Quantum three-body calculation of the nonresonant triple-α reaction rate at low temperatures

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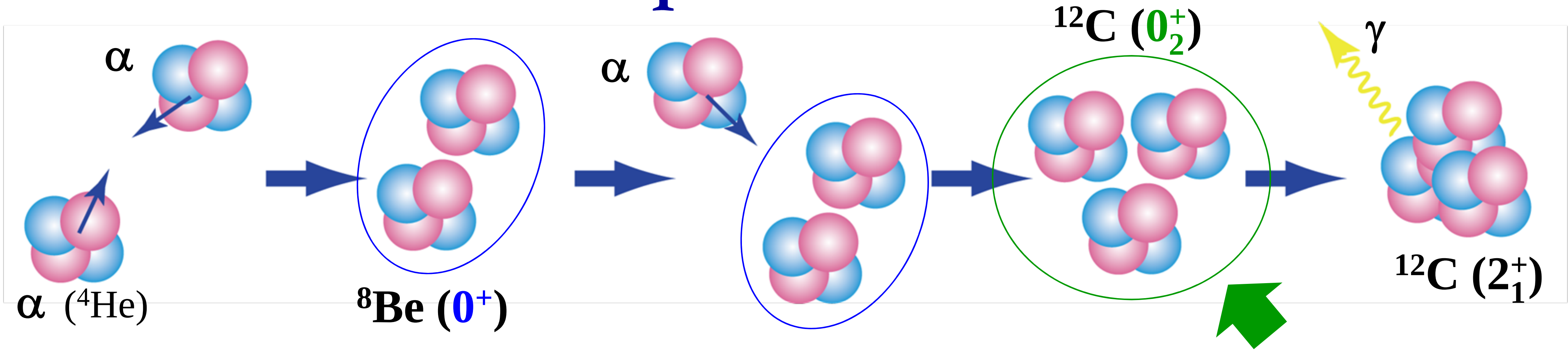
The triple α reaction



$(\tau \sim 10^{-17}$ sec$)$

9.641 3_1^-

I think, therefore it is!

α + α

0.0918 0^+

^{8}Be

7.3666

^{8}Be + α

7.6542 0_2^+

Hoyle state

4.4389 2_1^+

^{12}C 0_1^+

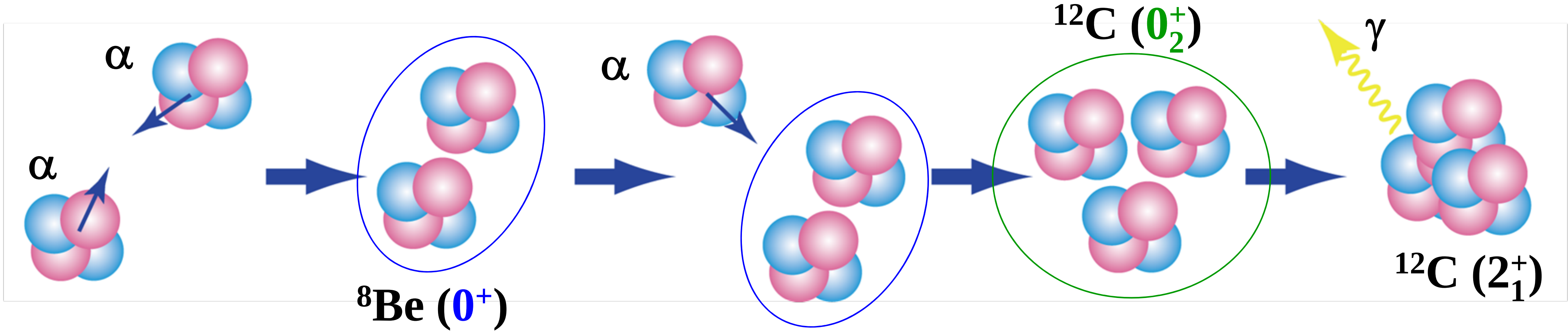
Fred Hoyle

Question:

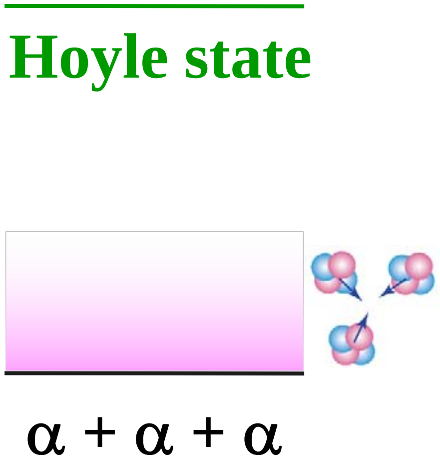
How is this picture of the triple alpha reaction accurate?

