

# 超新星物質の状態方程式と原始中性子星

親松和浩（愛知淑徳大学現代社会学部）

## 内容

超新星爆発と原始中性子星誕生の概略  
（住吉氏の春の学会講演のおさらい）

超新星物質EOSの種類

超新星物質EOSによる爆発計算結果の違い

状態方程式を制限していく試み

非一様物質計算の改善の試み

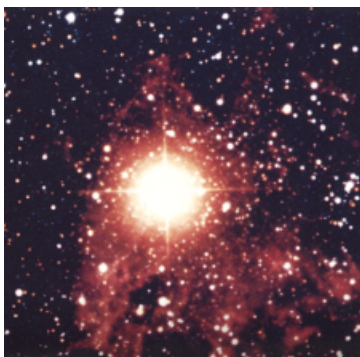
今後の課題

# 超新星爆発の概略

住吉氏 春の学会シンポジウム講演スライド

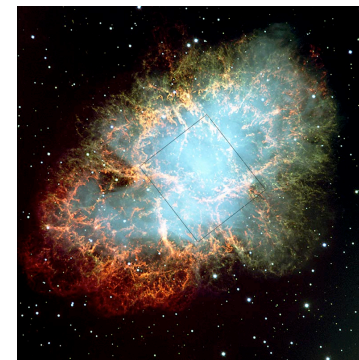
# 超新星爆発とは？

重い星( $\sim 20M_{\text{sun}}$ )の進化の最期に起こる大爆発



A.K.Mann "Shadow of a Star"

SN1987A



From <http://hubblesite.org>

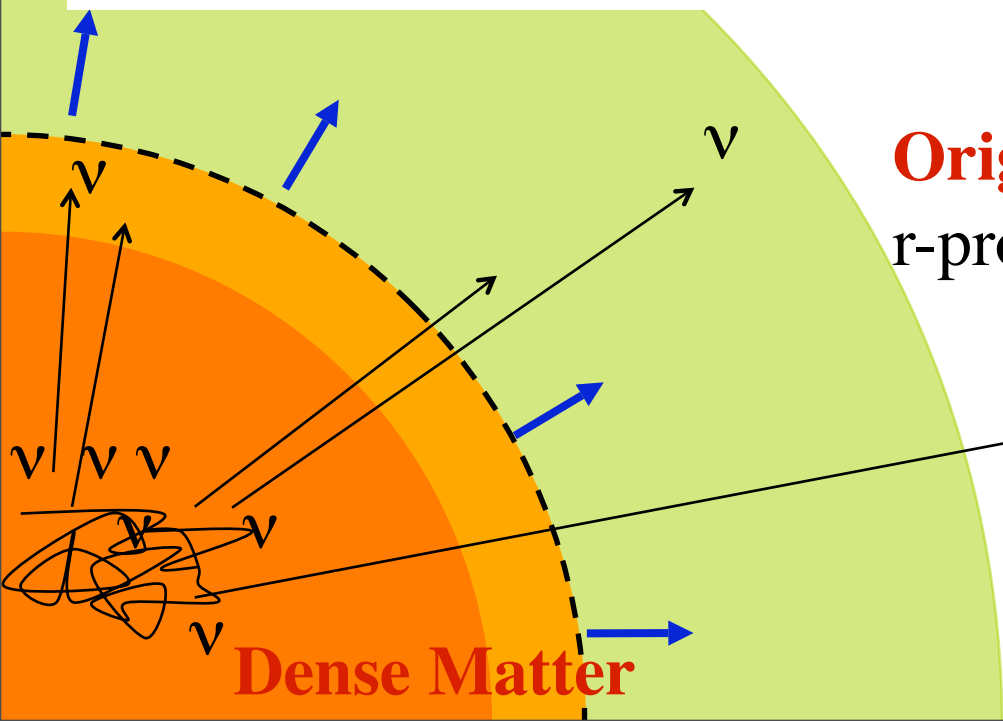
Crab nebula (SN1054)

# 重力崩壊型超新星爆発

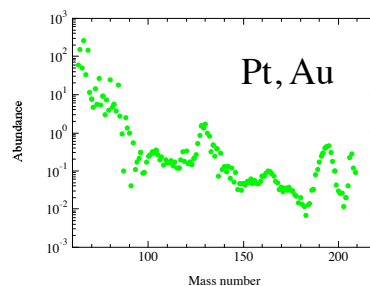
- 中性子星・ブラックホールの誕生
- 重元素の起源:rプロセス、爆発的元素合成
  - 銀河の進化、宇宙線、ガンマ線バースト、重力波
- 爆発メカニズムの解明:重力崩壊・コアバウンス

## Explosion Dynamics

Trapping, heating



Dense Matter



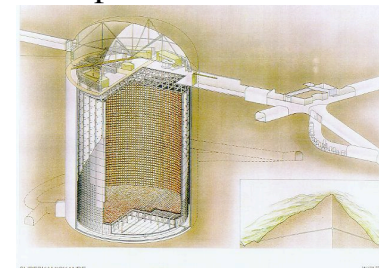
## Origin of Elements

r-process,  $\nu$ -process



<http://subarutelescope.org/>

Super-Kamiokande



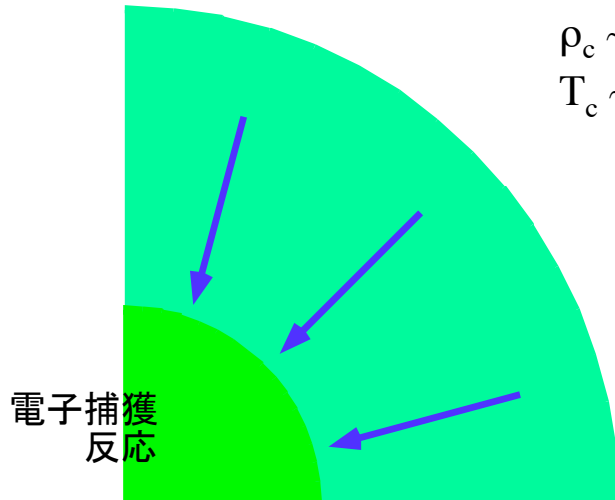
<http://www-sk.icrr.u-tokyo.ac.jp/>

Astronomy

Probe inside

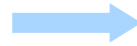
# 重力崩壊型超新星爆発

鉄のコア



**重力崩壊**

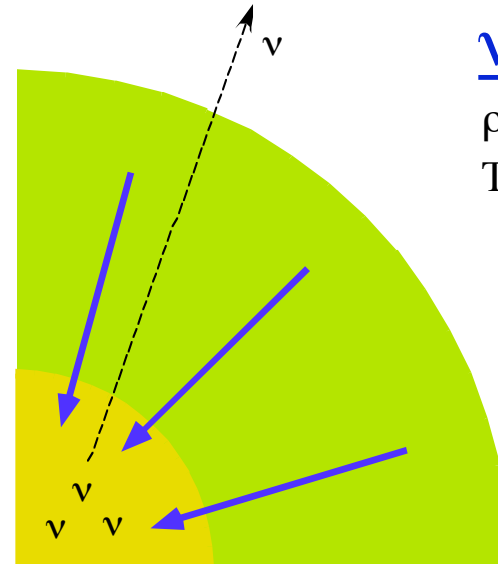
$\rho_c \sim 10^{10} \text{ g/cm}^3$   
 $T_c \sim 1 \text{ MeV}$



**$\nu$ -閉込め**

$\rho_c \sim 10^{12} \text{ g/cm}^3$   
 $T_c \sim 2 \text{ MeV}$

$\nu$  反応



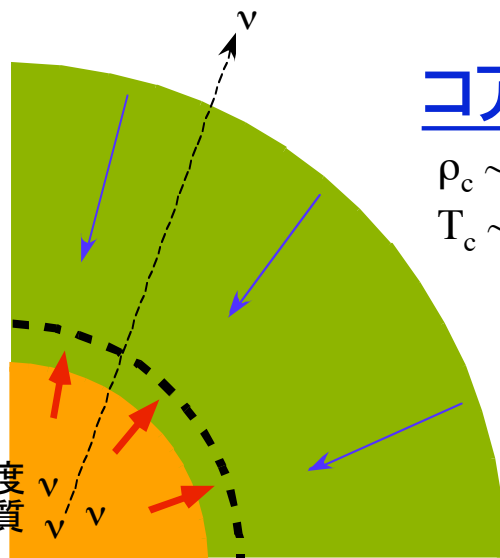
1000 km

**コアバウンス**

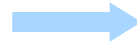
$\rho_c \sim 3 \times 10^{14} \text{ g/cm}^3$   
 $T_c \sim 5 \text{ MeV}$



高温高密度核物質

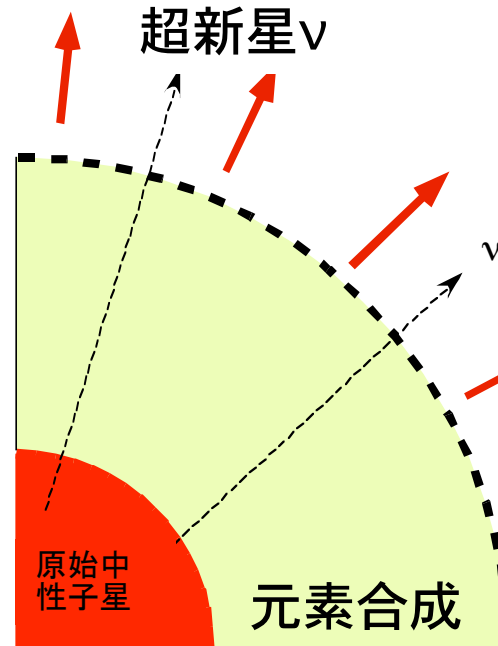


衝撃波

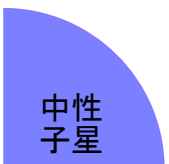
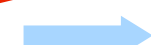


**爆発**

$\rho_c \sim 4 \times 10^{14} \text{ g/cm}^3$   
 $T_c \sim 10 \text{ MeV}$



~10 km



中性子星

$T \sim 0 \text{ MeV}$   
 $Y_e < 0.1$

# 超新星爆発解明の鍵

- 鉄のコアから中性子星へ
- 解放される重力エネルギー
- 爆発エネルギー

$$M_{\text{Fe}} \sim M_{\text{NS}} \sim 1.4 M_{\text{solar}}, R_{\text{Fe}} \sim 10^4 \text{ km} \rightarrow R_{\text{NS}} \sim 10 \text{ km}$$

$$\Delta E_{\text{Grav}} = - \left( \frac{GM^2}{R_{\text{Fe}}} - \frac{GM^2}{R_{\text{NS}}} \right) \sim 10^{53} \text{ erg}$$

