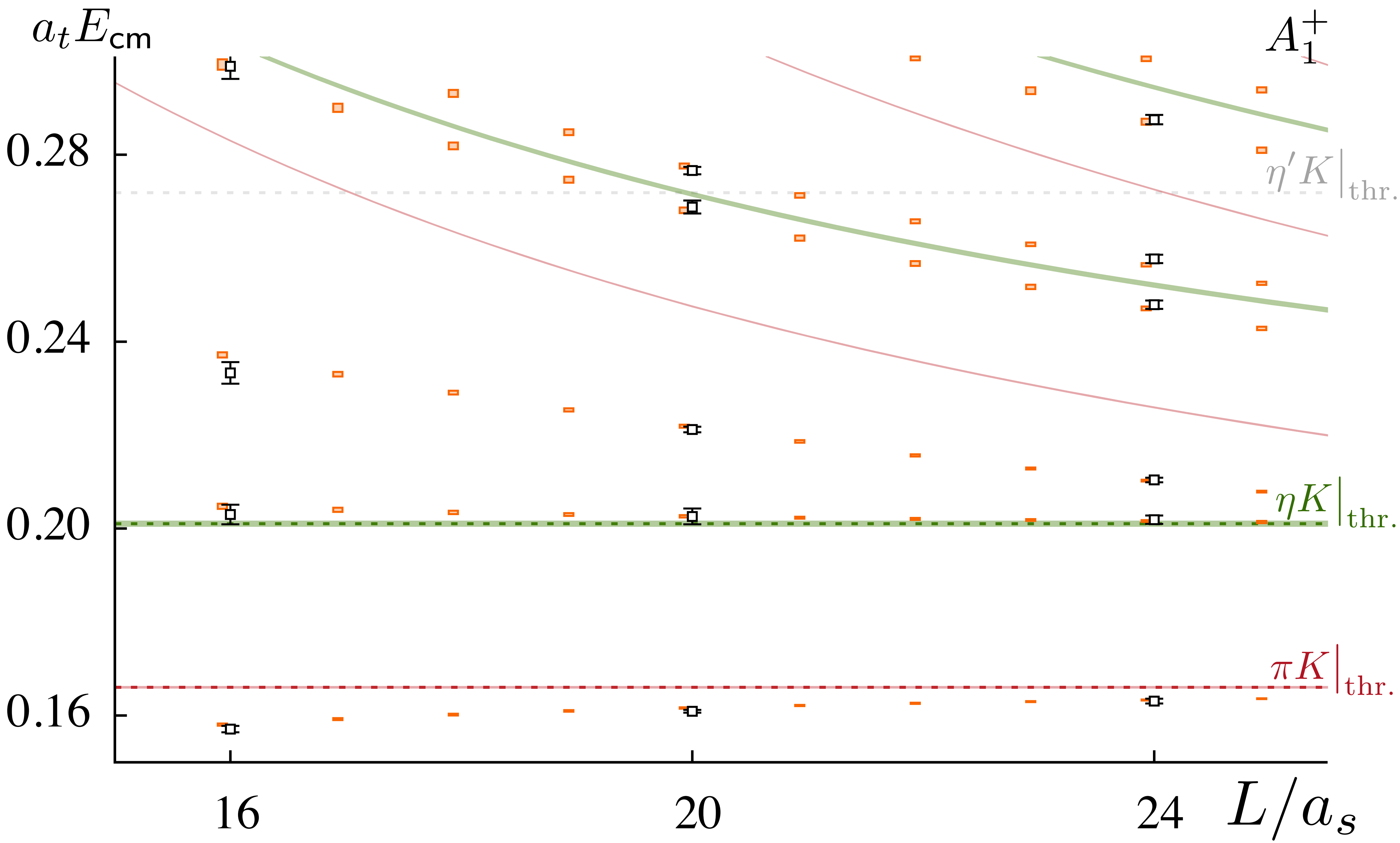


$\pi K/\eta K$ scattering

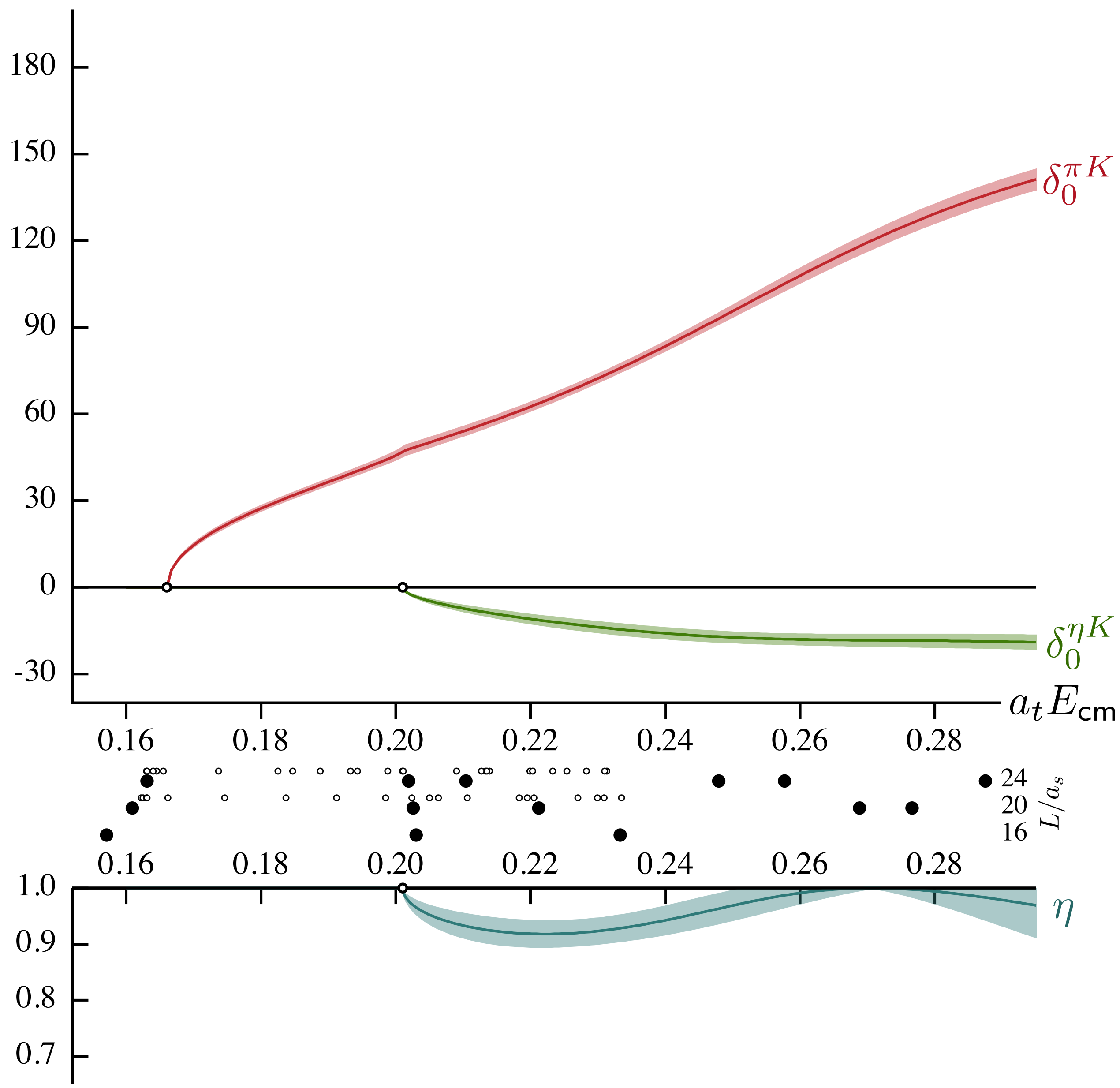
$m_\pi \sim 391\,\mathrm{MeV}$

- Describe all the finite-volume spectra