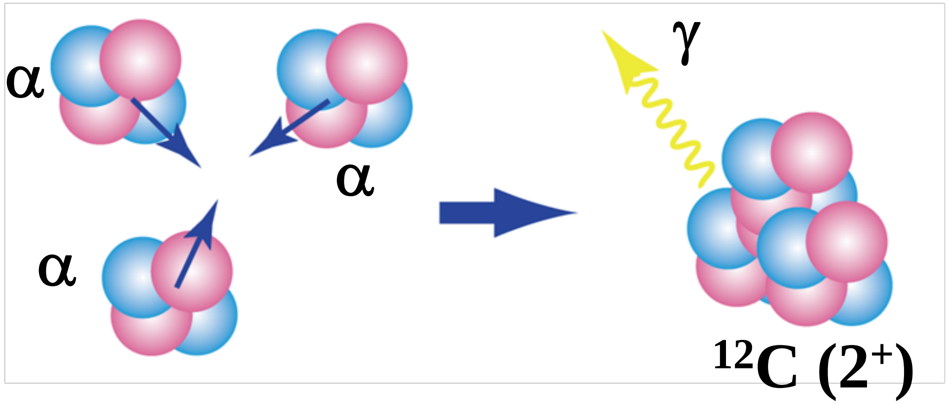
The resonant and nonresonant 3α process

☐ *T* > a few 10^8 K: **resonant** capture is dominant.



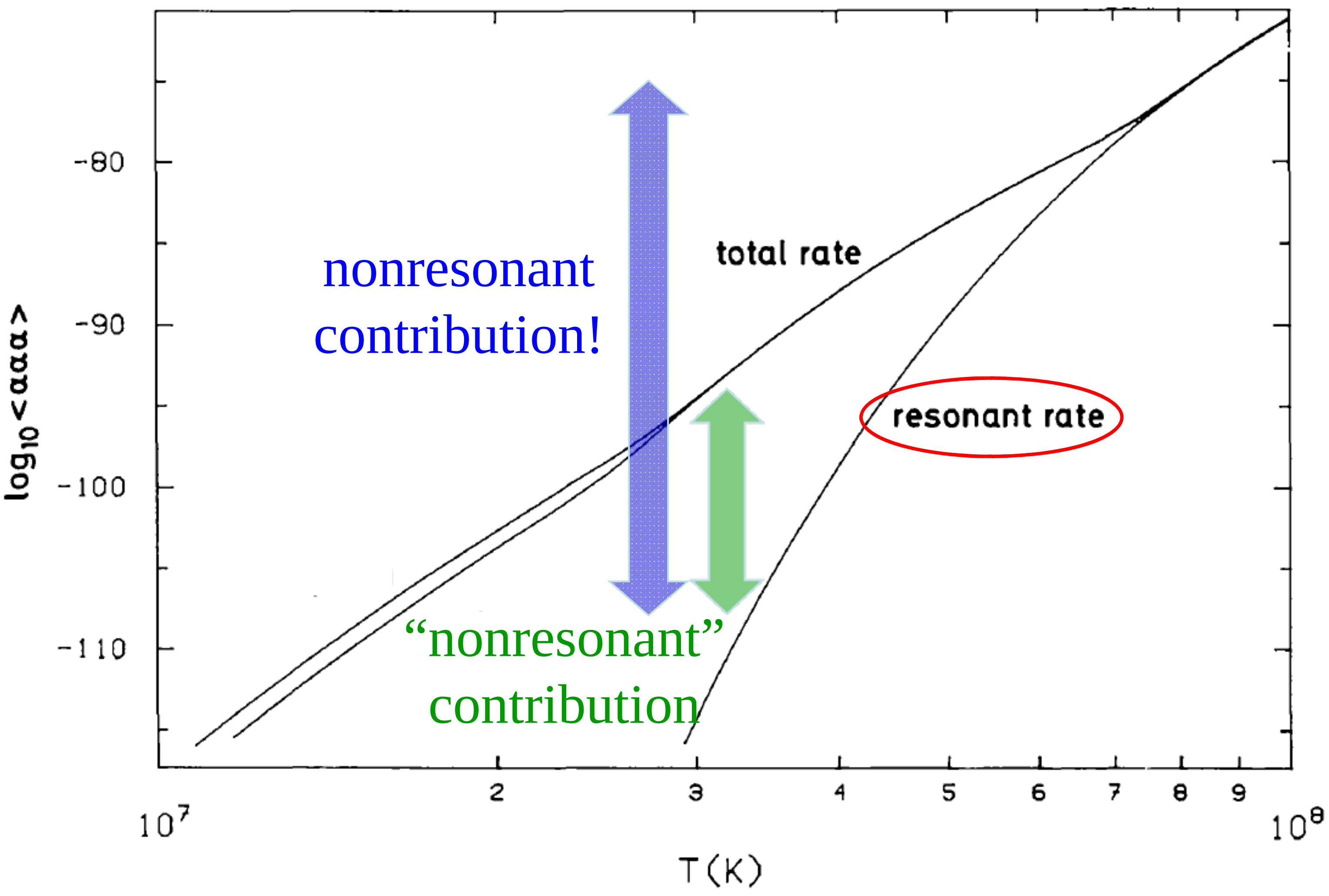
☐ *T* < 10^8 K: **nonresonant** capture is important.