・放出のエネルギー

$$E_{\text{exp}} \sim 10^{51} \text{ erg}$$

$$E_{\nu} \sim 10^{53} \text{ erg}$$

- 約1%が爆発に使われる: ほとんどはニュートリノが持ち去る
- ニュートリノと物質の相互作用が本質的: 精密科学

## • Macrophysics

- 流体力学
- ニュートリノ輻射輸送
- 回転、磁場



## • Microphysics (核物理)

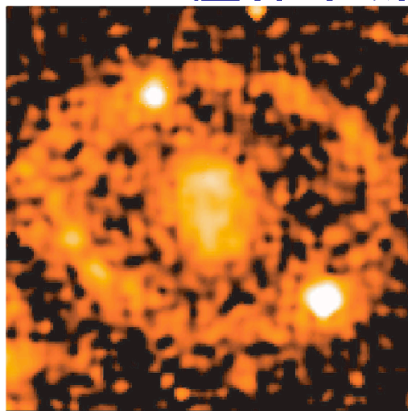
- 状態方程式
- ニュートリノ反応率
- 電子捕獲反応

# 多次元流体力学計算

超新星爆発多次元流体計算の例

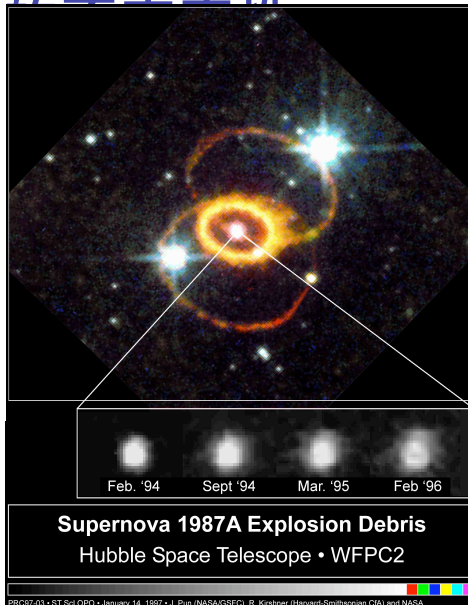
2D calc with rotation Shen-EOS

- 超新星爆発残骸
  - 形状, 元素合成, 偏光
  - 中性子星キック
- 非対称性
  - 球対称: 1D
  - (非)軸対称: 2D, 3D
- 回転, 磁場



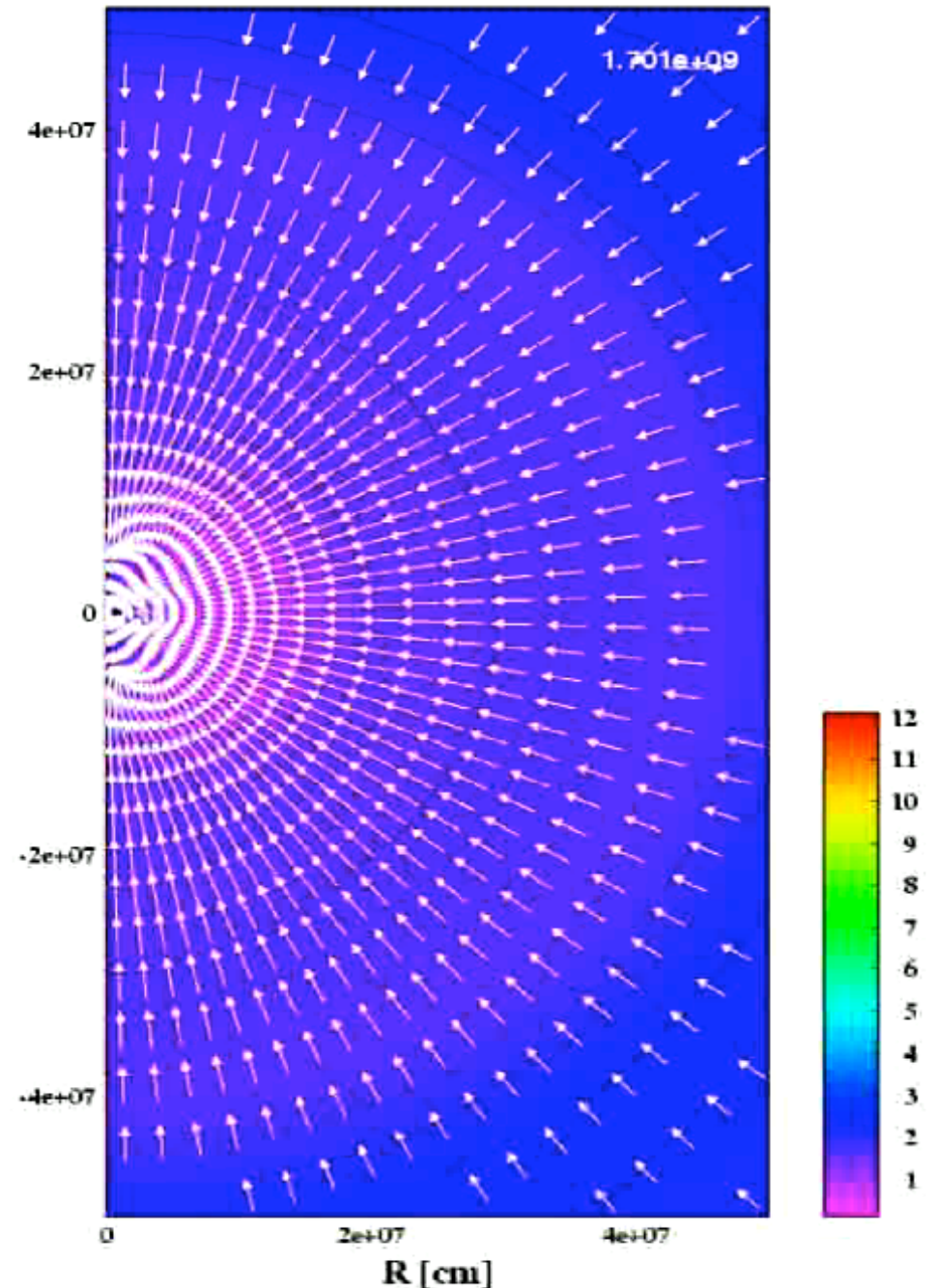
Jun. 2000

Wang 2002



Supernova 1987A Explosion Debris  
Hubble Space Telescope • WFPC2

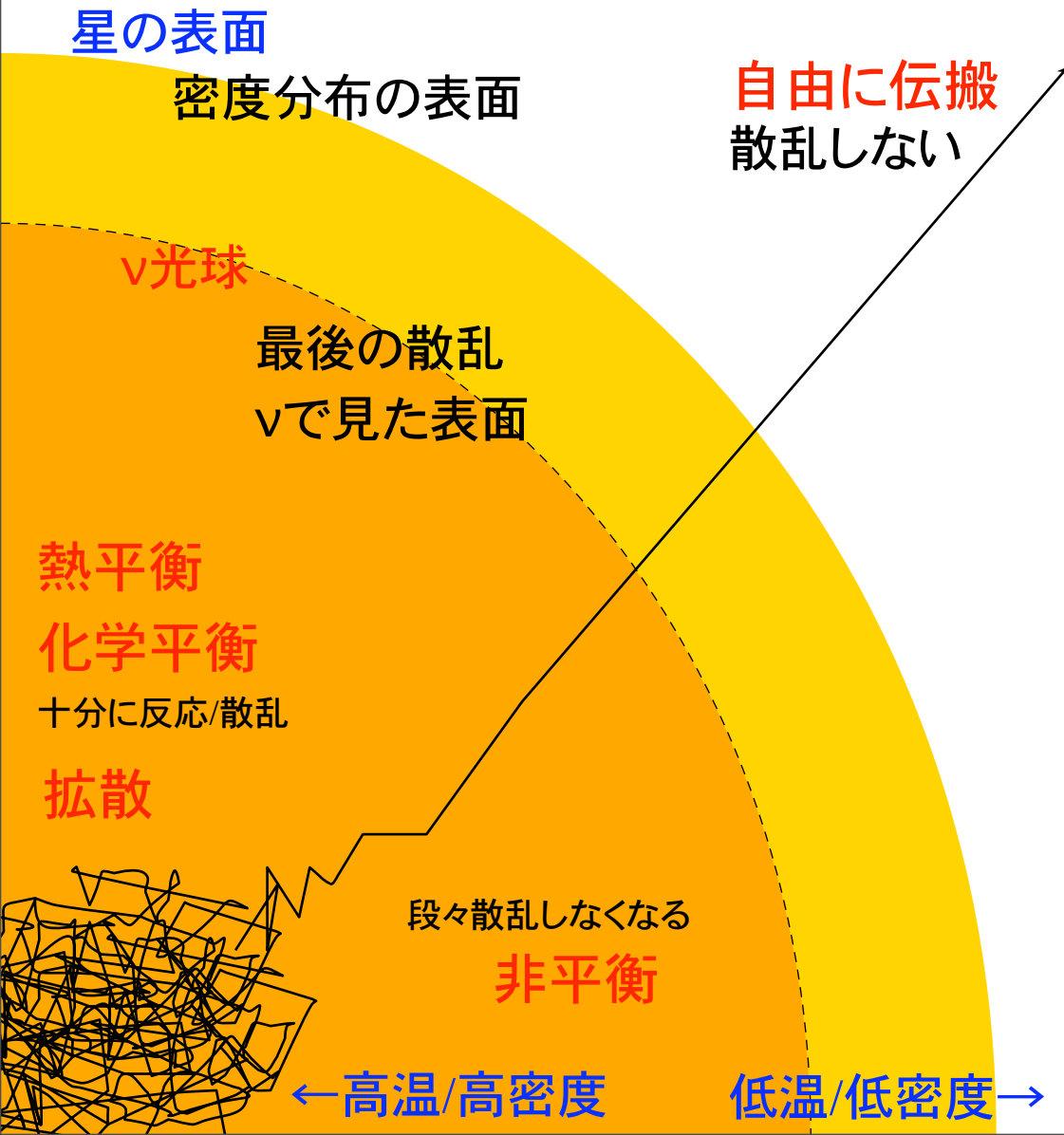
Jan. 1997



GAIT 2004 JUL - 23 09:38 000000000000

# $\nu$ 輻射輸送計算

- 星内部から $\nu$ が出てくるまで( $\nu$ 輻射輸送方程式)
  - $\nu$ の空間・運動量・方向分布を追う  $f(t,x,y,z,p_x,p_y,p_z)$ : 6+1次元問題
    - $\nu$ の伝搬を追う
    - $\nu$ 散乱・吸収・放出反応 (電子・陽子・中性子・原子核)を追う