$$\chi^2/N_{\mathrm{dof}} = \frac{49.1}{61-6} = 0.89$$

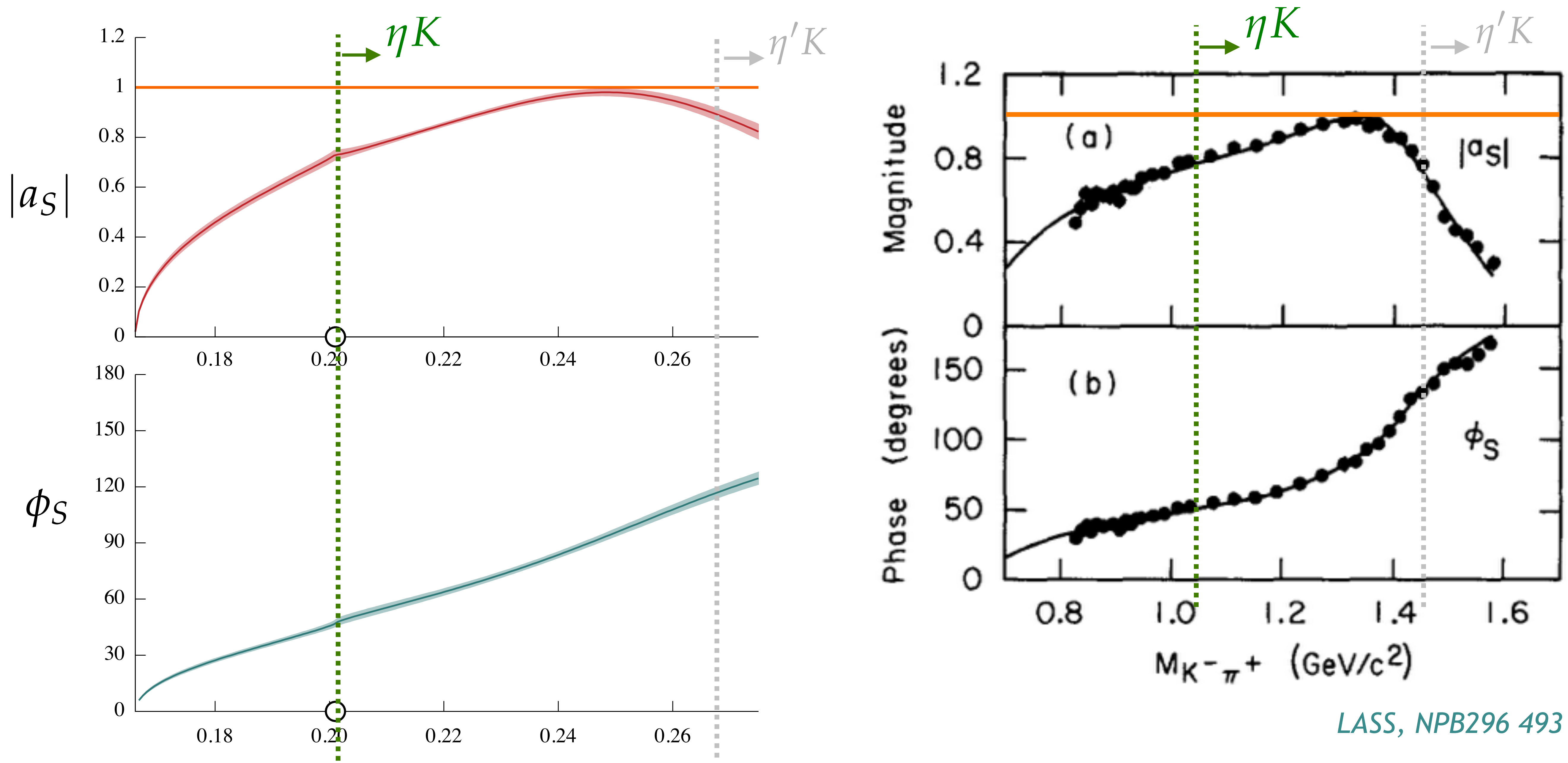
$$S_{\pi K,\pi K} = \eta\, e^{2i\delta^{\pi K}}$$
$$S_{\eta K,\eta K} = \eta\, e^{2i\delta^{\eta K}}$$