Results of Nomoto’s method

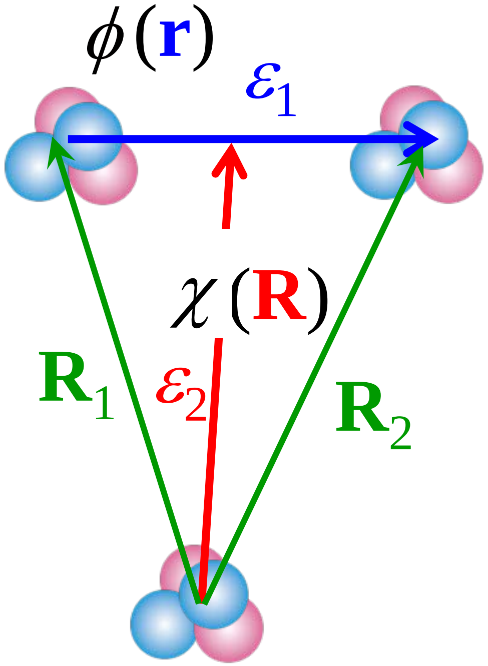
— *K. Nomoto et al., Astrophys. J. 149, 239 (1985).*



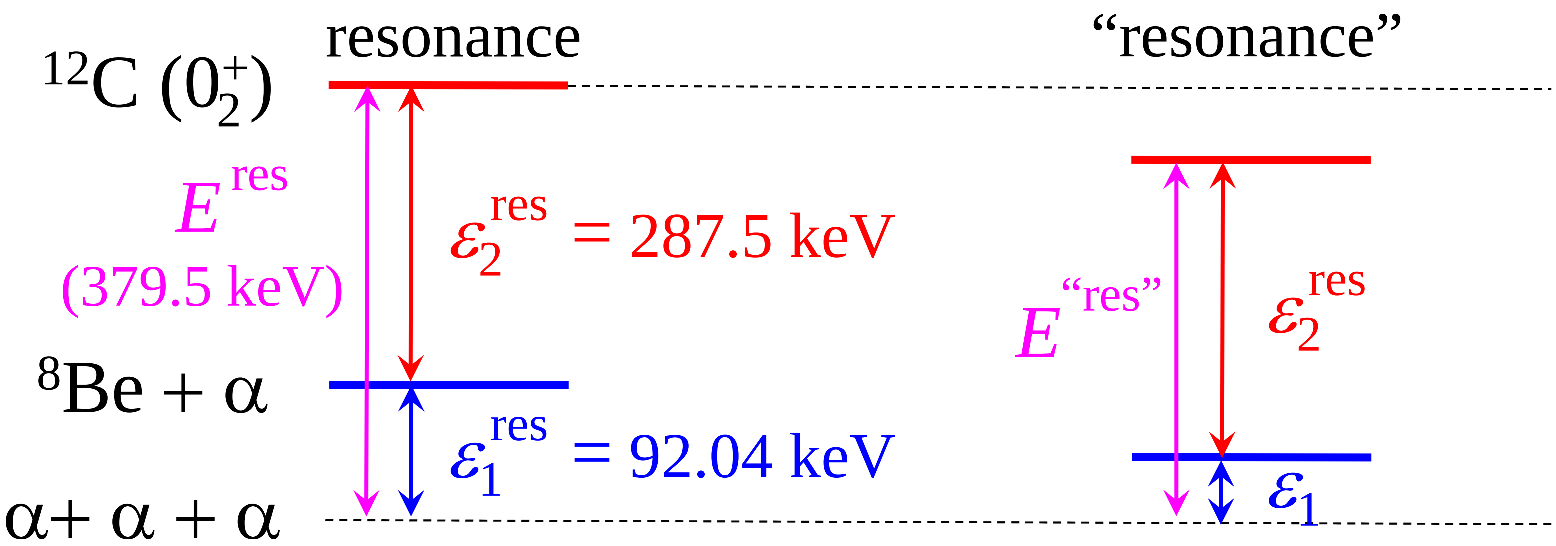
nonresonant contribution!

“nonresonant” contribution

Nomoto’s method for “nonresonant” capture



— *K. Nomoto et al., Astrophys. J. 149, 239 (1985).*



☐ **Schroedinger Eq. (1ch)**

$$\int \phi^*(\mathbf{r})\left[T_R + V_{\alpha\text{-}\alpha}(R_1) + V_{\alpha\text{-}\alpha}(R_2) + h_{\alpha\alpha} - E\right]\phi(\mathbf{r})\,\chi(\mathbf{R})\,d\mathbf{r} = 0$$

$$\Downarrow$$

$$\left[T_R + \boxed{V_{\alpha\alpha\text{-}\alpha}(R)} + \cancel{\varepsilon_1} - \left(\cancel{\varepsilon_1} + \varepsilon_2\right)\right]\chi(\mathbf{R}) = 0$$

Accurate only if αα-α interaction is independent of the αα states!

The nonresonant 3α reaction: now and past

☐ Preceding studies

✓ Pioneering study on nonresonant capture by Nomoto (*Nomoto's method*)

— *K. Nomoto, Astrophys. J. 253, 798 (1982);*
K. Nomoto et al., Astrophys. J. 149, 239 (1985).