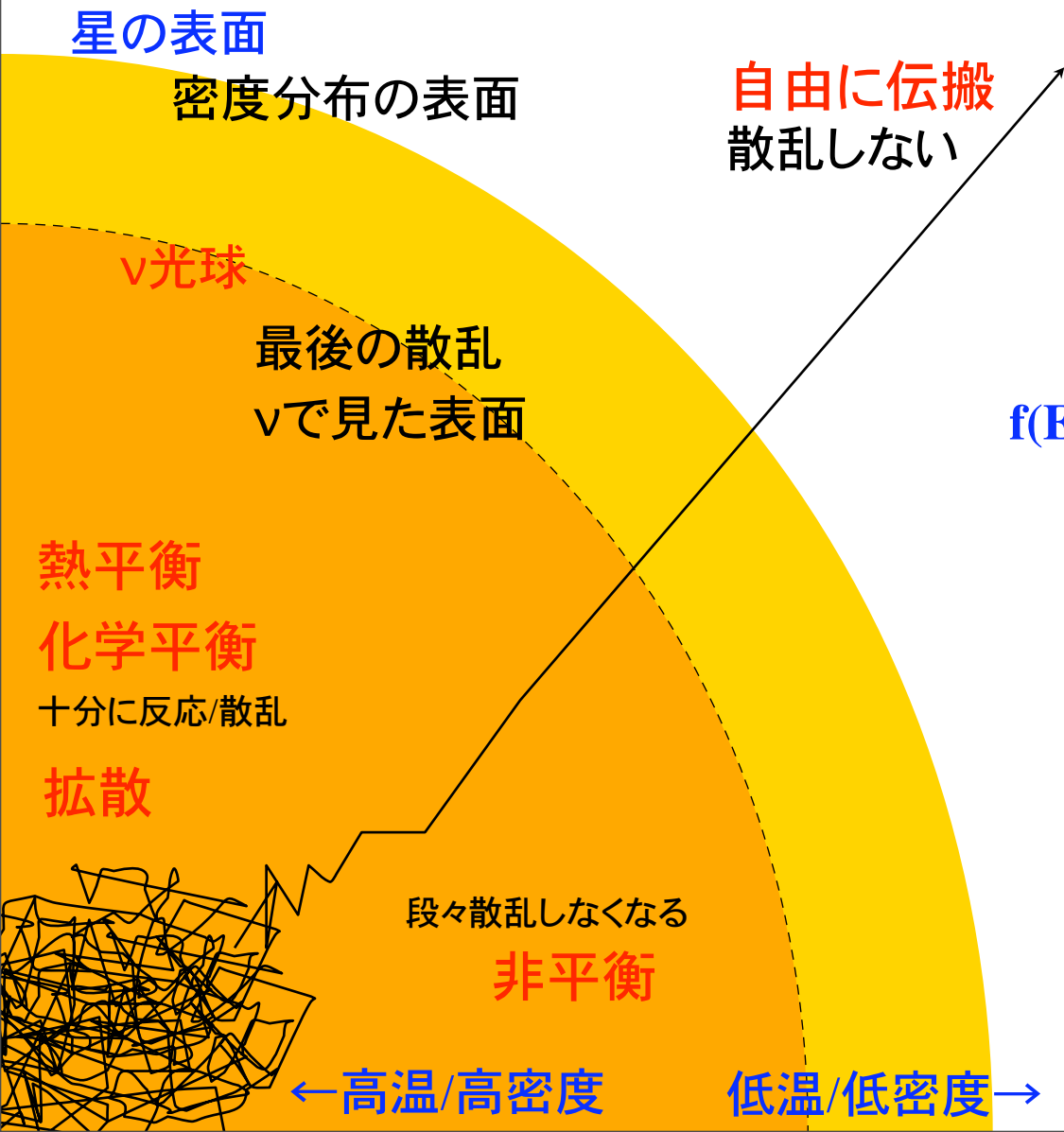
# $\nu$ 輻射輸送計算

- 星内部から $\nu$ が出てくるまで( $\nu$ 輻射輸送方程式)

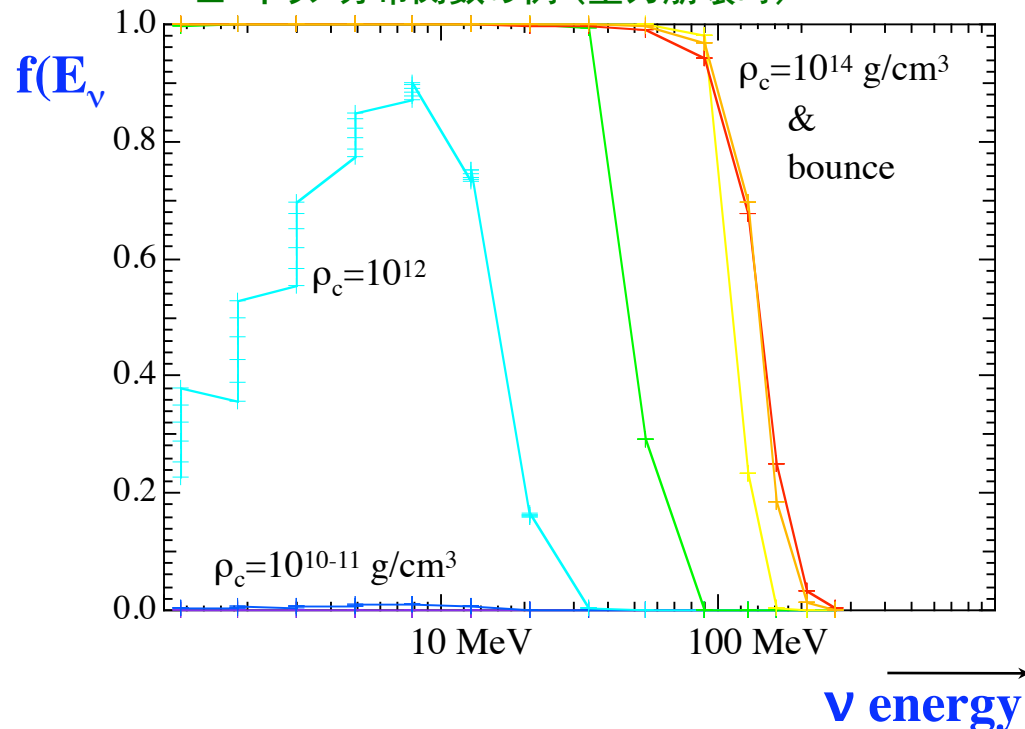
- $\nu$ の空間・運動量・方向分布を追う  $f(t, x, y, z, p_x, p_y, p_z)$ : 6+1次元問題

- $\nu$ の伝搬を追う

- $\nu$ 散乱・吸収・放出反応 (電子・陽子・中性子・原子核)を追う



ニュートリノ分布関数の例 (重力崩壊時)



# 基礎方程式系の例

## 電子数保存則

質量保存則

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho Y_e)}{\partial t} + \nabla \cdot (\rho Y_e \mathbf{v}) = -m_b \sum_f \int d\epsilon \left( \frac{S_\epsilon}{\epsilon} - \frac{\bar{S}_\epsilon}{\epsilon} \right)$$

エネルギー  
保存則

$$\frac{\partial E}{\partial t} + \nabla \cdot (E \mathbf{v}) + P \nabla \cdot \mathbf{v} = - \sum_f \int d\epsilon (S_\epsilon + \bar{S}_\epsilon)$$

状態方程式

$$E = f(T, \rho, Y_e)$$

$$P = g(T, \rho, Y_e)$$

重力ポテンシャル

$$\nabla^2 \Phi = 4\pi \rho G_N$$

運動量保存則

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla P + \rho \nabla \Phi + \nabla \cdot \left\{ \sum_f \int d\epsilon (\chi_\epsilon E_\epsilon + \bar{\chi}_\epsilon \bar{E}_\epsilon) \right\} = 0.$$

パウリ排他律

$$0 \leq E_\epsilon \leq \frac{\epsilon^3}{\alpha}$$

ニュートリノ  
輻射方程式

$$\frac{\partial E_\epsilon}{\partial t} + \nabla \cdot (E_\epsilon \mathbf{v}) - \nabla \cdot (D_\epsilon \nabla E_\epsilon) - \epsilon \frac{\partial}{\partial \epsilon} (\chi_\epsilon E_\epsilon) : \nabla \mathbf{v} = S_\epsilon$$

$$\frac{\partial \bar{E}_\epsilon}{\partial t} + \nabla \cdot (\bar{E}_\epsilon \mathbf{v}) - \nabla \cdot (\bar{D}_\epsilon \nabla \bar{E}_\epsilon) - \epsilon \frac{\partial}{\partial \epsilon} (\bar{\chi}_\epsilon \bar{E}_\epsilon) : \nabla \mathbf{v} = \bar{S}_\epsilon$$

$$0 \leq \bar{E}_\epsilon \leq \frac{\epsilon^3}{\alpha}$$

ニュートリノ  
反応率

$$S_\epsilon = S_\epsilon \left( 1 - \frac{\alpha}{\epsilon^3} E_\epsilon \right) - c \kappa_\epsilon^\alpha E_\epsilon + \left( 1 - \frac{\alpha}{\epsilon^3} E_\epsilon \right) c \int d\epsilon' \kappa^s(\epsilon', \epsilon) E_{\epsilon'}$$

$$- E_\epsilon c \int d\epsilon' \kappa^s(\epsilon, \epsilon') \left( 1 - \frac{\alpha}{\epsilon'^3} E_{\epsilon'} \right) + \left( 1 - \frac{\alpha}{\epsilon^3} E_\epsilon \right) \epsilon \int d\epsilon' G(\epsilon, \epsilon') \left( 1 - \frac{\alpha}{\epsilon'^3} \bar{E}_{\epsilon'} \right)$$

$$\bar{S}_\epsilon = \bar{S}_\epsilon \left( 1 - \frac{\alpha}{\epsilon^3} \bar{E}_\epsilon \right) - c \bar{\kappa}_\epsilon^\alpha \bar{E}_\epsilon + \left( 1 - \frac{\alpha}{\epsilon^3} \bar{E}_\epsilon \right) c \int d\epsilon' \bar{\kappa}^s(\epsilon', \epsilon) \bar{E}_{\epsilon'}$$

$$- \bar{E}_\epsilon c \int d\epsilon' \bar{\kappa}^s(\epsilon, \epsilon') \left( 1 - \frac{\alpha}{\epsilon'^3} \bar{E}_{\epsilon'} \right) + \left( 1 - \frac{\alpha}{\epsilon^3} \bar{E}_\epsilon \right) \epsilon \int d\epsilon' G(\epsilon', \epsilon) \left( 1 - \frac{\alpha}{\epsilon'^3} E_{\epsilon'} \right)$$