S-WAVE πK/ηK SCATTERING



Versus experimental scattering

S-WAVE πK→πK AMPLITUDE



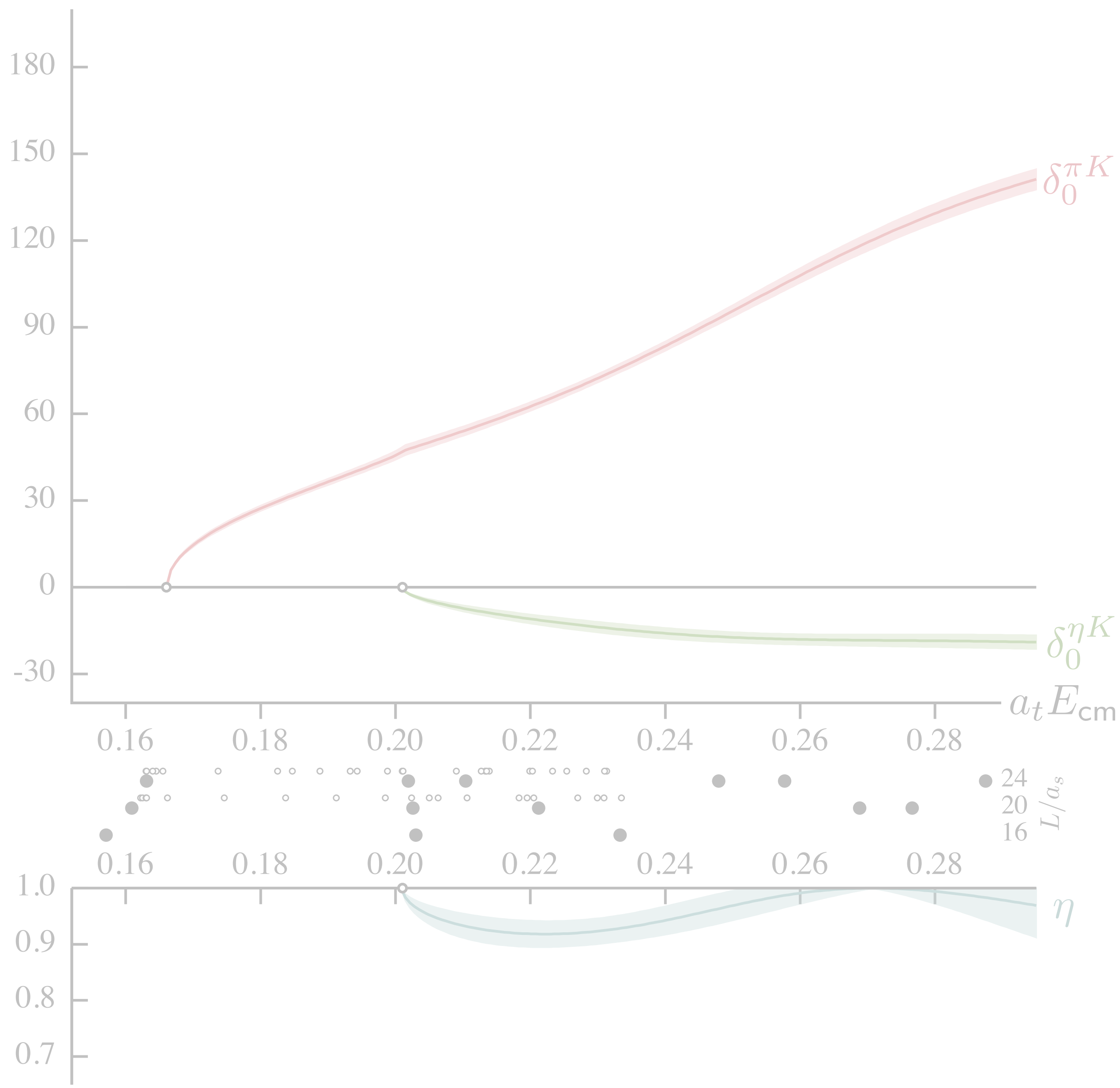
$m_\pi \sim 391$ MeV

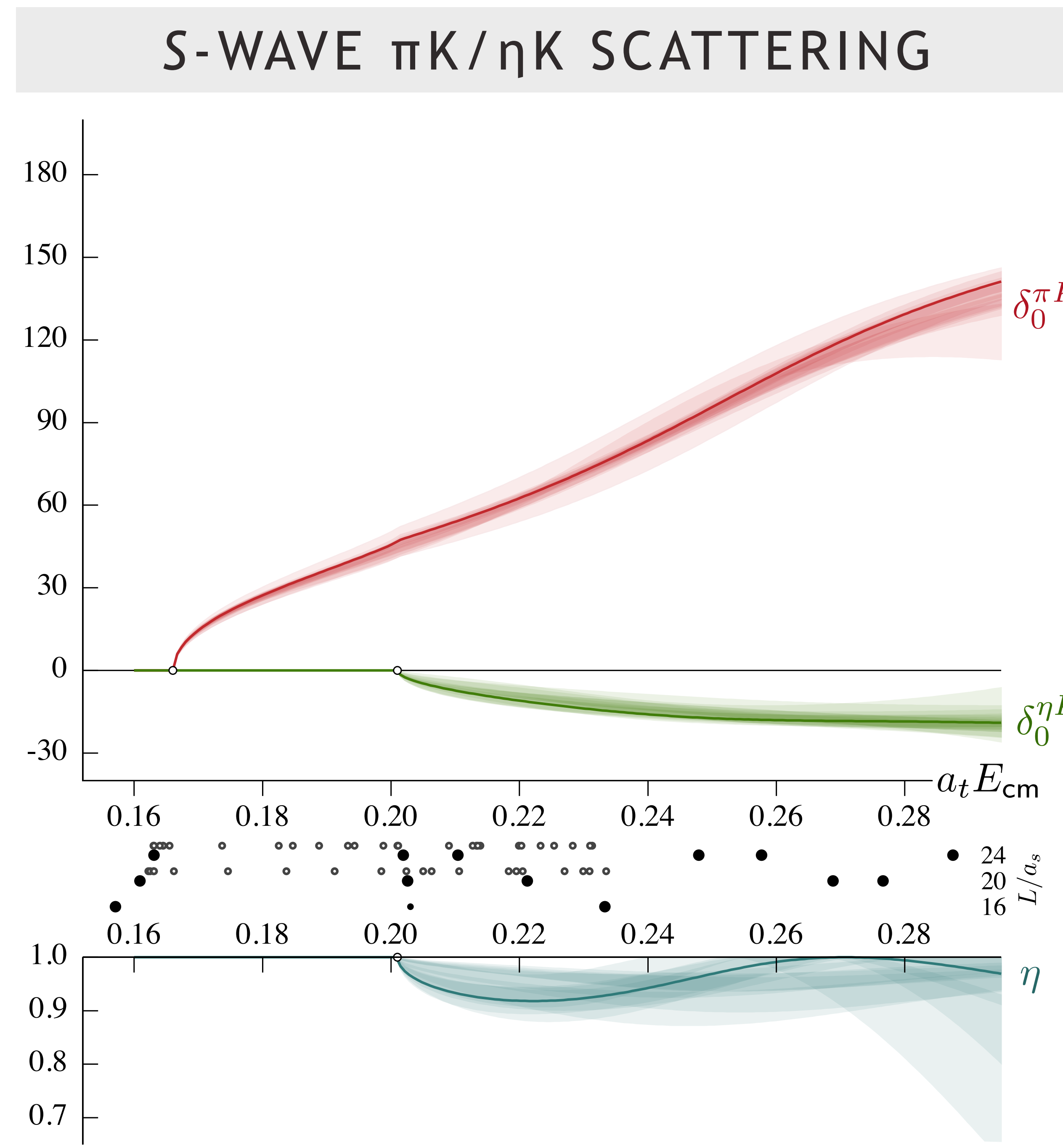
LASS, NPB296 493

$\pi K/\eta K$ scattering

$m_\pi \sim 391\,\mathrm{MeV}$

- Are the result parameterization dependent ?
  - Try a range of parameterizations …