✓ Potential model by Langanke

— *K. Langanke et al., Z. Phys. A 324, 147 (1986).*

⇨ Still based on the resonance picture with an "energy shift" of the Hoyle state as a correction

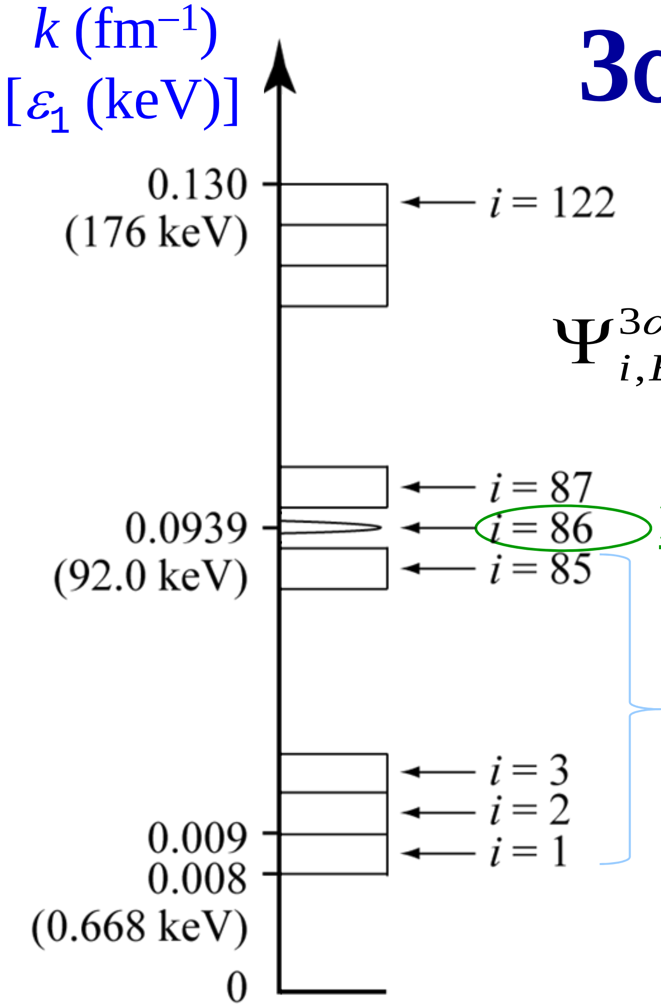
☐ This work

Accurate description of the three-body reaction treating the resonant and nonresonant processes on the same footing.

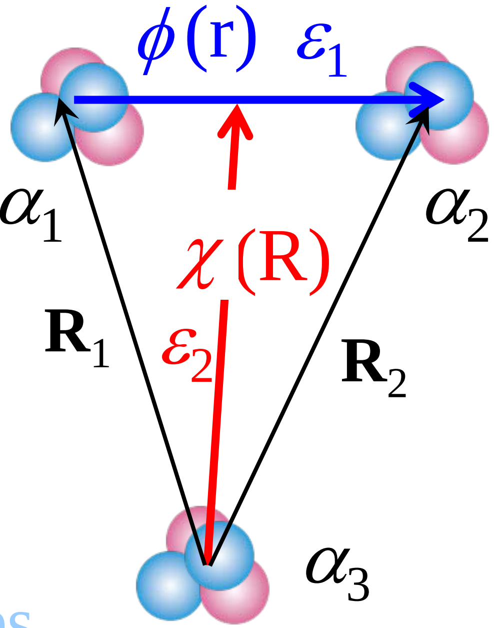
c.f. M. Kamimura and Y. Fukushima, Proceedings of the INS International Symposium on Nuclear Direct Reaction Mechanism, Shikanoshima, Fukuoka, Japan, 1978, p. 409.
P. Descouvemont and D. Baye, Phys. Rev. C ***36****, 54 (1987).*

3α wave function

α_1-α_2 states: ϕ



$$\Psi^{3\alpha}_{i,E}(\mathbf{r},\mathbf{R}) = \phi_i(\mathbf{r})\,\chi_{i,E}(\mathbf{R})$$



resonance

low-energy nonresonant states

play essential roles, but were naively neglected in the previous calculations.*

*M. Kamimura and Y. Fukushima, *Proc. INS Int. Symp. on Nuclear Direct Reaction Mechanism*, p. 409; P. Descouvemont and D. Baye, *PR C**36**, 54.*

(α_1-α_2)-α_3 states: χ

$$\left[T_R + V^i_{\alpha 1\alpha 2\text{-}\alpha 3}(R) + \varepsilon_2\right]\chi_{i,E}(\mathbf{R}) = 0,$$

$$V^i_{\alpha 1\alpha 2\text{-}\alpha 3}(R) = \left\langle \phi_i(\mathbf{r}) \middle| V^{\mathrm{N+C}}_{\alpha 1\alpha 3}(R_1) + V^{\mathrm{N+C}}_{\alpha 2\alpha 3}(R_2) \middle| \phi_i(\mathbf{r}) \right\rangle_{\mathbf{r}}.$$

Constraints on $V_{\alpha\alpha}^{\rm N}$

$V_{\alpha\alpha}^{\rm N}$: 2-range Gaussian (with repulsive part simulating the Orthogonal Condition Model; OCM)