さらに  
一般相対論

Fig. 7

# 極限状態での物質の性質

## 核物質密度よりも高く、中性子過剰 状態方程式(Equation of State; EOS)

### 1. 圧力-密度

- 星の構造、ダイナミクス

### 2. 温度(エントロピー)

$\nu$ -エネルギー分布, 平均エネルギー

### 3. 組成(陽子, 中性子, ヘリウム, 原子核)

- 原子核反応、ニュートリノ反応率

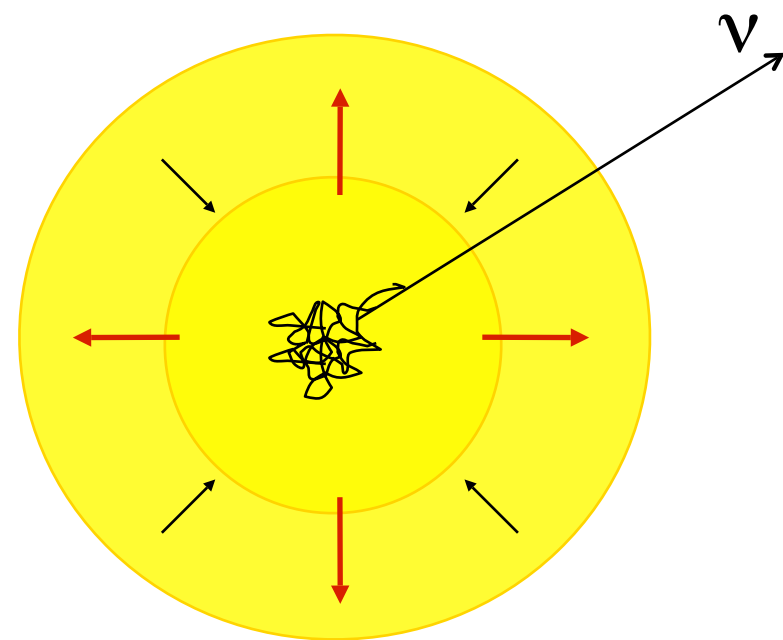
### ● 状態方程式データテーブル

- 密度:  $10^0 \sim 10^{15} \text{ g/cm}^3$

1. 陽子の割合:  $0 \sim 0.6$

2. 温度:  $0 \sim 100 \text{ MeV}$

- $(\rho, T, Y_e) : \varepsilon, p, S, \mu_n, \mu_p, X_i, m^*, \dots$



ちなみに

原子核物質密度:  $n_0 = 0.17 \text{ fm}^{-3}$   
( $\rho_0 = 3 \times 10^{14} \text{ g/cm}^3$ )

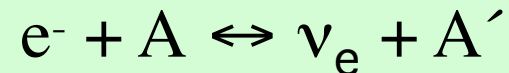
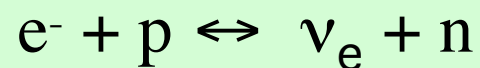
温度:  
 $1 \text{ MeV} \sim 10^{10} \text{ K}$

# 超新星コアでのニュートリノ反応

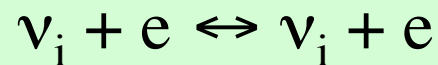
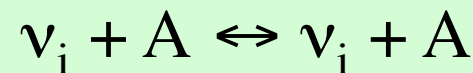
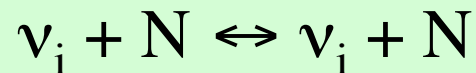
原子核の構造を反映:  $\sigma \sim A^2$

輸送方程式の散乱項:  $\nu$ 個数・エネルギー変化  $\rightarrow$  物質の加熱冷却

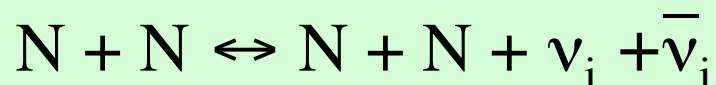
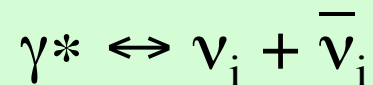
- 放出/吸収:



- 散乱:



- 対生成・消滅:



$i=e, \mu, \tau$

# 超新星爆発の核物理の進展

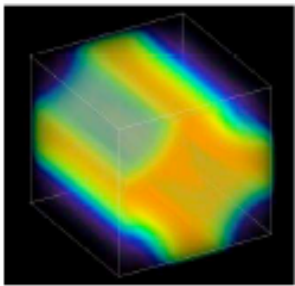
## 状態方程式データテーブル

- Lattimer-Swesty EOS (1991)
  - *Skyrme-Hartree-Fock approach*
- Relativistic EOS (Shen-Oyamatsu-Toki-Sumiyoshi, 1998)
  - *Relativistic Mean Field approach*
- Hyperon EOS (Ishizuka-Ohnishi, 2006)
- Variational Approach (Kanzawa-Takano, 2005~)

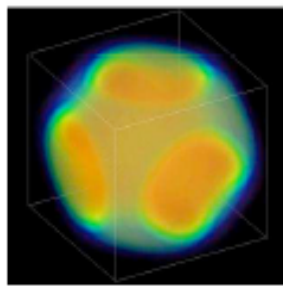
- 広い範囲の密度・温度・組成
- 一貫した枠組みで取り扱う
- 実験データによるチェック

## - 3D Skyrme Hartree-Fock EOS

(Newton, Stone Mezzacappa, 2007)



A=460



A=1400

( $n_b=0.08 \text{ fm}^{-3}$ ,  $T=2.5 \text{ MeV}$ ,  $Y_p=0.3$ )

## 電子捕獲反応

- Fuller, Fowler & Newman, Bruenn (1989)
- Langanke-Pinedo ( $45 < A < 112$ )
- T. Otsuka & T. Suzuki, Fujita-RCNP

## ニュートリノ反応

- Reddy, Horowitz, Yamada-Toki,...

# 超新星爆発の概略

## おわり

ここまでの提供

住吉光介氏（沼津高専）

# 超新星物質EOSの現状

以下では主にn,p,eだけからなる物質を考える

(原始) 中性子星を考えなければ  $\rho < 2\rho_0$

原子核モデルの精密化が鍵

(高密度状態方程式は2IpSAで)

( $T=0$ の核物質EOS) + (温度効果)

# Approaches to obtain the EOS

approach	starts from	ingredients	Theory/Model
empirical	the parametrized EOS	nuclear mass, size, ...	Liquid-Drop Model Droplet Model Thomas-Fermi Theory .....
Phenomenological	effective NN int.	nuclear mass, size, ...	Skyrme HF RMF AMD .....
many-body	bare NN int. (AV18, Bonn, Paris,...)	NN scattering, ...	Variational Calc. DBHF .....



# SN-EOS for supernova simulations

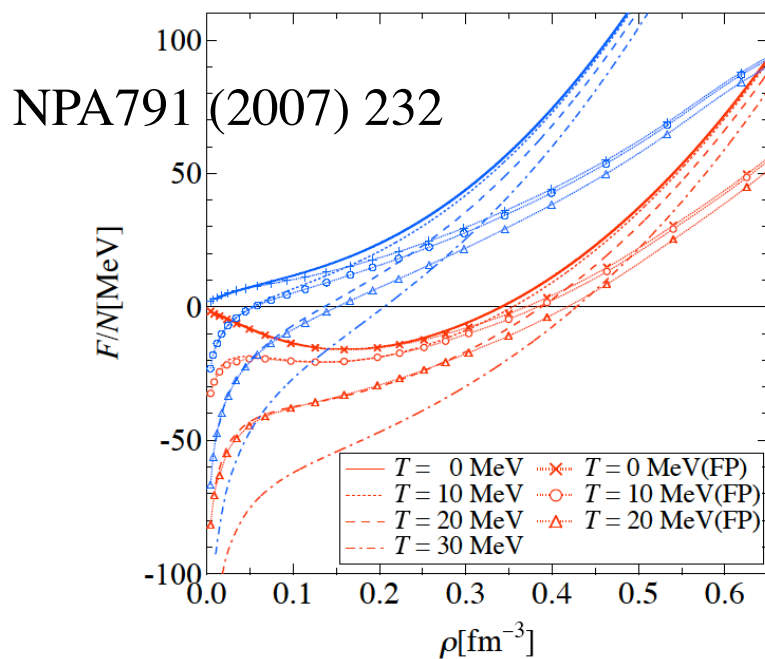
- Empirical
  - J.M. Lattimer, F.D. Swesty NPA535(1991).
- Phenomenological (RMF-TMI)
  - H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi, NPA637(1998), PTP100(1998).
- many-body (AV18) under construction (2IpSA-II)
  - H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791(2007)
  - Mukherjee, Pandharipande, Phys. Rev. C 75, 035802 (2007).

# The Nuclear EOS for Supernova Simulations based on the Realistic Nuclear Potential

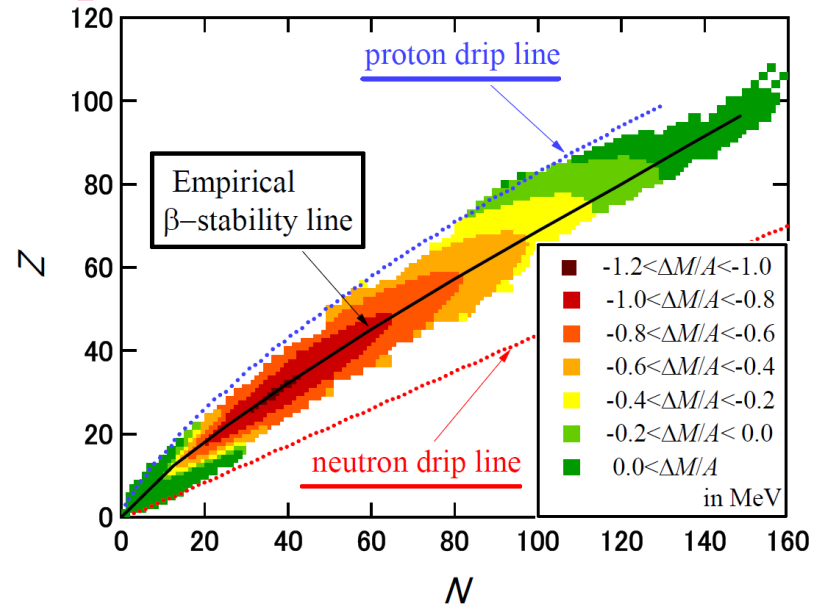
H. Kanzawa<sup>a</sup>, K. Oyamatsu<sup>b</sup>, K. Sumiyoshi<sup>c</sup> and M. Takano<sup>a</sup>

<sup>a</sup>Waseda Univ., <sup>b</sup>Aichi Shukutoku Univ., <sup>c</sup>Numazu CT

1) The uniform EOS: AV18+UIX,  
Variational many-body calculations



2) The Thomas-Fermi calculations  
for atomic nuclei reproduce their  
empirical data.