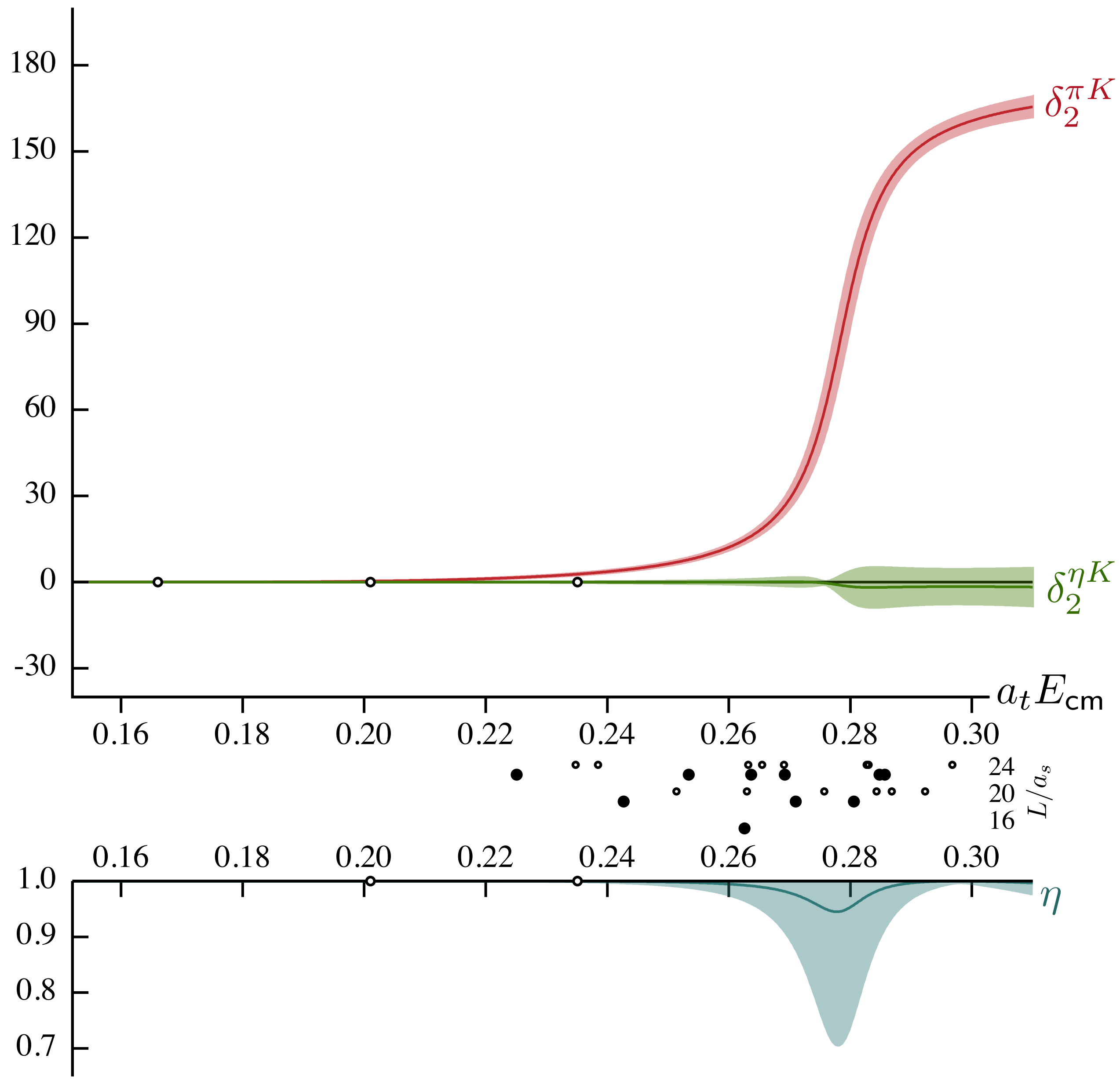


  - gross features are robust

πK/ηK scattering

- Clear narrow resonance in *D*-wave scattering

D-WAVE πK/ηK SCATTERING



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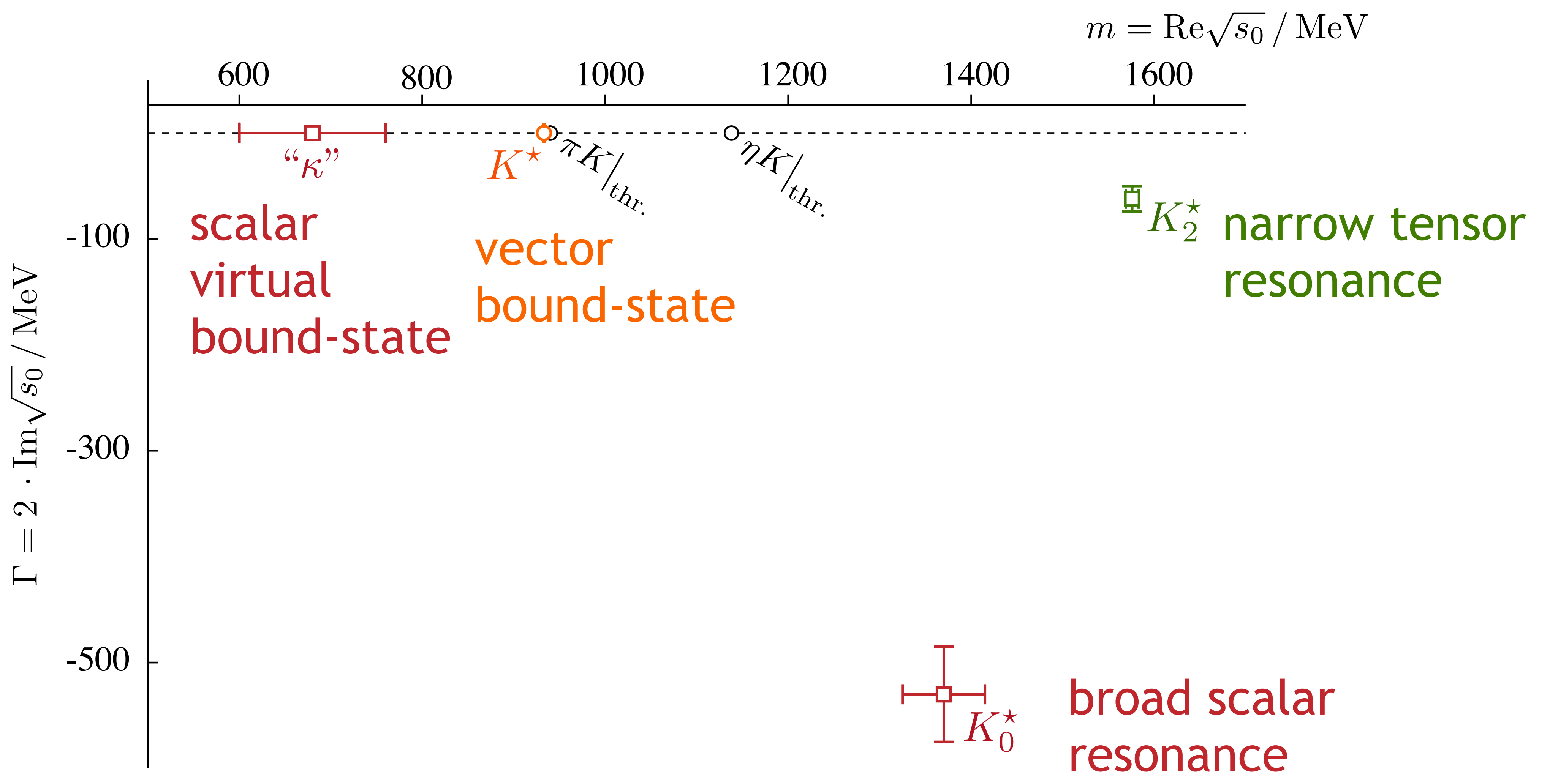
Singularity content

- ***t*-matrix poles** as least model-dependent characterization of resonances



$m_\pi \sim 391\,\mathrm{MeV}$

PRL 113 182001
PRD 91 054008

Scattering with external currents ?