1. ^{8}Be resonance properties

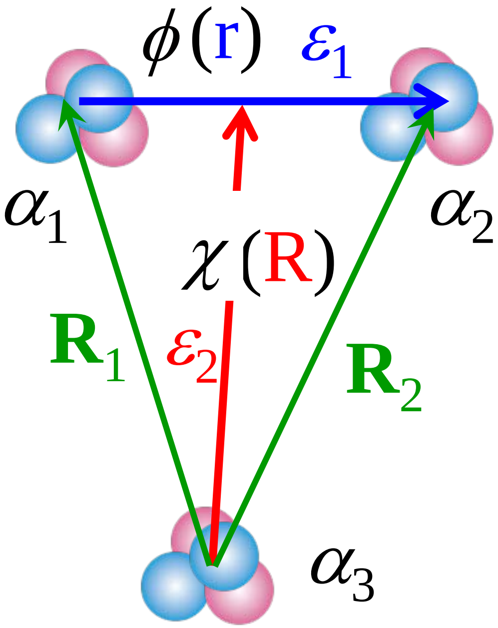
$\varepsilon_{\rm 1res}$ = 92.0 keV, Γ = 4.8 eV

exp. 92.04+/–0.05 5.57+/–0.25

2. Hoyle resonance properties (for *i* = 86)

$\varepsilon_{\rm 2res}$ = 287.5 keV, Γ = 4.0 eV

exp. 287.5 8.5+/–1.0



Achieved by reducing $V_{\alpha\alpha}^{\rm N}$ by only 1.5% in

$$V_{\alpha 1\alpha 2\text{-}\alpha 3}^{ij}(R) = \left\langle \phi_i(\mathbf{r}) \middle| V_{\alpha 1\alpha 3}^{\rm N+C}(R_1) + V_{\alpha 2\alpha 3}^{\rm N+C}(R_2) \middle| \phi_j(\mathbf{r}) \right\rangle_{\mathbf{r}} .$$

Reaction rate of the 3α reaction

☐ E2 transition from 3α scattering state

W.Fn obtained by Gaussian Expansion Method (GEM) with rearrangement

$$(\sigma v)_{\hat{k}_{i_0},E} = \frac{2\,(2\pi)^7}{75\hbar}\left(\frac{\hbar\omega}{\hbar c}\right)^5 \sum_M \left|\left\langle \Psi_M^{2^+} \middle| O_M^{\mathrm{E2}} \middle| \Psi_{i,E}^{3\alpha} \right\rangle\right|^2$$

☐ Reaction rate

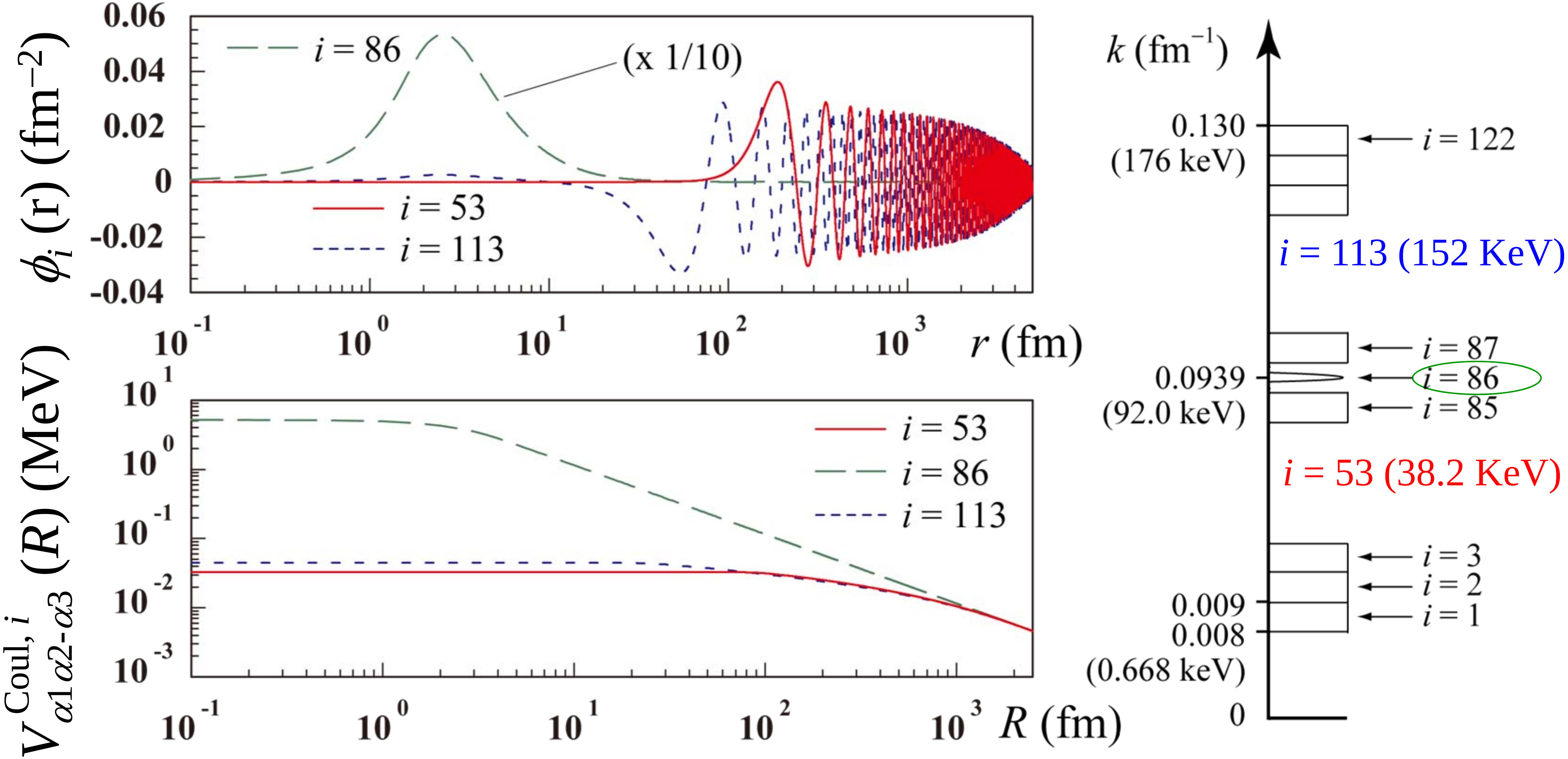
$$\langle\alpha\alpha\alpha\rangle(T) = 3N_{\mathrm{A}}^2 \frac{4}{\pi\,(k_{\mathrm{B}}T)^3}\int\left\{\sum_{i_0=1}^{122} w_{i_0}\,(\sigma v)_{\hat{k}_{i_0},E}\right\}\exp\left(-\frac{E}{k_{\mathrm{B}}T}\right)dE$$