3) The SN-EOS is under construction

# 状態方程式が違っていると

- Hix (Proc. Nuclei in the Cosmos X to be published)
  - Protoneutron starのsizeに20%違い
  - 200ms後のShock locationに20%違い
  - lepton, entropy gradientを変える
    - 低密度 ( $10^{10} \sim 10^{14}$  g/cm<sup>3</sup>程度) で成分に顕著な違い
- 住吉、中里
  - BH形成時に放出されるneutrino (21pSA9,10)

# 超新星ニュートリノ予測 (flux, spectra)

例: ブラックホール形成時のニュートリノ放出 ( $40M_{\text{solar}}$ )

Sumiyoshi et al., PRL (2006)

$\nu$ エネルギースペクトル

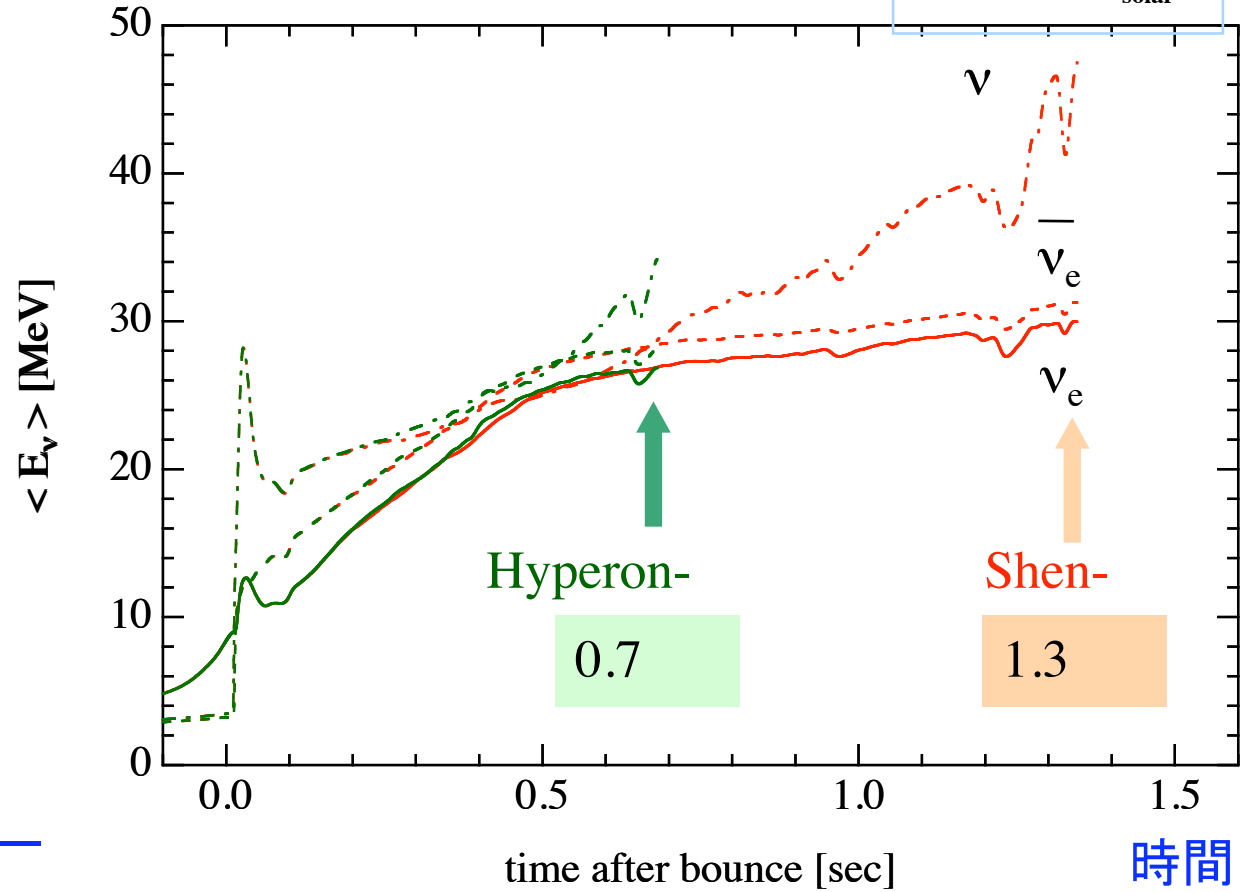
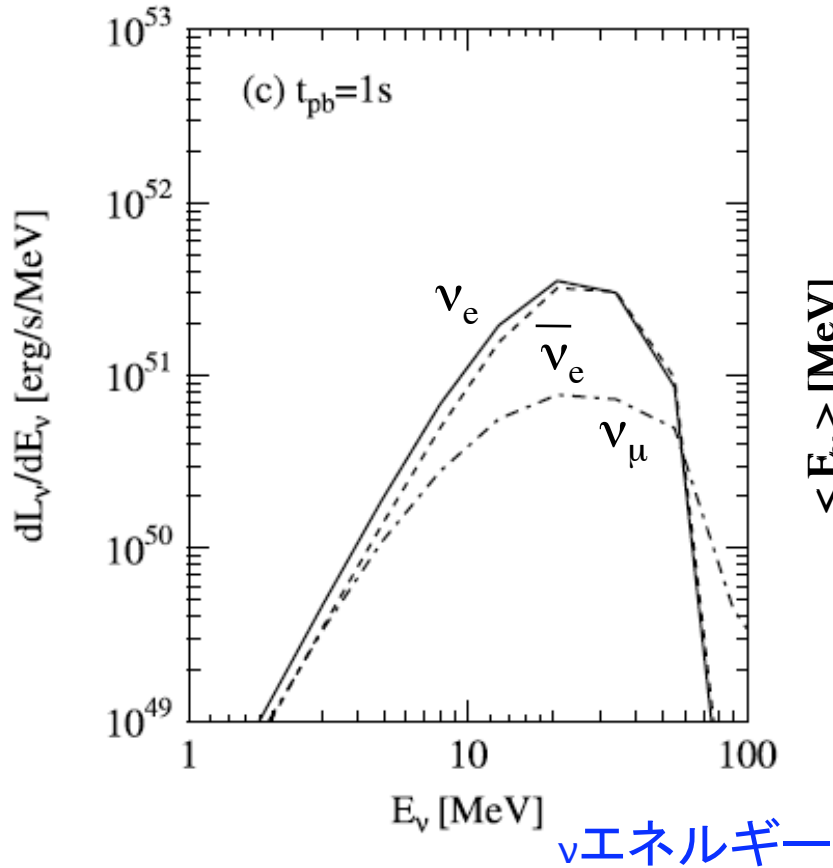
$\nu$ 平均エネルギー

$\nu$ 光度

Shen EOS

$\nu$ エネルギー cf. Hyperon

WW95:  $40M_{\text{solar}}$



# 状態方程式に対する制限

- $0.5 \rho_0 \sim \rho_0$ 
  - 不安定核質量、半径、中性子スキン、GDR
- $\rho_0 \sim 2-3 \rho_0$ 
  - Heavy Ion Collision
    - Bao et al., Phys. Rep.(2008) in press.(arXiv: 0804.3580)
- $\rho > 2-3 \rho_0$ 
  - Hyperon, Meson Condensate, Quark Matter
  - 中性子星の(質量、半径) 同時観測
    - 5つぐらいあるといい(Prakash)