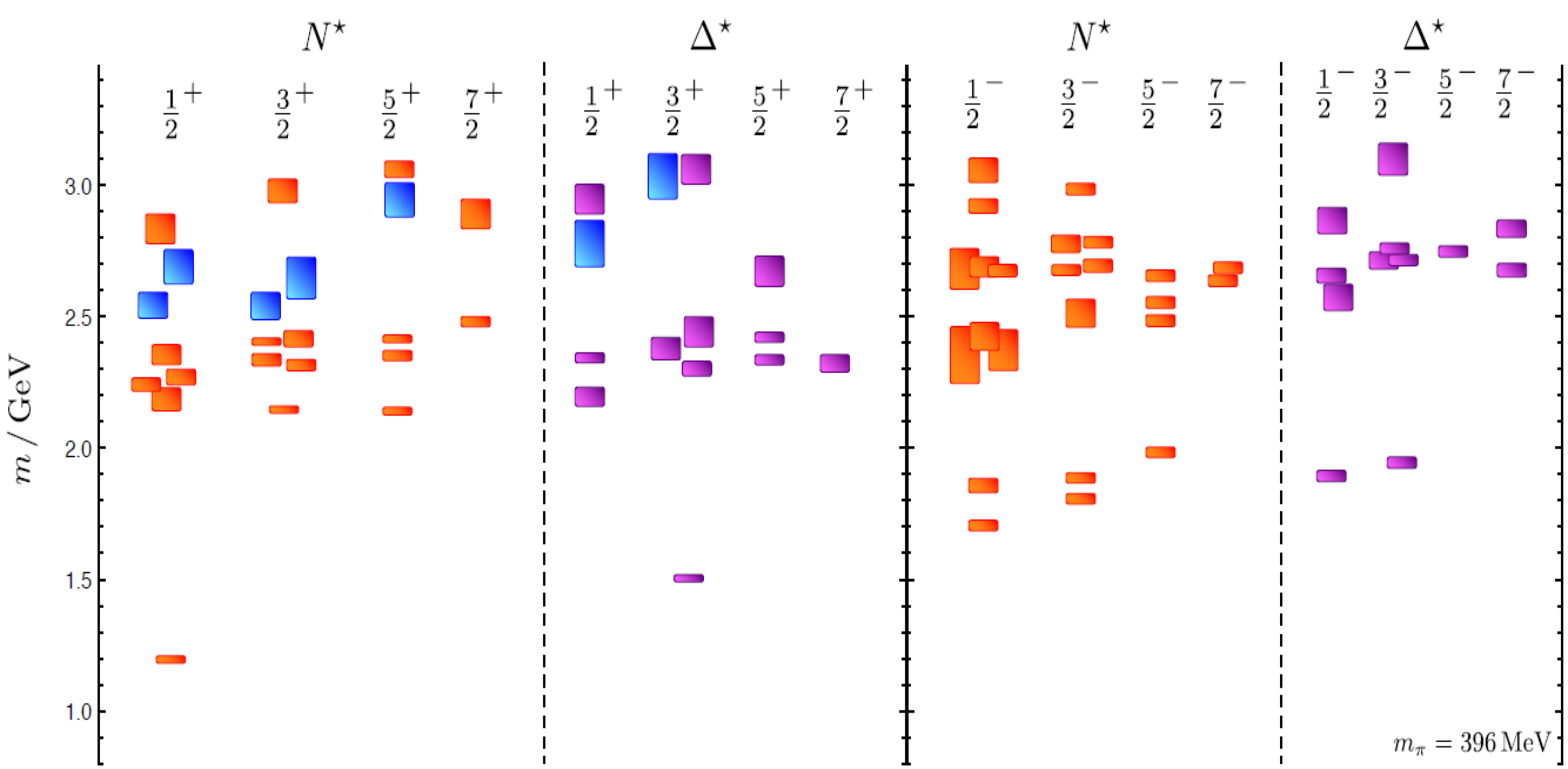
- E.g. $\pi\gamma \to \pi\pi$ in P-wave : the ρ appears as a resonance
    - The observables are the amplitudes $A_\ell(E_{\pi\pi}, Q^2)$

$\rho \to \pi\gamma$ form-factor
defined at the ρ-pole

$$A_1(s \to s_\rho) \sim \frac{c_{\rho\pi\pi}\, c_{\rho\pi\gamma}(Q^2)}{s - s_\rho}$$

Hey, this is NSTAR - where are the baryons?

