Violation of sum rules for twist-3 parton distributions in QCD

M. Burkardt[a] and Y. Koike[b]

[a] *Department of Physics, New Mexico State University, Las Cruces, NM 88003, USA*
[b] *Department of Physics, Niigata University, Niigata 950-2181, Japan*

There are well know sum rules for the twist-3 parton distributions $g_T(x,\mu^2) = g_1(x,\mu^2) + g_2(x,\mu^2)$ and $h_L(x,\mu^2)$:

$$\int_{-1}^{1} dx\, g_T(x,\mu^2) = \int_{-1}^{1} dx\, g_1(x,\mu^2), \tag{1}$$

$$\int_{-1}^{1} dx\, h_L(x,\mu^2) = \int_{-1}^{1} dx\, h_1(x,\mu^2), \tag{2}$$

where g_1 and h_1 are the twist-2 distrbutions. The first one is known as Burkhardt-Cottingham sum rule. These sum rules are the mere consequences of the rotational invariance and thus should be valid in QCD. The point $x = 0$, however, corresponds to the high energy limit and can never be achieved experimentally. (In DIS, x is identified as the Bjorken variable $x_B = Q^2/2\nu$.) Accordingly if g_T and h_L have a $\delta(x)$-function contribution at the orign $x = 0$ and g_1 and h_1 do not (which is a reasonable assumption for the twist-2 distributions), an experiment can never confirm the above sum rules, but would rather claim the violation of the sum rules.

In a recent paper [1] we examined the possibility of this $\delta(x)$ contribution in $g_T(x,\mu^2)$ and $h_L(x,\mu^2)$. From OPE, h_L can be decomposed as $h_L(x,\mu^2) = h_L^{WW}(x,\mu^2) + h_L^m(x,\mu^2) + h_L^3(x,\mu^2)$, where h_L^{WW} is written in terms of an integral of h_1 (Wandzura-Wilczek analogue), h_L^m is a quark-mass dependent piece and is expressed in terms of an integral of g_1, and h_L^3 is a gluon's field-strength dependent part. Model-independent analysis for h_L^m suggests a $\delta(x)$ term in it, and a one-loop calculation for a dressed massive quark target indeed confirms a $\delta(x)$ term in h_L^m. Construction of h_L^3 from the one-loop moment of a massive quark shows h_L^3 also has a $\delta(x)$ term and it adds up to $\delta(x)$ term in h_L^m. Since h_L^{WW} does not have such a singularity, $h_L(x,\mu^2)$ for a one-loop dressed massive quark has such $\delta(x)$-term. Direct light-cone calculation for $h_L(x,\mu^2)$ also confirmed this result. The same analysis for $g_T(x,\mu^2)$ did not find such a $\delta(x)$ contribution at least in the one-loop level, although general analysis of g_T^m (an analogue of h_L^m) does not exclude such a $\delta(x)$ term in g_T.

This perturbative calculation for a massive quark target shows a "violation" of the above sum rules for the twist-3 distributions, which is taken as an indication of potential failure of the sum rules nonperturbatively.

References

[1] M. Burkardt and Y. Koike, hep-ph/0111343, Nucl. Phys. B in press.