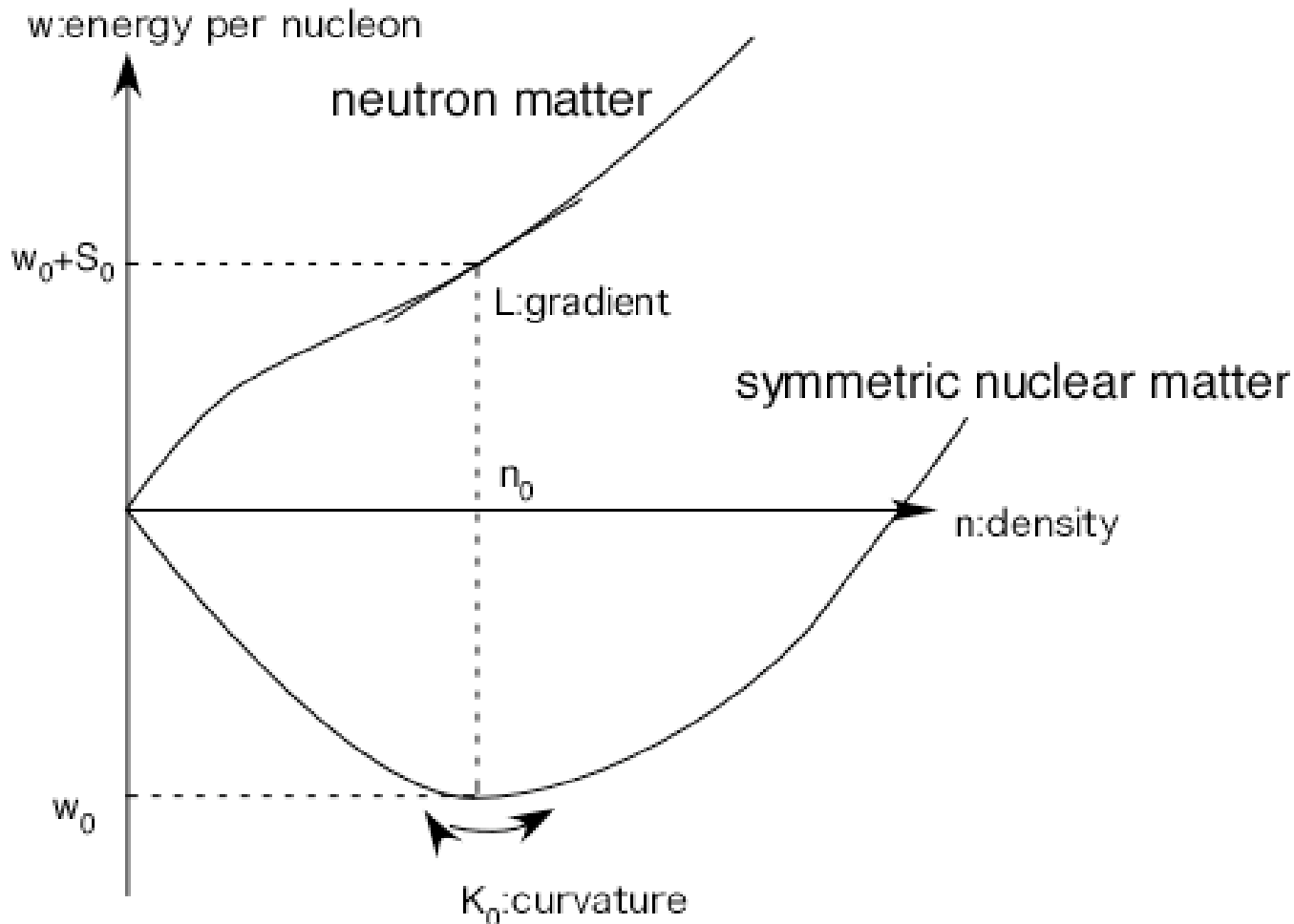
# T=0の核物質状態方程式のキーパラメータ

Energy per nucleon of nearly symmetric nuclear matter

$$w(n, x) \approx w_0 + \frac{K_0}{18n_0^2}(n - n_0)^2 + (1 - 2x)^2 \left[ S_0 + \frac{L}{3n_0}(n - n_0) \right]$$

$n_0$  : nuclear density,  $w_0$  : saturation energy,  $K_0$  : incompressibility

$S_0$  : symmetry energy at  $n=n_0$ , L: its density derivative coefficient



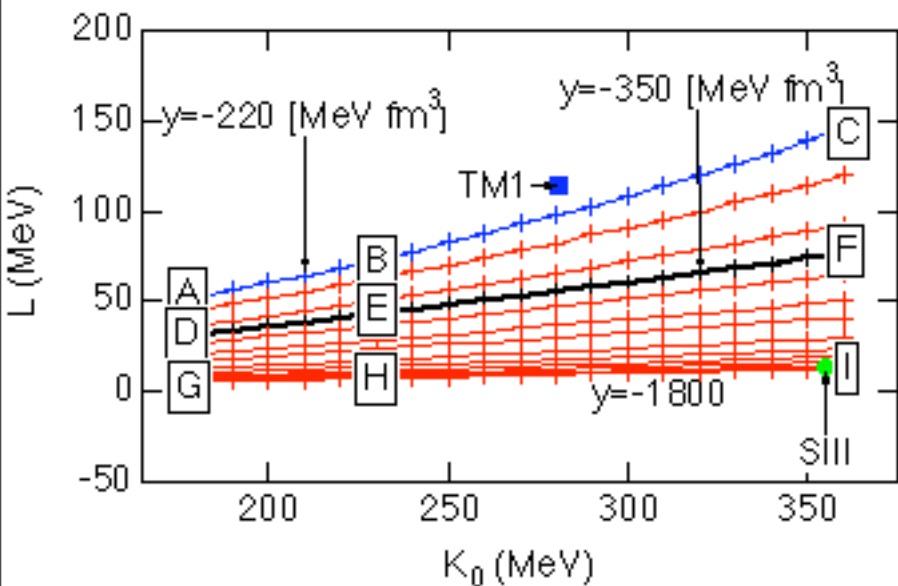
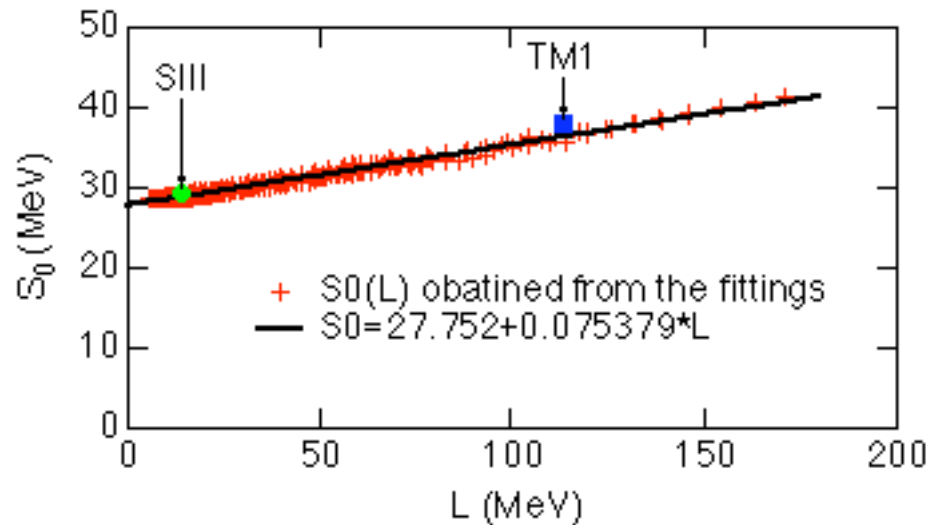
$$L = 3n_0 \left. \frac{dS(n)}{dn} \right|_{n=n_0}$$