- Initial determination of spectrum with only qqq style operators *PRD 84 & 85*
  - See rich spectrum, including hybrid-like states



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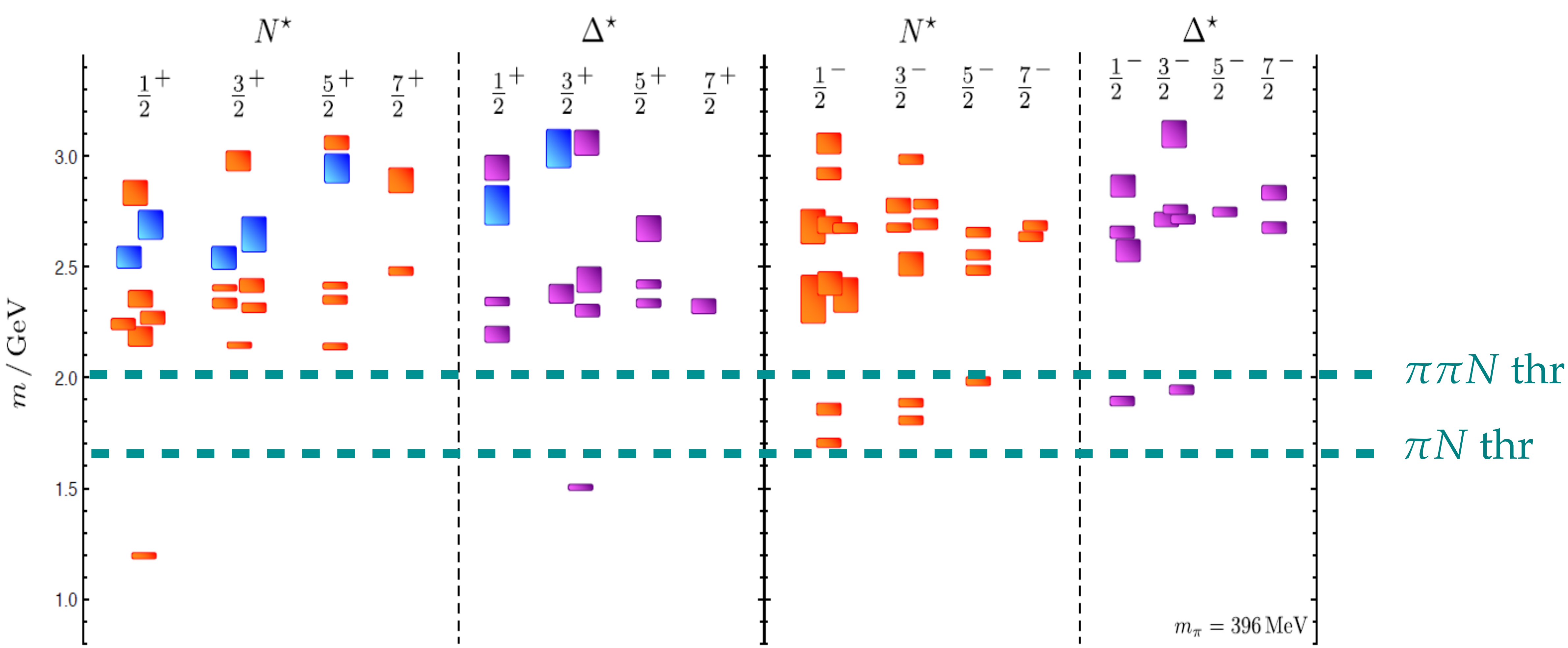

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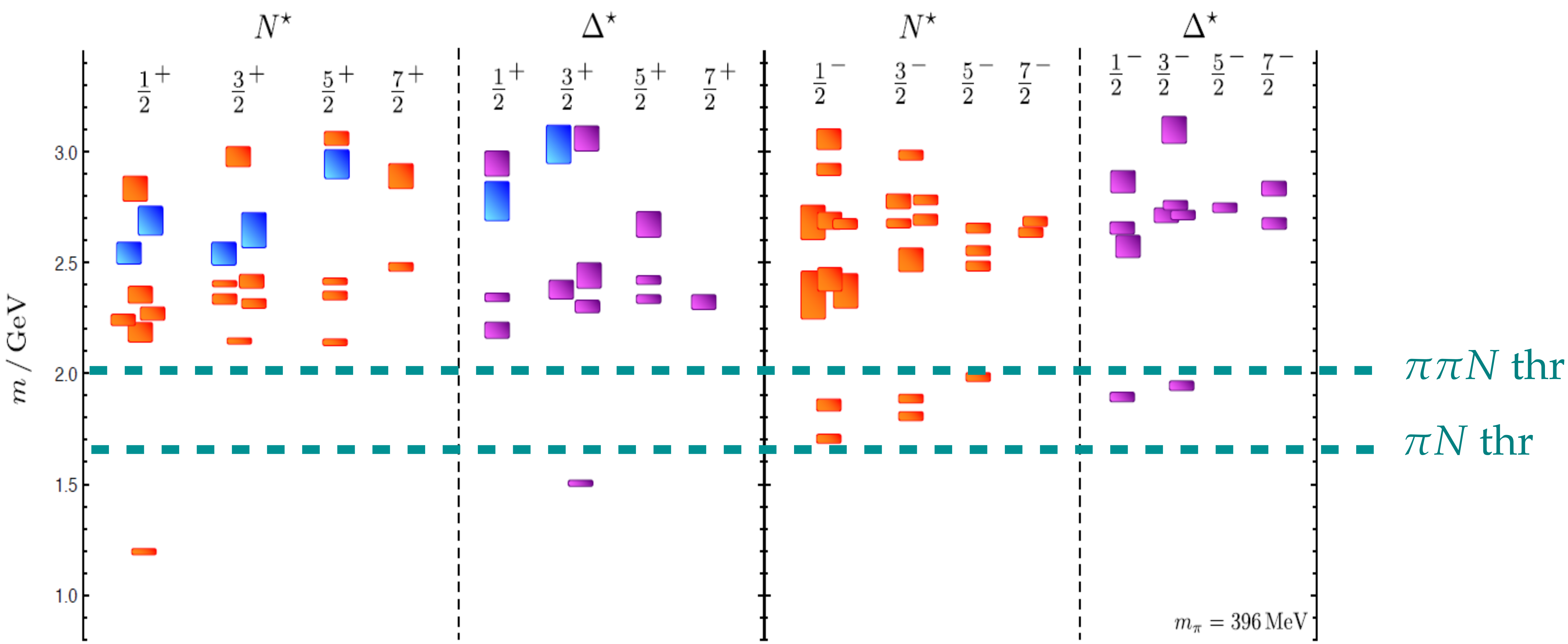