$$w_{i_0} = \frac{2\hat{\epsilon}_{12,i_0}}{\hat{k}_{i_0}}\sqrt{\hat{\epsilon}_{12,i_0}(E-\hat{\epsilon}_{12,i_0})}$$

☐ Correction with effective charge δe to reproduce Γ_γ

✓ We include $\delta e = 0.77\,e$ so that the B(E2) value obtained by the normalized 0_2^+ W.Fn. and the 2_1^+ W.Fn. reproduces the exp. value of 13.4 e^2 fm^4.

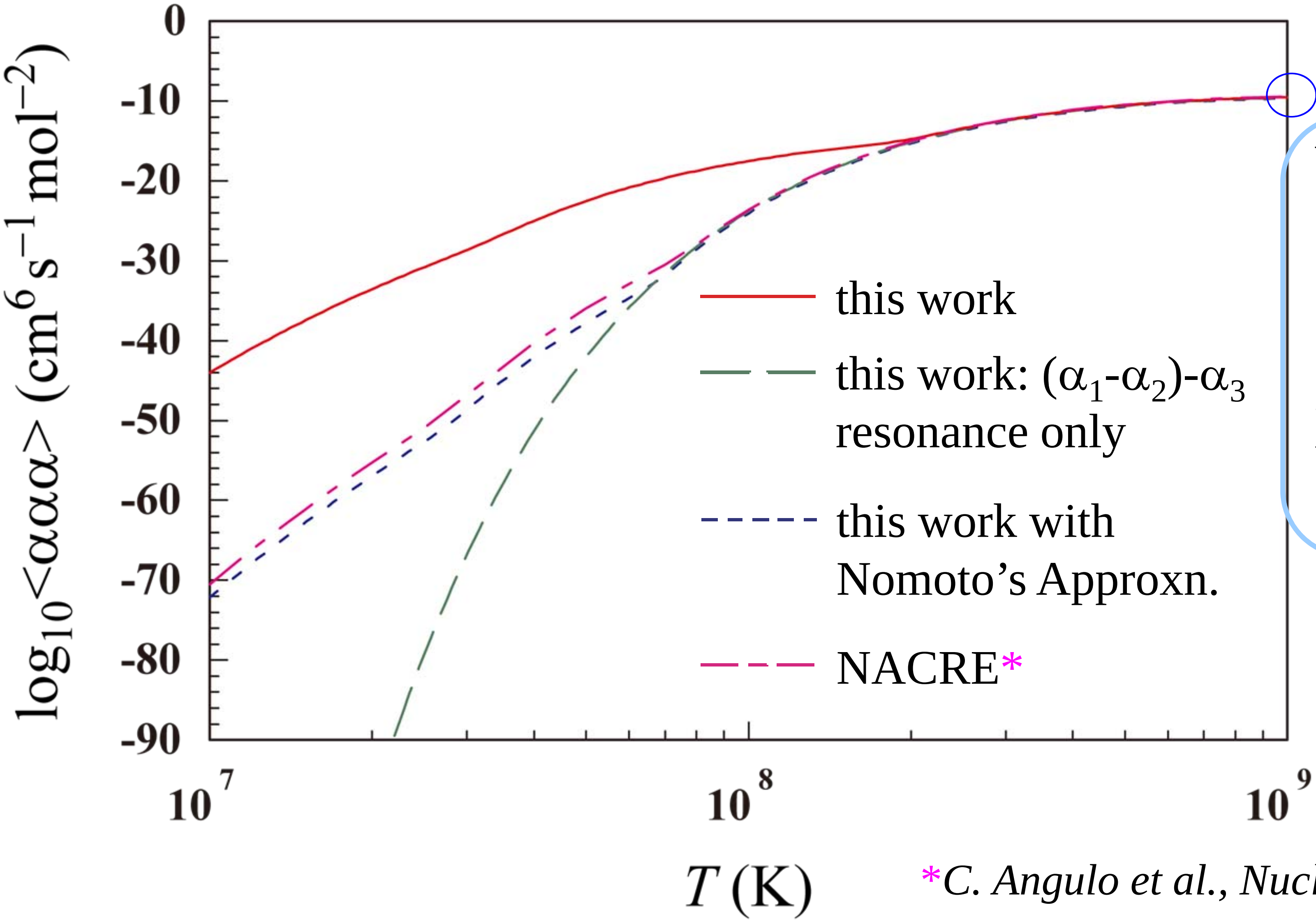
α_1-α_2 W.Fn. and (α_1-α_2)-α_3 Coulomb pot.