$$S_0 = S(n_0)$$

EOS parameter values obtained from stable nuclei

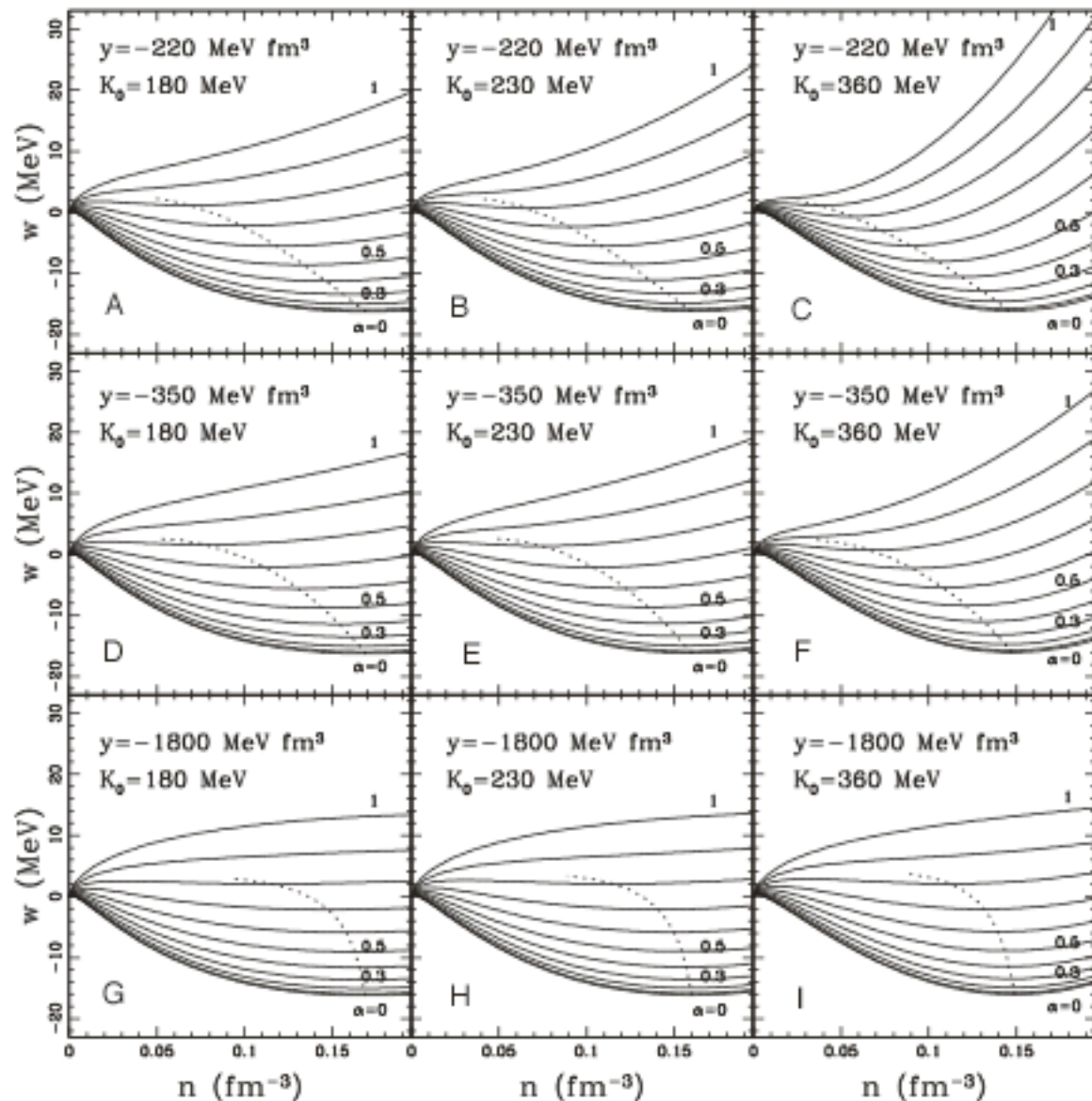
$S_0$ : symmetry energy

$L$ : density symmetry coefficient



$L, K_0$ の不定性  $\Rightarrow$  EOSの不定性

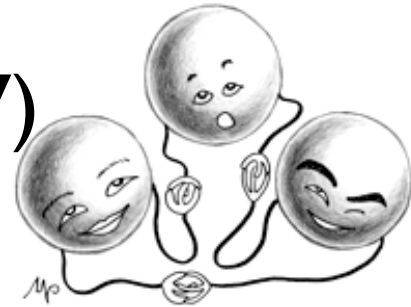
9 representative EOS A-I



K. Oyamatsu and K. Iida, PTP 109, 631-650, 2003.

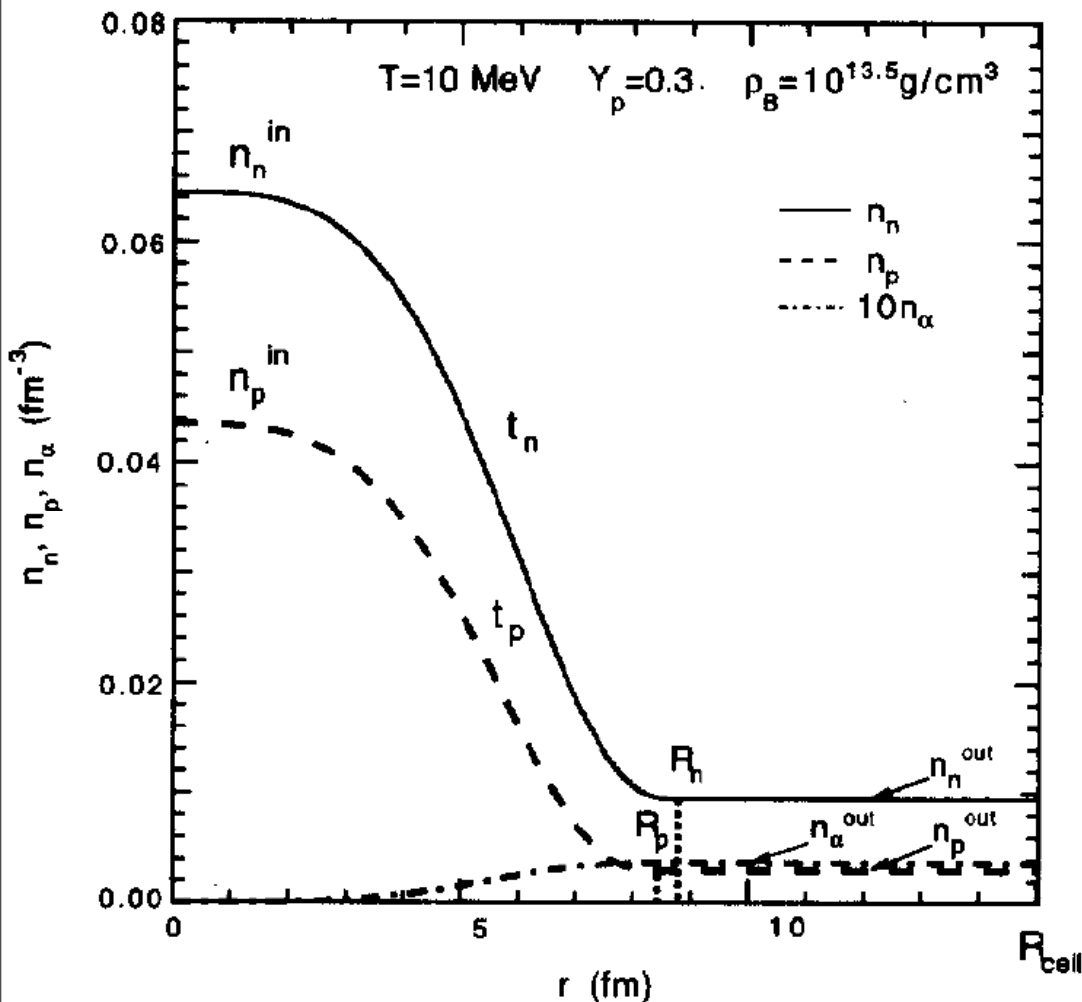
# Efforts being done

- Estimate L value from global behavior of nuclear mass and size in nuclear chart with Prof. Iida (Kouchi U.), Drs. Kohama(RIKEN), Koura(JAEA), Abu-Ibrahim(Cairo U.)
- For nuclear size, we need to directly compare calculations and cross section measurements.
  - Kurotama (Black sphere) model(2IaSB7)
  - Glauber calculation of cross sections
- Preliminary result from nuclear mass L value is relatively large.  
(closer to EOS C than to EOS G)



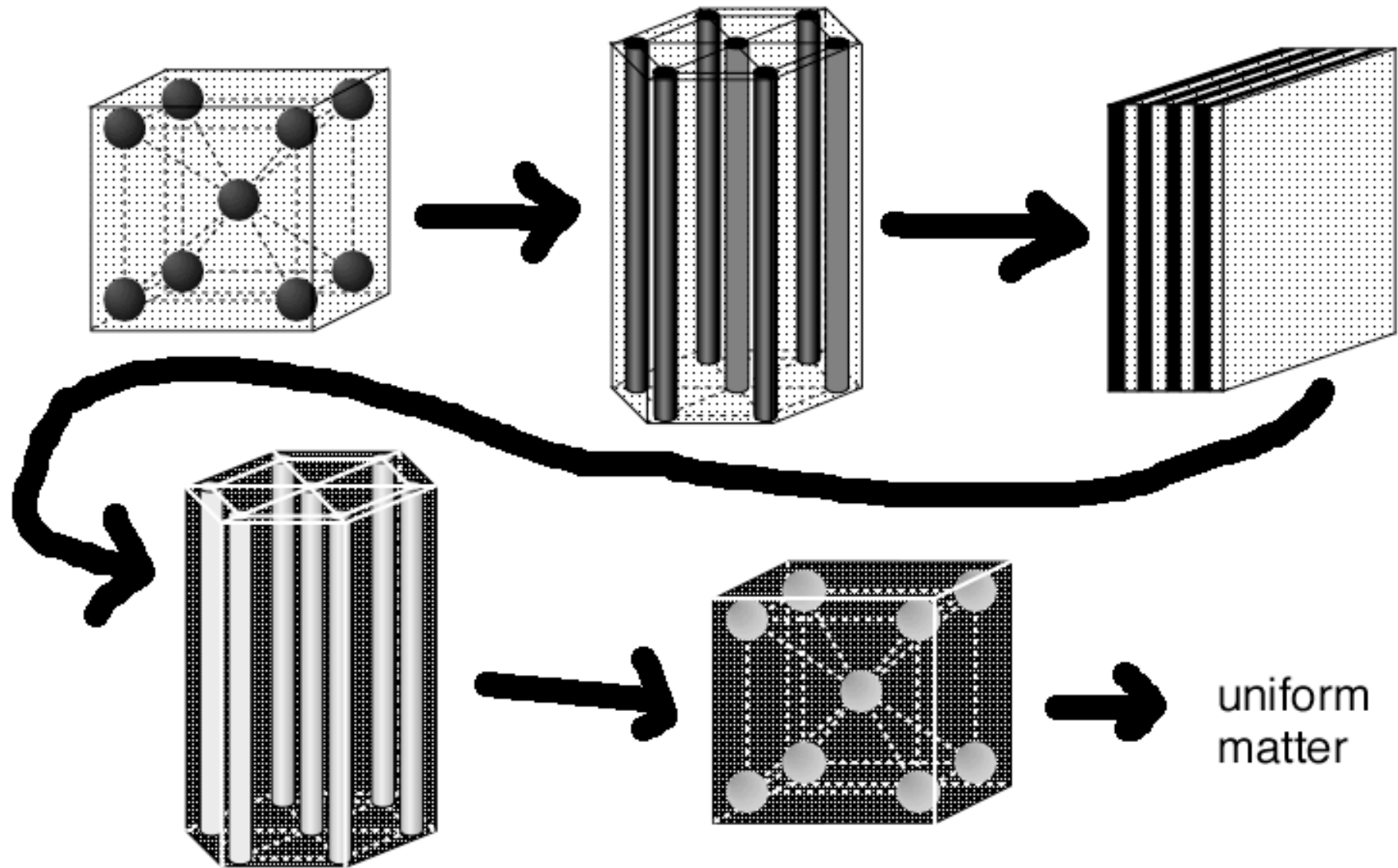


# 非一様物質 (Shen EOS)



- 球形のWigner-Seitz cell
- 原子核は球形とする
- 核種は1種類とする
- 成分は原子核とfree n, p,  $\alpha$
- Thomas-Fermi近似で解く
- 分布パラメータの最適化

# spherical nuclei and pasta nuclei



# From Microscales to Macroscales in 3D: Selfconsistent Equation of State for Supernova and Neutron Star Models

W G Newton<sup>1</sup>, J R Stone<sup>1,2,3</sup>, A Mezzacappa<sup>2</sup>

<sup>1</sup> Department of Physics, University of Oxford, Oxford OX1 3PU, United Kingdom

<sup>2</sup> Physics Division, Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, TN 37831, USA

<sup>3</sup> Department of Chemistry and Biochemistry, University of Maryland, College Park, MD 20742, USA

E-mail: [william.newton@seh.ox.ac.uk](mailto:william.newton@seh.ox.ac.uk), [stonejr@ornl.gov](mailto:stonejr@ornl.gov), [mezzacappaa@ornl.gov](mailto:mezzacappaa@ornl.gov)

**Abstract.** First results from a fully self-consistent, temperature-dependent equation of state that spans the whole density range of neutron stars and supernova cores are presented. The equation of state (EoS) is calculated using a mean-field Hartree-Fock method in three dimensions (3D). The nuclear interaction is represented by the phenomenological Skyrme model in this work, but the EoS can be obtained in our framework for any suitable form of the nucleon-nucleon effective interaction. The scheme we employ naturally allows effects such as (i) neutron drip, which results in an external neutron gas, (ii) the variety of exotic nuclear shapes expected for extremely neutron heavy nuclei, and (iii) the subsequent dissolution of these nuclei into nuclear matter. In this way, the equation of state is calculated across phase transitions without recourse to interpolation techniques between density regimes described by different physical models. EoS tables are calculated in the wide range of densities, temperature and proton/neutron ratios on the ORNL NCCS XT3, using up to 2000 processors simultaneously.

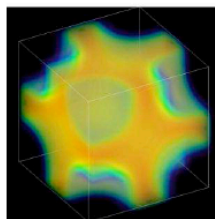
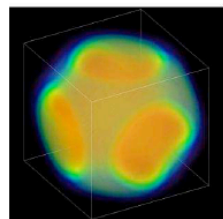
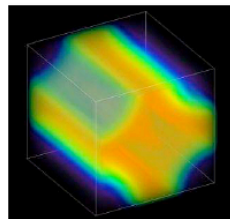
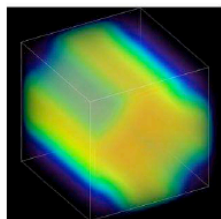
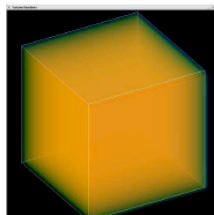
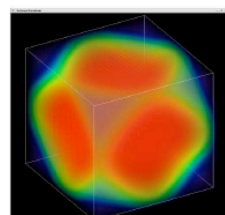
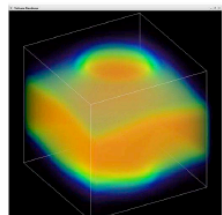
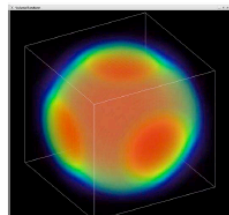


Figure 5. Neutron density distributions for  $A=180$ .

Figure 6. The same as Fig. 5 but for  $A=460$ .