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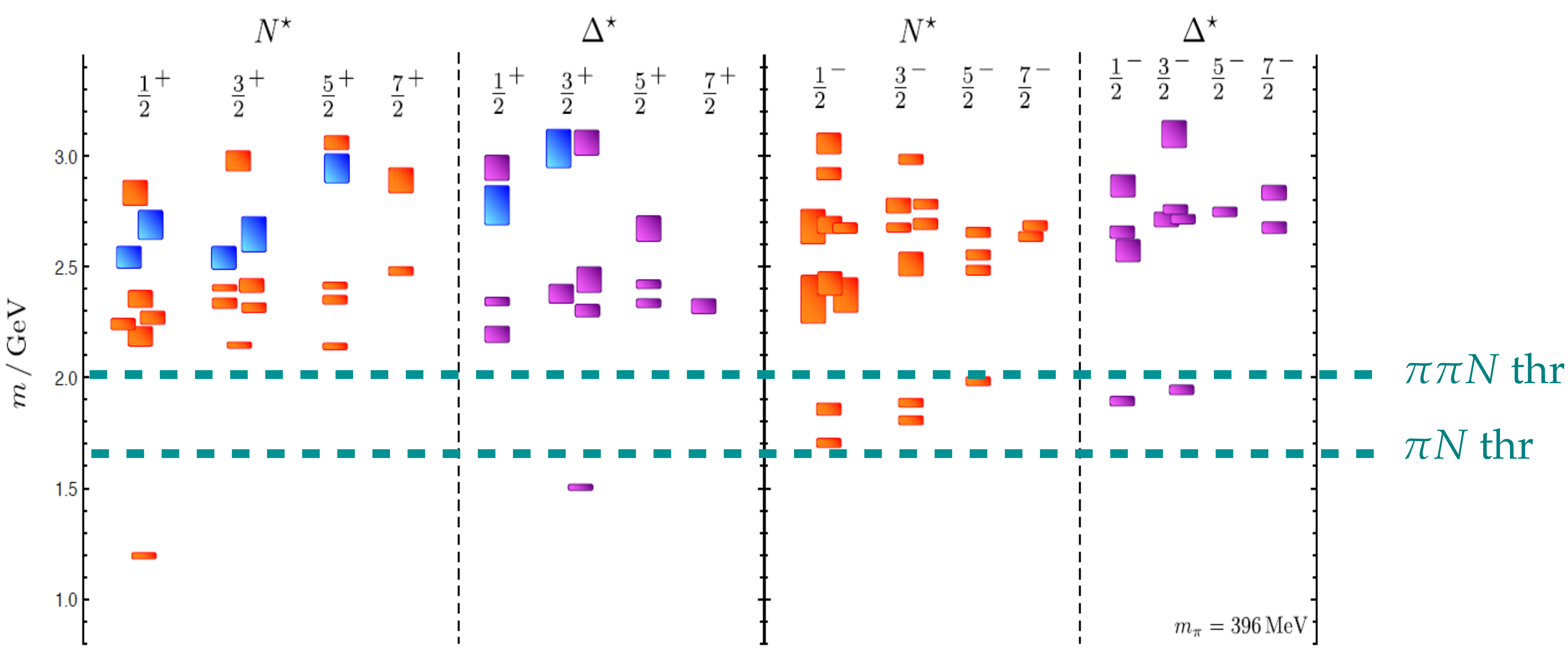
- Development of three-body formalism required *HANSEN & SHARPE - MUCH PROGRESS*



Hadron Spectrum Collaboration

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MESON SPECTRUM

PRL103 262001 (2009)	$I=1$
PRD82 034508 (2010)	$I=1,\ K^\star$
PRD83 111502 (2011)	$I=0$
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	$I=0$
JHEP05 021 (2013)	$D,\ D_s$

BARYON SPECTRUM

PRD84 074508 (2011)	$(N,\Delta)^\star$
PRD85 054016 (2012)	$(N,\Delta)_{\text{hyb}}$
PRD87 054506 (2013)	$(N\ldots\Xi)^\star$
PRD90 074504 (2014)	$\Omega^\star_{ccc}$
arXiv:1502.01845	$\Xi^\star_{cc}$

HADRON SCATTERING

PRD83 071504 (2011)	$\pi\pi\ I=2$
PRD86 034031 (2012)	$\pi\pi\ I=2$
PRD87 034505 (2013)	$\pi\pi\ I=1,\ \rho$
PRL113 182001 (2014)	$\pi K, \eta K$
PRD91 054008 (2015)	$\pi K, \eta K$

"TECHNOLOGY"

PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p}>0$

MATRIX ELEMENTS

arXiv:1501.07457	$M'\to\gamma M$
PRD90 014511 (2014)	$f_{\pi^\star}$

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- LQCD spectroscopy program maturing. First phase:
  - With only “single-hadron” operators obtain sketch of hadron spectrum
  - Suggests rich spectrum of mesons & baryons - exotic & non-exotic hybrids

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  - Demonstrated viability of finite-volume methods
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E.G., JOINT-PHYSICS ANALYSIS CENTER

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- Ultimately, determine underlying structure
  - Resonance scattering with external currents
    - Both vacuum overlaps (“wave-function”) & transition form-factors

SHULTZ 2015
BRICENO & HANSEN 2015