Resonant and nonresonant Coulomb potentials are completely different.
Nomoto's method neglects this difference and is a crude approximation.

The reaction rate

K.O., M. Kan, and M. Kamimura, Prog. Theor. Phys. ***122*** *(2009) 1055; arXiv:0905.0007 [astro-ph.SR].*

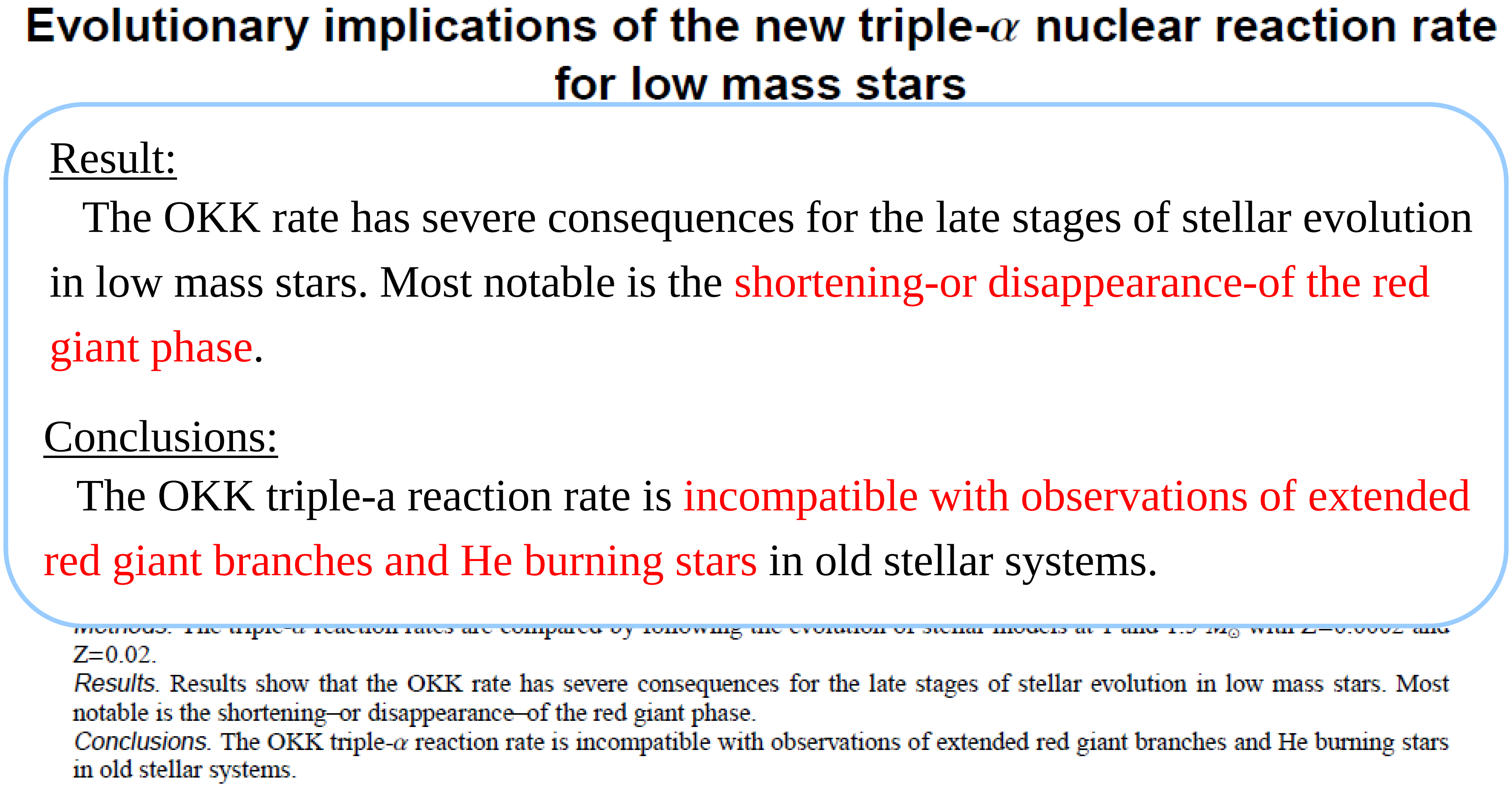


We have normalized our results to the rate of NACRE at 10^9 K. Normalization factor is 1.5 that indicates the uncertainty of our calculation.

*C. Angulo et al., Nucl. Phys. **A656** (1999), 3.*

Implication of the new reaction rate

—— *A. Dotter and B. Paxton, arXiv:0905.2397 [astro-ph.SR].*



Evolutionary implications of the new triple-α nuclear reaction rate for low mass stars

Result:

The OKK rate has severe consequences for the late stages of stellar evolution in low mass stars. Most notable is the shortening-or disappearance-of the red giant phase.

Conclusions:

The OKK triple-a reaction rate is incompatible with observations of extended red giant branches and He burning stars in old stellar systems.