Figure 7. The same as Fig. 5 but for  $A=1400$ .

Figure 8. The same as Fig. 5 but for  $A=2200$ .

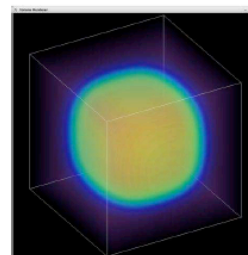


Figure 9. 3D neutron density distribution at  $n_b=0.04 \text{ fm}^{-3}$ .

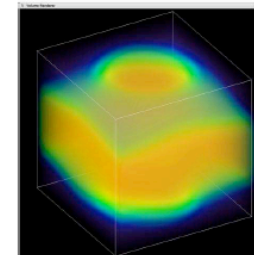


Figure 10. 3D neutron density distribution at  $n_b=0.08 \text{ fm}^{-3}$ .

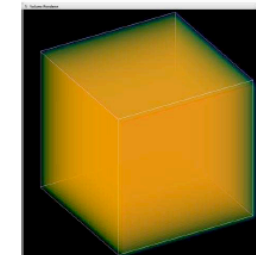


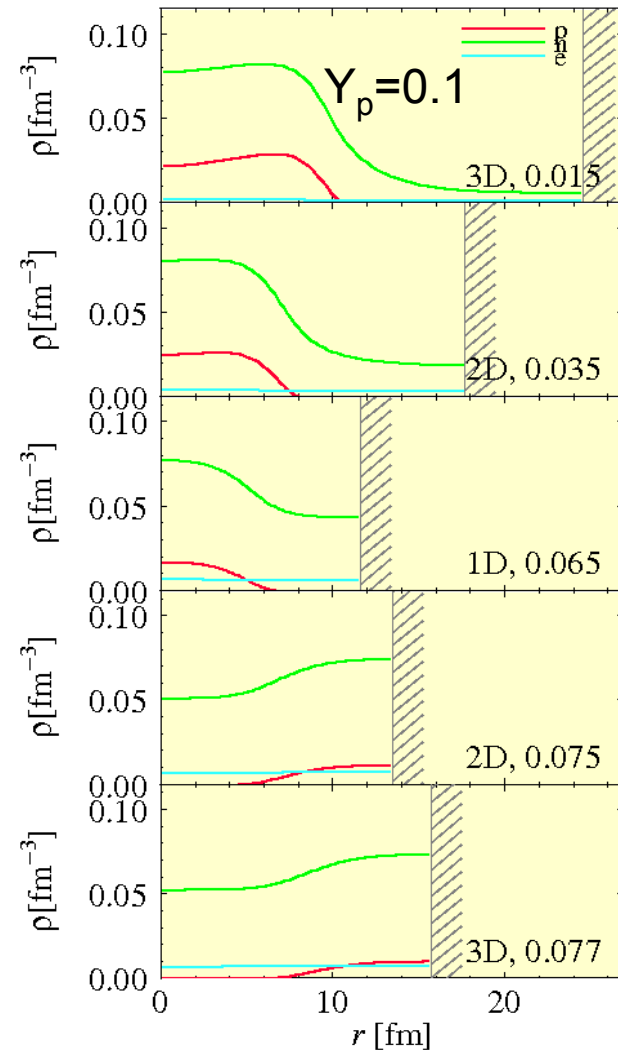
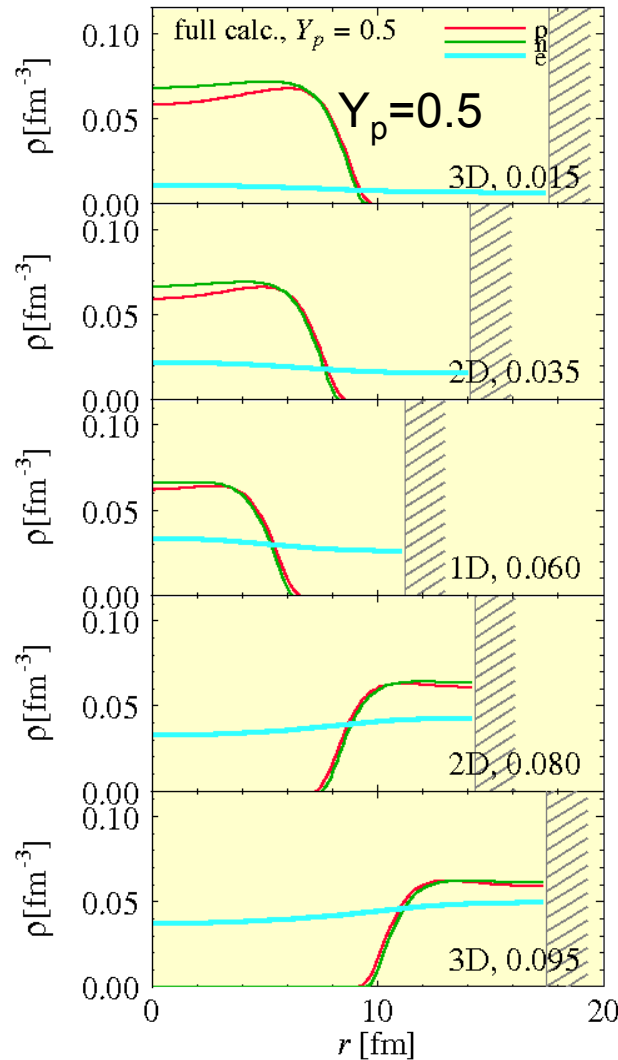
Figure 11. 3D neutron density distribution at  $n_b=0.12 \text{ fm}^{-3}$ .

# RMF calculation by Maruyama, Chiba, Tatsumi

Pasta structures in matter

Density profiles in WS cell

Pasta structures appear.



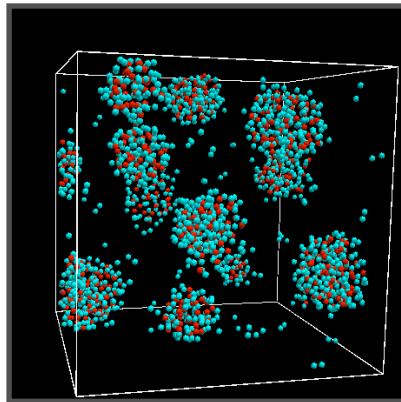
# QMD calculations by Sonoda, Watanabe, Sato, Takiwaki, Yasuoka, Ebisuzaki

## Pasta phases at zero temperature

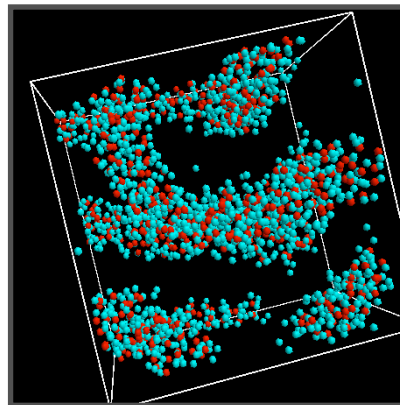
Cooling of Hot nuclear matter ( $\sim 10$  MeV) down to 0.1 MeV

Snapshots calculated for model 2

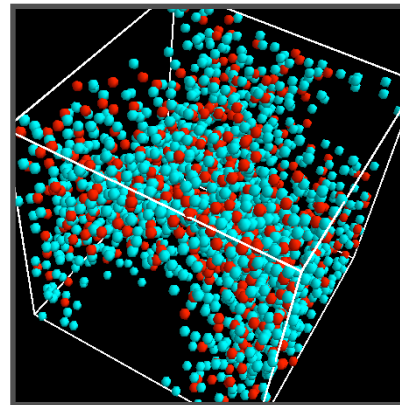
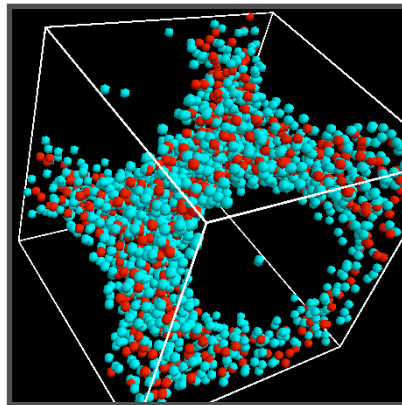
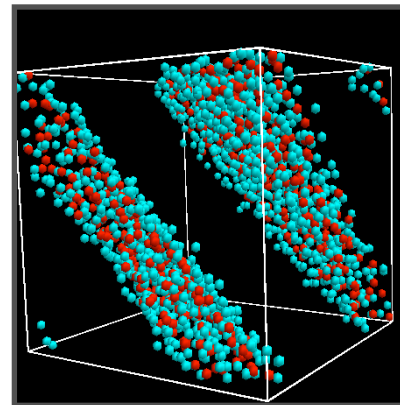
Sphere  
 $0.100\rho_0$



Rod  
 $0.200\rho_0$



Slab  
 $0.393\rho_0$



**Red**: Proton  
**Blue**: Neutron

$\rho_0 = 0.168 \text{ fm}^{-3}$   
(Nuclear density)

Rod-like Bubbles  $0.490\rho_0$

Spherical Bubbles  $0.575\rho_0$

# Shen EOS table size

- $\log_{10}(T(\text{MeV}))$      $-1 \sim 2$     31 grids
- $\log_{10}(Y_p)$      $-2 \sim -0.25$     71 grids
- $\log_{10}(\rho_B (\text{g/cm}^3))$      $5 \sim 15.4$     104 grids
- 総計     $31 \times 71 \times 104 = 228,904$  data points
  - 核図表の核種数はたかだか数千！

# 今後の課題 1 / 3

- 原子核密度近傍のEOSの経験的決定
  - L : 不安定核質量/半径、GDR
  - $K_0$  : Heavy ion collision
- 低密度物質の成分
  - Nuclear Statistical Equilibrium
  - 原子核パスタ
  - 半古典モデルを超えた取り扱い
    - Hartree-Fock、RMF、殻エネルギー評価

# 今後の課題 2 / 3

- 高密度物質
  - 中性子星（質量、半径）の組から逆問題を解く
  - 3体力の評価
  - hyperons, meson condensate, quark matter, ...



# 今後の課題 3 / 3

- EOSテーブルの精密化とサイズの増大
  - $T, \rho, Y_p$ の範囲の広領域化 Mega Grids以上
  - 成分の増加
- 超新星研究の推進（利用の拡大）
  - ファイル形式（コード）の標準化
    - 天体観測データ（会誌最新号）
    - [nucastrodata.org](http://nucastrodata.org), [bigbangonline.org](http://bigbangonline.org)