Z=0.02.

Results. Results show that the OKK rate has severe consequences for the late stages of stellar evolution in low mass stars. Most notable is the shortening–or disappearance–of the red giant phase.

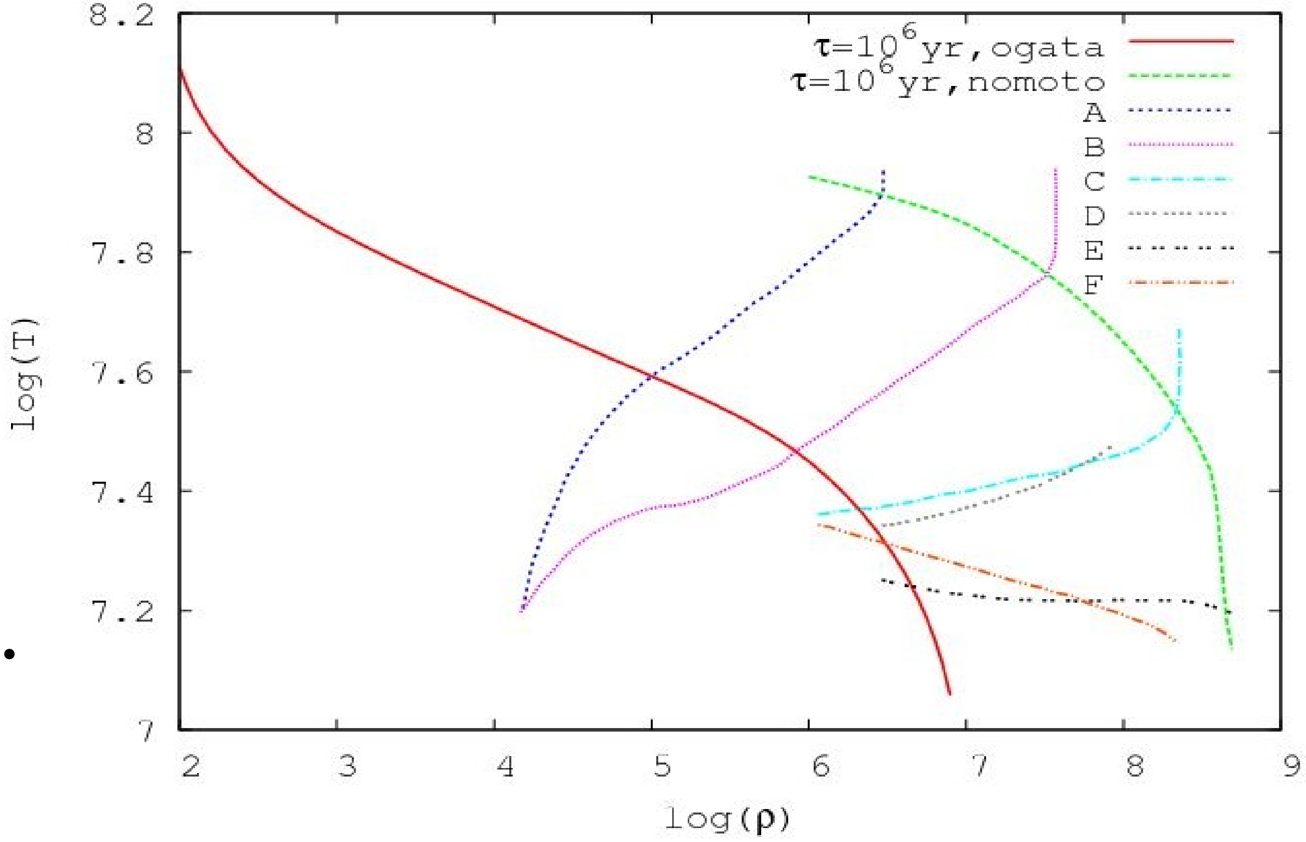
Conclusions. The OKK triple-α reaction rate is incompatible with observations of extended red giant branches and He burning stars in old stellar systems.

53 Effects of a new triple-alpha reaction rate on the helium ignition of accreting white dwarfs

M.Saruwatari , M. Hashimoto, R. Nakamura(Kyushu University)
S. Fujimoto (Kumamoto National College of Technology), K. Arai(Kumamoto University)

- **The helium ignitions occur in the low density by two orders of magnitude if the OKK rate is adopted.**

- **Nuclear flashes are triggered for all cases of A-F in the helium layers.**



Summary

☐ The triple-α reaction rate is reevaluated.

- ✓ The resonant and nonresonant processes are described on the same footing.
- ✓ The α_1-α_2 nonresonant states below the resonance are essentially important.
- ✓ The $(\alpha_1$-$\alpha_2)$-α_3 Coulomb barrier in the nonresonant capture process is much lower than that in the resonant process.
- ✓ We obtain a markedly larger reaction rate than NACRE below 10^8 K.
- ✓ Nomoto's method (used in many studies including NACRE) is shown to be a very *crude approximation* to the present three-body calculation.

☐ Future plan

- ✓ How can we resolve the inconsistency of a stellar evolution calculation with our new rate and observation?
- ✓ Systematic studies of ternary processes: $\alpha(\alpha n,\gamma)^9$Be, $n(p\alpha,{}^6$Li) etc.